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Contents

Preface xi

1. INTRODUCTION: ECONOMICS OF POWER GENERATION 1
   1.1 Introduction 1
   1.2 Load-Duration Curves 2
   1.3 Location of Power Plants 9
   1.4 Power Plant Economics 10
   1.5 Indian Energy Scenario 20
   1.6 Coal-fuelled Electricity Generating Unit 23
       Short-Answer Questions 37
       Problems 38
       Bibliography 40

2. ANALYSIS OF STEAM CYCLES 41
   2.1 Steam Power Plant 41
   2.2 Rankine Cycle 42
   2.3 Carnot Cycle 48
   2.4 Mean Temperature of Heat Addition 48
   2.5 Effect of Variation of Steam Condition
       on Thermal Efficiency of Steam Power Plant 51
   2.6 Reheating of Steam 53
   2.7 Regeneration 55
   2.8 Regenerative Feedwater Heating 58
   2.9 Feedwater Heaters 61
   2.10 Carnotization of Rankine Cycle 63
   2.11 Optimum Degree of Regeneration 65
   2.12 Supercritical Pressure Cycle 70
   2.13 Steam Power Plant Appraisal 70
   2.14 Deaerator 72
   2.15 Typical Layout of Steam Power Plant 74
   2.16 Efficiencies in a Steam Power Plant 76
   2.17 Cogeneration of Power and Process Heat 78
       Short-Answer Questions 96
       Problems 97
       Bibliography 102
3. COMBINED CYCLE POWER GENERATION 103

3.1 Flaws of Steam as Working Fluid in Power Cycle 103
3.2 Characteristics of Ideal Working Fluid for Vapour Power Cycle 104
3.3 Binary Vapour Cycles 105
3.4 Coupled Cycles 107
3.5 Combined Cycle Plants 109
3.6 Gas Turbine—Steam Turbine Power Plant 110

Short-Answer Questions 154
Problems 155
Bibliography 158

4. FUELS AND COMBUSTION 159

4.1 Coal 159
4.2 Coal Analysis 160
4.3 Fuel Oil 164
4.4 Natural and Petroleum Gas 165
4.5 Emulsion Firing 165
4.6 Coal–Oil and Coal–Water Mixtures 166
4.7 Industrial Wastes and Byproducts 166
4.8 Synthetic Fuels 168
4.9 Biomass 173
4.10 Thermodynamic View 175
4.11 Combustion Reactions 177
4.12 Mass Balance of a Steam Generator 186
4.13 Energy Balance of a Steam Generator 187
4.14 Draught (or Draft) System 189
4.15 Fans 197
4.16 Heat of Combustion 199
4.17 Heating Values: Enthalpy of Combustion 202
4.18 Theoretical Flame Temperature 204
4.19 Free Energy of Formation 205
4.20 Equilibrium Constant K_p 206
4.21 Effect of Dissociation 209

Short-Answer Questions 226
Problems 229
Bibliography 233

5. COMBUSTION MECHANISM, COMBUSTION EQUIPMENT AND FIRING METHODS 234

5.1 Kinetics of Combustion Reactions 235
5.2 Mechanism of Solid Fuel Combustion 237
5.3 Kinetic and Diffusion Control 238
5.4 Combustion Equipment for Burning Coal 241
5.5 Fuel Bed Combustion 241
5.6 Mechanical Stokers 244
5.7 Pulverized Coal Firing System 248
6. STEAM GENERATORS

6.1 Basic Types of Steam Generators 317
6.2 Fire-Tube Boilers 318
6.3 Water-Tube Boilers 321
6.4 Economisers 356
6.5 Superheaters 359
6.6 Reheaters 364
6.7 Steam Generator Control 365
6.8 Air Preheater 374
6.9 Fluidized Bed Boilers 379
6.10 Electrostatic Precipitator 397
6.11 Fabric Filters and Baghouses 400
6.12 Ash Handling System 402
6.13 Feedwater Treatment 403
6.14 Deaeration 407
6.15 Evaporation 409
6.16 Internal Treatment 409
6.17 Boiler Blowdown 410
6.18 Steam Purity 411

Short-Answer Questions 421
Problems 424
Bibliography 427

7. STEAM TURBINES

7.1 Introduction 428
7.2 Flow Through Nozzles 430
7.3 Turbine Blading 451
7.4 Electrical Energy Generation 508

Short-Answer Questions 553
Problems 555
Bibliography 561

8. CONDENSER, FEEDWATER AND CIRCULATING WATER SYSTEMS

8.1 Need of a Condenser 562
8.2 Direct Contact Condensers 563
8.3 Surface Condensers 565
# Contents

8.4 Feedwater Heaters 575  
8.5 Circulating Water System 579  
8.6 Cooling Towers 580  
8.7 Cooling Tower Calculations 586  
  Short-Answer Questions 598  
  Problems 600  
  Bibliography 601  

## 9. NUCLEAR POWER PLANTS 602

9.1 Structure of the Atom 602  
9.2 Chemical and Nuclear Reactions 604  
9.3 Nuclear Stability and Binding Energy 604  
9.4 Radioactive Decay and Half Life 606  
9.5 Nuclear Fission 610  
9.6 Chain Reaction 612  
9.7 Neutron Energies 613  
9.8 Nuclear Cross-Sections 616  
9.9 Neutron Flux and Reaction Rates 618  
9.10 Moderating Power and Moderating Ratio 619  
9.11 Variation of Neutron Cross-Sections with Neutron Energy 620  
9.12 Neutron Life Cycle 622  
9.13 Reflectors 624  
9.14 Heat Transfer and Fluid Flow in Nuclear Reactors 625  
9.15 Types of Reactors 632  
9.16 Pressurized Water Reactor (PWR) 633  
9.17 Boiling Water Reactor (BWR) 637  
9.18 Gas-Cooled Reactors 640  
9.19 Liquid Metal Fast Breeder Reactor 641  
9.20 Heavy Water Reactors 643  
9.22 Fusion Power Reactors 647  
  Short-Answer Questions 653  
  Problems 655  
  Bibliography 656

## 10. HYDROELECTRIC POWER PLANT 657

10.1 Introduction 657  
10.2 Advantages and Disadvantages of Water Power 658  
10.3 Optimization of Hydro-Thermal Mix 659  
10.4 Selection of Site for a Hydroelectric Plant 660  
10.5 Hydrological Cycle 661  
10.6 Hydrographs 662  
10.7 Storage and Pondage 666  
10.8 Essential Elements of a Hydroelectric Power Plant 667  
10.9 Classification of Hydroelectric Power Plants 676  
10.10 Hydraulic Turbines 679  
10.11 Turbine Size 682
11. DIESEL ENGINE AND GAS TURBINE POWER PLANTS 735

11.1 Applications of Diesel Engines in Power Field 735
11.2 Advantages and Disadvantages of Diesel Engine Power Plant 736
11.3 Types of Diesel Plants 736
11.4 General Layout 737
11.5 Combustion in a CI Engine 751
11.6 Performance Characteristics 754
11.7 Supercharging 761
11.8 Layout of a Diesel Engine Power Plant 765
11.9 Gas Turbine Power Plant 772
11.10 Components of Gas Turbine Plant 785
11.11 Gas Turbine Fuels 792
11.12 Gas Turbine Materials 793
11.13 Free Piston Engine Plant 794

12. ENERGY STORAGE 813

12.1 Pumped Hydro 816
12.2 Compressed Air Energy Storage (CAES) 818
12.3 Flywheel Energy Storage 822
12.4 Electrochemical Energy Storage 826
12.5 Magnetic Energy Storage 835
12.6 Thermal Energy Storage 837
12.7 Chemical Energy Storage 844
12.8 Hydrogen Energy 846
Short-Answer Questions  852
Problems  854
Bibliography  855

13. NON-CONVENTIONAL POWER GENERATION:
DIRECT ENERGY CONVERSION  856
13.1 Magnetohydrodynamic (MHD) Power Generation  856
13.2 Thermionic Power Generation  863
13.3 Thermoelectric Power Generation  869
13.4 Fuel Cells  879
13.5 Geothermal Energy  888
13.6 Hydrogen Energy System  889
Short-Answer Questions  893
Problems  894
Bibliography  895

14. ENVIRONMENTAL DEGRADATION AND USE
OF RENEWABLE ENERGY  896
14.1 Greenhouse Effect  897
14.2 Acid Rain  898
14.3 Smog  899
14.4 Nuclear Radiation  899
14.5 Solar Energy  899
14.6 Wind Energy  912
14.7 Ocean Energy Systems  917
14.8 Geothermal Energy  929
14.9 Small Hydro  934
14.10 Energy From Biomass  935
Short-Answer Questions  939
Bibliography  941

Appendix A  942
Appendix B  961
Appendix C  963
Index  966
Convinced by the suggestions made by the reviewers and users of the book, two new chapters, viz., Non-conventional Power Generation: Direct Energy Conversion, and Environmental Degradation and Use of Renewable Energy, have been added in this revised edition. Considering the current global environmental scenario, the last chapter is particularly important and considerable emphasis has been stressed on fuel cells and solar-hydrogen system, which seem to be the most significant future contributors in ridding us from the dual crises of energy and environment.

Fluidized beds, besides being one of the best Clean Coal Technologies, are also proving to be the most practical option for biomass conversion. The book boasts of providing an up-to-date information on fluidized bed technology, both in bubbling and circulating modes of combustion.

Meant for the undergraduate course on Power Plant Engineering studied by the mechanical engineering students, this book is a comprehensive and up-to-date offering on the subject. It has detailed coverage on hydroelectric, diesel engine and gas turbine power plants.

The book is divided into fourteen chapters. The first chapter introduces the economics of power generation. Chapter 2 is on the analysis of steam cycles. Chapters 3 and 4 discuss combined cycle power generation, and fuels and combustion respectively. Chapter 5 further discusses topics on combustion mechanism, combustion equipment and firing methods. Chapters 6 and 7 are on steam generators and steam turbines. Chapter 8 is on condenser, feedwater and circulating water systems. Chapter 9 is on nuclear power plants, Chapter 10 is on hydroelectric power plants and Chapter 11 is on diesel engine and gas turbine power plants. Chapter 12 discusses energy storage. As already mentioned, two new chapters have been added at the end. Chapter 13 is on non-conventional power generation and direct energy conversion while Chapter 14 discusses environmental degradation and use of renewable energy.

The book has about a hundred solved examples and two hundred practice problems. Moreover it has over 750 short-answer questions to give the students a thorough practice of the concepts learnt. The numerous self-explanatory illustrations makes this a very student-friendly text. The key features are as follows:
- In-depth coverage of Hydroelectric, Diesel Engine and Gas Turbine Power Plants
- Analytical and theoretical treatment of concepts with the right blend of theory, design and practice of power stations
- Unique chapter on Energy Storage Mechanisms
- Detailing of combined cycle mode of power generation and fluidized bed boilers

In addition, instructors can get help from the solutions manual and also from Power Point slides which can be used as teaching aides. These can be downloaded from the Online Learning Centre of the book.

The book has become quite voluminous and it would be difficult to complete it in one semester. The teacher has to adopt judiciously the portions necessary for the curriculum on the subject.

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The author would appreciate if readers point out any mistakes and graciously make suggestions for further improvement of the book, which would be gratefully acknowledged.

June, 2007

P. K. Nag
Introduction: Economics of Power Generation

1.1 INTRODUCTION

Electricity is the only form of energy which is easy to produce, easy to transport, easy to use and easy to control. So, it is mostly the terminal form of energy for transmission and distribution. Electricity consumption per capita is the index of the living standard of people of a place or country.

Electricity in bulk quantities is produced in power plants, which can be of the following types: (a) Thermal, (b) Nuclear, (c) Hydraulic, (d) Gas turbine and (e) Geothermal. Thermal, nuclear and geothermal power plants work with steam as the working fluid and have many similarities in their cycle and structure. Gas turbine plants are often used as peaking units. They run for short periods in a day to meet the peak load demand. They are, however, being increasingly used in conjunction with a bottoming steam plant in the mode of combined cycle power generation. Hydraulic power plants are essentially multipurpose. Besides generating power, they also cater for irrigation, flood-control, fisheries, afforestation, navigation, etc. They are, however, expensive and take long time to build. There is also considerable opposition against their erection due to the ecological imbalance they produce. Geothermal power plants can be built only in certain geographical locations.

Thermal power plants generate more than 80% of the total electricity produced in the world. Fossil fuels, viz. coal, fuel oil and natural gas are the energy source, and steam is the working fluid. Steam is also required in many industries for process heat. To meet the dual need of power and process heat, cogeneration plants are often installed.

There has been an exponential growth in the production of electricity. If the electricity production $E$ increases at the same fractional rate, $i$, each year, the rate of change of electricity production per year becomes
\[ \frac{dE}{dt} = Ei \]  \hspace{1cm} (1.1)

After integration,

\[ \ln \frac{E}{E_0} = i(t - t_0) \]

or,

\[ E = E_0 e^{i(t-t_0)} \]  \hspace{1cm} (1.2)

where \( E_0 \) is the electricity production in the base year \( t_0 \). This exponential behaviour gives rise to the term called “doubling time” which indicates the time required for electrical power production to double. If \( E_1 \) is the power generation at time \( t_1 \) and \( E_2 \) is the power generation at time \( t_2 \), then from Eq. (1.2).

\[ \frac{E_2}{E_1} = e^{i(t_2 - t_1)} \]  \hspace{1cm} (1.3)

If we denote the doubling time by \( t_d \), then

\[ t_d = t_2 - t_1 \quad \text{and} \quad \frac{E_2}{E_1} = 2 \]

Therefore,

\[ \ln 2 = it_d \quad \text{or} \quad t_d = \frac{0.693}{i} \]  \hspace{1cm} (1.4)

Using this simple expression, for various rates of increase in electricity production per year, the doubling time can be estimated. From the trend of production, it is observed that \( i = 6.2\% \) and so the doubling time \( t_d \) is about 11.2 years.

It is found that the demand for electricity bears a linear relationship with the gross national product (GNP) of a country. Projection of future demand of electricity is thus tied to estimates of economic growth of the concerned region. With the increase in economic growth, the consumption of electricity also increases.

### 1.2 LOAD-DURATION CURVES

When planning a power plant, the two basic parameters to be decided are:

1. Total power output to be installed (kW\(_{\text{inst}}\)).
2. Size of the generating units.

The total installed capacity required can be determined from:

1. First demand (kW\(_{\text{max}}\)) estimated.
2. Growth of demand anticipated.
3. Reserve capacity required.

The size of the generating units will depend on
1. Variation of load (load curve) during 24 hours (summer, winter, weekdays, holidays).
2. Total capacity of units connected to the electric grid.
3. Minimum start-up and shut-down periods of the units.
4. Maintenance programme planned.
5. Plant efficiency vs. size of unit.
6. Price and space demand per kW vs. size of unit.

For estimating the expected maximum load, the most useful tool is the construction of a hypothetical load curve. Figure 1.1 shows the typical hourly load curve in a metropolitan area, from which the maximum demand can be estimated. Several factors used in the electricity supply industry in this regard are defined below:

**Fig. 1.1** Typical hourly variation in energy demand for different types of electrical load and curves of total load in a metropolitan area

1. Load factor,
   
   $$ m = \frac{\text{average load over a given time interval}}{\text{peak load during the same time interval}} $$
Thus, if a plant has a peak load of 1000 MW but the average annual load is 350 MW, its annual load factor would be only 0.35. Most of its capacity remains unutilized for major part of the year, and so the cost would be high.

The average load is estimated by dividing the area under the daily load curve by the time period considered (Fig. 1.2).

\[
\text{Average load} = \frac{\text{area under load curve (kWh)}}{24 \text{ (h)}}
\]

![Load curve diagram]

**Fig. 1.2 Load curve**

2. Capacity factor (or plant factor),

\[
n = \frac{\text{average load}}{\text{rated capacity of the plant}} = \frac{\text{kWh generated in a year}}{\text{kW}_{\text{inst}} \times 24 \times 365} = \frac{\text{kWh}_{\text{gen}} \text{ (year)}}{\text{kW}_{\text{inst}} \times 8760}
\]

If the rated capacity of the plant is equal to the peak load, then the load factor and capacity factor will be numerically equal.

The difference between load factor and capacity factor is an indication of the reserve capacity.

3. Reserve factor,

\[
r = \frac{\text{kW}_{\text{inst}}}{\text{kW}_{\text{max}}} = \frac{m}{n} = \frac{\text{load factor}}{\text{capacity factor}}
\]
Each consumer has a "connected load" which is the sum of the continuous ratings of all the equipment and outlets on the consumer's circuits. The maximum demand is the maximum load which a consumer uses at any time. It is always less than, or equal to, the connected load.

4. Demand factor,

\[
\text{dem} = \frac{\text{actual maximum demand}}{\text{total connected demand}} = \frac{kW_{\text{max}}}{kW_{\text{conn}}}
\]

The time distribution of maximum demands for similar types of consumers is measured by a term called "diversity factor". It is the ratio of the sum of the maximum demands of the individual consumers and the simultaneous maximum demand of the whole group during a particular time.

5. Diversity factor,

\[
\text{div} = \frac{\text{sum of individual consumer groups}}{\text{actual peak load of the system}}
\]

The diversity factor is illustrated in Fig. 1.3(a). It is given to be

\[
\text{div} = \frac{a + b + c}{d}
\]

Diversity helps to improve the load factor and economic operation of the power plant.

Figure 1.3(b) shows the load curves of an industrialized country, the annual peak occurring in winter. The seasonal variation in the load influences the maintenance programme of the power plants. Major maintenance work (cleaning, overhauling etc.) has to be done during the off-peak season.

A high load factor is, in general, an indication of balanced load curve with relatively small load changes.

High values of demand factor, load factor, diversity factor and capacity factor are desired for economic operation of the plant and to produce electricity at less cost.

Plant use factor, \( u = \frac{kWh_{\text{gen}}}{kW_{\text{inst}} \times \text{operating hours}} \)

It is the ratio of energy produced in a given time to the maximum possible energy that could have been produced during the same time of operation. If the operating time is 1 year or 8760 hrs, the plant use factor is equal to the capacity factor (i.e., \( u = n \))

As the plant-use factor approaches 1, it indicates the need for additional capacity of the plant. The plant capacity is always designed to be greater than the peak load to take extra loads coming in future. The high value of the plant use factor indicates that the plant is operating quite efficiently. In some interconnected systems, the plant use factor may exceed unity (e.g., 1.1 or 1.2) indicating that the loads carried are in excess of the rated capacity since an equipment is always designed to take 10 to 20% more load than rated. The different factors are indicated in Fig. 1.3 (c).
Fig. 1.3  (a) Diversity of loads of consumer groups  
(b) Load curves of an industrialized country on typical winter and summer week-days

Load factor = \( \frac{\text{Average load}}{\text{Peak load}} \)

Use factor = \( \frac{\text{Peak load}}{\text{Plant capacity}} \)

Capacity factor = \( \frac{\text{Average load}}{\text{Plant capacity}} \)

(c) Representation of different factors

It can be seen that

Load factor \( \times \) Use factor = Capacity factor

Larger the unit size, less is the cost of electricity produced. Therefore, larger units are more economical than smaller units as regards investment per kW and operating expenses per kWh. Investigations should be made in each individual case, considering all operational, technical and economic factors involved, to determine the maximum unit size admissible. A reserve factor of 1.2 to 1.25 is often found to be satisfactory. The unit size must not exceed 20 to 25% of the total generating capacity of the plant.
In order to obtain a basis for economical operation of the various types of generating units normally connected to a grid and also for planning of tariffs, it is useful to construct load duration curves for typical days which indicate the power demands at different intervals of time. There can be residential loads (domestic lights, heaters, refrigerator, TV, audio and video cassette recorders, etc.), commercial loads (shops, offices, advertisements, restaurants, etc.), industrial loads (various mills and factories), municipal loads (street lighting, power for water supply and drainage), irrigation loads and traction loads (railways, trams, trolleys, etc.).

The construction of a load duration curve for one day from the chronological load curve of this day is illustrated in Fig. 1.4(b). It rearranges the load elements of a chronological curve in the descending order of magnitude. The area under the curve is

\[
\text{kWh} = \int_{0}^{24} \text{kW} \, dt
\]
during the day. The load duration curve indicates for how many hours a certain load has been required in the course of the day. By compiling the load hours on various days, the load duration curves for one month (720 hr) or one year (8760 hr) can be obtained (Fig. 1.4). Daily, monthly and yearly load duration curves aid better planning for economical utilization of the power plants, existing or planned.

Figure 1.5 shows the construction of a load-duration curve using the daily load curve. The area under the annual load duration curve represents the total energy supplied by the utility’s generating system during the year. It is usually divided into three parts:

1. Base load
2. Intermediate load
3. Peaking load

The base load is the load below which the demand never falls and is supplied 100% of the time. The peaking load occurs for about 15% of the time. The intermediate load represents the remaining load region.

Since peaking load plants are used only for a small fraction of time, the fuel cost is not of major importance. Minimum capital cost should be the criterion.

The base load plants are plants which are loaded very heavily. Operating costs of such plants are very important. A high capital cost is permissible if low operating costs can be maintained (e.g. new, large coal and nuclear power stations). Intermediate load plant can be somewhat smaller than older units. By judicious combination of all the three types of generation—peak, intermediate and base—maximum economy can be achieved.

The variable load problem affects power plant design and operation as well as the cost of generation. A careful study of the load duration curve helps to decide the capacity of the base load plant and also of the peak load plant. The base load plant should be run at high load factor. The peak load plant should be of smaller capacity to reduce the cost of generation. It could be a gas turbine.
Fig. 1.4 Load-duration curve constructed from the chronological load curve of a day

(a) Load curves
(b) Load-duration curves

Day
Month
Year

N_max
8.6h
2.6h
6h
Fig. 1.5 Construction of the load-duration curve

unit, pumped hydro-system, compressed air energy storage system or a diesel engine depending on the size and scope of availability. If the whole of load is to be supplied by the same power plant then the prime movers and generators should act fairly quickly and take up or shed load without variation of the voltage or frequency of the system. It is the function of the governor to control the supply of fuel to the prime mover according to load. The capacity of the generators should be so chosen as to suit and fit into the portions of the predicted load curve. If the load conditions differ too much from this capacity, the cost of energy increases.

1.3 LOCATION OF POWER PLANTS

The location of hydroelectric power plants is usually predetermined by the availability of water and the water head which is utilized. For conventional base load thermal power plants, the following factors are to be considered:

1. Availability of cooling water (if cooling towers are used the possibility of adequate make-up water).
2. Availability of fuel (water, rail or pipe connection to the fuel source, and the cost of fuel transport).
3. Distance from the centre of gravity of load demand.
4. Cost of land (including space for extension, maintenance workshop and storage yard).
5. Character of soil.
6. Main wind direction and water currents in cooling water source (sea, lake or river) in order to minimize air and water pollution, and other ecological considerations.
7. With coal-fired stations, disposal of ash.
8. If the plant is erected far from a town, accommodation for staff.
9. Rail and road connections.
10. Security considerations.
For nuclear power plants two additional factors are to be considered.

11. Density of population in the vicinity.
12. Danger of earthquakes.

1.4 POWER PLANT ECONOMICS

A power plant should provide a reliable supply of electricity at minimum cost to the consumer. The cost per kWh\textsubscript{net} is determined by:

1. Fixed costs (FC), mainly interest, depreciation, insurance, taxes, depending on the capital invested, i.e. on the construction costs of the plant including the cost of the land.
2. Operation and maintenance (O & M) costs covering salaries and wages, overhauling of equipment, repairs including spare parts, water, lubricating oil, chemicals and miscellaneous expenses.
3. Fuel costs, dependent on the amount of electricity generated.
4. kWh\textsubscript{net} of electricity sent out per year.

The total annual costs ($C_t$) in a power plant can be calculated from

$$C_t = \frac{I + D + T}{100} C_c + (W + R + M) + C_f$$

where $I$ is the interest, %; $D$ is depreciation, %; $T$ is taxes and insurance, %; $C_c$ the construction cost; $W$ is wages and salaries; $R$ is repairs (maintenance); $M$ is miscellaneous; and $C_f$ the fuel cost.

The annual amount of electricity sent out by a power plant (kWh\textsubscript{net}) is given by

$$\text{kWh}_{\text{net}} = \text{kW}_{\text{inst}} \times 8760 \times \left(1 - \frac{L_{\text{aux}}}{100}\right) \times n$$

where kW\textsubscript{inst} is the rated (installed) output of generators; $L_{\text{aux}}$ the power consumption by the auxiliaries, %; $n$ the plant capacity factor, and 8760 = 24 × 365 hours per year.

In order to calculate the electric power cost to a consumer, in addition to the production cost (fixed cost, operation and maintenance, and fuel cost), the transmission cost, distribution cost, administrative expenses, and return or profit on the investment have to be taken into consideration.

A measure for the reliability of a power plant is the forced outage rate defined by the annual ratio of

$$\frac{\text{forced outage hours}}{\text{service hours} + \text{forced outage hours}}$$

As forced outages raise operation and maintenance costs, proven reliability of equipment and regular preventive maintenance work are essential, not only to guarantee electric supply but also for the overall economy of the power plant.
The costs, however, have a continuous upward trend due to monetary inflation, rising fuel prices and increased demand for measures protecting the environment.

The cost of power generation can be reduced by
(a) selecting equipment of longer life and proper capacities,
(b) running the power station at high load factor,
(c) increasing the efficiency of the power plant,
(d) carrying out proper maintenance of power plant equipment to avoid plant breakdowns,
(e) keeping proper supervision, since good supervision is reflected in lesser breakdowns and extended plant life,
(f) using a plant of simple design that does not need highly skilled personnel.

1.4.1 Construction Costs

The construction costs of a plant consist of expenses for:

![Construction Costs Graph]

1. Jet type gas-turbine plants without recuperator
2. Jet type gas-turbine plants with recuperator
3. Industrial type gas-turbine plants with recuperator
4. Diesel plants for continuous service
5. Conventional steam power plants, oil fired
6. Conventional steam power plants, bituminous-coal fired
7. Nuclear LWR steam power plants

**Fig. 1.7** Approximate construction cost of power plants vs unit size (Aschner, 1978)
(i) planning and design;
(ii) land and preparation of site, including road (and rail) connections;
(iii) buildings and machinery foundations;
(iv) plant equipment, including transport to site;
(v) erection and testing of equipment;
(vi) interest during construction (for payments made before commissioning
of the plant); and
(vii) administrative work.

In all types of power plants there is an economy of scale, i.e. the construction
costs per kW decrease with size (Fig. 1.7). This effect varies with the type of
equipment. The economy of scale of the construction costs $C_c$ of two parts of
equipment of the same type can be defined by

$$C_{c2} = C_{c1} \left(\frac{R_2}{R_1}\right)^k$$

(1.7)

where $C_{c1}$ and $C_{c2}$ are for parts with rated outputs of $R_1$ and $R_2$, respectively, $k < 1$. Average values of $k$ for conventional steam generators and for steam turbo-
generators and centrifugal pumps are 0.8, whereas $k = 0.6$ for nuclear steam
supply systems and $k = 0.9$ for condensers and feedwater heaters (Aschner,
1978).

### 1.4.2 Fixed Costs and Depreciation

Fixed costs consist of annual costs for interest, depreciation, insurance and taxes
on property and estimated income calculated as a percentage of the capital
investment.

Interest rates $I$ depend on general financial conditions at the time of
construction and on the source of funding international, national, public, private
or bond issues. Because of inflation, i.e., decrease in the value of the money, the
concept of real and charged interest rates has been proposed. If, for instance, the
real rate is fixed at 10% and a price inflation of 7% per year is forecast, the
charged interest rate goes up to 17%.

Depreciation rates $D$ are determined by the life expectancy of the equipment
and the method used for calculating the depreciation.

The power plant and equipment in the station will have a certain period of
useful life. After years of use, the equipment loses its efficiency or becomes
obsolete and needs replacement. Sometimes, equipment may have to be changed
even when fairly new, if more efficient equipment has come into the market. To
enable this to be done when necessary, some money is put aside annually, and is
known as the depreciation fund.

There are two methods of accumulating the money for the depreciation fund:

1. Straight-line method
2. Sinking-fund method

The straight-line method is based on the assumption that depreciation occurs
uniformly every year according to a straight-line law. The money saved neglects
any interest. If $A$ is the capital cost of the equipment and $G$ is its salvage value after $N$ useful years, then the depreciation charge per year is

$$D = (A - G)/N$$

Figure 1.8 illustrates this simple, straight-line method of accumulating the depreciation reserve. It is seen that the value of the equipment, which is 100% initially, depreciates to only 15% at the end of its 100% useful years of life. The straight-line $BE$ indicates the depreciation and the slope of the line indicates the rate at which the plant depreciates. The depreciation at the end of 50% of useful life is $FG$, and the total depreciation at the end of 100% of useful life is $CE$. The line $AD$ represents the depreciation reserve to be accumulated so that at the end of useful life the total is $DJ$, which is equal to the total depreciation $CE$.

![Diagram](image)

**Fig. 1.8 Straight-line method**

In the sinking-fund method, a sum of money is set aside every year for $N$ years and invested to earn compound interest. Let us suppose $P$ is the annual deposit and $i$ is the interest compounded annually when the deposit is invested.

At the end of the first year, Rs. $P$ is deposited. After $(N-1)$ years this will be worth (compounded annually)

$$Rs. P(1 + i)^{N-1}$$

At the end of the second year, Rs. $P$ is deposited. After $(N-2)$ years this will be worth

$$Rs. P(1 + i)^{N-2}$$
At the end of \((N-1)\) years, Rs. \(P\) is deposited. After 1 year this will be worth
Rs. \(P(1 + i)\)

At the end of \(N\)th year, Rs. \(P\) is deposited and this will be worth Rs. \(P\)
At the end of \(N\) years, the value of all the sums deposited will be

\[
P(1 + i)^{N-1} + P(1 + i)^{N-2} + \ldots + P(1 + i) + P
\]

\[
= P + P(1 + i) + P(1 + i)^2 + \ldots + P(1 + i)^{N-1}
\]

This is a geometric progression with a common ratio of \(r = 1 + i\).
Therefore, the sum is

\[
S = P \frac{(1 + i)^{N-1}}{i}
\]

This sum is equal to \(A' = A - G\) = capital cost – salvage value

\[
P = \frac{A'i}{(1 + i)^{N-1}} \quad (1.8)
\]

The sinking-fund method can also be represented by a diagram as shown in
Fig. 1.9. The curve \(AHD\) shows the sinking fund accumulating at a certain rate
of compound interest. The total amount at the end of the useful life is given by
\(JD\), which is the initial cost of the plant minus its salvage value. The rate at
which the sum is saved annually is not, however, the same as in the straight-line
method, as the final sum consists of annual instalments plus the compound
interest on all instalments at the end of the whole period \(JD = CE\). The
depreciation at the end of 50% of useful life is shown by \(FG\), while the sinking-
fund saved at the end of 50% of useful life is shown by \(HI\).

The amount which should be set aside annually is given by Eq. (1.8),

\[\text{Fig. 1.9 Sinking fund method}\]
\[ P = \text{annual payment to sinking fund} \]
\[ = (\text{initial value} - \text{salvage value}) \frac{i}{(1 + i)^{N-1}}, \]

where \( i \) is the annual rate of compound interest and \( N \) is the number of years over which the total amount of depreciation is to be saved.

### 1.4.3 Fuel Cost

The fuel consumption depends on the amount of electrical energy produced. As load on the prime movers increases, the fuel consumption increases and so does the cost. The efficiency of the prime mover is the highest at the rated load. At lower loads, the efficiency decreases and so the fuel consumption increases. Therefore, the cost of electricity generation is the minimum at rated load and it increases as load decreases. The cost of fuel also depends on the calorific value, availability and transportation charges.

### 1.4.4 Present-Worth Concept

In the evaluation of power plant development plans or alternate payment schemes (cash flow schemes) for financing a power plant project, the influence of interest on the value of funds to be spent at a later date leads to the present-worth concept. The present worth is the value of a sum of money at the present time that, with compound interest, will have a specified value at a certain time in the future.

Compound interest payment at the interest rate \( i \) will increase the value of a fund by \((1 + i)^N\) within \( N \) years. The present worth (PW) of a payment \( S \) to be made after \( N \) years is therefore,

\[
PW = \frac{S}{(1 + i)^N} \quad (1.9)
\]

The term \((1 + i)^{-N}\) is frequently referred to as the single payment present worth factor (PWF).

If \( i = 10\% \) and \( N = 4 \) years,

\[
PW = 0.683 \times S
\]

Also, if \( i = 10\%, N = 1 \) year and \( S = \text{Rs. 110}, \) then \( PW = \text{Rs. 100}. \)

On many occasions, equal amounts of annual expenses are required. Then the present worth of a uniform annual series of payments \( P \) after \( N \) years is calculated from Eqs (1.8) and (1.9),

\[
PW = P \frac{(1 + i)^{N-1}}{i} \times \frac{1}{(1 + i)^N}
\]
\[ \frac{P}{i(1+i)^N} \left[ (1+i)^N - 1 \right] = \left( \frac{P}{0.10(1.1)^Y} \right) \left[ (1.1)^Y - 1 \right] = 6.8136 \ P \]

1.4.5 Incremental Heat Rate

The performance of a generating plant can be expressed in terms of

Plant net heat rate (PNHR) (expressed in kJ/kWh) = \( \frac{\text{heat input} (I)}{\text{net kW output} (L)} \)

and

Plant net efficiency \( (n_{th}) \) = \( \frac{\text{net output} (L)}{\text{heat input} (I)} \)

Table 1.1 indicates typical performance of various generating systems. The given efficiencies are achieved only when the plants operate at full loads. The incremental heat rate (IR) is defined as

\[ IR = \frac{dL}{dL} \]

It represents the amount of energy needed to generate an additional unit of output at any given load. In Figure 1.10, it is seen that as the plant output decreases, the plant net heat rate increases, whereas the incremental heat rate decreases.

![Graph](image)

**Fig. 1.10** Plant net heat rate and incremental heat rate
Table 1.1 Typical conversion efficiency of various generating systems (Li and Priddy, 1985)

<table>
<thead>
<tr>
<th>Generation type</th>
<th>Unit size (MW)</th>
<th>Thermal efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steam</td>
<td>200 – 800</td>
<td>30 – 40</td>
</tr>
<tr>
<td>Nuclear (BWR &amp; PWR)</td>
<td>500 – 1100</td>
<td>31 – 34</td>
</tr>
<tr>
<td>Gas turbine</td>
<td>50 – 100</td>
<td>22 – 28</td>
</tr>
<tr>
<td>Combined gas turbine/steam turbine</td>
<td>300 – 600</td>
<td>36 – 50</td>
</tr>
<tr>
<td>Diesel engine</td>
<td>10 – 30</td>
<td>27 – 30</td>
</tr>
</tbody>
</table>

1.4.6 Economic Scheduling Principle

An utility system has always more than one generating unit. Proper distribution of load among the generating units is a problem frequently encountered by engineers. If the load is not properly distributed, it will result in a decrease of the thermal efficiency as a whole. According to the economic scheduling principle, the load is so distributed that at any moment all generating units will have the same incremental heat rates. As the system load increases, the incremental heat rate of each unit will increase. This principle is illustrated by the example involving two generating units in the network.

Let $I_c$ be the combined input to units 1 and 2, and $L_c$ be the combined output of units 1 and 2. The combined input, $I_c$, is a function of either $L_1$ or $L_2$ for a given combined output $L_c$. When the combined input $I_c$ is at a minimum, it must hold

\[
\frac{dI_c}{dL_1} = 0
\]

Since

\[
I_c = I_1 + I_2
\]

we have

\[
\frac{dI_1}{dL_1} + \frac{dI_2}{dL_1} = 0
\]

Now,

\[
\frac{dI_2}{dL_1} = (\frac{dI_2}{dL_2}) \times (\frac{dL_2}{dL_1})
\]

Since,

\[
L_c = L_1 + L_2
\]

\[
\frac{dL_2}{dL_1} = -1
\]

\[
\frac{dI_2}{dL_1} = - \frac{dI_2}{dL_2}
\]

From Eqs. (1.13) and (1.14),

\[
\frac{dI_1}{dL_1} = \frac{dI_2}{dL_2}
\]

Thus, the combined input $I_c$ is a minimum only if the incremental heat rate of unit 1 is equal to that of unit 2. This principle can be extended to the network of multiple units.

Figure 1.11 represents the input-output curves of two generating units operating in parallel and supplying a common load. The corresponding heat rate curves are shown in Fig. 1.12. In dividing the load between these units to achieve maximum fuel economy, the principle of equal incremental heat rate is being used. Starting at zero load, the turbine $B$ picks up the load up to $L_{B1}$, while
the turbine $A$ remains at zero load. When combined load is greater than $L_{B1}$, the turbine $A$ starts to pick up load until it reaches $L_{A1}$. At the combined load $L_{A1}+$
The second valve of turbine starts to open. As the load increases, the turbine B picks up the additional load, while the turbine A remains at the load $L_{A1}$. For any increase beyond $L_{A1} + L_{B2}$, the turbine A starts to open the second valve and later the third valve. At the combined load $L_{A3} + L_{B2}$, the turbine A would be at a full load $L_{A3}$ and the turbine B at the load $L_{B2}$. If the combined load increases beyond this level, the turbine B starts to open the last valve to meet the additional demand.

In a utility system with many units, to maintain continuity of service, the generating capacity in operation must be greater than the system load. The difference between these two is defined as the spinning reserve. The minimum magnitude of spinning reserve varies from one utility to another, but it is usually equal to or greater than the capacity of the largest unit in operation. To minimize the total system fuel cost, the principle of equal incremental heat rate is to be observed in distributing the system load among the operating units.

**1.4.7 Effect of Load Factor on Cost per kWh**

The fixed costs are proportional to the MW capacity of a plant, while the operating costs are proportional to the total energy produced by the station annually. The fixed costs remain the same whether the station is running or idle. The fuel consumption and operating costs will, however, vary with the load. At 100% load factor the installed capacity is utilized to the best possible extent and the maximum possible amount of energy is generated during the year. The cost per kWh is thus a minimum in this case. When the load factor is lower than 100%, less energy is produced giving a higher cost per kWh, since the non-varying fixed cost and lower operating costs are distributed over fewer kWh generated. As the load factor decreases, the cost per kWh increases (Fig. 1.13).

![Diagram](https://via.placeholder.com/150)

**Fig. 1.13** Effect of load factor on cost of energy
1.5 INDIAN ENERGY SCENARIO

Economic growth in India, being dependent on the power sector, has necessitated an enormous growth in electricity demand over the last two decades. In 1947, the total power generation capacity was only 1360 MW, and by 1991 it grew to 65,000 MW, of which 69% (45,000 MW) was generated in thermal plants.

Hydro developments, as stated are not so environmentally benign as was earlier believed. They can cause conflict over usage of lands and the damage to ecology. The total Indian hydro-potential, as assessed by the Central Electricity Authority, is 84,000 MW. The installed capacity is 18,443 MW (March, 1991) compared to 200 kW in 1897 (Darjeeling) and 508 MW at the time of independence. Most of the turbinomachinery have been indigenously produced by the Bharat Heavy Electricals Ltd., the largest unit being 165 MW. However, the pace of hydro development has considerably slowed down recently for many reasons, and the focus is more on mini or small hydel plants in the mountainous regions. Tentatively, in the order of 26,000 MW of hydro capacity would be added by 2002 during the 9th five-year plan, as envisaged.

Nuclear power promotion has all but ceased in many areas of the world as a consequence of growing public concern regarding safety and regulatory hurdles. Nonetheless, in India, there appears to be a comparatively energetic nuclear programme. Presently, in India, about 2.3% of generating capacity is nuclear based. This corresponds to about 1500 MW of the installed capacity comprising 8 units. An additional 1320 MW is under construction, based on 220 MW units, and should be on line by 1997. According to the development plan, the total installed capacity by 2002 will be 5700 MW, including the first Indian 500 MW PHWR unit in Rajasthan. It is anticipated that following on the tail of the natural uranium fuelled reactor programme, a fast breeder programme using plutonium will be established, followed by a thorium based programme, thereby taking advantage of a large indigenous resource of thorium.

The state-of-the-art technology for the renewable energy utilization (i.e. solar, wind, biomass, tidal, OTEC, etc.) is such that it will hardly make any contribution to the total energy production. However, construction of a 100 MW ocean thermal energy conversion (OTEC) project has commenced in 1994, 40 km offshore from Kulasenkarapatanam in Tamil Nadu.

Coal-fired power plants are now getting maximum attention, since coal is abundantly available and the implementation time is relatively short. Tentatively, the additional thermal power generation capacity during the 8th Plan is 28,000 MW and during the 9th Plan (by 2002) further 32,000 MW has been added as shown in Table 1.2.

It has been planned to generate an additional 73,000 MW of power by the end of the 11th Plan (2007-2012) which excludes the proposed eight ultra megapower projects with a capacity of 4000 MW each (i.e. 105,000 MW total). The per capita energy consumption will be still 600 MW in India, while it is 12000 MW in China (in 2006).
Table 1.2  Indian generation capacity (in MW)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal</td>
<td>45,000</td>
<td>28,000(a)</td>
<td>32,000</td>
<td>58,000</td>
<td>1,63,000</td>
</tr>
<tr>
<td></td>
<td>(69.3%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydro</td>
<td>18,443</td>
<td>8,680</td>
<td>26,000</td>
<td>23,000</td>
<td>76,123</td>
</tr>
<tr>
<td></td>
<td>(28.5%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nuclear</td>
<td>1,500</td>
<td>1,320</td>
<td>2,880</td>
<td>–</td>
<td>5,700</td>
</tr>
<tr>
<td></td>
<td>(2.2%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Addition</td>
<td>38,000</td>
<td>61,000</td>
<td>81,000</td>
<td></td>
<td>–</td>
</tr>
<tr>
<td>Total</td>
<td>65,000</td>
<td>1,03,000</td>
<td>1,64,000</td>
<td>2,45,000</td>
<td>2,45,000</td>
</tr>
</tbody>
</table>

* includes 7500 MW of gas-based combined cycle plant.

As in other parts of the world, natural gas-based combined cycle power plant is gaining prominence. Consequently, the 8th Plan envisages installation of 7500 MW capacity, using indigenous gas reserves.

Table 1.3 gives the growth pattern in unit sizes and steam parameters in coal-based power plants. In the 70s, the focus was on 100 and 200 MW units, while in the 80s it was on 210 and 500 MW units. Consideration is now being given to increasing the size of future units to 660,800 and 1000 MW to reduce installation cost per kW.

Table 1.3  Growth pattern in unit sizes and steam parameters

<table>
<thead>
<tr>
<th>Unit size (MW)</th>
<th>Collaboration</th>
<th>Steam parameters</th>
<th>Station first installed</th>
<th>Year</th>
<th>No. of units till March, 1991</th>
</tr>
</thead>
<tbody>
<tr>
<td>10–15</td>
<td></td>
<td>27.5 bar 400°C</td>
<td></td>
<td>1947–50</td>
<td>N.A.</td>
</tr>
<tr>
<td>30–37.5</td>
<td></td>
<td>59 bar, 482°C</td>
<td></td>
<td>1951–60</td>
<td>62</td>
</tr>
<tr>
<td>60–100</td>
<td></td>
<td>72.4–91 bar, 496–535°C</td>
<td></td>
<td>1961–76</td>
<td>56</td>
</tr>
<tr>
<td>100</td>
<td>USSR</td>
<td>91 bar, 535°C</td>
<td>Patratu</td>
<td>Dec. 71</td>
<td>79</td>
</tr>
<tr>
<td>200</td>
<td>LMW, USSR</td>
<td>132 bar, 535°C/535°C</td>
<td>Obra (UPSEB)</td>
<td>Dec. 77</td>
<td>12</td>
</tr>
<tr>
<td>210</td>
<td>KWU, Siemens</td>
<td>152 bar, 535°C/535°C</td>
<td>Korba (NTPC)</td>
<td>March 83</td>
<td>89</td>
</tr>
<tr>
<td>500</td>
<td>KWU, Siemens</td>
<td>172 bar, 538°C/538°C</td>
<td>Trombay-5 (Tata)</td>
<td>Dec. 84</td>
<td>13</td>
</tr>
</tbody>
</table>

With the liberalization of Indian economy, the power sector has been made open to the private sector, and it is expected that the rate of growth of generation capacity will be faster than that envisaged in the five-year plans (Table 1.2).
Fig. 1.14  Modern Pulverized Coal Fuelled Electricity Generating Unit
1.6 COAL-FUELLED ELECTRICITY GENERATING UNIT

Figure 1.14 depicts a modern pulverized coal-fired electricity generation facility that uses the most commonly employed Rankine-based thermodynamic cycle. Steam at the desired pressure, temperature and mass flow is produced in a steam generator and expanded through a turbine generator. The exhaust steam is then condensed in a condenser and the condensed liquid is used again in the steam generator.

Coal is delivered to the facility by railway wagons, barges or trucks. The coal handling system unloads the coal, stocks, reclaiming, crushes and conveys it to storage silos. Coal from the silos is then pulverized to a fine powder and blown into the steam generator, where it is mixed with air and combusted to release energy for the generation of steam. The steam generator produces, superheats and reheats steam as it proceeds through the cycle.

The steam turbine generator converts the thermal energy (enthalpy rise) of the superheated and reheated steam to electrical energy. Steam exhausted from the turbine is condensed to liquid in the condenser. The condensate pumps feed the water through the l.p. regenerative feedwater heaters to a deaerator. Boiler feed pumps move the deaerated liquid through the h.p. feedwater heaters back to the steam generator.

Forced draught (or draft) fans supply combustion air to the steam generator and the primary air fans transport pulverized coal into the burners. Induced draught fans remove the flue gases from the furnace and exhaust them through the stack into the atmosphere.

Cooling water for the condenser is supplied by the circulating water system, which takes the heat removed from the condenser and rejects it to cooling towers or another heat sink such as a cooling lake, river or sea.

Combustion gases coming out of the steam generator require additional treatment for removal of flyash particulates, sulphur dioxide and nitrogen oxides before the gases are released through the exhaust stack.

Example 1.1 A power station supplies the following loads to the consumers:

<table>
<thead>
<tr>
<th>Time in hours</th>
<th>0–6</th>
<th>6–10</th>
<th>10–12</th>
<th>12–16</th>
<th>16–20</th>
<th>20–22</th>
<th>22–24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load in MW</td>
<td>30</td>
<td>70</td>
<td>90</td>
<td>60</td>
<td>100</td>
<td>80</td>
<td>60</td>
</tr>
</tbody>
</table>

(a) Draw the load curve and estimate the load factor of the plant. (b) What is the load factor of a standby equipment of 30 MW capacity if it takes up all loads above 70 MW? What is its use factor?

Solution (a) The load curve is drawn in Fig. E1.1.

Energy generated = area under the load curve

\[ = 30 \times 6 + 70 \times 4 + 90 \times 2 + 60 \times 4 + 100 \times 4 + 80 \times 2 + 60 \times 2 \]
Fig. E1.1

\[ = 180 + 280 + 180 + 240 + 400 + 160 + 120 \]
\[ = 1560 \text{ MWh} \]

Average load = \( \frac{1560 \text{ MWh}}{24 \text{ h}} \) = 65 MW

Load factor = \( \frac{\text{average load}}{\text{peak load}} \) = \( \frac{65}{100} \) = 0.65 \( \text{Ans.} \)

(b) If the load above 70 MW is supplied by a standby unit of 30 MW capacity, the energy generated by it
\[ = 20 \times 2 + 30 \times 4 + 10 \times 2 \]
\[ = 40 + 120 + 20 = 180 \text{ MWh} \]

Time during which the standby unit remains in operation
\[ = 2 + 4 + 2 = 8 \text{ h} \]

Average load = 180 MWh/8h = 22.5 MW
Load factor = 22.5/30 = 0.75 \( \text{Ans.} \)

Use factor = \( \frac{\text{energy generated}}{\text{plant capacity} \times \text{operating hours}} \)
\[ = \frac{180 \times 10^3 \text{ kWh}}{30 \times 10^3 \text{ kW} \times 8 \text{ h}} = 0.75 \text{ Ans.} \]
**Example 1.2** The peak load on a power plant is 60 MW. The loads having maximum demands of 30 MW, 20 MW, 10 MW and 14 MW are connected to the power plant. The capacity of the power plant is 80 MW and the annual load factor is 0.50. Estimate (a) the average load on the power plant, (b) the energy supplied per year, (c) the demand factor, (d) the diversity factor.

**Solution**

(a) Load factor = \( \frac{\text{average load}}{\text{peak load}} \)

\[
0.5 = \frac{\text{average load}}{60 \text{ MW}}
\]

Average load = 30 MW \( \text{Ans.} \)

(b) Energy supplied per year

\[
= \text{average load} \times 8760 \text{ h} = 30 \times 8760 \text{ MWh} = 262.8 \times 10^6 \text{ kWh} \quad \text{Ans.}
\]

(c) Demand factor = \( \frac{\text{maximum demand}}{\text{connected load}} = \frac{60}{30 + 20 + 10 + 14} \)

\[
= \frac{60}{74} = 0.811 \quad \text{Ans.}
\]

(d) Diversity factor = \( \frac{\text{sum of individual maximum demands}}{\text{simultaneous maximum demand}} \)

\[
= \frac{30 + 20 + 10 + 14}{60} = \frac{74}{60} = 1.233 \quad \text{Ans.}
\]

**Example 1.3** A thermal power plant of 210 MW capacity has the maximum load of 160 MW. Its annual load factor is 0.6. The coal consumption is 1 kg per kWh of energy generated and the cost of coal is Rs 450.00 per tonne. Calculate (a) the annual revenue earned if energy is sold at Re 1 per kWh and (b) the capacity factor of the plant.

**Solution** Annual load factor = (average load)/(peak load)

Average load = 0.6 \times 160 = 96 MW

Energy generated per year

\[
= 96 \times 8760 \text{ MWh} = 840,960 \times 10^3 \text{ kWh}
\]

Coal required per year = 840,960 \times 10^3 \text{ kg}

\[
= 840,960 \text{ tonnes}
\]

Cost of coal per year = 840,960 \times 450 = \text{Rs} \ 378,432 \times 10^6

Cost of energy sold = \text{Rs} \ 840,960 \times 10^3

\[
= \text{Rs} \ 840.96 \times 10^6
\]
(a) Revenue earned by the power plant per year

\[ = \text{Rs } 840.96 \times 10^6 - \text{Rs } 378.432 \times 10^6 \]

\[ = \text{Rs } 462.528 \times 10^6 = \text{Rs } 46.25 \text{ crore} \quad \text{Ans. (a)} \]

(b) Capacity factor = \frac{\text{average load}}{\text{capacity of plant}} = \frac{96 \text{ MW}}{210 \text{ MW}}

\[ = 0.457 \quad \text{Ans. (b)} \]

**Example 1.4** A power plant has the following annual factors: load factor = 0.75, capacity factor = 0.60, use factor = 0.65. Maximum demand is 60 MW. Estimate (a) the annual energy production, (b) the reserve capacity over and above the peak load, and (c) the hours during which the plant is not in service per year.

**Solution**

(a) Load factor = \frac{\text{average load}}{\text{peak load}}

Average load = 0.75 \times 60 = 45 \text{ MW}

Annual energy production = 45,000 \times 8760 = 394,200,000 \text{ kWh}

\[ = 394.2 \times 10^6 \text{ kWh} \quad \text{Ans.} \]

(b) Capacity factor = \frac{\text{average load}}{\text{plant capacity}}

Plant capacity = 45/0.6 = 75 \text{ MW}

Reserve capacity over and above the peak load

\[ = 75 - 60 = 15 \text{ MW} \quad \text{Ans.} \]

(c) Use factor = \frac{\text{energy generated per year}}{\text{plant capacity} \times \text{hours in operation}}

Hours in operation = \frac{394.2 \times 10^6 \text{ kWh}}{75 \times 10^3 \text{ kW} \times 0.65} = 8086 \text{ hrs}

Hours not in service in a year = 8760 - 8086 = 674 \text{ hrs} \quad \text{Ans.}

**Example 1.5** It is proposed to supply a load with a maximum demand of 500 MW and load factor of 70%. Choice is to be made from a nuclear power plant, a hydraulic power plant and a steam power plant. Calculate the overall cost per kWh in case of each scheme as given below:

<table>
<thead>
<tr>
<th>Cost</th>
<th>Steam power plant</th>
<th>Hydroelectric power plant</th>
<th>Nuclear power plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Capital cost per MW installed</td>
<td>Rs 3 crore</td>
<td>Rs 4 crore</td>
<td>Rs 5 crore</td>
</tr>
<tr>
<td>2. Interest</td>
<td>6%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>3. Depreciation</td>
<td>6%</td>
<td>4%</td>
<td>5%</td>
</tr>
<tr>
<td>4. Operating cost (including fuel) per kWh</td>
<td>30 paise</td>
<td>5 paise</td>
<td>15 paise</td>
</tr>
<tr>
<td>5. Transmission and distribution cost per kWh</td>
<td>2 paise</td>
<td>3 paise</td>
<td>2 paise</td>
</tr>
</tbody>
</table>
**Solution**

(a) Steam power plant:

Capital cost = Rs 3 × 500 crore = Rs 1500 × 10^7
Interest = 0.06 × 1500 × 10^7 = Rs 90 × 10^7
Depreciation = 0.06 × 1500 × 10^7 = Rs 90 × 10^7

Annual fixed cost (interest + depreciation) = Rs 180 × 10^7

Energy generated per year = average load × 8760
= Load factor × maximum demand × 8760
= 0.7 × 500 × 10^3 × 8760
= 306.6 × 10^7 kWh

Running cost/kWh = (operating cost / kWh)
+ (transmission and distribution cost/kWh)
= 30 + 2 = 32 paise

Overall cost/kWh = fixed cost/kWh + running cost/kWh

= \( \frac{\text{Rs} \ 180 \times 10^7}{306.6 \times 10^7 \ \text{kWh}} + 0.32 \)

= 0.59 + 0.32 = 0.91 or 91 paise

(b) Hydroelectric power plant:

Capital cost = Rs 4 × 500 crore = Rs 2000 × 10^7
Interest = 0.05 × 2000 × 10^7 = Rs 100 × 10^7
Depreciation = 0.04 × 2000 × 10^7 = Rs 80 × 10^7

Annual fixed cost = interest + depreciation = Rs 180 × 10^7

Fixed cost/kWh = \( \frac{\text{Rs} \ 180 \times 10^7}{306.6 \times 10^7} \) = 0.59

Running cost/kWh = 5 + 3 = 8 paise
Overall cost/kWh = 0.59 + 0.08 = Re 0.67

(c) Nuclear power plant:

Capital cost = Rs 5 × 500 crore = Rs 2500 × 10^7
Interest = 0.05 × 2500 × 10^7 = Rs 125 × 10^7
Depreciation = 0.05 × 2500 × 10^7 = Rs 125 × 10^7

Annual fixed cost = Rs 250 × 10^7

Fixed cost/kWh = \( \frac{\text{Rs} \ 250 \times 10^7}{306.6 \times 10^7 \ \text{kWh}} \) = 0.82

Running cost/kWh = 15 + 2 = 17 paise
Overall cost/kWh = 0.82 + 0.17 = Re 0.99

Therefore, the overall cost/kWh is minimum in case of hydroelectric power plant.
Example 1.6 A power plant of 210 MW installed capacity has the following particulars:

- Capital cost = Rs 18,000/kW installed
- Interest and depreciation = 12%
- Annual load factor = 60%
- Annual capacity factor = 54%
- Annual running charges = Rs \(200 \times 10^6\)
- Energy consumed by power plant auxiliaries = 6%

Calculate (a) the cost of power generation per kWh, and (b) the reserve capacity

Solution

\[
\frac{\text{Load factor}}{\text{Capacity factor}} = \frac{\text{average load}}{\text{maximum demand}} \times \frac{\text{capacity of the plant}}{\text{average load}}
\]

\[
\frac{0.60}{0.54} = \frac{210 \text{ MW}}{\text{maximum demand}}
\]

Maximum demand = \((210 \times 0.54)/0.60 = 189 \text{ MW}\)

Reserve capacity = \(210 - 189 = 21 \text{ MW}\) \(\text{Ans. (b)}\)

Average load = load factor \times maximum demand

= 0.6 \times 189 = 113.4 \text{ MW}

Energy produced per year

= 113.4 \times 10^3 \times 8760 = 993.384 \times 10^6 \text{ kWh}

Net energy delivered = 0.94 \times 993.384 \times 10^6

= 933.781 \times 10^6 \text{ kWh}

Annual interest and depreciation (fixed cost)

= 0.12 \times 18000 \times 210 \times 10^3 = Rs 453.6 \times 10^6

Total annual cost = fixed cost + running cost

= 453.6 \times 10^6 + 200 \times 10^6

= Rs 653.6 \times 10^6

Cost of power generation = \[
\frac{\text{Rs 653.6} \times 10^6}{933.781 \times 10^6 \text{ kWh}}
\]

= Rs 0.70 or 70 paise \(\text{Ans. (a)}\)

Example 1.7 The incremental fuel costs for two generating units \(a\) and \(b\) of a power plant are given by the following relations,

\[
dF_a/dP_a = 0.065\ P_a + 25
\]

\[
dF_b/dP_b = 0.08\ P_a + 20
\]

where \(F\) is the fuel cost in rupees per hour and \(P\) is the power output in MW. Estimate (a) the economic loading of the two units when the total load supplied by the power plants is 200 MW, (b) the loss in fuel cost per hour if the load is equally shared by both units.
Solution  (a) The given data are

\[ P_a + P_b = 200 \]  

(1)

The condition for economic loading demands equal incremental heat rates of the two units which leads to \( \frac{dF_a}{dP_a} = \frac{dF_b}{dP_b} \)

\[ 0.065 P_a + 25 = 0.08 P_b + 20 \]  

(2)

Solving Eq. (1) and (2), we get

\[ P_a = 75.86 \text{ MW} \quad \text{and} \quad P_b = 124.14 \text{ MW} \]

(b) If the load is equally shared by the units, i.e. \( P_a = P_b = 100 \text{ MW} \), then the increase in cost of fuel for unit \( a \) is given by

\[
\int \frac{dF_a}{dp_a} dp_a = \int \frac{(0.065 P_a + 25)}{75.86} \frac{dp_a}{dp_a} = \left[ 0.065 \left( \frac{P_a^2}{2} + 25 P_a \right) \right]_{75.86}^{100} = 741.47
\]

i.e., Rs 741.47 per hour.

The increase in cost for unit \( b \) is

\[
\int \frac{dF_b}{dp_b} dp_b = \int \frac{(0.08 P_b + 20)}{124.14} \frac{dp_b}{dp_b} = \left[ 0.08 \left( \frac{P_b^2}{2} + 20 P_b \right) \right]_{124.14}^{100} = -699.23
\]

that is, the cost of fuel for unit \( b \) decreases by Rs 699.23 per hour.

Therefore, the net increase in fuel cost due to departure from economic distribution of load

\[ = 741.47 - 699.23 = \text{Rs 42.24 per hour} \quad \text{Ans.} \]

Example 1.8  Calculate the cost of generation per kWh for a power station having the following data:

- Installed capacity of the plant = 200 MW
- Capital cost = Rs 400 crores
- Rate of interest and depreciation = 12%
- Annual cost of fuel, salaries and taxation = Rs 5 crores
- Load factor = 50%
- Also estimate the saving in cost per kWh if the annual load factor is raised to 60%.

Solution  Assuming the maximum demand equal to the capacity of the power plant,

\[ \text{Load factor} = \frac{\text{average load}}{\text{maximum demand}} \]
\[ 0.5 = \frac{\text{average load}}{200} \]

Average load = 200 \times 0.5 = 100 \text{ MW}

Energy generated per year = 100 \times 10^3 \times 8760 = 87.6 \times 10^7 \text{ kWh}

Interest and depreciation (fixed cost)

\[ = 0.12 \times 400 \times 10^7 = \text{Rs} \ 48 \times 10^7 \]

Total annual cost = 48 \times 10^7 + 5 \times 10^7 = \text{Rs} \ 53 \times 10^7

Cost per kWh = \frac{\text{Rs} \ 53 \times 10^7}{87.6 \times 10^7 \text{ kWh}} = \text{Rs} \ 0.61 \ \text{ or } \ 61 \text{ paise} \quad \text{Ans.}

When the load factor is raised to 60%,

Average load = 0.60 \times 200 = 120 \text{ MW}

Energy produced per year = 120 \times 10^3 \times 8760

= 105.12 \times 10^7 \text{ kWh}

Total annual cost will remain the same.

Cost per kWh = \frac{\text{Rs} \ 53 \times 10^7}{105.12 \times 10^7} = \text{Re} \ 0.50 \ \text{ or } \ 50 \text{ paise}

Saving in cost per kWh = 61 - 50 = 11 \text{ paise} \quad \text{Ans.}

**Example 1.9**  A 300 MW thermal power station is to supply power to a system having maximum and minimum demand of 240 MW and 180 MW respectively in a year. Assuming the load duration curve to be a straight line, estimate the (a) load factor, (b) capacity factor.

**Solution**  The load duration curve is shown in Figure E1.9.

Energy supplied per year

\[ = (180 \text{ MW} \times 8760 \text{ h}) + \frac{1}{2} (240 - 180) \text{ MW} \times 8760 \text{ h} \]

\[ = 210 \times 8760 \text{ MWh} = 183.96 \times 10^7 \text{ kWh} \]

Average load = \frac{183.96 \times 10^7 \text{ kWh}}{8760 \text{ h}} = 210 \times 10^3 \text{ kW}

Load factor = \frac{\text{average load}}{\text{maximum demand}} = \frac{210 \text{ MW}}{240 \text{ MW}} = 0.875 \quad \text{Ans.}

Capacity factor = \frac{210 \times 8760}{300 \times 8760} = 0.70 \quad \text{Ans.}
**Example 1.10**  The maximum load on a thermal power plant of 60 MW capacity is 50 MW at an annual load factor of 60%. The coal consumption is 1 kg per unit of energy generated and the cost of coal is Rs 600 per tonne of coal. Find (a) the annual revenue earned if the energy is sold at Rs 2 per kWh and (b) the capacity of the plant.

**Solution**  
Average load = Peak load x Load factor  

\[ = 50 \times 0.6 = 30 \text{ MW} \]

Energy generated per year = \(30 \times 10^3 \times 8760 = 262.8 \times 10^6 \text{ kWh}\)

Coal required per year = \(\frac{(262.8 \times 10^6) \times 1}{1000} = 262.8 \times 10^3 \text{ tonnes}\)

Cost of coal per year = \(262.8 \times 10^3 \times 600 = 15,768 \times 10^4 \text{ rupees}\)

Cost of energy sold = \(262.8 \times 10^6 \text{ kWh} \times 2 = \text{Rs } 525.6 \times 10^6 \)

Revenue earned by the power plant per year  

\[ = \text{Rs } 525.6 \times 10^6 - \text{Rs. } 157.68 \times 10^6 \]

\[ = \text{Rs } 367.92 \times 10^6 \quad \text{Ans.}\]

**Example 1.11**  A thermal power plant consists of two 60 MW units, each running for 8000 hours, and one 30 MW unit running for 2000 hours per year. The energy produced by the plant is \(876 \times 10^6 \text{ kWh per year}\). Determine the plant load factor and plant use factor. Consider the maximum load as equal to the plant capacity.

**Solution**  
Plant capacity = \(60 \times 2 + 30 = 150 \text{ MW}\)

Average load = \(\frac{\text{Energy produced per year}}{8760}\)

\[ = \frac{876 \times 10^6}{8760} = 10^5 \text{ kW} = 100 \text{ MW}\]

Load factor = \(\frac{100}{150} = 0.67 \quad \text{Ans.}\)

Plant use factor = \(\frac{\text{Actual energy produced}}{\text{Max. possible energy that can be produced}}\)
Max. possible energy that can be produced

\[ = 60 \times 2 \times 8000 + 30 \times 1 \times 200 \text{MWh} \]

\[ = 1020 \times 10^6 \text{kWh} \]

\[ \therefore \quad \text{Plant use factor} = \frac{876 \times 10^6}{1020 \times 10^6} = 0.86 \quad \text{Ans.} \]

**Example 1.12** The following loads are connected to a power plant:

<table>
<thead>
<tr>
<th>Type of load</th>
<th>Max. demand (MW)</th>
<th>Diversity factor</th>
<th>Demand factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>15</td>
<td>1.25</td>
<td>0.70</td>
</tr>
<tr>
<td>Commercial</td>
<td>25</td>
<td>1.20</td>
<td>0.90</td>
</tr>
<tr>
<td>Industrial</td>
<td>50</td>
<td>1.30</td>
<td>0.98</td>
</tr>
</tbody>
</table>

If the overall diversity factor is 1.5, determine (a) the maximum load, and (b) the connected load of each type.

**Solution** The maximum demand of the plant

\[ = 15 + 25 + 50 = 90 \text{MW} \]

Since the overall diversity factor is 1.5, the maximum load of the system

\[ = \frac{90}{1.5} = 60 \text{MW} \quad \text{Ans. (a)} \]

Max. domestic demand = 15 \times 1.25 = 18.75 MW

Connected domestic load = \[ \frac{18.75}{0.70} = 26.78 \text{ MW} \]

Similarly, connected commercial load

\[ = \frac{25 \times 1.2}{0.9} = 33.33 \text{ MW} \]

Connected industrial load = \[ \frac{50 \times 1.3}{0.98} = 66.33 \text{ MW} \]

\[ \therefore \quad \text{Total connected load of the plant} = 26.78 + 33.33 + 66.33 = 126.44 \text{ MW} \quad \text{Ans.} \]

**Example 1.13** A power station has to supply load as follows:

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0-6</th>
<th>6-12</th>
<th>12-14</th>
<th>14-18</th>
<th>18-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load (MW)</td>
<td>30</td>
<td>90</td>
<td>60</td>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

(a) Draw the load curve. (b) Draw the load duration curve. (c) Select suitable generating units to supply the load. (d) Calculate the load factor. (e) Calculate the capacity of the plant and the plant capacity factor.
Solution  The load curve and load duration curve have been drawn in Fig. E1.13.

Energy generated = $30 \times 6 + 90 \times 6 + 60 \times 2 + 100 \times 4 + 50 \times 6$

$= 1540 \text{ MWh} = 1540 \times 10^3 \text{ kWh}$

Average load $= \frac{1540 \times 10^3}{24} \text{ kW}$

Maximum demand $= 100 \times 10^3 \text{ kW}$

Load factor $= \frac{1540 \times 10^3}{24 \times 100 \times 10^3} = 0.64 \quad \text{Ans. (d)}$

To supply the load, three generating units, each of 30 MW capacity, and one generating unit of 10 MW capacity will be selected. One additional unit will be kept as standby. Its capacity will be equal to that of the largest unit, i.e. 30 MW.
Load duration curve will indicate the operational schedule of different generating units, which will be as follows:

(i) One unit of 30 MW will run for 24 hours  
(ii) Second unit of 30 MW will run for 18 hours  
(iii) Third unit of 30 MW will run for 10 hours  
(iv) Fourth unit of 10 MW will run for 4 hours  

\[ \text{Plant capacity} = 30 \times 4 + 10 \times 1 = 130 \text{ MW} \]  
\[ \text{Ans. (c)} \]

\[ \text{Capacity factor} = \frac{\text{energy generated}}{\text{capacity} \times \text{operating time}} \]

\[ = \frac{1540 \times 10^3 \text{ kWh}}{130 \times 10^3 \times 24 \text{ kWh}} = 0.494 \]  
\[ \text{Ans. (e)} \]

**Example 1.14**  
A generating unit of 10 MW capacity supplies the following loads:

(a) Domestic consumers with a maximum demand of 6 MW at a load factor of 20%.  
(b) Small industrial load with a maximum demand of 3.6 MW at a load factor of 50%.  
(c) Street-light load with a maximum demand of 400 kW at 30% load factor.  

Find the overall cost of energy per kWh for each type of consumer using the following data:

Capital cost of the plant = Rs 10,000 per kW  
Total running cost = Rs 36,00,000 per year  
Annual rate of interest and depreciation on capital cost = 10%  

**Solution**  
Energy supplied per year to all three consumers

\[ = [(600 \times 0.2) + (3600 \times 0.5) + (400 \times 0.3)] \times 8760 \]

\[ = 3120 \times 8760 \text{ kWh} \]

Operating charges per kWh = \[\frac{36,00,000}{3120 \times 8760} \]

\[ = 0.132 \text{ or } 13.2 \text{ paise} \]

Capital cost of the plant = \[10 \times 10^3 \times 10^4 = \text{Rs} 10^8\]

Fixed charges per year = \[0.1 \times 10^8 = \text{Rs} 10^7\]

Fixed charges per kW = \[(\text{Rs} 10^7)/(10^4 \text{ kW}) = \text{Rs} 1000\]

(a) For domestic consumers,

\[ \text{total charges} = 1000 \times 6000 + 0.132 \times 6000 \times 0.2 \times 8760 \]

\[ = \text{Rs} 7,387,584 \]

Overall cost per kWh = \[\frac{\text{Rs} 7,387,584}{1200 \times 8760 \text{ kWh}} \]
(b) For industrial consumers,

\[ \text{total charges} = 1000 \times 3600 + 0.132 \times 3600 \times 0.5 \times 8760 \]
\[ = \text{Rs} \ 5,681,376 \]

Overall cost per kWh = \[ \frac{\text{Rs} \ 5,681,376}{1800 \times 8760} = \text{Rs} \ 0.36, \text{ or } 36 \text{ paise} \quad \text{Ans.} \]

(c) For street-lighting load,

\[ \text{total charges} = 1000 \times 400 + 0.132 \times 400 \times 0.3 \times 8760 \]
\[ = \text{Rs} \ 5,387,584 \]

Overall cost per kWh = \[ \frac{\text{Rs} \ 5,387,584}{120 \times 8760} = \text{Rs} \ 0.51 \text{ or } 51 \text{ paise} \quad \text{Ans.} \]

**Example 1.15** The capital cost of a power generating equipment in a steam power plant is \( \text{Rs} \ 80 \times 10^6 \). The useful life of the plant is 30 years and its salvage value is 5% of the capital cost. Determine by the sinking-fund method the amount of money to be saved annually for replacement if the yearly rate of compound interest is 6%.

**Solution** From Eq. (1.8), the amount of money to be kept aside each year is given by

\[
P = \frac{A' i}{(1 + i)^N - 1}
\]

Here,

\[ A' = \text{capital cost} - \text{salvage value} \]
\[ = 0.95 \times 80 \times 10^6 = \text{Rs} \ 76 \times 10^6 \]
\[ N = 30 \text{ years; } i = 0.06 \]

\[ P = \frac{76 \times 10^6 \times 0.06}{(1.06)^{30} - 1} = \frac{4560000}{4.7435} = \text{Rs} \ 9,61,317 \quad \text{Ans.} \]

**Example 1.16** The initial investment in a power plant is \( \text{Rs} \ 4000 \text{ crore} \). After 4 years, a further \( \text{Rs} \ 1200 \text{ crore} \) have to be paid and from the 5th onwards to the 12th year \( \text{Rs} \ 400 \text{ crore per year} \) are paid. The salvage value of the plant after 30 years will be \( \text{Rs} \ 600 \text{ crore} \). The interest rate is 10%. Estimate the present worth of these payments at the time of commissioning.

**Solution** By using Eqs. (1.9) and (1.10), the present worth of all payments at the time of commissioning is

\[
\text{PW} = 4000 + 1200 \left( \frac{1}{(1 + 0.1)^4} + \frac{400}{0.1(1 + 0.1)^{12}} \left[ (1.1)^{12} - 1 \right] \right)
\]
\[ - \frac{400}{0.1(1 + 0.1)^5} \left[ (1.1)^5 - 1 \right] - 600 \left( \frac{1}{(1 + 0.1)^{30}} \right) \]
\[ = 4000 + 819.62 + 2725.44 - 1516.31 - 34.39 \]
\[ = \text{Rs} \ 5994.36 \text{ crores} \quad \text{Ans.} \]
Example 1.17  Two coal-fired generating units $P$ and $Q$ have the incremental heat rate defined by

$$(IR)_P = 0.4818 \times 10^{-7} L_P^4 - 0.9089 \times 10^{-4} L_P^3 + 0.6842 \times 10^{-1} L_P^2 - 0.2106 \times 10^2 + 9860$$

for unit $P$, and

$$(IR)_Q = 0.9592 \times 10^{-7} L_Q^4 - 0.7811 \times 10^{-4} L_Q^3 + 0.2625 \times 10^{-1} L_Q^2 - 0.2189 \times 10 L_Q + 9003$$

for unit $Q$; (IR) is in kJ/kWh and $L$ is in MW. Determine the incremental heat rate at which the combined output of these two units is 1000 MW.

**Solution**

$$L = L_P + L_Q$$

where, $L_P$ = load of unit $P$ and $L_Q$ = load of unit $Q$

$L_P + L_Q = 1000$

By making $(IR)_P = (IR)_Q$, and solving the above three equations by a numerical method such as Newton-Raphson algorithm, we get

$L_P = 732.5$ MW

$L_Q = 267.5$ MW

$IR = (IR)_P = (IR)_Q = 9292$ kJ/kWh  \(\text{Ans.}\)

Example 1.18  The fixed costs of a thermal station are Rs. 2700 per kW of installed capacity per year. The fuel and operating costs are 40 paise per kWh generated. Find the cost of electrical energy per kWh at station load factors of 100%, 75%, 50% and 25%.

**Solution**  Fixed costs per kW per hour = \(\frac{\text{Rs 2700}}{8760}\) = 31 paise

At 100% load factor, 1 kW plant running for 1 hour produces 1 kWh. If run at 75% load factor, it can produce 0.75 kWh and so on. The cost per kWh at different load factors can therefore be found as follows:

<table>
<thead>
<tr>
<th>Load factor (%)</th>
<th>Energy produced in 1 hr with 1-kW plant (kWh)</th>
<th>Fixed cost per hr (paise)</th>
<th>Fuel and operating cost (paise)</th>
<th>Total cost per hr (paise)</th>
<th>Cost per kWh (paise)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1</td>
<td>31</td>
<td>40</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>75</td>
<td>0.75</td>
<td>31</td>
<td>30</td>
<td>61</td>
<td>81</td>
</tr>
<tr>
<td>50</td>
<td>0.50</td>
<td>31</td>
<td>20</td>
<td>51</td>
<td>102</td>
</tr>
<tr>
<td>25</td>
<td>0.25</td>
<td>31</td>
<td>10</td>
<td>41</td>
<td>164</td>
</tr>
</tbody>
</table>
SHORT-ANSWER QUESTIONS

1.1 Why is electricity the most convenient form of energy?
1.2 What are the different types of power plants where electricity is produced in bulk quantities?
1.3 What is doubling time in electricity production?
1.4 How is the demand for electricity related to the GNP of a country?
1.5 What are the two basic parameters to decide while planning a power plant?
1.6 How is the total installed capacity of a power plant decided?
1.7 On what factors does the unit size of a power plant depend?
1.8 What do you understand by load factor and capacity factor? When are they numerically equal?
1.9 Define reserve factor. What is its usual value? How does it fix the maximum unit size?
1.10 What do you mean by diversity factor?
1.11 How is the load–duration curve constructed?
1.12 Explain what you understand by base load and peaking load. Why are base load plants loaded heavily?
1.13 What are the considerations to be made while selecting the suitable site for a thermal and a nuclear power plant?
1.14 What are the elements which contribute to the cost of electricity?
1.15 How is the total annual cost of electricity estimated?
1.16 What is the forced outage rate?
1.17 How can the cost of power generation be reduced?
1.18 What are the expenses which constitute the construction cost of a power plant?
1.19 How does the economy of scale affect the construction cost?
1.20 What are fixed costs?
1.21 What do you understand by depreciation? How is the depreciation rate determined?
1.22 What is the straight-line method of determining the depreciation charge?
1.23 What do you mean by the sinking-fund method of accumulating the money for the depreciation fund?
1.24 How does the fuel cost relate to the load and the cost of power generation?
1.25 What do you understand by the present-worth concept? How is it estimated?
1.26 What is incremental heat rate?
1.27 Explain the principle of economic scheduling.
1.28 Show that for two units running in parallel, the combined energy input will be minimum if the incremental heat rate of unit 1 is equal to that of unit 2.
1.29 Explain what you mean by spinning reserve.
1.30 Explain the effect of load factor on the cost of electricity generated.
1.1 The loads on a power plant with respect to time for 24 hours are given as follows:

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0–6</th>
<th>6–8</th>
<th>8–12</th>
<th>12–14</th>
<th>14–18</th>
<th>18–22</th>
<th>22–24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load (MW)</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>50</td>
<td>70</td>
<td>80</td>
<td>40</td>
</tr>
</tbody>
</table>

Draw the load curve and find out the load factor of the power station. If the loads above 60 MW are taken by a standby unit of 20 MW capacity, find out the load factor and the use factor of the standby unit.

[Ans. 0.71, 0.75 and 0.60]

1.2 A central power station has annual load factor, capacity factor and use factor as 60%, 40% and 45%, respectively. The maximum demand is 15 MW. Estimate (a) the annual energy production and revenue earned if the cost of energy is Re 1/kWh, (b) the reserve capacity over and above the peak load, and (c) the hours per year the station is not in service.

[Ans. (a) 78.84 × 10^6 kWh, Rs 78.84 × 10^6, (b) 7.5 MW, (c) 974 h]

1.3 A power station of 30 MW capacity has the maximum annual demand of 25 MW. It supplies loads having maximum demands of 10 MW, 8.5 MW, 5 MW and 4.5 MW. The annual load factor is 45%. Estimate (a) the average load, (b) the energy supplied per year, (c) the diversity factor, and (d) the demand factor.

[Ans. (a) 11.25 MW, (b) 98.55 × 10^6 kWh, (c) 1.12, (d) 0.89]

1.4 A power station is said to have a use factor of 47% and capacity factor of 40%. For how many hours in a year was the power station not in service?

[Ans. 1305 h]

1.5 Determine the generating cost per unit of 80 MW power station with the following data:

Capital cost = Rs 160 × 10^7
Annual cost of fuel = Rs 32 × 10^6
Annual wages and taxes = Rs 36 × 10^6
Interest and depreciation = 10% of capital cost
Annual load factor = 45%

[Ans. 72.3 paise]

1.6 The incremental fuel costs for two generating units A and B of a power plant are given by the following relations:

\[ \frac{dF_A}{dP_A} = 0.06 \ P_A + 11.4 \]
\[ \frac{dF_B}{dP_B} = 0.07 \ P_B + 10 \]

where \( P \) is the power in MW and \( F \) is the fuel cost in rupees per hour.

(a) Find the economic loading of the two units when the total load to be supplied by the power station is 150 MW.
(b) Find the net increase in fuel cost per hour if the load is equally shared by the two units.

[Ans. (a) \( P_A = 70 \text{ MW} \), \( P_B = 80 \text{ MW} \), (b) Rs 1.63 per hour]

1.7 The following data pertain to a power plant:
Installed capacity = 200 MW; Capital cost = Rs 4000 \times 10^6; Annual cost of fuel, taxes and salaries = Rs 600 \times 10^6; Rate of interest = 5% of the capital; Rate of depreciation = 6% of the capital; Annual load factor = 65%; Capacity factor = 55%; Energy used in running the plant auxiliaries = 4% of total units generated. Determine (a) the reserve capacity, and (b) the cost of power generation.

[Ans. (a) 31 MW]

1.8 A power plant has the following annual factors:
Load factor = 70%; Capacity factor = 50%; Use factor = 60%; Maximum demand = 20 MW.
Estimate (a) annual energy production, (b) reserve capacity over and above the peak load, and (c) the hours per year during which the plant is not in service.

[Ans. (a) \( 122.8 \times 10^6 \text{ kWh} \), (b) 8 MW, (c) 2060 h]

1.9 A 60 MW turbogenerator set has an overall efficiency of 25%. The calorific value of coal used is 24 MJ/kg. Estimate the consumption of coal per kWh and also per day of 24 h if the load factor is 30%.

[Ans. 0.6 kg, 259.2 tonnes]

1.10 Find the cost of generation per kWh from the following data:
Capacity of the plant = 150 MW; Capital cost = Rs 20,000 per kW installed; Interest and depreciation = 10% on capital; Fuel consumption = 1.2 kg/kWh; Fuel cost = Rs 400 per tonne; Salaries, wages, repair and maintenance = Rs 150 \times 10^6 per year; Maximum demand = 120 MW; Load factor = 50%.

1.11 A plant costing Rs 65,000 has a useful life of 15 years. Find the amount which should be saved annually to replace the equipment at the end of that time (a) by the straight-line method, and (b) by the sinking-fund method if the annual rate of compound interest is 5%. Assume that the salvage value of the equipment is Rs 5000.

[Ans. (a) Rs 4000, (b) Rs 2780]

1.12 The connected loads of a consumer consist of 10 electric bulbs of 60 W each and 2 electric heaters of 1000 W each. His maximum demand is 1500 W. On an average, he uses 8 bulbs for 5 hours a day and one heater for 3 hours a day. Estimate his average load, monthly energy consumption and load factor.

[Ans. 350 W, 252 kWh, 23.33%]

1.13 A power generating station has a maximum demand of 10,000 kW and the daily load on the station is as follows:
(a) Draw the load curve and the load duration curve.
(b) Choose the size and number of generating units.
(c) Draw up the operating schedule of the units.
(d) What reserve plant would be necessary?
(e) Calculate the load factor, plant capacity factor and plant use factor of the station.

[Ans. (b) Three units of 5000, 2500 and 2500 kW capacity, (d) 5000 kW, (e) 55%, 36.66%, 86.5%]

1.14 Explain the method of constructing a load duration curve using a load curve. The following data were collected from the daily load curves of a power system during a year:

<table>
<thead>
<tr>
<th>Load (kW)</th>
<th>Duration (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15,000</td>
<td>87</td>
</tr>
<tr>
<td>12,000</td>
<td>876</td>
</tr>
<tr>
<td>10,000</td>
<td>1,752</td>
</tr>
<tr>
<td>8,000</td>
<td>2,628</td>
</tr>
<tr>
<td>6,000</td>
<td>4,380</td>
</tr>
<tr>
<td>4,000</td>
<td>7,000</td>
</tr>
<tr>
<td>2,000</td>
<td>8,760</td>
</tr>
</tbody>
</table>

Construct the annual load duration curve and find the load factor of the system.

[Ans. 46.8%]

1.15 A power station has an installed capacity of 210 MW. The cost of the station is Rs 2 crore per MW. The fixed costs are 14% of the cost of investment. On full load at 100% load factor, the variable costs of the station per year are 1.5 times the fixed costs. Assume that there is no reserve capacity of the plant and that the variable costs are proportional to the energy production. Find the cost of generation per kWh at load factors of 100%, 80%, 60%, 40% and 20%. Plot the curve.

**BIBLIOGRAPHY**

2.1 STEAM POWER PLANT

A steam power plant continuously converts the energy stored in fossil fuels (coal, oil, natural gas) or fissile fuels (uranium, thorium) into shaft work and ultimately into electricity. The working fluid is water which is sometimes in the liquid phase and sometimes in the vapour phase during its cycle of operations. Figure 2.1 illustrates a fossil fuelled power plant as a bulk energy converter from fuel to electricity using water as the working medium. Energy released by the burning of fuel is transferred to water in the boiler (B) to generate steam at a high pressure and temperature, which then expands in the turbine (T) to a low pressure to produce shaft work. The steam leaving the turbine is condensed into water in the condenser (C) where cooling water from a river or sea circulates carrying away the heat released during condensation. The water (condensate) is then fed back to the boiler by the pump (P), and the cycle goes on repeating itself. The working substance, water, thus follows along the B–T–C–P path of the cycle interacting externally as shown. Since the fluid is undergoing a cyclic process, there will be no net change in its internal energy over the cycle (\( \oint dE = 0 \)), and consequently, the net energy transferred to the unit mass of the fluid as heat during the cycle must equal the net energy transfer as work from the fluid.

\[
\sum_{\text{cycle}} Q_{\text{net}} = \sum_{\text{cycle}} W_{\text{net}}
\]

or,

\[
Q_1 - Q_2 = W_T - W_P
\]  \(2.1\)

where

- \(Q_1\) = heat transferred to the working fluid, kJ/kg
- \(Q_2\) = heat rejected from the working fluid, kJ/kg
- \(W_T\) = work transferred from the working fluid, kJ/kg
- \(W_P\) = work transferred into the working fluid, kJ/kg

The efficiency of the vapour power cycle would thus be
\[ \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \] (2.2)

**Fig.2.1 Steam power plant—bulk energy converter from fuel to electricity**

### 2.2 RANKINE CYCLE

For each process in the vapour power cycle, it is possible to assume a hypothetical or ideal process which represents the basic intended operation and does not produce any extraneous effect (like heat loss). For the steam boiler, this would be a reversible constant pressure heating process of water to form steam, for the turbine the ideal process would be a reversible adiabatic expansion of steam, for the condenser it would be a reversible constant pressure heat rejection as the steam condenses till it becomes saturated liquid, and for the pump the ideal process would be the reversible adiabatic compression of this liquid ending at the initial pressure. When all these four processes are ideal, the cycle is an ideal cycle, called a Rankine cycle. This is a reversible cycle. Figure 2.2 shows the flow diagram of the Rankine cycle, and in Fig. 2.3, the cycle has been plotted on \( p - v, T - s \) and \( h - s \) coordinates. Applying the steady flow energy equation (SFEE) to each of the processes on the basis of a unit mass of fluid and neglecting changes in kinetic and potential energy, the work and heat interactions can be evaluated in terms of the properties of the fluid.

For 1 kg fluid, the SFEE for the boiler as the control volume gives

\[ h_4 + Q_1 = h_1 \]

\[ \therefore \quad Q_1 = h_1 - h_4 \] (2.3)

The SFEE for the turbine as the control volume gives

\[ h_1 = W_T + h_2 \]

\[ \therefore \quad W_T = h_1 - h_2 \] (2.4)
Similarly, the SFEE for the condenser is
\[ h_2 = Q_2 + h_3 \]
\[ \therefore \quad Q_2 = h_2 - h_3 \quad (2.5) \]

and the SFEE for the pump gives
\[ h_3 + W_p = h_4 \]
\[ \therefore \quad W_p = h_4 - h_3 \quad (2.6) \]

The efficiency of the Rankine cycle is then given by
\[ \eta = \frac{W_{\text{net}}}{Q_i} = \frac{W_T - W_p}{Q_i} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} \quad (2.7) \]

The pump handles liquid water which is incompressible, i.e. its density or specific volume undergoes little change with an increase in pressure. For reversible adiabatic compression, by the use of the property relation
\[ T \, ds = dh - vdp \]
Since \( ds = 0 \),
\[ \int_3^4 dh = \int_3^4 vdp \]
\[ h_4 - h_3 = v_3(p_4 - p_3) = W_p \quad (2.8) \]

The pump work is usually small compared to the turbine work, and is often neglected.

The capacity of a steam plant is often expressed in terms of steam rate or specific steam consumption (s.s.c). It is defined as the rate of steam flow (kg/s) required to produce unit shaft output (1 kW).

\[ \therefore \quad \text{Steam rate (S.R.)} = \frac{1}{W_{\text{net}}} \, \frac{\text{kg}}{\text{kW} \cdot \text{s}} \quad (2.9) \]

The cycle efficiency is sometimes expressed alternatively as heat rate which is the rate of heat input (kJ/s) required to produce unit shaft output (1 kW).

\[ \text{Heat rate (H.R.)} = \frac{Q_i}{W_T - W_p} = \frac{1}{\eta} \, \frac{\text{kJ}}{\text{kW} \cdot \text{s}} \quad (2.10) \]

\section*{2.2.1 Economiser, Evaporator and Superheater}

Heat transfer to water in the steam generator takes place in the three different regimes (Figs 2.2 and 2.3). Water is first heated sensibly in the economiser in the liquid phase at a certain pressure from state 4 to state 5 till it becomes saturated liquid.

In the evaporator or the boiler proper, there is phase change or boiling with the state changing from 5 to 6 by absorbing the latent heat of vaporization at that pressure. The saturated vapour at state 6 is further heated at constant pressure in the superheater to state 1 in the vapour or gaseous phase. For unit mass of fluid, heat transfer in these three types of heat exchangers is given by
\[
\begin{align*}
Q_{Eco} &= h_5 - h_4 \\
Q_{Eva} &= h_6 - h_5 = h_{fg} \\
Q_{SH} &= h_1 - h_6
\end{align*}
\]

\( (2.11) \)

**Fig. 2.2** A simple steam plant representing Rankine cycle

**Fig. 2.3** Rankine cycle on \( p-v \), \( T-s \) and \( h-s \) coordinates (a, b and c)
It is demonstrated in Fig. 2.4 by the areas under the respective processes at the particular isobar. The fractions of the total heat transfer absorbed in the economiser, evaporator and superheater are given by

\[
\begin{align*}
\frac{Q_{\text{Eco}}}{Q_1} &= \frac{h_5 - h_4}{h_1 - h_4} = \frac{\text{area under 4 - 5}}{\text{area under 4 - 1}} \\
\frac{Q_{\text{Eva}}}{Q_1} &= \frac{h_6 - h_5}{h_1 - h_4} = \frac{\text{area under 5 - 6}}{\text{area under 4 - 1}} \\
\frac{Q_{\text{SH}}}{Q_1} &= \frac{h_1 - h_6}{h_1 - h_4} = \frac{\text{area under 6 - 1}}{\text{area under 4 - 1}}
\end{align*}
\]  

(2.12)

\[\text{Fig. 2.4 Fraction of total heat transfer absorbed in the economiser, evaporator and superheater}\]

It may be noted that as the pressure increases, the latent heat decreases and so the heat absorbed in the evaporator decreases and the fraction of the total heat absorbed in the superheater increases. In high pressure boilers, more than 40% of the total heat is absorbed in the superheaters. For steam generators operating above the critical pressure (approx 221 bar) there is no evaporator or boiling section. However, there is a transition zone where all the liquid on being heated suddenly flashes into vapour (Fig. 2.5).

\[\text{Fig. 2.5 Rankine cycle with supercritical boiler pressure}\]
2.2.2 Internally Irreversible Rankine Cycle

Internal irreversibility of Rankine cycle is caused by fluid friction, throttling and mixing. As the flow rates in the steam turbine as well as in the pumps are large, and the expansion and compression processes are quite rapid, the heat loss per unit mass may be considered negligible. Though the assumption of adiabatic flow in them is still valid, due to fluid friction the expansion and compression processes are not reversible and entropy of the fluid in both increases (Fig. 2.6). The internal or isentropic efficiency ($\eta_T$) of the turbine is given by

$$\eta_T = \frac{h_1 - h_2}{h_1 - h_{2s}}$$  \hspace{1cm} (2.13)

The pumping process, being adiabatic and irreversible, also results in an increase in entropy, and the isentropic efficiency of the pump is given by

$$\eta_p = \frac{h_{4s} - h_3}{h_4 - h_3}$$  \hspace{1cm} (2.14)

The actual pump work would thus be

$$W_p = \frac{h_{4s} - h_3}{\eta_p} = \frac{v_3(p_4 - p_1)}{\eta_p}$$ \hspace{1cm} (2.15)

One thus pays a penalty for irreversibility: the turbine produces less work and the pump absorbs more work.

The liquid leaving the pump must be at a higher pressure than at the turbine inlet because of the pressure drops due to friction etc. in boiler heat exchangers, feedwater heaters, pipes, bends, valves etc. Thus, $p_4$ represents the exit pump pressure, $p_1$ the turbine inlet pressure, and $p_5$ the steam generator exit pressure (Fig. 2.6). Steam leaves the boiler at state 5 and enters the turbine at state 1. The pressure of steam drops from $p_5$ to $p_{5'}$ (or $p_1$) due to friction in the pipeline and entropy decreases from 5' to 1 due to heat loss.

Fig. 2.6 Internally irreversible Rankine cycle
2.2.3 Externally Irreversible Rankine Cycle

External irreversibility of the Rankine cycle is caused due to the temperature differences between the combustion gases and the working fluid on the source side, and the temperature differences between the condensing working fluid and the condenser cooling water on the sink side (Figs 2.2 and 2.7). The products of combustion or flue gases get cooled from \( a \) to \( d \) and the working fluid temperature rises from 4 to 1 in counterflow heat exchangers. The minimum temperature differences between the two fluids are \( c - 5 \) and \( a - 1 \), and the points where these occur are called \textit{pinch points}. Too small a pinch point temperature difference causes a lower thermal (external) irreversibility and an increase in surface area resulting in a large expensive steam generator, whereas a large pinch-point temperature difference results in a small, inexpensive steam generator but with a reduced plant efficiency due to a large thermal irreversibility. The most economical pinch-point temperature difference is obtained by optimization which takes into account the fixed charges (based on capital costs) and operating costs (based on efficiency, and hence, fuel costs).

Line \( e-f \) represents the rise in temperature of the cooling water in the condenser, whereas the working fluid temperature remains constant during condensation process \( 2-3 \). When one of the fluids undergoes phase change, the direction of flow of the other fluid, whether parallel or counter-flow, is immaterial so far as the size of the heat exchanger and thermal irreversibility are concerned.

![Diagram of Externally Irreversible Rankine Cycle](image)

\( \text{Fig. 2.7 External irreversibility with Rankine cycle} \)
CARNOT CYCLE

The Carnot cycle is an ideal but non-practical cycle giving the maximum possible thermal efficiency for a cycle operating on selected maximum and minimum temperature ranges. For the Carnot cycle 1–2–3–4 in Fig. 2.8, compressing a very wet steam at state 3 would require a compressor of size and cost comparable with the turbine; it would absorb work comparable to that developed by the turbine and its life would be short because of blade erosion and cavitation problems due to excessive moisture. For the cycle 1–2–5–6–1, the pump work \((h_6 - h_5)\) is again very high and it is impossible to supply heat at infinite pressures and at constant temperature from state 6 to state 4. So, the Carnot cycle cannot be realised in practice, but it sets the upper limit to which the cycle efficiency of any thermal plant can be raised. For both the cycles 1–2–3–4 and 1–2–5–6, the cycle efficiency is given by

\[
\eta_{\text{max}} = 1 - \frac{T_2}{T_1} = \eta_{\text{Carnot}}
\]  (2.16)

MEAN TEMPERATURE OF HEAT ADDITION

In the Rankine cycle, heat is added reversibly at a constant pressure but at infinite temperatures. If \(T_{m_1}\) is the mean temperature of heat addition as shown in Fig. 2.9, so that the area under 4 and 1 is equal to the area under 5 and 6, then heat added is

\[
Q_1 = h_1 - h_4 = T_{m_1} (s_1 - s_4)
\]

\[
\therefore \quad T_{m_1} = \text{mean temperature of heat addition} = \frac{h_1 - h_4}{s_1 - s_4}
\]  (2.17)
Since $Q_2 = \text{heat rejected} = h_2 - h_3 = T_2(s_1 - s_4)$,

$$\therefore \quad \eta_{\text{Rankine}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_{m_1}} \frac{(s_1 - s_4)}{(s_1 - s_4)} = 1 - \frac{T_2}{T_{m_1}} \quad (2.18)$$

where $T_2$ is the temperature of heat rejection. The lower is the $T_2$ for a given $T_{m_1}$, i.e. lower is the condenser pressure, the higher will be the efficiency of the Rankine cycle. But, the lowest practicable temperature of heat rejection is the temperature of the surroundings, $T_o$. The saturation pressure corresponding to this temperature $T_o$ is the minimum pressure to which steam can be expanded in the turbine. This being fixed by the ambient conditions,

$$\eta_{\text{Rankine}} = f(T_{m_1}) \quad \text{only} \quad (2.19)$$

*The higher the mean temperature of heat addition, the higher will be the cycle efficiency.*

### 2.4.1 Effect of Superheat

The effect of increasing the initial temperature at constant pressure on cycle efficiency is shown in Fig. 2.10. When the initial state changes from 1 to 1', $T_{m_1}$ between 1 and 1' is higher than $T_{m_1}$ between 4 and 1. So an increase in the superheat at constant pressure increases the mean temperature of heat addition and hence, the cycle efficiency. Moreover, with increase in superheat, the expansion line of steam in the turbine shifts to the right, as a result of which the quality of steam at turbine exhaust increases and performance of the turbine improves, as explained later. If hot flue gas is the primary fluid or heat source for steam generation in the power cycle, the use of superheat also reduces the thermal irreversibility. Since $c_p = T \left( \frac{\partial s}{\partial T} \right)_p$, we have the slope $\left( \frac{\partial T}{\partial s} \right)_p = \frac{T}{c_p}$. Now $(c_p)_{\text{water}} > (c_p)_{\text{gas}}$, therefore $\left( \frac{\partial T}{\partial s} \right)_{\text{water}} > (c_p)_{\text{gas}}$. If hot pressurized water is the heat source for steam generation, as in a pressurized water reactor (PWR), the superheat may not be practical, since the temperature differences between $m-p$ and 5–1 vary little (Fig. 2.11). If the temperature at state 1 is fixed, to use superheat the pressure has to be reduced from $p$ to $p'$. This increases the overall temperature difference resulting in reducing rather than increasing the cycle efficiency.
Fig. 2.10 Effect of superheat on the mean temperature of heat addition

Fig. 2.11 Superheat may not be practical with hot pressurized water as heat source as in a PWR

Therefore, fossil fuel steam generators as well as gas-cooled and liquid metal cooled nuclear power plants employ superheat, while PWR power plants do not use superheat.

2.4.2 Effect of Inlet Pressure

The maximum temperature of steam that can be used is fixed from metallurgical considerations, i.e. the materials used for the manufacture of the components which are subjected to the high-pressure high-temperature steam like the superheaters, valves, pipelines, inlet stages of the turbine and so on. It is called the metallurgical limit. When the maximum temperature is fixed by this limit, as the operating steam pressure at which heat is added in the boiler increases from $p_1$ to $p_2$ (Fig. 2.12), the mean temperature of heat addition increases since $T_{\text{ml}}$ between states 7 and 5 is higher than that between states 4 and 1. But when the turbine inlet pressure increases from $p_1$ to $p_2$, the ideal expansion line of steam shifts to the left and the moisture content at the turbine exhaust increases (because $x_6 < x_2$). If the moisture content of steam in the later stages of the turbine is high, the entrained water particles along with the vapour coming out
of the nozzles with high velocity strike the blades and erode their edges, as a result of which the life of the blades decreases. From the consideration of the erosion of blades in the later stages of a turbine, the maximum moisture content at the turbine exhaust is not allowed to exceed 12%, or the quality of steam to fall below 88%. It is desirable that most of the turbine expansion should take place in the single phase or vapour region.

Therefore, with the maximum steam temperature at the turbine inlet, the minimum temperature of heat rejection, and the minimum quality of steam at the turbine exhaust being fixed by the materials used, the ambient conditions, and turbine blade erosion, respectively, the maximum steam pressure at the turbine inlet also gets fixed (Fig. 2.13). The vertical line drawn from state 2 fixed by $T_2$ and $x_2$, intersects the $T_{\text{max}}$ line at state 1, which gives the maximum steam pressure at the turbine inlet. The irreversibility in the expansion process has, however, not been considered.

Fig 2.12 Effect of increase of pressure on Rankine cycle

Fig. 2.13 Fixing of maximum steam pressure at turbine inlet

2.5 EFFECT OF VARIATION OF STEAM CONDITION ON THERMAL EFFICIENCY OF STEAM POWER PLANT

The variation of Rankine efficiency with the inlet steam pressure at a constant steam temperature of 470 °C and at three condenser pressures is shown in Fig. 2.14. It is seen that for inlet steam pressures above 100 bar, there is a continuing but decreasing rate of improvement of cycle efficiency. The increase in steam pressure is limited by considerations of mechanical stresses and the ensuing higher cost of equipment.

Figure 2.14 also demonstrates that there is a considerable improvement in cycle efficiency with the decrease of condenser pressure. Such a decrease mainly depends on the available cooling water temperature ($t_c$) and thus on the climatic condition of the place. A lower cooling water temperature gives lower condenser pressure (higher vacuum). It follows that with identical steam
conditions, and cycle and similar equipment, the thermal efficiency of a condensing steam power plant will be less in a warm region than in a cold region.

![Graph showing the effect of inlet steam pressure and condenser pressure on Rankine efficiency with constant inlet steam temperature of 470 °C](image)

**Fig. 2.14** Effect of inlet steam pressure (p₁) and condenser pressure on Rankine efficiency with constant inlet steam temperature of 470 °C

An increase in inlet steam temperature, i.e. an increase in superheat at constant inlet steam pressure and condenser pressure gives a steady improvement in cycle efficiency and lowers the heat rate due to the increase in $T_{m1}$, as discussed in the earlier section. Raising the inlet steam temperature also reduces the wetness of the steam in the later stages of the turbine and improves the turbine internal efficiency. However, the increase in steam temperature is limited by the properties of the construction materials of boilers and turbines. The ultimate strength of unalloyed steels drops by about 30% as the steam temperature is raised from 400 to 500 °C. Alloying with chromium and molybdenum and eventually, the use of austenitic instead of ferritic steels increases the strength at high temperatures. Steam temperatures up to 620 °C have been used in some plants. The operating experience with the expensive high-temperature austenitic steels has, however, not been uniformly satisfactory. Recent practice in steam power plants generally limits steam temperatures to 538°C and in a few cases to 565 °C.

The maximum steam pressures (throttle pressures) that can be used at three different condenser pressures and two inlet steam temperatures are shown in Table 2.1 with turbine internal efficiency of 85% and quality at turbine exhaust of 88%. For other values of $\eta_T$, $t_1$ and $x_2$, the pressure limits can be readily determined by drawing the corresponding expansion line of the turbine on a Mollier diagram (Fig. 2.13).
Table 2.1 *Maximum inlet steam pressure (without reheating)*
with $\eta_T = 0.85$ and $x_2 = 0.88$

<table>
<thead>
<tr>
<th>Condenser pressure</th>
<th>Turbine inlet steam temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_j = 500 , ^\circ\text{C}$</td>
</tr>
<tr>
<td>$p_2 = 25 , \text{mm Hg abs}$</td>
<td>$(p_1)_{\text{max}} = 60 , \text{bar}$</td>
</tr>
<tr>
<td>$p_2 = 50 , \text{mm Hg abs}$</td>
<td>$(p_1)_{\text{max}} = 80 , \text{bar}$</td>
</tr>
<tr>
<td>$p_2 = 75 , \text{mm Hg abs}$</td>
<td>$(p_1)_{\text{max}} = 95 , \text{bar}$</td>
</tr>
</tbody>
</table>

### 2.6 REHEATING OF STEAM

If a steam pressure higher than $(p_1)_{\text{max}}$ is used (Fig. 2.13), in order to limit the quality to 0.88 at turbine exhaust, reheating of steam has to be adopted. In that case, all the steam after partial expansion in the turbine is brought back to the boiler, reheated by combustion gases and then fed back to the turbine for further expansion. The flow, $T$-$s$ and $h$-$s$ diagrams for the ideal Rankine cycle with reheat are shown in Fig. 2.15. In the *reheat cycle* the expansion of steam from the initial state 1 to the condenser pressure is carried out in two or more steps depending upon the number of reheats used. In the first step, steam expands in the high pressure (H.P.) turbine from the initial state to some intermediate pressure (process 1–2s). The steam is resuperheated (or reheated) at constant pressure in the boiler (process 2s–3) and the remaining expansion (process 3–4s) of steam is carried out in the low pressure (L.P.) turbine. For 1 kg of steam,
Fig. 2.15  Reheat Cycle

\[
Q_1 = h_1 - h_{6s} + h_3 - h_{2s} \quad Q_2 = h_{4s} - h_5
\]

\[
W_T = h_1 - h_{2s} + h_3 - h_{4s} \quad W_p = h_{6s} - h_5
\]

\[
\eta = \frac{W_T - W_p}{Q_1} = \frac{(h_1 - h_{2s} + h_3 - h_{4s}) - (h_{6s} - h_5)}{h_1 - h_{6s} + h_3 - h_{2s}}
\]  

(2.20)

Steam rate = \(\frac{3600}{W_{net}}\) kg/kWh  

Heat rate = \(\frac{3600}{\eta}\) kJ/kWh  

(2.21)  (2.22)

Had the high pressure \(p_1\) been used without reheat (Fig. 2.15), the cycle would have been 1–4’ s–5–6s with lot of moisture at turbine exhaust having quality \(x_{4's}\). With the use of reheat, the area 2s–3–4s–4’s has been added to the basic cycle. It is seen that the net work output of the plant increases with reheat because \((h_3 - h_{4s})\) is greater than \((h_{2s} - h_{4's})\), and hence the steam rate decreases. Reheating also improves the quality at turbine exhaust from \(x_{4's}\) to \(x_{4s}\). Whether
the cycle efficiency improves with reheat depends upon whether the mean temperature of heat addition in process 2s–3 is higher than that in process 6s–1.

By increasing the number of reheat, still higher steam pressures could be used, but the mechanical stresses increase in much higher proportion than the pressure because of the prevailing high temperature. The cost and fabrication difficulties will also increase. In that way the maximum steam pressure gets fixed, and more than two reheat have not yet been used. The use of more than two reheat results in cycle complication and increases capital costs that are not justified by improvement in the cycle efficiency.

The cycle efficiency in a single reheat plant is influenced by the pressure \( p_{rh} \) at which steam is reheated. The change in cycle efficiency \( \Delta \eta \) per cent as a function of the ratio of reheat pressure to initial pressure \( p_{rh}/p_1 \) is plotted in Fig. 2.16, for the cycle with initial steam at 172 bar and 538 °C and steam reheat at 538 °C. For \( p_{rh}/p_1 = 1 \), \( \Delta \eta = 0 \) since no reheat is used. A reheat pressure too close to the initial pressure results in little improvement in cycle efficiency because only a small portion of additional heat is added at high temperature. The efficiency increases as the reheat pressure \( p_{rh} \) is lowered and reaches a peak at a pressure ratio \( p_{rh}/p_1 \) between 0.20 and 0.25. Lowering the reheat pressure further causes the temperature differences between the primary fluid (flue gases) and the working fluid to increase, and brings down the efficiency again since the mean temperature of heat addition during reheat is less than that of the basic cycle. Too low a reheat pressure (for \( p_{rh}/p_1 < 0.025 \)) results in a negative \( \Delta \eta \). The optimum reheat pressure for most of the modern power plants is 0.2 to 0.25 of the initial steam pressure. It may also be noted in Fig. 2.16 that as the reheat pressure decreases, the quality of steam at turbine exhaust \( (x_d) \) increases. For too low a reheat pressure the exhaust steam may even be in the superheated state, which is not good for the condenser.

2.7 REGENERATION

In order to increase the mean temperature of heat addition \( T_{ml} \) and reduce the external thermal irreversibility, attention was so far confined to increasing the amount of heat supplied at high temperatures, such as increasing superheat, using higher pressure and temperature of steam, and using reheat. The mean temperature of heat addition can also be increased by reducing the amount of heat added at low temperatures (in the liquid phase) in the economiser section of the steam generator with the attention focused in the process of heat transfer between the flue gas and feedwater, \( c-d \) and 4–5 in Fig. 2.7, and \( e-b \) and 6s–7 in Fig. 2.15(b). This irreversibility could be entirely eliminated if the feedwater could be entered into the steam generator at saturated liquid state 5 (Fig. 2.7) rather than at 4. This is possible by the process of regeneration in which energy is exchanged internally between the expanding fluid in the turbine and the compressed fluid before heat addition.
Fig. 2.16 Effect of reheat-to-initial pressure ratio on cycle efficiency, h.p. turbine exit temperature and l.p. turbine exhaust quality with initial steam at 172 bar, 538 °C and steam reheat to 538 °C

2.7.1 Stirling Cycle

A well known gas cycle that uses regeneration is the Stirling cycle comprising two reversible isotherms and two reversible isochores. Heat addition at constant temperature $T_1$ from an external source and heat rejection at constant temperature $T_2$ to an external sink take place in the processes 4–1 and 2–3 respectively. Regeneration or heat exchange occurs reversibly between the constant volume processes 1–2 and 3–4 (Fig. 2.17). The areas under 1–2 and 3–4 denoting heat lost by the expanded fluid and gained by the compressed fluid are equal. Therefore, all the heat is added reversibly at $T_1$ and all the heat is rejected reversibly at $T_2$. So, the ideal Stirling cycle has the same efficiency as the Carnot cycle.

![Fig. 2.17 Regenerative Stirling cycle with reversible heat transfer between processes 1–2 and 3–4](image)

2.7.2 Ideal Regenerative Cycle

Following the Stirling cycle, in the ideal regenerative cycle (Fig. 2.18) the condensate after leaving the pump circulates around the turbine casing so that
heat is transferred from the vapour expanding in the turbine to the condensate circulating around it. It is assumed that this heat transfer process is reversible, i.e. at each point the temperature of the vapour is only infinitesimally higher than the temperature of the liquid. The process 1–2 thus represents reversible expansion of steam in the turbine with reversible heat rejection to the surrounding liquid heated reversibly in the process 4s–5. For any small step,

\[ \Delta T \text{(water)} = -\Delta T \text{(steam)} \quad \text{and} \quad \Delta s \text{ (water)} = -\Delta s \text{ (steam)} \]

The slopes of the lines 1–2 and 5–4s (Fig. 2.18) are thus identical in contour. Areas 4s–5–b–a–4s and 1–d–c–2–1 are not only equal but also congruous. Therefore, all the heat added from an external source \( (Q_1) \) is at constant temperature \( T_1 \), and all the heat rejected \( (Q_2) \) is at constant temperature \( T_2 \), both being reversible. Then

\[
Q_1 = h_1 - h_5 = T_1(s_1 - s_5) \\
Q_2 = h_2 - h_3 = T_2(s_2 - s_3)
\]

Since for reversible heat transfer,

\[
\Delta s_{\text{uni}} = \Delta s_{\text{water}} + \Delta s_{\text{steam}} = 0 \\
\Delta s_{\text{water}} = -\Delta s_{\text{steam}} \quad \text{or} \quad s_5 - s_3 = s_1 - s_2
\]

\[
\therefore \quad s_1 - s_5 = s_2 - s_3
\]

\[
\therefore \quad \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}
\]

The efficiency of the ideal regenerative cycle like the Stirling cycle is thus equal to that of the Carnot cycle. Writing the steady flow energy equation for the turbine,

\[
h_1 - W_T - h_2 + h_{4s} - h_5 = 0
\]

\[
\therefore \quad W_T = (h_1 - h_2) - (h_5 - h_{4s})
\]

The pump work remains the same as in the Rankine cycle, i.e.,

\[
W_p = h_{4s} - h_3
\]
The net work output of the ideal regenerative cycle is thus less and hence, its steam rate will be more, although it is more efficient, compared to the Rankine cycle. However, the cycle is not practicable because:
(a) reversible heat transfer cannot be realized in finite time,
(b) heat exchanger in the turbine is mechanically impracticable, and
(c) the moisture content of the steam in the turbine is high, which leads to excessive erosion of turbine blades.

2.8 REGENERATIVE FEEDWATER HEATING

In the practical regenerative cycle, the feedwater enters the boiler at a temperature between states 5 and 4 s (Fig. 2.18), and it is heated by steam extracted or bled from intermediate stages of the turbine. The flow, T-s and h-s diagrams with saturated steam at turbine inlet and two regenerative direct-contact feedwater heaters are shown in Fig. 2.19. For 1 kg of steam at turbine inlet, $m_1$ kg of steam is extracted at pressure $p_2$ to mix adiabatically in the heater 1 with the $(1 - m_1)$ kg of feedwater as shown. The remaining $(1 - m_1)$ kg steam expands reversibly to pressure $p_3$ when $m_2$ kg steam is extracted to mix with $(1 - m_1 - m_2)$ kg feedwater in the heater 2. The remaining steam, $(1 - m_1 - m_2)$ kg, expands reversibly to the condenser pressure $p_4$. The heat and work transfer quantities of the cycle are

$$
W_T = 1(h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4)
$$

$$
W_p = (1 - m_1 - m_2)(h_6 - h_7) + (1 - m_1)(h_8 - h_9) + 1(h_{10} - h_9)
$$

$$
Q_1 = 1(h_1 - h_{10}); \quad Q_2 = (1 - m_1 - m_2)(h_4 - h_3)
$$

$$
\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{W_T - W_p}{Q_1}
$$

(2.24)
Fig. 2.19 Regenerative cycle with two direct-contact feedwater heaters

In the Rankine cycle operating at the given boiler and condenser pressures $p_1$ and $p_4$, the heat addition would have been from state 6 to state 1. By using two stages of regenerative heating, feedwater enters at state 10 instead of state 6, and heat addition from an external heat source is now from state 10 to state 1. Therefore,

\[
(T_{m1}) \text{ with regeneration} = \frac{h_1 - h_{s0}}{s_1 - s_{s0}} \\
(T_{m1}) \text{ without regeneration} = \frac{h_1 - h_{s6}}{s_1 - s_{s6}}
\]  \hspace{1cm} (2.25)

Since $(T_{m1})$ with regeneration $>$ $(T_{m1})$ without regeneration, the efficiency of the regenerative cycle is higher than that of the Rankine cycle.

The energy balance for heater 1 gives

\[ m_1 h_2 + (1 - m_1)h_8 = 1 h_9 \]

\[ \therefore \quad m_1 = \frac{h_9 - h_8}{h_2 - h_8} \]  \hspace{1cm} (2.26)

The energy balance for heater 2 gives

\[ m_2 h_3 + (1 - m_1 - m_2)h_6 = (1 - m_1)h_7 \]

\[ \therefore \quad m_2 = (1 - m_1) \frac{h_7 - h_6}{h_3 - h_6} \]  \hspace{1cm} (2.27)
The path 1–2–3–4 in Fig. 2.19 (b) represents the states of a decreasing mass of fluid. For 1 kg of steam, the path would be represented by 1–2'–3'–3''–4' as shown in Fig. 2.20. From Eq. (2.24),

\[ W_1 = h_1 - h_2 + (1 - m_1) (h_2 - h_3) + (1 - m_1 - m_2) (h_3 - h_4) \]
\[ = (h_1 - h_2) + (h_2' - h_3') + (h_3'' - h_4') \] (2.28)

where 
\( (1 - m_1) (h_2 - h_3) = 1 (h_2' - h_3') \) and 
\( (1 - m_1 - m_2) (h_3 - h_4) = 1 (h_3'' - h_4') \)

The heat released by steam condensing from 2 to 2' is utilized in heating up the water from 8 to 9.

\[ 1 (h_2 - h_2') = 1(h_9 - h_8) \]

Similarly,

\[ 1(h_3' - h_3'') = 1(h_7 - h_6) \] (2.29)

From Eqs (2.27) to (2.29)

\[ W_1 = (h_1 - h_4') - (h_2 - h_2') - (h_3' - h_3'') \]
\[ = (h_1 - h_4') - (h_9 - h_8) - (h_7 - h_6) \] (2.30)

The similarity of Eq. (2.23) and (2.30) can be noticed. It is seen that the stepped cycle 1–2–2'–3'–3''–4'–5–6–7–8–9–10 approximates the ideal regenerative cycle in Fig. 2.18, and a greater number of stages would give a closer approximation.

The heat rejected \( Q_2 \) in the cycle decreases from \((h_4 - h_3)\) to \((h_4' - h_3)\). There is also a loss in work output by the amount (Area under 2–2' + area under 3'–3'' – area under 4 – 4'), as shown by the hatched area in Fig. 2.20. So, the steam rate increases by regeneration, i.e. more steam has to circulate per hour to produce unit shaft output. It increases the boiler size and hence the capital cost. It reduces the operating cost due to higher cycle efficiency.

![Diagram](image)

**Fig. 2.20 Regenerative cycle for unit mass of fluid**

From Eqs (2.26) and (2.27), \( m_1 \) and \( m_2 \) can be evaluated. Eqs (2.26) and (2.27) can also be written alternatively as

Energy given off by extracted steam in condensation = energy gain of feedwater

or

\[ m_1 (h_2 - h_9) = (1 - m_1) (h_9 - h_8) \]

\[ m_2 (h_3 - h_7) = (1 - m_1 - m_2) (h_7 - h_6) \]

Heaters have been assumed to be adequately insulated so that there is no heat loss to the surroundings.
The effects of regenerative feedwater heating for the same turbine output may be summarized as follows:
1. It significantly increases the cycle efficiency and reduces the heat rate (reducing operating cost).
2. It increases the steam flow rate (requiring bigger boiler).
3. It reduces the steam flow to the condenser (needing smaller condenser).
4. If there is no change of boiler output, the turbine output drops.

**2.9 FEEDWATER HEATERS**

Feedwater heaters are of two types, viz., open heaters and closed heaters. In an open or contact type heater, the extracted steam is allowed to mix with feedwater and both leave the heater at a common temperature (Fig. 2.19a). In a closed heater, the fluids are kept separate and are not allowed to mix together (Fig. 2.21). Closed heaters are shell-and-tube heat exchangers where the feedwater flows through the tubes and the extracted steam condenses outside the tubes in the shell. The heat released by condensation is transferred through the walls of the tubes. The condensate (saturated water at the steam extraction pressure), sometimes called the heater drip, then passes through a trap into the next lower pressure heater. This, to some extent, reduces the steam required by that heater. The trap passes only liquid and no vapour. The drip from the lowest pressure heater could similarly be trapped to the condenser, but this would be throwing away energy to the condenser cooling water. To avoid this waste, a drip pump feeds the drip directly into the feedwater stream.

If the Fig. 2.19 and 2.21 are compared, it may be noted that the feedwater inlet temperature, \( Q_1, Q_2, W_T \) and so on would be marginally affected with the heaters being either open or closed. For the heaters, the energy balance gives
\[ m_1(h_2 - h_{11}) = 1(h_{12} - h_{10}) = 1c_{pw} (t_{12} - t_{10}) \]
\[ m_2(h_3 - h_8) + m_1(h_{11} - h_8) = (1 - m_1 - m_2) (h_7 - h_6) \]
\[ = (1 - m_1 - m_2) c_{pw} (t_7 - t_6) \]  

(2.31)

where \( c_{pw} \) is the specific heat of water.

Considering the \( l - p \) heater of Fig. 2.21, wet steam at state 3 is used to heat up the h.p. subcooled feedwater at state 6. The temperature–length diagram is shown in Fig. 2.22(a). The water exit temperature at 7 cannot reach the inlet bleed steam temperature at 3. A terminal temperature difference (TTD) is defined for all closed feedwater heaters as

\[ TTD = \text{saturation temperature of bleed steam} - \text{exit water temperature} \]  

(2.32)

The value of TTD varies with heater pressure. For \( l - p \) heaters receiving wet steam, the TTD is positive and often of the order of 3 °C. Too small a value, although good for plant efficiency, would require a larger heater. Too large a value would reduce the cycle efficiency.

If the extracted steam upon condensation gets subcooled, a drain cooler may be used. The heater would then have two sections, a condensing section and a draincooler section [Fig. 2.22(b)].
Fig. 2.22 Temperature-length diagram for l.p. and h.p. feedwater heaters

For the h.p. heater receiving superheated steam [Fig. 2.22(c)] bled from the turbine at state 2, the steam is first desuperheated, then condensed and finally subcooled to state 11, whereas the feedwater gets heated from 9 to 12. It may be noted that the exit water temperature \( t_{12} \) is higher than the saturation temperature at \( p_2 \), and the TTD is here negative. The heater is then composed of a desuperheating section, a condensing section and a drain cooler section.

The advantages of the open heater are simplicity, lower cost, and high heat transfer capacity. The disadvantage is the necessity of a pump at each heater to handle the large feedwater stream.

A closed heater requires only a single pump for the main feedwater stream regardless of the number of heaters. The drip pump, if used, is relatively small. Closed heaters are costly and may not give as high a feedwater temperature as do open heaters. In most steam power plants, closed heaters are favoured but at least one open heater is used, primarily for the purpose of feedwater deaeration. The open heater in such a system is called the deaerator. Closed heaters are mostly horizontal. Sometimes, they are made vertical to reduce the floor areas needed for their installations.

### 2.10 CARNOTIZATION OF RANKINE CYCLE

Figure 2.23 shows the flow diagram of a condensing steam power plant worked with saturated steam at the turbine inlet and four steam extractions. In the \( T-S \) diagram, abscissa represents the total entropy of the turbine steam. Without
Fig. 2.23  Regenerative feed cycle with four extractions

extraction the ideal cycle is $AFCDEA$. After the throttle steam has expanded from $A$ to 1, some steam is extracted so that the total entropy is reduced from 1 to 2. The heat given up by this steam is added to the feedwater, heating it from 11 to 12. This process is continued in four steps to the last extraction which heats the feedwater from $D$ to 9. With an infinite number of extraction stages, $AB$ would be parallel and equal in length to $DE$. The irreversible process $DE$—the heating from condenser to boiler saturation temperature—could thus be made reversible. The area of the parallelogram $CEAG$, which represents the cycle output, would be equal to the area of the rectangle $AFGE$ which represents the output of the Carnot cycle. Regenerative feedwater heating by turbine extraction is, therefore, also termed the carnontization of the Rankine cycle. A regenerative feed heating cycle with an infinite number of feedwater heaters has thus an efficiency equal to that of Carnot cycle.
2.11 OPTIMUM DEGREE OF REGENERATION

Complete Carnotization of Rankine cycle is not possible with a finite number of heaters. If there is one feedwater heater used, \( m \) kg of steam is extracted from the turbine for each kg of steam entering it to heat the feedwater from state 5 to state 6 (Fig. 2.24) so that by energy balance,

\[
m(h_2 - h_6) = (1 - m)(h_6 - h_5) \quad \text{or} \quad m = \frac{h_6 - h_5}{h_2 - h_5} = \frac{h_6 - h_4}{h_2 - h_4}
\]

Therefore, the thermal efficiency of the cycle is

\[
h = 1 - \frac{(1-m)(h_3 - h_4)}{h_1 - h_6} = 1 - \frac{\left(1 - \frac{h_6 - h_4}{h_2 - h_4}\right)(h_3 - h_4)}{h_1 - h_6}
\]

\[
= 1 - \frac{(h_2 - h_6)(h_3 - h_4)}{(h_2 - h_4)(h_1 - h_6)} \quad (2.33)
\]

Following Salisbury [4], Horlock [1] and Haywood [5], it may be approximately assumed that the turbine expansion line follows a path on the diagram such that \((h - h_f) = \text{constant} = \beta\), where \( h \) is the local enthalpy on the expansion line at a given pressure, and \( h_f \) is the enthalpy of saturated water at that pressure. Therefore, as seen in Fig. 2.24, \( h_1 - h_8 = h_2 - h_6 = h_3 - h_4 = \beta = \text{constant} \). Let the enthalpy rise of feedwater in the heater is \( \gamma \), which is equal to \((h_6 - h_4)\). Now,

\[ h_2 - h_4 = h_2 - h_6 + h_6 - h_4 = \beta + \gamma \]

If the total enthalpy rise of feedwater is equal to \( \alpha = h_8 - h_4 \), then

\[ h_1 - h_6 = h_1 - h_8 + h_8 - h_4 + h_4 - h_6 = \beta + \alpha - \gamma \]

Therefore, Eq. (2.33) can be written in the form
\[ \eta = 1 - \frac{\beta^2}{(\beta + \gamma)(\alpha + \beta - \gamma)} \]  

(2.34)

Here, \( \alpha \) and \( \beta \) are fixed and \( \gamma \) is variable. So, there is an optimum value of \( \gamma \) for which \( \eta \) is a maximum. On differentiation,

\[ \frac{d\eta}{d\gamma} = \beta^2 [((\alpha + \beta - \gamma) - (\beta + \gamma)) = 0 \quad \therefore \quad \gamma = \frac{\alpha}{2} \]  

(2.35)

The cycle efficiency is maximum when the total enthalpy rise of feedwater \((h_8 - h_4)\) from the condenser temperature to the boiler saturation temperature is divided equally between the feedwater heater and the economiser (i.e. \(h_8 - h_6 = h_6 - h_4\)) in a single bleed cycle. So, the temperature rise of feedwater in the heater is

\[ \Delta t = \frac{1}{2} (t_{\text{boiler saturation}} - t_{\text{condenser}}) \]

and the corresponding cycle efficiency is

\[ \eta = 1 - \frac{\beta^2}{\left(\beta + \frac{\alpha}{2}\right)\left(\alpha + \beta - \frac{\alpha}{2}\right)} = 1 - \frac{\beta^2}{\left(\beta + \frac{\alpha}{2}\right)^2} = \frac{\alpha^2 + 4\alpha\beta}{(\alpha + 2\beta)^2} \]  

(2.36)

For a non-regenerative cycle,

\[ \eta_0 = 1 - \frac{h_3 - h_4}{h_1 - h_4} \]

\[ h_3 - h_4 = \beta \quad \text{and} \quad h_1 - h_4 = h_1 - h_8 + h_8 - h_4 = \beta + \alpha \]

\[ \therefore \quad \eta_0 = 1 - \frac{\beta}{\alpha + \beta} = \frac{\alpha}{\alpha + \beta} \]  

(2.37)

The efficiency gain due to regeneration

\[ \Delta \eta = \eta - \eta_0 = \frac{\alpha^2 + 4\alpha\beta}{(\alpha + 2\beta)^2} - \frac{\alpha}{\alpha + \beta} = \frac{\alpha^2 \beta}{(\alpha + \beta)(\alpha + 2\beta)^2} \]  

(2.38)

This is positive. This shows that the cycle efficiency has improved due to regeneration.

In the heater train, the feedwater enters the economiser section of the boiler at state \( F \) (Fig. 2.25), where feedwater is heated to the saturation temperature \((G)\) at the boiler pressure. Assuming the economiser also as a feedwater heater (where feedwater is heated by the outgoing flue gases, instead of by the bled turbine steam, the total enthalpy rise \((h_G - h_3)\) or temperature rise from the condenser to the boiler saturation is divided equally among the feedwater heaters for maximum gain in the efficiency. The enthalpy rise per heater (including the economiser) is thus,
\[ \Delta h_{\text{per heater}} = \frac{h_G - h_3}{n + 1} \]  
(2.39)

where \( n \) is the number of heaters and 1 stands for the economiser. Therefore, the total enthalpy rise of feedwater for \( n \) heaters by regenerative feedheating is

\[ \Delta h_{\text{total}} = \frac{n}{n + 1} (h_G - h_3) \]  
(2.40)

Thus, the total temperature rise of feedwater, \( \Delta t_{fw} \), due to regeneration for the maximum cycle efficiency is given by

\[ \Delta t_{fw} = \frac{n}{n + 1} \Delta t_{OA} \]  
(2.41)
where the overall temperature difference,
\[ \Delta t_{OA} = \text{boiler saturation temperature} - \text{condenser temperature} \]

More is the number of heaters, more is the total temperature rise of feedwater, \( \Delta t_{fw} \), by regeneration, less becomes the heat addition to water in the boiler, more becomes the mean temperature of heat addition, and more is the cycle efficiency. From Eq. (2.41),

If \( n = 0 \), \[ \Delta t_{fw0} = 0 \]
If \( n = 1 \), \[ \Delta t_{fw1} = \frac{1}{2} \Delta t_{OA} \]
If \( n = 2 \), \[ \Delta t_{fw2} = \frac{2}{3} \Delta t_{OA} \]
If \( n = 3 \), \[ \Delta t_{fw3} = \frac{3}{4} \Delta t_{OA} \] and so on.

By the use of the first heater, the gain is
\[ \Delta t_{fw1} - \Delta t_{fw0} = \frac{1}{2} \Delta t_{OA} \]

By the use of the second heater, the gain over the first heater would be
\[ \Delta t_{fw2} - \Delta t_{fw1} = \frac{2}{3} \Delta t_{OA} - \frac{1}{2} \Delta t_{OA} = \frac{1}{6} \Delta t_{OA} \]

By the use of the third heater, the gain over the second heater would be
\[ \Delta t_{fw3} - \Delta t_{fw2} = \frac{3}{4} \Delta t_{OA} - \frac{2}{3} \Delta t_{OA} = \frac{1}{12} \Delta t_{OA} \]

Employing the fourth heater, the gain over the third heater becomes
\[ \Delta t_{fw4} - \Delta t_{fw3} = \frac{4}{5} \Delta t_{OA} - \frac{3}{4} \Delta t_{OA} = \frac{1}{20} \Delta t_{OA} \] and so on.

Since the gain in cycle efficiency is proportional to the gain in feedwater temperature, the efficiency gain follows the law of diminishing return with the increase in the number of heaters. In fact, the greatest increment in efficiency is brought by the first heater. The increments for each additional heater thereafter successively diminish (Fig. 2.26). The number of heaters is fixed up by the energy balance of the whole plant when it is found that the cost of adding another heater does not justify the saving in heat supply \( Q_1 \) or the marginal increase in cycle efficiency. An increase in feedwater temperature \( t_{fw} \) reduces the heat absorption from the outgoing flue gases in the economiser and may cause a reduction in boiler efficiency. The number of heaters and hence, the degree of regeneration thus get optimized. Five to seven points of extraction are often used in practice. Some cycles use as many as nine.
**Fig. 2.26** Efficiency gain (Δη) successively diminishes with the increase in the number of heaters

Figure 2.27 shows the optimum locations of the heaters for a non-reheat steam power plant.

**Fig. 2.27** Optimum locations of heaters

(a) For one heater, $\Delta t_{fw} = \frac{1}{2} \Delta t_{OA}$; (b) For two heaters, $\Delta t_{fw} = \frac{2}{3} \Delta t_{OA}$; and (c) For three heaters, $\Delta t_{fw} = \frac{3}{4} \Delta t_{OA}$
Figure 2.28 shows the reduction in heat rate by regenerative feedheating with different number of extractions for the same throttle steam conditions. With the use of four extractions the gain in heat rate is about 11%. The temperatures at which the heaters operate for maximum reduction in heat rate have been indicated in the figure and joined by a dotted line. The curve for a single feedwater heater peaks at a temperature rise of about halfway or 120 °C.

![Graph showing reduction in heat rate with number of extraction stages.]

**Fig. 2.28 Reduction of heat rate by regenerative feedwater heating**

For two feedwater heaters, the peak occurs at $\frac{2}{3} \times 220$ or about 142 °C and so on. Small departures from the optimum values have no serious effect on the heat rate.

### 2.12 SUPERCritical PRESSURE CYCLE

Figure 2.29 shows the plant arrangement and $T$–$s$ diagram for a supercritical steam cycle. Steam is generated in a “once-through” boiler at a pressure above the critical point of 221.2 bar. If the plant incorporates reheat and several stages of feedheating, there is about a 2% gain in thermal efficiency compared with the corresponding subcritical cycle. However, such an increment is gained only at the expense of increased cost and complexity of the plant. Double reheat needs to be incorporated to prevent the 1.p. turbine exhaust wetness from being excessive.

### 2.13 STEAM POWER PLANT APPRAISAL

Table 2.2 shows the past improvements in performance of steam power plants. These have been brought about by increasing the maximum pressure and temperature of the cycle, lowering the minimum pressure and introducing reheat and feedwater heating.
Table 2.2 Approximate performance data of various steam power plants (Aschner)

<table>
<thead>
<tr>
<th></th>
<th>Watt's beam engine</th>
<th>1 MW locomotive steam engine</th>
<th>2 MW marine steam engine</th>
<th>30 MW(e) steam power plant</th>
<th>660 MW(e) steam power plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal efficiency (%)</td>
<td>2</td>
<td>7</td>
<td>20</td>
<td>35</td>
<td>44</td>
</tr>
<tr>
<td>Initial pressure (bar)</td>
<td>2</td>
<td>15</td>
<td>15</td>
<td>40</td>
<td>160</td>
</tr>
<tr>
<td>Initial temperature (°C)</td>
<td>100</td>
<td>300</td>
<td>250</td>
<td>450</td>
<td>540</td>
</tr>
<tr>
<td>Reheat pressure (bar)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>40</td>
</tr>
<tr>
<td>Reheat temperature (°C)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>540</td>
</tr>
<tr>
<td>Exhaust pressure (bar)</td>
<td>near 1</td>
<td>1.2</td>
<td>0.1</td>
<td>0.045</td>
<td>0.045</td>
</tr>
<tr>
<td>Exhaust condition</td>
<td>wet</td>
<td>near sat</td>
<td>wet</td>
<td>0.9, dry</td>
<td>0.9, dry</td>
</tr>
<tr>
<td>Feedwater heaters</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

Fig. 2.29 Supercritical steam cycle with double reheat
Units above 150 MW output are now most frequently built for h.p. steam conditions of 142 bar, 538 °C or 170 bar, 538 °C with reheating up to 538 °C. At supercritical pressures for more than 600 MW size, h.p. steam is often at 270 bar, 538 °C with double reheating to 538 °C. H.P. steam and reheat temperatures up to 565 °C have also been used. A landmark station, built in 1959, is the 325 MW Eddystone, Unit I of Philadelphia Electric Company, a double reheat plant with supercritical steam (325 bar, 610 °C/565 °C/565 °C) which had the highest steam conditions and lowest heat rate for any plant in the world.

Common unit size of a steam power plant is now 500 MW(e). Plant sizes, however, will continue to increase and 1300 MW(e) turbo generators are likely to be the standard unit sizes in the power station in near future. Large improvements through raising the metallurgical limit are unlikely in the near future because it would then have to use prohibitively expensive austenitic steels or complicated turbine blade cooling. The thermodynamic design of the steam cycle for a 1300 MW(e) turbo generator would have steam conditions similar to those of the present generation of 660 MW(e) sets, but the plant sizes would be correspondingly larger. Even larger unit sizes of 2000 and 3000 MW(e) have been proposed but the physical construction and operation of such plants would pose serious problems like the flexibility of the longer rotor shafts, construction of the huge generator stator, effect on the electricity grid system due to shutdown of one of these very large sets, and high loss of revenue due to a shutdown.

2.14 DEAERATOR

One of the feedwater heaters is a contact-type open heater, known as deaerator, others being closed heaters. It is used for the purpose of deaerating the feedwater.

The presence of dissolved gases like oxygen and carbon dioxide in water makes the water corrosive, as they react with the metal to form iron oxide. The solubility of these gases in water decreases with increase in temperature and becomes zero at the boiling or saturation temperature. These gases are removed in the deaerator, where feedwater is heated to the saturation temperature by the steam extracted from the turbine. Feedwater, after passing through a heat exchanger, called vent condenser, is sprayed from the top so as to expose large surface area, and the bled steam from the turbine is fed from the bottom (Fig. 2.30). By contact the steam condenses and the feedwater is heated to the saturation temperature. Dissolved oxygen and carbon dioxide gases get released from the water and leave along with some vapour, which is condensed back in the vent condenser, and the gases are vented out.

To neutralize the effect of residual dissolved oxygen and carbon dioxide gases in water, sodium sulphite (\(\text{Na}_2\text{SO}_3\)) or hydrazine (\(\text{N}_2\text{H}_4\)) is injected in suitable calculated doses into the feedwater at the suction of the boiler feed pump (BFP).
During suction of the BFP, some of the saturated feedwater may flash into vapour due to reduction in pressure causing vapour lock and cavitation problems in the pump. To prevent this from occurring and to provide a net positive suction head (NPSH) for the pump, the deaerator is located at a sufficient height ($H$) from the basement where the pump is installed so that the pressure before suction is ($P_{\text{sat}} + \rho_f gH$). When this water is sucked by the pump, the pressure does not fall below $P_{\text{sat}}$ and there is no flashing of any water into vapour, which protects the BFP from any damage due to vapour lock and cavitation.

The deaerator is usually placed near the middle of the feedwater system so that the total pressure difference between the condenser and the boiler is shared equitably between the condensate pump and boiler feed pump. The feedwater heaters before the deaerator are often termed as high pressure (h.p.) heaters and those after the deaerator are termed as low pressure (l.p.) heaters.

The deaerator is not used in water-cooled-and-moderated nuclear power plants because of the concern regarding radioactivity release with deaeration.
Figure 2.31(a) shows the typical layout of a steam power plant demonstrating all the relevant components in it. The heat balance diagram of a 215 MW reheat power plant is shown in Fig. 2.31(b).
Heat rate of turbo-generator

\[ h_{TG} = \frac{635,510(819 - 246) + 565,468(845 - 737)}{214,000} = 1,987 \text{ kcal/kWh} \]

Fig. 2.31 (b)  Heat balance diagram of a 214 MW reheat unit (1 kcal = 4.187 kJ)
2.16 EFFICIENCIES IN A STEAM POWER PLANT

A steam power plant is a bulk energy converter from fuel to electricity (Fig. 2.32). The overall efficiency of a power plant is defined as

\[
\eta_{\text{overall}} = \frac{\text{power available at the generator terminals}}{\text{rate of energy release by the combustion of fuel}} = \frac{\text{MWe} \times 10^3}{w_t \times \text{C.V.}} \quad (2.42)
\]

where \(w_t\) is the fuel burning rate and C.V. is the calorific value of fuel. The boiler (or the steam generator) efficiency is defined by

\[
\eta_{\text{boiler}} = \frac{\text{rate of energy absorption by water to form steam}}{\text{rate of energy release by the combustion of fuel}} = \frac{w_s (h_i - h_4)}{w_t \times \text{C.V.}} \quad (2.43)
\]

where \(w_s\) is the steam generation rate.

The cycle efficiency is given by

\[
\eta_{\text{cycle}} = \frac{h_i - h_2}{h_i - h_4} \quad (2.44)
\]

The pump work has here been neglected. The mechanical efficiency of the turbine will be

\[
\eta_{\text{turbine(mech)}} = \frac{\text{brake output of the turbine}}{\text{internal output of the turbine}}
\]
\[ \eta_{\text{generator}} = \frac{\text{electrical output at generator terminals}}{\text{brake output of the turbine}} = \frac{\text{MWe} \times 10^3}{\text{brake output in kWe}} \]  

Multiplying Eq. (2.43) to Eq. (2.46),
\[ \eta_{\text{boiler}} \times \eta_{\text{cycle}} \times \eta_{\text{turbine(mech)}} \times \eta_{\text{generator}} \]
\[ = \frac{w_s (h_1 - h_4)}{w_f \times \text{C.V.}} \times \frac{w_s (h_1 - h_2)}{w_s (h_1 - h_4)} \times \frac{\text{brake output}}{w_s (h_1 - h_2)} \times \frac{\text{MWe} \times 10^3}{\text{brake output}} \]
\[ = \frac{\text{MWe} \times 10^3}{w_f \times \text{C.V.}} = \eta_{\text{overall}} \]

Therefore,
\[ \eta_{\text{overall}} = \eta_{\text{boiler}} \times \eta_{\text{cycle}} \times \eta_{\text{turbine(mech)}} \times \eta_{\text{generator}} \]  

There are certain auxiliary equipment in the power plant like F.D. and I.D. fans, pulverizers, crushers, conveyors and so on which are driven by electricity taken from the generated power of the plant. The net power transmitted from the generator will be the gross power produced minus the power consumed by the internal auxiliaries of the plant. The efficiency of the auxiliaries, \( \eta_{\text{aux}} \), is given by
\[ \eta_{\text{aux}} = \frac{\text{net power transmitted by the generator}}{\text{gross power produced by the plant}} \]

Therefore, the overall efficiency of the plant is the product of five component efficiencies as given by
\[ \eta_{\text{overall}} = \eta_{\text{boiler}} \times \eta_{\text{cycle}} \times \eta_{\text{turbine(mech)}} \times \eta_{\text{generator}} \times \eta_{\text{aux}} \]

For a modern power plant, the typical values are
\[ \eta_{\text{boiler}} = 0.92, \eta_{\text{cycle}} = 0.44, \eta_{\text{turbine(mech)}} = 0.95, \eta_{\text{generator}} = 0.93, \eta_{\text{aux}} = 0.95, \eta_{\text{overall}} = 0.92 \times 0.44 \times 0.95 \times 0.93 \times 0.95 = 0.34 \]

Only 34% of the energy in fuel is converted to electricity and 66% of the energy is lost. The maximum loss of energy takes place in the condenser where heat is rejected to cooling water. This is the loss due to heat to work energy conversion in the cycle or the loss due to the second law.

The parameter which readily reflects the fuel economy is the heat rate, which is inversely proportional to the efficiency and hence, the lower its value the better. It broadly indicates the heat added per unit of work produced. There are various heat rates corresponding to the work used in the denominator. Thus,
Net cycle heat rate (HR) = \frac{\text{rate of heat addition to cycle}}{\text{net cycle work output}} = \frac{Q_1}{W_{\text{net}}}

Gross cycle HR = \frac{\text{rate of heat addition}}{\text{turbine output}} = \frac{Q_1}{W_T}

Net station (HR) = \frac{\text{rate of heat addition to boiler}}{\text{net station output}} = \frac{Q_1}{\text{net station output}}

Gross station (HR) = \frac{Q_1}{\text{gross generation output}}

(2.50)

#### 2.17 COGENERATION OF POWER AND PROCESS HEAT

There are several industries such as paper mills, textile mills, chemical factories, jute mills, sugar factories, rice mills and so on where saturated steam at the desired temperature is required for heating, drying etc. For constant temperature heating (or drying), steam is a very good medium since isothermal condition can be maintained by allowing saturated steam to condense at that temperature and utilizing the latent heat released for heating purposes. Apart from the process heat, the factory also needs power to drive various machines, for lighting and other purposes.

Earlier, steam for power purposes was generated at a moderate pressure and saturated steam for process work was generated separately at a pressure which gave the desired heating temperature. Having two separate units for process heat and power is wasteful, for of the total heat supplied to the steam generator for power purposes, a greater part will normally be carried away by the cooling water in the condenser.

#### 2.17.1 Back Pressure Turbine

By modifying the initial steam pressure and exhaust pressure, it is possible to generate the required power and make available the required quantity of exhaust steam at the desired temperature for process work. In Fig. 2.33, the exhaust steam from the turbine is utilized for process heating, the process heater replacing the condenser of the ordinary Rankine cycle. The pressure at exhaust from the turbine is the saturation pressure corresponding to the temperature desired in the process heater. Such a turbine is called a back pressure turbine. A plant producing both electrical power and process heat simultaneously is called a cogeneration plant. When the process steam is the basic need, and the
power is produced incidentally as a by-product, the cycle is often called a by-product power cycle. Figure 2.34 shows the $T$–$s$ plot for such a cycle. If $W_T$ is the turbine output in kW, $Q_H$ is the process heat required in kJ/h, and $w_s$ is the steam flow rate in kg/h.

\[ W_T \times 3600 = w_s(h_1 - h_2) \quad \text{and} \quad Q_H = w_s(h_2 - h_3) \]

\[ W_T \times 3600 = \frac{Q_H}{h_2 - h_3}(h_1 - h_2) \]

or,

\[ Q_H = \frac{W_T \times 3600 \times (h_2 - h_3)}{h_1 - h_2} \quad \text{kJ/h} \quad (2.51) \]

Of the total energy input $Q_1$ (as heat) to the cogeneration plant, $W_T$ part of it only is converted into shaft work or electricity. The remaining energy $(Q_1 - W_T)$, which would otherwise have been a waste, as in the Rankine cycle, by second law, is utilized as process heat.

The cogeneration plant efficiency $\eta_{co}$ is given by

\[ \eta_{co} = \frac{W_T + Q_H}{Q_1} \quad (2.52) \]
For separate generation of electricity and steam, the heat added per unit total energy output is

\[
\frac{1}{\eta_e} + \frac{1-e}{\eta_h}
\]

where \( e \) = electricity fraction of total energy output

\[
= \frac{W_T}{W_T + Q_H}
\]

\( \eta_e \) = electric plant efficiency

\( \eta_h \) = steam (or process heat) generator efficiency

The combined efficiency \( \eta_c \) for separate generation is therefore given by

\[
\eta_c = \frac{1}{\eta_e + \frac{1-e}{\eta_h}}
\]  \hspace{1cm} (2.53)

Cogeneration is beneficial if the efficiency of the cogeneration plant, Eq. (2.52), is greater than that of separate generation, Eq. (2.53).

Back pressure turbines are quite small with respect to their power output because they have no great volume of exhaust to cope with, the density being high. They are usually single cylinder and hence, cheap in terms of cost per MW compared to condensing sets of the same power. Besides their use in process industries and petrochemical installations, back pressure turbines are used for desalination of sea-water, district heating, and also for driving compressors and feed pumps.

2.17.2 Pass-Out Turbine

In many cases, the power available from the back pressure turbine through which the whole of the heating steam flows is appreciably less than that required in a factory. This may be due to relatively high back pressure, or small heating requirement or both. Pass-out turbines are employed in these cases, where a certain quantity of steam is continuously extracted from the turbine at an intermediate stage for heating purposes at the desired temperature and pressure (Fig. 2.35).
Fig. 2.35 Cogeneration plant with a pass-out turbine

\[
\begin{align*}
Q_1 &= w_s (h_1 - h_8); & Q_2 &= (w_s - w) (h_3 - h_4) \\
Q_H &= w(h_2 - h_6); & W_T &= w_s(h_1 - h_2) + (w_s - w) (h_2 - h_3) \\
W_p &= (w_s - w) (h_5 - h_4) + w(h_7 - h_6); & wh_7 + (w_s - w)h_5 &= w_s h_8
\end{align*}
\]

where \( w_s \) is the boiler capacity and \( w \) is the steam flow rate required at the desired temperature for process heating.

2.17.3 Process Heat Unit Topping the Power Cycle

In some plants, such as a high temperature cement kiln, the primary heat is used directly for process requirements. The low grade waste heat from the process heater is then used to generate electricity, obviously at low efficiency. Such a cycle has a combined efficiency lying below that given by Eq. (2.53) and therefore, is of less thermodynamic or economic interest.
A cogeneration plant is only advisable from an economic viewpoint if the cost of electricity generated by it is less than that purchased from a utility system. If a utility is not available cogeneration becomes necessary irrespective of the cost of generation. In general, very low fraction of electric to total energy (e) are not considered economical for cogeneration.

**Example 2.1**

Steam at 40 bar, 500°C flowing at the rate of 5500 kg/h expands in a h.p. turbine to 2 bar with an isentropic efficiency of 83%. A continuous supply of steam at 2 bar, 0.87 quality and a flow rate of 2700 kg/h is available from a geothermal energy source. This steam is mixed adiabatically with the h.p. turbine exhaust steam and the combined flow then expands in a l.p. turbine to 0.1 bar with an isentropic efficiency of 78%. Determine the power output and the thermal efficiency of the plant. Assume that 5500 kg/h of steam is generated in the boiler at 40 bar, 500°C from the saturated feedwater at 0.1 bar.

Had the geothermal steam not been added, what would have been the power output and efficiency of the plant? Neglect pump work.

**Solution**

With reference to Fig. E2.1

\[ h_1 = 3445.3 \text{ kJ/kg} \]
\[ s_1 = 7.0901 \text{ kJ/kg K} \]
\[ K = 1.5301 + x_{2s} \times 5.5970 \]

\[ x_{2s} = \frac{5.5600}{5.5970} = 0.9934 \]

\[ h_{2s} = 504.7 + 0.9934 \times 2201.9 = 2692.04 \text{ kJ/kg} \]

\[ h_1 - h_2 = 0.83(3445.3 - 2692.04) = 625.21 \text{ kJ/kg} \]

\[ h_2 = 3445.3 - 625.21 = 2820.09 \text{ kJ/kg} \]

\[ h_3 = 504.7 + 0.87 \times 2201.9 = 2420.4 \text{ kJ/kg} \]

\[ 2700h_3 + 5500h_2 = (2700 + 5500)h_4 \]

\[ h_4 = \frac{2700 \times 2420.4 + 5500 \times 2820.09}{8200} = 796.96 + 1891.52 \]

\[ = 2688.48 \text{ kJ/kg} \]
Fig. E2.1

\[ h_4 = 504.7 + x_4 \times 2201.9 = 2688.48; \quad x_4 = \frac{2183.78}{2201.9} = 0.9917 \]

\[ s_4 = 1.5301 + 0.9917 \times 5.5970 = 7.0806 \text{ kJ/kg-K} \]

\[ = s_{ss} = 0.6493 + x_{ss} \times 7.5009 \]

\[ x_{ss} = 0.8574; \quad h_{ss} = 191.84 + 0.8574 \times 2392.8 = 2243.44 \text{ kJ/kg} \]

\[ h_4 - h_5 = 0.78(2688.48 - 2243.44) = 347.1 \text{ kJ/kg} \]

\[ h_6 = 191.83 \text{ kJ/kg} \]

\[ \dot{W} = 5500(h_1 - h_2) + 8200(h_4 - h_3) = 5500 \times 625.21 + 8200 \times 347.1 \]

\[ = 6284875 \text{ kJ/h} = 1745.8 \text{ kW} \quad (Ans.) \]

\[ Q_1 = 5500(h_1 - h_6) = 5500(3445.3 - 191.8) \times \frac{1}{3600} = 4970.63 \text{ kW} \]

\[ \eta_{cycle} = \frac{1745.8}{4970.63} = 0.353 \quad \text{or} \quad 35.3\% \]

Without geothermal heat supply:

\[ W_T = 5500 \,(h_1 - h_2) = 955.18 \text{ kW} \quad (Ans.) \]

\[ Q_1 = 5500 \,(h_1 - h_6) = 4970.63 \text{ kW} \]

\[ \eta_{cycle} = \frac{955.18}{4970.63} = 0.1922 \quad \text{or} \quad 19.22\% \quad (Ans.) \]

**Example 2.2** A steam power plant with inlet steam to the h.p. turbine at 90 bar and 500 °C, and condensation at 40 °C produces 500 MW. It has one stage of reheat optimally placed which raises the steam temperature back to 500 °C. One closed feedwater heater with drains cascaded back to the condenser receives bled steam at the reheat pressure, and the remaining steam is reheated and then expanded in the l.p. turbine. The h.p. and l.p. turbines have isentropic efficiencies of 92% and 90%, respectively. The isentropic efficiency of the pump is 75%. Calculate (a) the mass flow rate of steam at turbine inlet in kg/s, (b) the cycle efficiency, and (c) the cycle work ratio. Use TTD = −1.6 °C.
Solution  The flow and $T$-$s$ diagrams are shown in Fig. E2.2. The optimum reheat pressure is taken to be 20% of the boiler pressure, which becomes $0.2 \times 90 = 18$ bar. Now, $h_1 = 3386.1 \text{ kJ/kg}, s_1 = 6.6576 \text{ kJ/kg-K} = s_{2s}, h_{2s} = 2915 \text{ kJ/kg}, h_3 = 3469.8 \text{ kJ/kg}.$

\[ s_3 = 7.4825 = s_{4s} = s_f + x_{4s} s_{fg} \]

\[ x_{4s} = \frac{7.4825 - 0.5725}{7.6845} = 0.8992 \]

\[ h_{4s} = 167.57 + 0.8992 \times 2406.7 = 2331.7 \text{ kJ/kg} \]

\[ h_5 = 167.57 \text{ kJ/kg}, \quad h_7 = 883.42 \text{ kJ/kg} \]

\[ W_{Ps} = \int \dot{v} dp = 0.001008 \times 90 \times 10 = 9.072 \text{ kJ/kg} \]

\[ h_{6s} = 176.64 \text{ kJ/kg} \]

\[ h_1 - h_2 = 0.92 (3386 - 2915) = 433.3 \text{ kJ/kg}, \quad \text{or} \quad h_2 = 3386.1 - 433.3 = 2952.8 \text{ kJ/kg} \]

\[ h_3 - h_4 = 0.9(3469.8 - 2331.7) = 1024.29 \text{ kJ/kg} \]

\[ h_4 = 3469.8 - 1024 = 2445.5 \text{ kJ/kg} \]

\[ W_p = h_6 - h_5 = \frac{9.072}{0.75} = 12.1 \text{ kJ/kg}, \quad \text{or} \quad h_6 = 167.57 + 12.1 = 179.67 \text{ kJ/kg} \]

\[ t_{sat} \text{ at } 18 \text{ bar} = 207.15 \degree C \quad \therefore t_9 = 207.15 + 1.6 = 208.75 \degree C \]

\[ h_9 = 875 \text{ kJ/kg} \]

\[ 1(h_9 - h_6) = m(h_2 - h_7), \quad m = \frac{875 - 179.67}{2952.8 - 883.4} = 0.336 \text{ kg} \]

\[ W_T = (h_1 - h_2) + (1-m)(h_3 - h_4) = 433.3 + 0.664 \times 1024.3 = 1113.435 \text{ kJ/kg} \]

\[ W_{net} = W_T - W_p = 1113.435 - 12.1 = 1101.335 \text{ kJ/kg} \]

\[ w_s = \frac{500 \times 10^3}{1101.335} = 454 \text{ kg/s} \quad \text{Ans. (a)} \]

\[ Q_1 = h_1 - h_9 + (1 - \eta)(h_3 - h_2) \]
Fig. E2.2
\[ 3386.1 - 875 + 0.664(3469.8 - 2952.8) = 2854.3 \text{ kJ/kg} \]

\[ \eta_{\text{cycle}} = \frac{1101.335}{2854.3} = 0.3858, \text{ or } 38.58\% \quad \text{Ans.(b)} \]

Work ratio \[ \frac{W_{\text{net}}}{W_T} = \frac{1101.335}{1113.435} = 0.989 \quad \text{Ans. (c)} \]

**Example 2.3** An ideal steam power plant operates between 70 bar, 550 °C and 0.075 bar. It has seven feedwater heaters. Find the optimum pressure and temperature at which the heaters operate.

**Solution** With reference to Fig. E2.3,

![T-s diagram](image)

**Fig. E2.3**

\[ T_B = \text{saturation temperature at 70 bar = 285.9 °C} \]

\[ T_C = \text{saturation temperature at 0.075 bar = 40.3 °C} \]

The temperature rise per heater for maximum cycle efficiency

\[ = \frac{285.9 - 40.3}{7 + 1} = 30.7 °C \]

*Heater 1*: \( t_1 = 285.9 - 30.7 = 255.2 °C \)

*Heater 1*: \( p_1 = 4.33 \text{ MPa (from steam table)} \)

*Heater 2*: \( t_2 = 255.2 - 30.7 = 224.5 °C; \quad p_2 = 2.5318 \text{ MPa} \)

*Heater 3*: \( t_3 = 193.8 °C; \quad p_3 = 1.367 \text{ MPa} \)

*Heater 4*: \( t_4 = 163.1 °C; \quad p_4 = 0.6714 \text{ MPa} \)

*Heater 5*: \( t_5 = 132.4 °C; \quad p_5 = 0.2906 \text{ MPa} \)

*Heater 6*: \( t_6 = 101.7 °C; \quad p_6 = 0.108 \text{ MPa} \)

*Heater 7*: \( t_7 = 71.0 °C; \quad p_7 = 32.65 \text{ kPa} \)
Example 2.4 In a power plant, the efficiencies of the electric generator, turbine (mechanical), boiler, cycle and the overall plant are 0.97, 0.95, 0.92, 0.42 and 0.33, respectively. What percentage of the total electricity generated is consumed in running the auxiliaries?

Solution

\[ \eta_{\text{plant}} = \eta_{\text{Boiler}} \times \eta_{\text{Turbine(mech)}} \times \eta_{\text{Generator}} \times \eta_{\text{Cycle}} \times \eta_{\text{Auxiliaries}} \]

\[ \therefore \eta_{\text{Auxiliaries}} = \frac{0.33}{0.97 \times 0.95 \times 0.92 \times 0.42} = 0.9268 \]

\[ 1 - 0.9268 = 0.0732 \]

or 7.32% of total electricity generated is consumed by the auxiliaries.

Example 2.5 A steam generator comprises a boiler, a superheater, an economiser and an air preheater. The feedwater enters the economiser at 140 °C and leaves as saturated liquid. Air is preheated from a temperature of 25 °C to 250 °C. Steam leaves the boiler drum at 60 bar, 0.98 dry and leaves the superheater at 450 °C. When using coal with a calorific value of 25.2 MJ/kg, the rate of evaporation is 8.5 kg steam per kg coal and the air fuel ratio is 15 : 1 by mass. Neglecting heat losses and pressure drops, estimate the heat transfer per kg fuel in each component and the efficiency of the steam generator. What are the percentages of the total heat absorption taking place in the economiser, boiler and the superheater, respectively? Assume \( c_p \) of air and water as 1.005 and 4.2 kJ/kg K, respectively.

Solution With reference to Fig. E2.5, \( h_1 = 140 \times 4.2 = 588 \text{ kJ/kg} \)

\[ h_f = 1213.35 \text{ kJ/kg}, \quad h_{fg} = 1571.0 \text{ (at 60 bar)} \]

\[ h_4 = 3301.8 \text{ kJ/kg}, \]

Fig. E2.5
\[ h_3 = 1213.35 + 0.98 \times 1571.0 = 2752.93 \text{ kJ/kg} \]

\[ \eta_{\text{St.gen}} = \frac{w_s (h_4 - h_1)}{w_t \times \text{C.V.}} = \frac{8.5 \times 2713.8}{25.2 \times 1000} = 0.9154 \text{ or } 91.54\% \]

Heat transfer in the economiser
\[ = \frac{w_s (h_2 - h_1)}{w_t} = 8.5 \times 625.35 \times 10^{-3} = 5.3155 \text{ MJ/kg} \]

Heat transfer in the boiler
\[ = \frac{w_s (h_3 - h_2)}{w_t} = 8.5 \times 1539.58 \times 10^{-3} = 13.086 \text{ MJ/kg} \]

Heat transfer in the superheater
\[ = \frac{w_s (h_4 - h_3)}{w_t} = 8.5 \times 548.87 \times 10^{-3} = 4.665 \text{ MJ/kg} \]

Heat transfer in the air pre-heater
\[ = \frac{w_a c_p \Delta t}{w_t} = 15 \times 1.005 \times (250 - 25) \times 10^{-3} = 3.392 \text{ MJ/kg} \]

Percentage of total heat absorbed in the economiser
\[ = \frac{h_2 - h_1}{h_4 - h_1} \times 100 = \frac{625.35}{2713.8} \times 100 = 23.04\% \]

Percentage of total heat absorbed in the boiler
\[ = \frac{h_3 - h_2}{h_4 - h_1} \times 100 = \frac{1539.58}{2713.8} \times 100 = 56.73\% \]

Percentage of total heat absorbed in the superheater
\[ = \frac{h_4 - h_3}{h_4 - h_1} \times 100 = \frac{548.87}{2713.8} \times 100 = 20.23\% \]

**Example 2.6** Steam at 150 bar, 550 °C is expanded in an h.p. turbine to 20 bar when it is reheated to 500 °C and expanded in i.p. and 1.p. turbines to condenser pressure of 0.075 bar. There are five feedwater heaters, one extraction from h.p. turbine at 50 bar, 3 from i.p. turbine at 10 bar, 5 bar and 3 bar, and one from 1.p. turbine at 1.5 bar. The middle heater is the deaerator and all others are closed heaters. Assuming ideal conditions, determine (a) the cycle efficiency, (b) the feedwater temperature at inlet to the steam generator, (c) the steam rate, (d) the heat rate, (e) the quality of steam at turbine exhaust, and (f) the power output if the steam flow rate is 300 t/h. Take TTD = 0 for all the heaters.

**Solution** With reference to Fig. E2.6,
\[ h_1 = 3448.6 \text{ kJ/kg}, \quad s_1 = s_2 = s_3 = 6.5199 \text{ kJ/kg K} \]
\[ h_4 = 3467.6 \text{ kJ/kg}, \quad s_4 = s_5 = s_6 = s_7 = s_8 = s_9 = 7.4317 \text{ kJ/kg K} \]
Fig. E2.6 (a)
\[ t_2 = 370 \, ^\circ \text{C}, \quad h_2 = 3112 \, \text{kJ/kg} \]
\[ t_3 = 245 \, ^\circ \text{C}, \quad h_3 = 2890 \, \text{kJ/kg} \]
\[ t_5 = 400 \, ^\circ \text{C}, \quad h_5 = 3250 \, \text{kJ/kg} \]
\[ t_6 = 300 \, ^\circ \text{C}, \quad h_6 = 3050 \, \text{kJ/kg} \]
\[ t_7 = 225 \, ^\circ \text{C}, \quad h_7 = 2930 \, \text{kJ/kg} \]
\[ t_8 = 160 \, ^\circ \text{C}, \quad h_8 = 2790 \, \text{kJ/kg} \]

\[ 7.4317 = 0.5764 + x_9 \times 7.6751 \]

\[ x_9 = 0.8932 \]

---

**Fig. E2.6 (b)**

\[ h_9 = 168.79 + 0.8932 \times 2406.0 = 2317.83 \, \text{kJ/kg} \]
\[ h_{10} = 168.79 \, \text{kJ/kg} \]
\[ h_{11} = 168.79 + 0.001 \times 5 \times 100 = 169.29 \, \text{kJ/kg} \]
\[ h_{12} = 467.11 \, \text{kJ/kg}, \quad t_{14} = 111.37, \quad h_{14} = 467 \, \text{kJ/kg} \]
\[ h_{12} = h_{13} = h_{14} = h_{15} = 467 \, \text{kJ/kg} \]
\[ h_{16} = h_{17} = 561.47 \, \text{kJ/kg}, \quad h_{18} = 640.23 \, \text{kJ/kg} \]
\[ h_{19} = 640.23 + 0.001 \times 145 \times 100 = 654.63 \, \text{kJ/kg} \]
\[ h_{20} = 762.8 \, \text{kJ/kg} = h_{21}, \quad h_{22} = h_{22} = h_{23} = 1154.23 \, \text{kJ/kg} \]

**Heater 1**

\[ m_1(h_2 - h_{22}) = 1(h_{23} - h_{21}) \]

\[ m_1 = \frac{1154.23 - 762.8}{3112 - 1154.23} = \frac{391.43}{1957.77} = 0.2 \, \text{kg} \]
Heater 2

\[ m_2(h_5 - h_{20}) + m_1(h_{22} - h_{20}) = 1(h_{21} - h_{19}) \]
\[ m_2(3250 - 762.8) + 0.2(1154.23 - 762.8) = (762.8 - 654.6) \]
\[ m_2 \times 2487.2 + 78.29 = 108.2 \]
\[ m_2 = 0.012 \text{ kg} \]

Heater 3

\[ m_3(h_6 - h_{18}) + (m_1 + m_2)(h_{20} - h_{18}) = (1 - m_1 - m_2 - m_3)(h_{18} - h_{17}) \]
\[ m_3(3050 - 640.2) + 0.212(762.8 - 640.2) = (0.788 - m_3)(640.2 - 561.5) \]
\[ m_3 \times 2409.8 + 26.00 = 62.02 - 78.7m_3 \]
\[ m_3 = 36.02/2488.5 = 0.0145 \text{ kg} \]

Heater 4

\[ m_4(h_7 - h_{16}) = (1 - m_1 - m_2 - m_3)(h_{17} - h_{15}) \]
\[ m_4(2930 - 561.5) = 0.7735(561.5 - 467) = 73.096 \]
\[ m_4 = 0.031 \text{ kg} \]

Heater 5

\[ m_5(h_8 - h_{12}) + m_4(h_{16} - h_{12}) = (1 - m_1 - m_2 - m_3 - m_4 - m_5)(h_{14} - h_{11}) \]
\[ m_5(2790 - 467) + 0.031(561.5 - 467) = (0.7425 - m_5)(467 - 169.3) \]
\[ m_5 \times 2323 + 2.93 = 221.04 - 297.7m_5 \]
\[ m_5 = \frac{218.11}{2620.7} = 0.0832 \text{ kg} \]

\[
W_T = 1(h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1)(h_4 - h_5) \\
+ (1 - m_1 - m_2)(h_5 - h_6) + (1 - m_1 - m_2 - m_3)(h_6 - h_7) \\
+ (1 - m_1 - m_2 - m_3 - m_4)(h_7 - h_8) \\
+ (1 - m_1 - m_2 - m_3 - m_4 - m_5)(h_8 - h_9) \\
= (3448.6 - 3112) + 0.8(3112 - 2890) + 0.8(3467.6 - 3250) \\
+ 0.788(3250 - 3050) + 0.7735(3050 - 2930) \\
+ 0.7425(2930 - 2790) + 0.6593(2790 - 2317.8) \\
= 336.6 + 177.6 + 174.1 + 157.6 + 92.8 + 104.0 + 311.3 \\
= 1354.0 \text{ kJ/kg} \\
W_p = 0.5 + 14.5 + 0.15 = 15.15 \text{ kg} \\
W_{net} = 1354 - 15.15 = 1338.85 \text{ kJ/kg} \\
Q_1 = 1(h_1 - h_{23}) + (1 - m_1)(h_4 - h_3) \\
= 3448.6 - 1154.2 + 0.8(3467.6 - 2890) = 2756.48 \text{ kJ/kg} \\
\therefore \quad n_{cycle} = \frac{1338.85}{2756.48} = 0.4857, \quad \text{or} \quad 48.57\% \quad \text{Ans. (a)}
\[ t_{23} = \text{feedwater temperature at inlet to the steam generator} \]

\[ = 264 \, ^\circ\text{C} \quad \text{Ans. (b)} \]

\[ \text{Steam rate} = \frac{3600}{W_{\text{net}}} = \frac{3600}{1338.85} = 2.69 \, \text{kJ/kWh} \quad \text{Ans. (c)} \]

\[ \text{Heat rate} = \frac{Q_1}{W_{\text{net}}} \times 3600 = \frac{3600}{0.4857} = 7412 \, \text{kJ/kWh} \quad \text{Ans. (d)} \]

\[ \text{Quality of steam at turbine exhaust} = x_9 = 0.8932 \quad \text{Ans. (e)} \]

\[ \text{Power output} = \frac{1338.85 \times 300 \times 10^3}{3600} \, \text{kW} = 111.57 \, \text{MW} \quad \text{Ans. (f)} \]

**Example 2.7** A textile factory requires 10 t/h of steam for process heating at 3 bar saturated and 1000 kW of power, for which a back pressure turbine of 70% internal efficiency is to be used. Find the steam condition required at inlet of the turbine.

![Fig. E2.7](image)

**Solution** If \( w = \text{mass flow rate of steam} \),

\[ w(h_1 - h_2) = 1000 \, \text{kW} \]

\[ \therefore \quad h_1 - h_2 = \frac{1000 \times 3600}{10,000} = 360 \, \text{kJ/kg} \]

With reference to Fig. E2.7, \( h_2 = 2725.3 \)

\[ \therefore \quad h_1 = 3085.3 \, \text{kJ/kg} \]

Now,

\[ h_1 - h_{2s} = \frac{360}{0.7} = 514.286 \, \text{kJ/kg} \]

\[ \therefore \quad h_{2s} = 3085.3 - 514.286 = 2571.014 \, \text{kJ/kg} \]

\[ h_{2s} = h_t + x_{2s} h_{fg} = 561.47 + x_{2s} \times 2163.8 = 2571.014 \]

\[ \therefore \quad x_{2s} = 0.9287 \]
\[ s_{2s} = 1.6718 + 0.9287(6.9919 - 1.6718) = 6.6125 \text{ kJ/kg K} \]

\[ = s_1 \]

Corresponding to \( h_1 = 3085.3 \text{ kJ/kg} \) and
\[ s_1 = 6.6125 \text{ kJ/kg K, from the Mollier diagram}, \]
\[ p_1 = 37.3 \text{ bar, } t_1 = 344 \text{ °C}, \] which is the state of steam at turbine inlet.

**Example 2.8** In a cogeneration plant, the power load is 5.6 MW and the heating load is 1.163 MW. Steam is generated at 40 bar and 500 °C and is expanded isentropically through a turbine to a condenser at 0.06 bar. The heating load is supplied by extracting steam from the turbine at 2 bar, which condensed in the process heater to saturated liquid at 2 bar and then pumped back to the boiler. Compute (a) the steam generation capacity of the boiler in t/h, (b) the heat input to the boiler in kW, (c) the fuel burning rate of the boiler in t/h if a coal of calorific value 25 MJ/kg is burned and the boiler efficiency is 88%, (d) the heat rejected to the condenser, (e) the rate of flow of cooling water in the condenser if the temperature rise of water is 6°C. Neglect pump work.

**Solution** With reference to Fig. E2.8,
\[ h_1 = 3445.3 \text{ kJ/kg}, \quad s_1 = 7.0901 = s_2 = s_3 \]
\[ 7.0901 = 1.5301 + x_2 \times 5.5970 \quad \text{or} \quad x_2 = \frac{5.56}{5.597} = 1.0 \]

![Fig. E2.8](image)

\[ \therefore h_2 = h_g = 2706.7 \text{ kJ/kg}, \quad h_2 - h_6 = h_g = 2201.9 \text{ kJ/kg} \]

If \( w \) is the rate of steam extraction for process heating,
\[ w(h_2 - h_6) = 1.163 \times 10^3 \]
\[ w = \frac{1.163 \times 10^3}{2201.9} = 0.528 \text{ kg/s} = 1901.4 \text{ kg/h} \]
\[ s_1 = 7.0901 = s_f + x_3 s_{fg} = 0.52 + x_3 \times 7.815 \]
\[ x_3 = 0.84 \]
\[ h_3 = 149.79 + 0.84 \times 2416 = 2180.59 \text{ kJ/kg} \]

Total work output, \( W_T = w_s (h_1 - h_2) + (w_s - w) (h_2 - h_3) \)
\[ 5.6 \times 10^3 = w_s \times 738.6 + w_s \times 526.11 - 277.8 \]

\[ w_s = \frac{5877.8}{1264.7} = 4.648 \text{ kg/s} = 16731 \text{ kg/h} = 16.73 \text{ t/h} \quad \text{Ans. (a)} \]
\[ h_f = 504.7 + 1.061 \times 10^{-3} (40 - 2) \times 100 = 508.73 \text{ kJ/kg} \]
\[ h_s = 149.79 + 1.006 \times 100 \times 40 \times 10^{-3} = 153.8 \text{ kJ/kg} \]
\[ Q_1 = (w_s - w) (h_1 - h_5) + w(h_1 - h_f) \]
\[ = (4.648 - 0.528) (3445.3 - 153.8) + 0.528 (3445.3 - 508.73) \]
\[ = 4.120 \times 3291.5 + 0.528 \times 2936.57 = 15111.5 \text{ kJ/s} \]
\[ = 15.111 \text{ MW} \quad \text{Ans. (b)} \]

\[ h_{boiler} = \frac{Q_1}{w_f \times \text{C.V.}} = \frac{15.111}{w_f \times 25} = 0.88 \]
\[ w_f = 0.687 \text{ kg/s} = 2473.2 \text{ kg/h} = 2.473 \text{ t/h} \quad \text{Ans. (c)} \]
\[ Q_2 = (w_s - w) (h_3 - h_4) = 4.12 \times 2030.8 = 8367 \text{ kW} \]
\[ = 8.367 \text{ MW} \quad \text{Ans. (d)} \]

If \( w_c = \) water flow rate in the condenser,
\[ Q_2 = w_c c_p (t_2 - t_1) \]
\[ \therefore \quad w_c = \frac{8367}{4.187 \times 6} = 333.05 \text{ kg/s} = 0.333 \text{ m}^3/\text{s} \quad \text{Ans. (e)} \]

**Example 2.9** In a combined power and process plant the boiler generates 21,000 kg/h of steam at a pressure of 17 bar and temperature 230 °C. A part of the steam goes to a process heater which consumes 132.56 kW, the steam leaving the process heater 0.957 dry at 17 bar being throttled to 3.5 bar. The remaining steam flows through an h.p. turbine which exhausts at a pressure of 3.5 bar. The exhaust steam mixes with the process steam before entering the l.p. turbine which develops 1337.5 kW. At the exhaust, the pressure is 0.3 bar and the steam is 0.912 dry. Draw the line and \( T-s \) diagrams of the plant and determine (a) the steam quality at the exhaust of the h.p. turbine, (b) the power developed by the h.p. turbine, and (c) the isentropic efficiency of the h.p. turbine.
Solution  The flow and $T$–$s$ diagrams of the plant are shown in Fig. E2.9.

\[ \omega_k = 21,000 \text{ kg/h} \]

\[ \omega_k - \omega \]

Boiler

17 bar, 230 °C

17 bar, 0.957 dry

3.5 bar

\[ Q_H = 132.56 \text{ kW} \]

3.5 bar

L.P. Turbine

0.3 bar, 0.912 dry

\[ \omega_k \]

1337.5 kW

(a)

\[ h_1 = 2869.7 \text{ kJ/kg}, s_1 = 6.5408 \text{ kJ/kg K} \]

\[ h_2 = 870.44 + 0.957 \times 1924.7 = 2712.38 \text{ kJ/kg} \]

\[ h_2 = h_3 \]

\[ w_s (h_5 - h_6) = 1337.5 \text{ kW} \]

\[ h_5 - h_6 = \frac{1337.5 \times 3600}{21000} = 229.29 \text{ kJ/kg} \]

(b)

Fig. E2.9
\[ h_6 = 289.23 + 0.912 \times 2336.1 = 2419.75 \text{ kJ/kg} \]
\[ h_5 = 2649.04 \text{ kJ/kg} \]
\[ s_1 = s_{4s} = 6.5408 = 1.7275 + x_{4s} \times 5.2130 \]
\[ x_{4s} = 0.923 \]
\[ h_{4s} = 584.33 + 0.923 \times 2148.1 = 2567.03 \text{ kJ/kg} \]
\[ w(h_1 - h_2) = 132.56 \text{ kW} \]
\[ w = \frac{132.56}{2867.7 - 2712.38} = 0.843 \text{ kg/s} \]
\[ w_s = \frac{21000}{3600} = 5.833 \text{ kg/s}, \quad w_s - w = 4.990 \text{ kg/s} \]
\[ wh_3 + (w_s - w) h_4 = w_s h_5 \]
\[ 0.843 \times 2712.38 + 4.99 \times h_4 = 5.833 \times 2649.04 \]
\[ 2286.54 + 4.99 h_4 = 15451.85 \]
\[ h_4 = 2638.33 \text{ kJ/kg} = 584.33 + x_4 \times 2148.1 \]
\[ x_4 = 0.956 \quad \text{Ans. (a)} \]
\[ (W_T)_{HP} = (w_s - w)(h_1 - h_4) = 4.99 \times 231.37 = 1154.54 \text{ kW} \quad \text{Ans. (b)} \]
\[ (h_w)_{HP} = \frac{h_1 - h_4}{h_1 - h_{4s}} = \frac{2869.7 - 2638.33}{2869.7 - 2567.03} = 302.67 \]
\[ = 0.7644 \quad \text{or,} \quad 76.44\% \quad \text{Ans. (c)} \]

**SHORT-ANSWER QUESTIONS**

2.1 What do you understand by externally irreversible and internally irreversible Rankine cycle?

2.2 What is a pinch point? What is its effect on the size and efficiency of the steam generator?

2.3 What is the mean temperature of heat addition? What is its effect on cycle efficiency?

2.4 Explain the different methods by which the mean temperature of heat addition can be increased.

2.5 How is the maximum pressure of a steam cycle fixed up?

2.6 When does reheating of steam become necessary? Explain the effect of reheat on cycle output and efficiency.

2.7 How is the optimum reheat pressure arrived at?

2.8 Why are more than two reheat not used in practice?

2.9 Why is the thermal efficiency of a condensing steam power plant less in a warm region than in a cold region?

2.10 Why is the ideal regenerative cycle not practicable?
2.11 Explain the effect of regeneration on steam cycle output and efficiency.
2.12 The use of regenerative feedwater heating increases the capital cost but reduces the operating cost of a steam power plant. Explain.
2.13 Give a comparative estimate of open and closed feedwater heaters.
2.14 What is heater drip?
2.15 What is the function of a steam trap?
2.16 What do you understand by terminal temperature difference? Can it be negative? Explain.
2.17 What is the effect of TTD on heater size and cycle efficiency?
2.18 What is a drain cooler?
2.19 Why is one of the feedwater heaters always an open heater? What is it called?
2.20 Explain the function of the deaerator. Why is deaerator installed at a large height from the basement?
2.21 Why is hydrazine injected at the suction of the boiler feed pump?
2.22 Explain the optimum degree of regeneration. How is it arrived at?
2.23 Show that regenerative feedwater heating improves the cycle efficiency.
2.24 What is a supercritical steam cycle?
2.25 What is the common unit size of a steam power plant? What will be its projected size in the next decade?
2.26 What is boiler efficiency?
2.27 Show that the overall plant efficiency is a product of five component efficiencies.
2.28 What do you understand by the efficiency of the auxiliaries, \( n_{\text{aux}} \)?
2.29 What is heat rate? What is the difference between net cycle heat rate and gross cycle heat rate?
2.30 What do you understand by cogeneration of power and process heat? Explain its thermodynamic advantage.
2.31 What is a back pressure turbine? What are its applications?
2.32 Explain cogeneration plant efficiency.
2.33 What is a pass-out turbine and when is it used?

**PROBLEMS**

2.1 A cyclic steam power plant is to be designed for a steam temperature at turbine inlet of 360 °C and an exhaust pressure of 0.08 bar. After isentropic expansion of steam in the turbine, the moisture content at the turbine exhaust is not to exceed 15%. Determine the maximum allowable steam pressure at the turbine inlet, and calculate the Rankine cycle efficiency for these steam conditions. Estimate also the mean temperature of heat addition.

[Ans. 16.83 bar, 31.7%, 187.5 °C]

2.2 A steam power station uses the following cycle: Steam at boiler outlet: 150 bar, 550 °C. Reheat at 40 bar to 550 °C. Condenser at 0.1 bar. Assuming ideal processes find (a) the quality at turbine exhaust, (b) the cycle efficiency, and (c) the steam rate.
2.3 In a single-heater regenerative cycle the steam enters the turbine at 30 bar, 400 °C and the exhaust pressure is 0.10 bar. The feedwater heater is a direct contact type which operates at 5 bar. Find (a) the efficiency and the steam rate of the cycle, and (b) the increase in mean temperature of heat addition, efficiency and steam rate, as compared to the Rankine cycle (without regeneration). Neglect pump work.

[Ans. (a) 0.88, (b) 43.9%, (c) 2.18 kg/kWh]

2.4 A simple steam power cycle uses solar energy for the heat input. Water in the cycle enters the pump as a saturated liquid at 40 °C, and is pumped to 2 bar. It then evaporates in the boiler at this pressure, and enters the turbine as saturated vapour. At the turbine exhaust the conditions are 40 °C and 10% moisture. The flow rate is 150 kg/h. Determine (a) the turbine isentropic efficiency, (b) the net work output, (c) the cycle efficiency, and (d) the area of the solar collector needed if the collectors pick up 0.58 kW/m².

[Ans. (a) 35.36%, (b) 3.93 kg/kWh, (b) 27.4 °C, 1.18%, 0.47 kg/kWh]

2.5 In a nuclear power plant heat is transferred in the reactor to liquid sodium. The liquid sodium is then pumped to a heat exchanger where heat is transferred to steam. The steam leaves this heat exchanger as saturated vapour at 50 bar, and is then superheated in an external gas fired superheater to 600 °C. The steam enters the turbine, which has one extraction point at 4 bar, where steam flows to an open feedwater heater. The turbine efficiency is 75% and the condenser temperature is 40 °C. Determine the heat transfer in the reactor and in the superheater to produce a power output of 80 MW, and the thermal efficiency.

[Ans. 178.5 MW, 71.1 MW, 0.32]

2.6 In a reheat cycle, the initial steam pressure and the maximum temperature are 150 bar and 550 °C respectively. If the condenser pressure is 0.1 bar and the moisture at the condenser inlet is 15%, and assuming ideal processes, determine (a) the reheat pressure, (b) the cycle efficiency, and (c) the steam rate.

[Ans. (a) 13.5 bar, (b) 43.6%, (c) 2.05 kg/kWh]

2.7 In a reheat steam cycle, the maximum steam temperature is limited to 500 °C. The condenser pressure is 0.1 bar and the quality at turbine exhaust is 0.8778. Had there been no reheat the exhaust quality would have been 0.7592. Assuming ideal processes, determine (a) the reheat pressure, (b) the boiler pressure, (c) the cycle efficiency, and (d) the steam rate.

[Ans. (a) 30 bar, (b) 150 bar, (c) 50.51%, (d) 1.9412 kg/kWh]

2.8 In a reheat cycle steam at 500 °C expands in an h.p. turbine till it is saturated vapour. It is reheated at constant pressure to 400 °C and then expands in an l.p. turbine to 40 °C. If the maximum moisture content at the turbine exhaust is limited to 15%, find (a) the reheat pressure, (b) the boiler pressure, (c) the net specific work output, (d) the cycle efficiency, and (e) the steam rate. Assume all processes ideal.
What would have been the quality at turbine exhaust, the net work output and the cycle efficiency, without the reheating of steam?

2.9 A regenerative cycle operates with steam supplied at 30 bar and 300 °C, and the condenser pressure is 0.08 bar. The extraction points for two heaters (one closed and one open) are 3.5 bar and 0.7 bar, respectively. Calculate the thermal efficiency of the plant neglecting pump work.

[Ans. 36%]

2.10 The net power output of an ideal reheat regenerative steam cycle is 80 MW. Steam enters the h.p. turbine at 80 bar, 500 °C and expands till it becomes saturated vapour. Some of the steam then goes to an open feedwater heater and the balance is reheated to 400 °C, after which it expands in an l.p. turbine to 0.07 bar. Compute (a) the reheat pressure, (b) the steam flow rate to the h.p. turbine, (c) the cycle efficiency, and (d) the rate of flow of cooling water in the condenser if the temperature rise of water is 8 °C, (e) If the velocity of steam flowing from the turbine to the condenser is limited to 130 m/s, find the diameter of the connecting pipe.

[Ans. (a) 6.5 bar, (b) 58.4 kg/s, (c) 43.7% (d) 3146.5 kg/s, (e) 2.97 m]

2.11 Steam is generated at 70 bar, 500 °C and expands in a turbine to 30 bar with an isentropic efficiency of 77%. At this condition, it is mixed with steam twice its mass at 30 bar, 400 °C. The mixture then expands with an isentropic efficiency of 80% to 0.06 bar. At a point in the expansion where the pressure is 5 bar, steam is bled for feedwater heating in a direct contact heater, which raises the feedwater to the saturation temperature of the bled steam. Calculate the mass of steam bled per kg of high pressure steam and the cycle efficiency. Assume that the L.P. expansion condition line is straight.

[Ans. 0.53 kg, 31.9%]

2.12 A certain chemical plant requires heat from process steam at 120 °C at the rate of 5.83 MW and power at the rate of 1 MW from the generator terminals. Both the heat and power requirements are met by a back pressure turbine of 80% brake efficiency (brake output/isentropic output) and 85% internal efficiency, which exhausts steam at 120 °C, dry and saturated. All the latent heat released during condensation is utilized in the process heater. Find the pressure and temperature of steam at inlet to the turbine. Assume 90% efficiency for the electric generator.

[Ans. 22.5 bar. 360 °C]

2.13 A 10 MW steam turbine operates with steam at 40 bar, 400 °C at the inlet and exhausts at 0.1 bar. 10,000 kg/h of steam at 3 bar are to be extracted for process work. The turbine has 75% isentropic efficiency throughout. Find the boiler capacity required.

[Ans. 13.74 kg/s = 49.46 t/h]

2.14 A pass-out two-stage turbine receives steam at 50 bar, 350 °C. At 1.5 bar, the h.p. steam exhausts and 12,000 kg of steam per hour are taken at this stage for process purposes. The remainder is reheated at 1.5 bar to 250 °C and then
expanded through the l.p. turbine to a condenser pressure of 0.05 bar. The power output from the turbine unit is 3750 kW. Take isentropic efficiency for h.p. and i.p. stages as 0.84 and 0.81 respectively. Calculate the boiler capacity required.

[Ans. 18.5 t/h]

2.15 A 850 MW steam power plant operates with turbine inlet at 100 bar, 550 °C and condenser pressure at 0.05 bar. There are three feedwater heaters placed optimally as follows: (i) the h.p. heater is of the closed type with drains cascaded backward, (ii) the i.p heater is of the open type, and (iii) the l.p. heater is of the closed type with drains pumped forward. Each of the turbine sections has the same isentropic efficiency of 90%. The pumps have isentropic efficiencies of 80%. Calculate (a) the mass flow rate of steam at turbine inlet, (b) the mass flow rate of cooling water in the condenser, if it undergoes a 10 °C temperature rise, (c) the cycle efficiency, and (e) the cycle heat rate.

2.16 A coal-fired power plant has a turbine generator rated at 1000 MW gross. The plant requires 9% of this power for its internal operations. It uses 9800 tonnes of coal of heating value 26 MJ/kg per day. The steam generator efficiency is 86%. Calculate the gross and net station heat rates and efficiencies.

[Ans. 10.616 MJ/kWh, 11.667 MJ/kWh, 33.9%, 30.86%]

2.17 Steam is supplied to a pass-out turbine at 40 bar, 400 °C and dry saturated process steam is required at 4 bar. The l.p. stage exhausts at 0.1 bar and the condition line for the turbine expansion may be assumed to be straight. If the power load is 1000 kW and the maximum process load is 1.4 kW, estimate the maximum steam flow through the high and low pressure stages. Assume that the steam just condenses in the process plant.

[Ans. 3837.6 kg/h, 3635.3 kg/h]

2.18 A steam power plant has the boiler efficiency of 92%, turbine (mechanical) efficiency of 96%, and electric generator efficiency of 97%. If 7% of the power generated is consumed in running the auxiliaries of the plant and the overall plant efficiency is 34%, find the percentage of energy lost in the condenser.

[Ans. 57.4%]

2.19 An ideal steam power plant operates between 150 bar, 550 °C and 0.075 bar. It has seven feedwater heaters. Find the optimum pressures and temperatures at which the heaters operate.

2.20 In a cogeneration plant, $10^6$ kg/h of steam at 80 bar, 480 °C expands in the h.p. turbine to 10 bar. From the exhaust $4 \times 10^5$ kg/h of steam is extracted for process heating. The remaining steam expands in the l.p. turbine to 0.08 bar. Saturated liquid at 0.08 bar leaving the condenser is pumped to 9.5 bar where it mixes with the condensate from the process heater leaving at 9.5 bar, 120°C. The entire flow is then pumped to 80 bar. The isentropic efficiencies of the turbines and the pumps are 86% and 80%, respectively. Determine (a)
the heating load, in kJ/h, (b) the power developed by the turbines, in kW, and (c) the rate of heat transfer in the steam generator, in kJ/h.

[Ans. (a) 9.53 kJ/h, (b) 236500 kW, (c) 3.032 × 10^9 kJ/h]

2.21 In a reheat-regenerative steam cycle, steam enters the h.p. turbine at 80 bar, 480 °C and expands to 7 bar. The steam is then reheated to 440 °C before entering the l.p. turbine, where it expands to the condenser pressure of 0.08 bar. Steam extracted from the h.p. turbine at 20 bar is fed to a closed feedwater heater, from which the drain at 205 °C is cascaded to an open feedwater heater through a trap. Steam extracted from the l.p. turbine at 3 bar is also fed into the open feedwater heater, from which the total flow is pumped into the steam generator. The net power output of the cycle is 100 MW. Assuming ideal processes, determine (a) the cycle efficiency, and (b) the rate of steam generation in kg/h.

[Ans. (a) 43%, (b) 2.8 × 10^5 kg/h]

2.22 A pressurized water reactor power plant operates at pressures and temperatures as shown in Fig. P2.22. The steam leaving the heat exchanger is at 50 bar, dry saturated and the condensate is saturated to the saturation temperature of the bled steam at 5 bar in an open feed heater. Assuming all processes to be ideal and neglecting pump work estimate (a) the ratio of the working fluids in the two circuits, (b) the fraction of the steam supply which is bled from the turbine, and (c) the cycle efficiency.

[Ans. (a) 10 :1, (b) 0.22, (c) 39%]

![Fig. P2.22](image-url)
3.1 FLAWS OF STEAM AS WORKING FLUID IN POWER CYCLE

There are certain drawbacks with steam as the working substance in a power cycle. The maximum temperature gained in steam cycles using the best available material is about 600 °C, while the critical temperature of steam is 375°C, which necessitates large superheating and permits the addition of only an infinitesimal amount of heat at the highest temperature.

High moisture content is involved during expansion of steam in going to higher steam pressures in order to obtain higher mean temperature of heat addition ($T_{m_1}$). The use of reheat thus becomes necessary. Since reheater tubes are costly and the steam plant layout becomes complex, the use of more than two reheat is hardly recommended. Also, as pressure increases, the metal stresses increase, and the increase in thicknesses of the walls of boiler drums, tubes, pipe-lines and so on is not in proportion to the pressure increase, but much faster because of the prevalence of high temperature.

It may be noted that high $T_{m_1}$ is desired for high cycle efficiency. The need for high pressures is only forced due to weak characteristics of steam.

If the lower limit is now considered, it is seen that at the heat rejection temperature of 40 °C, the saturation pressure of steam is 0.075 bar, which is considerably lower than atmospheric pressure. The temperature of heat rejection can still be lowered using some refrigerant as a coolant in the condenser. The corresponding vacuum will be still higher, and to maintain such high vacuum in the condenser is a big problem.

It is the low temperature of heat rejection which is of real interest. The necessity for vacuum is a disagreeable characteristic of steam.

The saturated vapour line in the $T$–$s$ diagram of steam is sufficiently inclined, so that when steam is expanded to lower pressures (for higher turbine
output as well as cycle efficiency), it involves more moisture content, which is not desired from the consideration of the erosion of turbine blades in later stages.

### 3.2 CHARACTERISTICS OF IDEAL WORKING FLUID FOR VAPOUR POWER CYCLE

The desirable characteristics of the working fluid in a vapour power cycle to obtain the best thermal efficiency are given below:

(a) The fluid should have a high critical temperature so that the saturation pressure at the maximum permissible temperature (metallurgical limit) is relatively low. It should have a large enthalpy of vaporization at that pressure.

(b) The saturation pressure at the temperature of heat rejection should be above the atmospheric pressure so as to avoid the necessity of maintaining vacuum in the condenser.

(c) The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to the boiling point.

(d) The saturated vapour line of $T$–$s$ diagram should be steep, very close to the turbine expansion process so that excessive moisture does not appear during expansion.

(e) The freezing point of the fluid should be below the room temperature, so that it does not get solidified while flowing through the pipelines.

(f) The fluid should be chemically stable and should not contaminate the materials of construction at any temperature.

(g) The fluid should be non-toxic, non-corrosive, not excessively viscous, and low in cost.

The characteristics of such a hypothetical working fluid are approximated in the $T$–$s$ diagram as shown in Fig. 3.1. Some superheat is desired to reduce piping losses and improve turbine efficiency. The bounded area of the cycle is almost a rectangle, and its thermal efficiency is very close to the Carnot efficiency.

![T-s diagram of an ideal working fluid for a vapour power cycle](image-url)

**Fig. 3.1** $T$–$s$ diagram of an ideal working fluid for a vapour power cycle
3.3 BINARY VAPOUR CYCLES

No single fluid can meet all the requirements as mentioned above. Although in the overall evaluation, water is better than any other working fluid, at high temperatures, however, there are a few better fluids, and notable among them are: (a) diphenyl ether, \((C_6H_5)_2O\), (b) aluminium bromide \(\text{AlBr}_3\) and (c) liquid metals like mercury, sodium, potassium and so on. Among these, only mercury has actually been used in practice. Diphenyl ether could be considered but it has not yet been used because like most organic substances, it decomposes gradually at high temperatures. Aluminium bromide is a possibility and yet to be considered.

As at pressure of 12 bar, the saturation temperatures for water, aluminium bromide, and mercury are 187°C, 482.5°C and 560°C, respectively. The highest cyclic temperature consistent with the best available material for use in power plant is about 560°C. Therefore, mercury is a better working fluid in the high temperature range because at 560°C, its vaporization pressure is relatively low. Its critical pressure and temperature are 1080 bar and 1460 °C, respectively.

But in the low temperature range, mercury is unsuitable because its saturation pressure becomes exceedingly low, and it would be impractical to maintain such a high vacuum in the condenser. At 30 °C, the saturation pressure of mercury is only \(2.7 \times 10^{-4}\) cm Hg. Its specific volume at such a low pressure is very large, and it would be difficult to accommodate such a large volume flow.

For this reason, to take advantage of the beneficial features of mercury in the high temperature range and to get rid of its deleterious effects in the low temperature range, mercury vapour leaving the mercury turbine is condensed at a higher temperature and pressure, and the heat released during the condensation of mercury is utilized in evaporating water to form steam to operate on a conventional turbine.

Thus, in the binary (or two fluid) cycle, two cycles with different working fluids are coupled in series, the heat rejected by one being utilized in the other.

The flow diagram of mercury-steam binary cycle and the corresponding \(T\)-\(s\) diagram are given in Figs 3.2 and 3.3 respectively. The mercury cycle \(a\rightarrow b\rightarrow c\rightarrow d\) is a simple Rankine cycle using saturated vapour. The heat rejected by mercury during condensation (process \(b\rightarrow c\)) is transferred to boiler water and form saturated vapour (process \(5\rightarrow 6\)). The saturated vapour is heated from the external source (furnace) in the superheater (process \(6\rightarrow 1\)). Superheated steam expands in the turbine and is then condensed. The condensate is then pumped to the economiser where it is heated till it becomes saturated liquid by the outgoing flue gases (process \(4\rightarrow 5\)). The saturated liquid then goes to the mercury condenser–steam boiler, where the latent heat is absorbed. In an actual plant, the steam cycle is always a regenerative cycle with feedwater heating, but for the sake of simplicity, this complication has been omitted.

Let \(m\) represent the flow rate of mercury in the mercury cycle per kg of steam circulating in the steam cycle. Then, for 1 kg of steam,
\[ Q_1 = m(h_a - h_d) + (h_1 - h_6) + (h_5 - h_4), \quad Q_2 = h_2 - h_3 \]
\[ W_T = m(h_a - h_b) + (h_1 - h_2), \quad W_P = m(h_d - h_c) + (h_4 - h_3) \]
\[ h_{cycle} = \frac{Q_1 - Q_2}{Q_1} = \frac{W_T - W_P}{Q_1} \]  

\( Q_1, Q_2, W_T, W_P \)

**Fig. 3.2** Flow diagram of mercury-steam binary cycle

**Fig. 3.3** \(T\)-\(s\) diagram of mercury-steam binary cycle

and \(\text{steam rate (s.s.c)} = \frac{3600}{W_T - W_P} \text{ kg/kWh}\)

The energy balance of the mercury condenser-steam boiler gives

\[ m(h_b - h_c) = (h_6 - h_5) \]
\[ m = \frac{h_6 - h_5}{h_b - h_c} \text{ kg Hg/kg H}_2\text{O} \]  

(3.2)

To vaporize one kg of water, 7 to 8 kg of mercury must condense.

The mercury-steam cycle first proposed by Emmet in 1925 [1] was in actual commercial use for more than four decades. A series of such plants were developed by the General Electric of the USA in the forties culminating in the installation of the Schiller 40 MW plant at Portsmouth, New Hampshire, which went on load in 1950. The thermal efficiencies of these plants are high (above 50%) and mercury makes up the necessary thermodynamic constraints. However, such plants have never attained wide commercial acceptance because there has always been the possibility of improving steam cycles by increasing pressure and temperature, and by using reheat and regeneration. Over and above, mercury is expensive, limited in supply and highly toxic. Because of the low latent heat of mercury, high mercury flow rates (about 8 times the steam flow rate) are needed. Special precautions are necessary to prevent leakage of mercury. Thus, the capital costs are high. Although the New Hampshire plant has now been dismantled due to higher overall cost per unit of electricity generated, it was the first major combined cycle plant and represented a great engineering achievement.

3.4 COUPLED CYCLES

The mercury steam cycle represents the two-fluid cycles where two Rankine cycles have been coupled in series. The mercury cycle is called the topping cycle and the steam cycle is called the bottoming cycle. If a sulphur dioxide cycle is added to it in the low temperature range, so that the heat released during the condensation of steam is utilized in forming sulphur dioxide vapour which expands in another turbine, then the mercury–steam–sulphur dioxide cycle is a three fluid or tertiary cycle. Similarly, other liquid metals, apart from mercury, like sodium or potassium, may be considered for a working fluid in the topping cycle. Apart from SO₂, other refrigerants (ammonia, freons etc.) may be considered as working fluids for the bottoming cycle. Figure 3.4 shows a sodium–mercury–steam tertiary cycle on T–s coordinates, having thus three cycles coupled in series.

3.4.1 Thermodynamics of Rankine–Rankine Coupled Cycles

If two Rankine cycles with two different working fluids are coupled in series, the heat lost by one is absorbed by the other (Fig. 3.5), as in the mercury–steam binary cycle. Let \( \eta_1 \) and \( \eta_2 \) be the efficiencies of the topping and bottom cycles, respectively, and \( \eta \) be the overall efficiency of the combined cycle.
Fig. 3.4  Sodium–mercury–steam tertiary cycle

Fig. 3.5  Two vapour power cycles coupled in series

\[ \eta_1 = 1 - \frac{Q_2}{Q_1} \quad \text{and} \quad \eta_2 = 1 - \frac{Q_3}{Q_2} \]

or

\[ Q_2 = (1 - \eta_1)Q_1 \quad \text{and} \quad Q_3 = Q_2(1 - \eta_2) \]

Now,

\[ \eta = 1 - \frac{Q_3}{Q_1} = 1 - \frac{Q_2(1 - \eta_2)}{Q_1} = 1 - \frac{Q_1(1 - \eta_1)(1 - \eta_2)}{Q_1} \]

\[ \therefore \eta = 1 - (1 - \eta_1)(1 - \eta_2) \quad (3.3) \]

or

\[ 1 - \eta = (1 - \eta_1)(1 - \eta_2) \quad (3.4) \]
For \( n \) cycles coupled in series, the overall efficiency would be given by

\[
1 - \eta = (1 - \eta_1) (1 - \eta_2) \ldots (1 - \eta_n)
\]

or

\[
1 - \eta = \prod_{i=1}^{n} (1 - \eta_i)
\]

(3.5)

\[
\therefore \text{Total loss} = \text{product of losses in all the cycles in series}
\]

For two-cycles coupled in series, from Eq. (3.3),

\[
\eta = 1 - (1 - \eta_1 - \eta_2 + \eta_1 \eta_2)
\]

\[
\therefore \eta = \eta_1 + \eta_2 - \eta_1 \eta_2
\]

(3.6)

Thus, the overall efficiency of two cycles coupled in series equals the sum of the individual efficiencies minus their product.

By combining two cycles in series, even if individual efficiencies are low, it is possible to achieve a fairly high combined efficiency, which cannot be attained by a single cycle. For example, if \( \eta_1 = 0.50 \) and \( \eta_2 = 0.40 \),

\[
\eta = 0.5 + 0.4 - 0.5 \times 0.4 = 0.70
\]

It is almost impossible to achieve such a high efficiency in a single cycle.

For sodium–mercury–steam tertiary cycle, if \( \eta_1 = 0.5 \), \( \eta_2 = 0.4 \) and \( \eta_3 = 0.4 \), from Eq. (3.3),

\[
\eta = 1 - (1 - \eta_1) (1 - \eta_2) (1 - \eta_3) = 1 - 0.5 \times 0.6 \times 0.6 = 0.82
\]

Of course, there are other losses which have not been considered.

There is thus great scope of improving cycle efficiency and achieving fuel economy using multifluid coupled cycles and utilizing the favourable characteristics of the working fluids in different temperature ranges for bulk power generation.

### 3.5 COMBINED CYCLE PLANTS

The maximum steam temperature in a power cycle does not exceed 600 °C, although the temperature in a dry bottom pulverized coal furnace is about 1300 °C. Therefore, there is a great thermal irreversibility and a decrease of availability because of heat transfer from combustion gases to steam through such a large temperature difference. By superposing a high temperature power plant as a topping unit to the steam plant, a higher energy conversion efficiency from fuel to electricity can be achieved, since the combined plant operates through a higher temperature range. Combined plants may be of the following types:

(i) Gas turbine–steam turbine plant
(ii) MHD–steam plant
(iii) Thermionic–steam plant
(iv) Thermoelectric–steam plant

MHD-steam, thermionic-steam and thermoelectric-steam power plants have been discussed in Chapter 13.
3.6 GAS TURBINE—STEAM TURBINE POWER PLANT

The air standard cycle for a gas turbine power plant is the Brayton cycle which, like Rankine cycle, also consists of two reversible adiabatics and two reversible isobars, but unlike Rankine cycle the working fluid does not undergo phase change. The efficiency of the cycle is given by,

\[ n_{\text{Brayton}} = 1 - \frac{1}{r_p^{(\gamma - 1)/\gamma}} \]  

(3.7)

where \( r_p \) is the pressure ratio and \( \gamma \) is the specific heat ratio.

A gas turbine plant can be either open or closed. Simple, open gas turbine plant is shown in Fig. 3.6. Since the product of combustion is the working fluid which produces power by doing work on the blades of the gas turbine, it is an internal combustion plant. However, unlike the reciprocating internal combustion engine, the gas turbine is a steady flow device and the blades are always subjected to the highest gas temperature. To limit the maximum gas temperature to about 1200 K at inlet to the turbine consistent with the material used, a high air-fuel ratio is used. The disadvantages of a gas turbine power plant in a utility system are the following:

![Diagram of Gas Turbine Plant]

(a) Fig. 3.6 Open cycle gas turbine plant

1. Large compressor work input, since the power required to drive the compressor is considerably higher than that required by a pump for the same pressure rise. The compressor thus consumes a large part of the work produced by the turbine.
2. Large exhaust loss, since the exhaust gas temperature is quite high and also the mass flow rate of gas is large due to high air–fuel ratio used.
3. Machine inefficiencies, since with the decrease in compressor efficiency (∂e) the work input to the compressor increases and with the decrease in
turbine efficiency ($\eta_T$) the work output from the turbine decreases. At certain values of $\eta_c$ and $\eta_T$, a situation may arise when the compressor consumes more power than what the turbine develops. So, the machine efficiencies of the compressor and the turbine have to be high enough to yield justifiable net work output.

4. Low cycle efficiency, due to the large exhaust loss, large compressor work and machine inefficiencies.

5. Costly fuel, since the cost of kerosene and other fuels used is much higher than that of coal. Its availability is also not always guaranteed.

Due to the above factors, the cost of power generated by a stationary gas turbine plant for a utility system is high. However, a gas turbine plant offers certain advantages also, as given below:

1. Less installation cost
2. Less installation time
3. Quick starting and stopping
4. Fast response to load changes

So, a gas turbine plant is often used as a peaking unit for certain hours of the day when the energy demand is high. A large steam plant designed to meet peak loads would operate at an uneconomical load factor during most of the year.

To overcome its low cycle efficiency, a gas turbine may be used in conjunction with a steam turbine plant in an utility base load station, to offer the utilities the gas turbine advantages of quick starting and stopping and permit flexible operation of the combined plant over a wide range of loads.

3.6.1 Thermodynamics of Brayton–Rankine Combined Cycle Plant

Let us consider two cyclic power plants coupled in series, the topping plant operating on Brayton cycle and the bottoming one operating on Rankine cycle (Fig. 3.7). Helium gas may be the working fluid in the topping plant and water in the bottoming plant. As shown in Section 3.4.1, the overall efficiency of the combined plant is given by Eq. (3.6)

$$\eta = \eta_1 + \eta_2 - \eta_1 \eta_2$$

where $\eta_1$ and $\eta_2$ are the thermal efficiencies of the Brayton cycle and the Rankine cycle, respectively.

1. Heat loss between two plants in series In the previous section, it was assumed that all the heat rejected by the topping plant is absorbed by the bottoming one. However, there is always some heat loss and the heat absorbed is always less than the heat rejected.

Let $Q_L$ be the heat loss between the two plants (Fig. 3.8). The overall plant efficiency is

$$\eta = \frac{W_1 + W_2}{Q_L}$$
Now, \[ \eta_1 = \frac{W_1}{Q_1}, \quad \eta_2 = \frac{W_2}{Q_2}, \]
\[ Q_3 = Q_2 - Q_L = Q_1(1 - \eta_1) - Q_L. \]
Therefore,
\[ \eta = \eta_1 + \eta_2 \frac{Q_3}{Q_1} = \eta_1 + \eta_2 \]
\[ [(1 - \eta_1) - Q_L/Q_1] \]
\[ = \eta_1 + \eta_2 - \eta_1 \eta_2 - \eta_2 x_L \]  
(3.8)

where \( x_L \) is the fraction of the heat supplied which is lost \((Q_L/Q_1)\). The overall efficiency can also be expressed in a different way. The ratio of \( Q_3/Q_2 \) is the efficiency of the boiler in the bottoming plant, so that

**Fig. 3.8** Heat loss in between two plants in series
\[ \eta_B = \frac{Q_3}{Q_2} = 1 - \frac{Q_1}{Q_2} \]

Now,
\[
\eta = \frac{W_1 + W_2}{Q_1} = \eta_1 + \frac{\eta_2 Q_3}{Q_1} = \eta_1 + \eta_2 \eta_B \frac{Q_3}{Q_1}
= \eta_1 + \eta_2 \eta_B (1 - \eta_1) = \eta_1 + \eta_2 \eta_B - \eta_1 \eta_2 \eta_B
= \eta_1 + (\eta_2)_{oc} - h_1(\eta_o)_{oc}
\]

(3.9)

where \((\eta_o)_{oc} = \eta_B\). \(\eta_2\) is the overall efficiency of the bottoming plant.

2. **Two cyclic plants operating in parallel** Let us consider two cyclic plants operating in parallel, one on Brayton cycle and the other on Rankine cycle (Fig. 3.9). The total heat supplied \(Q_1\) is divided between the two plants \(Q_2\) and \(Q_4\), so that \(x_1 = \frac{Q_2}{Q_1} = \frac{Q_2}{Q_2 + Q_4}\) is the fraction of the total heat taken by the plant operating on Brayton cycle. The net works done by the two plants are

\[ W_1 = \eta_1 Q_2 \quad \text{and} \quad W_2 = \eta_2 Q_4 \]

The overall efficiency of the combined plant is

\[ \eta = \frac{W_1 + W_2}{Q_1} = \frac{W_1}{Q_2 + Q_4} + \frac{W_2}{Q_2 + Q_4} = \frac{\eta_1 Q_2 + \eta_2 Q_4}{Q_2 + Q_4} \]

\[
\begin{array}{c}
\text{Fig. 3.9} \quad \text{Two cyclic plants operating in parallel} \\
\end{array}
\]

\[ = \eta_1 x_1 + \eta_2 (1 - x_1) \]
\[ = \eta_1 x_1 + \eta_2 - \eta_2 x_1 \]
\[ = \eta_2 + x_1 (\eta_1 - \eta_2) \]  

(3.10)

If
\[ x_2 = \frac{Q_4}{Q_2 + Q_4} \quad \text{and} \quad 1 - x_2 = \frac{Q_2}{Q_2 + Q_4} \]

then
\[ \eta = \eta_1 (1 - x_2) + \eta_2 x_2 \]
\[ = \eta_1 - \eta_1 x_2 + \eta_2 x_2 \]
\[ = \eta_1 - x_2 (\eta_1 - \eta_2) \]  

(3.11)
If $\eta_1 > \eta_2$, then from Eq. (3.10), $\eta > \eta_2$, but from Eq. (3.11), $\eta < \eta_1$. Therefore, $\eta$ lies between $\eta_1$ and $\eta_2$. There is, thus, no advantage to the parallel system. If the cyclic plant 1 operating on Brayton cycle could absorb more heat, say, equal to $(Q_2 + Q_4)$, then it would be advantageous to use that plant alone.

3. **Series-parallel plants with two cyclic plants in series having supplementary firing** Let the fraction of the total heat supplied that is used for supplementary heating (Fig. 3.10) be

\[ x_2 = \frac{Q_4}{Q_1} \quad \text{so that} \quad 1 - x_2 = \frac{Q_2}{Q_1} = \frac{Q_2}{Q_2 + Q_4} \]

Now,

\[ W_1 = \eta_1 Q_2, \quad Q_3 = (1 - \eta_1)Q_2, \quad W_2 = \eta_2 Q_5 \]

Therefore, the overall plant efficiency is given by

\begin{align*}
\eta &= \frac{W_1 + W_2}{Q_1} = \frac{\eta_1 Q_2 + \eta_2 Q_5}{Q_2 + Q_4} \\
&= \eta_1 (1 - x_2) + \eta_2 \frac{Q_4 + Q_3}{Q_2 + Q_4} \\
&= \eta_1 (1 - x_2) + \eta_2 \left[ x_2 + (1 - \eta_1) \frac{Q_2}{Q_1} \right] \\
&= \eta_1 (1 - x_2) + \eta_2 [x_2 + (1 - \eta_1)(1 - x_2)] \\
&= \eta_1 + \eta_2 - \eta_1 \eta_2 - x_2 \eta_1 (1 - \eta_2) \quad (3.12)
\end{align*}

Therefore, the overall efficiency of a series–parallel plant is less than that of two coupled cycles in series since the last term in Eq. (3.12) is positive. In absence of supplementary heating, i.e. when $x_2 = 0$, the overall efficiency reduces to that of an ideal series plant.

4. **Series-parallel plants with two plants in series having supplementary heat supply and heat loss in between the two plants** Let $x_2$ and $x_L$ be the fractions of heat supply ($Q_1$) which represent supplementary heat supply ($Q_4$) and heat loss to the surroundings ($Q_L$), respectively (Fig. 3.10).

\[ x_2 = \frac{Q_4}{Q_1} = \frac{Q_4}{Q_2 + Q_4}, \quad x_L = \frac{Q_L}{Q_1} = \frac{Q_L}{Q_2 + Q_4}, \quad \eta_1 = 1 - \frac{Q_3}{Q_2}, \quad W_1 = \eta_1 Q_2 \]

\[ W_2 = \eta_2 Q_5, \quad Q_5 = Q_4 + Q_3 - Q_L, \quad Q_3 = (1 - x_2)(1 - \eta_1)Q_1 \]

So, the overall efficiency of the combined plant becomes

\begin{align*}
\eta &= \frac{W_1 + W_2}{Q_1} = \frac{\eta_1 Q_2 + \eta_2 Q_5}{Q_2 + Q_4} \\
&= \eta_1 (1 - x_2) + \eta_2 \frac{Q_4 + Q_3 - Q_L}{Q_2 + Q_4}
\end{align*}
\[ \eta = \eta_1(1 - x_2) + \eta_2 \left[ x_2 + \frac{(1 - \eta_1)Q_2}{Q_2 + Q_4} - x_L \right] \]

\[ = \eta_1(1 - x_2) + \eta_2[x_2 + (1 - \eta_1)(1 - x_2) - x_L] \]

\[ = \eta_1 + h_2 - \eta_1 \eta_2 - x_L \eta_2 - \eta_1 x_2(1 - \eta_2) \]

(3.13)

If \( x_2 = 0 \), \( \eta = \eta_1 + \eta_2 - \eta_1 \eta_2 - x_L \eta_2 \)

which is the same as Eq. (3.8) derived earlier. It can also be expressed in a different way by defining the boiler efficiency

\[ \eta_B = \frac{Q_5}{Q_5 + Q_i} = 1 - \frac{Q_L}{Q_5 + Q_i} = 1 - \frac{x_L Q_i}{Q_4 + Q_3 - Q_L + Q_i} \]
\[
\frac{x_l}{x_2 + (1 - x_2)(1 - \eta_1)} = 1 - \frac{x_l}{x_2 + (1 - x_2)(1 - \eta_1)}
\]
or
\[
x_l = (1 - \eta_B) [x_2 + (1 - x_2)(1 - \eta_1)]
\] (3.14)

Therefore, from Eq. (3.13) the overall plant efficiency of the combined plant becomes
\[
\eta = \eta_1 + \eta_2 - \eta_1 \eta_2 - \eta_2(1 - \eta_B) [x_2 + (1 - x_2)(1 - \eta_1)] - \eta_1 x_2(1 - \eta_2)
= \eta_1(1 - x_2) + (\eta_0)_2 [x_2 + (1 - x_2)(1 - \eta_1)]
\] (3.15)

where \((\eta_0)_2 = \eta_B \eta_2\) is the overall efficiency of the bottom cycle. In absence of supplementary heating, \(x_2 = 0\) and
\[
\eta = \eta_1 + (\eta_0)_2 (1 - \eta_1) = \eta_1 + (\eta_0)_2 - \eta_1 (\eta_0)_2
\]
which is the same as Eq. (3.9) derived earlier.

### 3.6.2 Effect of Supplementary Heating on the Overall Efficiency of the Combined Gas Turbine–Steam Turbine Power Plant

Some thermodynamic concepts of different arrangements of combined cycles have been discussed above. We would now consider a practical gas turbine–steam turbine (GT–ST) power plant where the exhaust from the gas turbine is utilized to generate steam and operate a conventional steam power plant. We will examine how the supplementary heating, i.e. burning additional fuel into the oxygen rich exhaust of the gas turbine, affects the overall efficiency of the combined plant.

Fuel is burnt partially first in the combustion chamber (CC) of the gas turbine plant \((Q_1)\) and then heat recovery steam generator (HRSG) \((Q_2)\). If the net work outputs of the GT and the ST plants be \(W_1\) and \(W_2\), respectively, and \(\eta_1\) and \(\eta_2\) are their corresponding efficiencies, then the overall efficiency is
\[
\eta = \frac{W_1 + W_2}{Q_1 + Q_2} = \frac{\eta_1 Q_1 + \eta_2 ((1 - \eta_1) Q_1 + Q_4)}{Q_1 + Q_2}
\]

Now,
\[
\frac{\partial \eta}{\partial Q_2} = \frac{1}{(Q_1 + Q_2)^2} \left[ (Q_1 + Q_2) \left\{ (1 - \eta_1) Q_1 \frac{\partial \eta_2}{\partial Q_2} + Q_2 \frac{\partial \eta_2}{\partial Q_2} + \eta_2 \right\} 
- \{ \eta_1 Q_1 + \eta_2 [(1 - \eta_1) Q_1 + Q_4]) \right]\]
\]
\[
= \frac{(1 - \eta_1) Q_1 \frac{\partial \eta_2}{\partial Q_2} + Q_2 \frac{\partial \eta_2}{\partial Q_2} + \eta_2}{Q_1 + Q_2} - \frac{\eta_1 Q_1 + \eta_2 [(1 - \eta_1) Q_1 + Q_4]}{(Q_1 + Q_2)^2}
\]

Additional firing of fuel in the HRSG improves the overall efficiency if \(\partial \eta/\partial Q_2 > 0\),
or \[ \{(1 - \eta_1)Q_1 + Q_2\} \frac{\partial \eta_2}{\partial Q_2} + \eta_2 = \eta_1 Q_1 + \eta_2 \left[ ((1 - \eta_1)Q_1 + Q_4 \right] \]

Since the right hand expression is the overall efficiency of the plant,

\[ \{Q_1(1 - \eta_1) + Q_2\} \frac{\partial \eta_2}{\partial Q_2} > (\eta - \eta_2) \]

Now, the left hand expression inside the bracket represents the total heat input to the HRSG, which is equal to \( \frac{W_2}{\eta_2} \), and the inequality becomes

\[ \frac{W_2}{\eta_2} \frac{\partial \eta_2}{\partial Q_2} > (\eta - \eta_2) \]

or

\[ \frac{\partial \ln \eta_2}{\partial Q_2} > \frac{\eta - \eta_2}{W_2} \] (3.16)

With additional firing of the fuel in the HRSG, the power output of the steam cycle \( W_2 \), as well as its efficiency \( \eta_2 \) increase and so the increase in the overall efficiency diminishes. This was also confirmed in Section 3.6.1 for a coupled Brayton–Rankine cycle plant. Therefore, supplementary firing is becoming less and less attractive. Generally it is more profitable to burn the fuel in the combustor of the gas turbine plant itself since the heat is supplied to the system at a temperature higher than that in the steam cycle.

For the compression process of air, (Fig. 3.11),

\[ \frac{T_b}{T_a} = \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\gamma} \] where \( \gamma = 1.4 \)

For the expansion of combustion gases in the gas turbine

\[ \frac{T_c}{T_d} = \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\gamma} \] where \( \gamma = 1.3 \) (may be assumed)

\[ W_1 = (W_{\text{net}})_{\text{GT}} - w_a [c_{pg} (T_c - T_d) - c_{pa}(T_b - T_a)] \] (3.17)

neglecting the mass of fuel (for a high air–fuel ratio), and \( w_a \) being the mass flow of air.

\[ (W_T)_{\text{ST}} = w_s(h_1 - h_2) \] (3.18)

where \( w_s \) is the steam flow rate.

By energy balance,

\[ w_a c_{pg} (T_c - T_d) = w_s(h_1 - h_4) \] (3.19)

Now,

\[ Q_1 = w_a c_{pg} (T_c - T_b + T_e - T_d) \] (3.20)

and the overall efficiency is

\[ \eta = \frac{(W_{\text{GT}})_{\text{net}} + W_2}{Q_1} \]

neglecting the pump work.
Fig. 3.11 GT-ST combined plant with supplementary combustion of fuel

As inlet temperatures to gas turbine keep increasing (due to the use of better material and blade cooling), the importance of supplementary firing diminishes further. However, supplementary firing may provide increased operating and fuel flexibilities in CC plants, which may fall into the following two categories.

1. Combined cycle plants with limited supplementary firing  Supplementary firing raises the temperature of the exhaust gas to 800 to 900 °C. Relatively high flue gas temperature raises the condition of steam (84 bar, 525 °C), thereby improving the efficiency of the steam cycle. Following is given an example (Kehlhofer 1991):
2. CC plants with maximum supplementary firing

Maximum supplementary firing refers to the maximum fuel that can be fired with the oxygen available in the GT exhaust. The steam cycle is conventional with reheat and regeneration. Hot GT exhaust is used as the combustion air. Regenerative air preheater is not required. However, a fresh air fan, which makes it possible to operate the steam plant even when the GT is not in operation, increases the availability of the unit.

The use of large supplementary firing in CC systems with high GT inlet temperatures causes the efficiency to drop. For this reason, CC plants with maximum supplementary firing are only of minimal importance today, in comparison to simple CC installations. They have, however, two advantages: (a) Coal can be burnt in the steam generator as supplementary fuel, (b) It has very good part-load efficiency.

3.6.3 Advantages of Combined Cycle Power Generation

The worldwide demand for combined cycle power plants is growing dramatically, with some experts forecasting explosive growth over the next decade [2]. In its basic form, a gas turbine exhausting into a heat recovery steam generator (HRSG) that supplies steam to a steam turbine cycle is the most efficient system of generating electricity today. Thanks to defence and aircraft research and development programs, the gas turbine technology has forged ahead considerably during the last two decades with unit plant efficiencies of 30 to 35%, comparable to the state-of-the-art fossil-fired power stations. The advantages of combined gas-steam cycles may be summarized as follows:

1. **High overall plant efficiency**  Efficiencies exceeding 50% can be attained.

2. **Low investment costs**  Because 2/3 of the output is produced in a GT and only 1/3 in the simple ST, the investment costs required are approximately 30% less than those for a conventional steam power plant.
3. Small amount of water required  The amount of cooling water required is only about 40 to 50% as much as for a steam plant.

4. Great operating flexibility  The simple steam cycle makes it possible to start-up and shut-down the plants quickly, which also affects efficiency in a positive direction (reducing start-up losses).

5. Phased installation  Because the gas turbines can go into operation much sooner than the steam plant, installation in stages is possible. The gas turbine plant can keep on generating power as the steam plant is under construction. This makes it possible to adjust the growth in demand for energy in a grid. Later, a coal gasification unit can be installed if there is too sharp an increase in the price of oil or gas.

6. Simplicity of operation  It is simpler to run than a conventional steam power plant. Moreover, because combined cycle plants are generally operated fully automatically, they are especially suitable for use where operating staff is less experienced.

7. Low environmental impact  Gas burning combined cycle (CC) plants in particular are ideally suitable for use in heavily populated regions because of their high efficiency and low emission levels of pollutants. In particular, very low NOx levels of clean CC plants are one of their most attractive features. Furthermore, gas-fired CC plants produce per kWh only 40% of the CO₂ produced by a coal-fired plant [25].

8. Advantages for cogeneration of heat and electricity

The good thermodynamic properties of CC plants are highly suitable for cogeneration of heat and electricity. Electrical yields of more than 40% are quite common in heating or industrial power plants with a back pressure turbine.

Large output combined with high cycle efficiency, low emission level and low investment cost are the main attractive features of the CC power generation.

By dividing the expansion process into two temperature ranges, 1100–550 °C in the gas turbine and 550 °C to ambient temperature in the steam turbine, high overall efficiencies exceeding 50% are achieved.

3.6.4 Dual Pressure Steam Cycle

Rankine cycle is required to be split in order to decrease the irreversibility involved in the heat transfer between the hot exhaust gas source and the water/steam. Figure 3.12(a) shows a $t, q$ diagram indicating heat flow from the flue gas to the water steam cycle. It can be seen that more heat can be extracted from the exhaust gas if the steam pressure is lower ($p₂$). The integral $\int(1 - T_o/T) \, dq$ describes the maximum exergy, where the ratio $T_o/T$ is a measure of the exergy loss. In Fig. 3.12(b), $T_o/T$ is plotted as a function of $q$. It is seen that at high steam pressure, the temperature difference or the irreversibility is reduced in the high temperature region (area C), and the losses are dominant in the low temperature region (area A). At low steam pressure, the losses in the high
temperature range (area $C$) prevail. Thus, there is an optimum steam pressure when the sum of areas $A + B + C$ is at a minimum, thus minimizing the exergy loss.

**Fig. 3.12** Effect of live steam pressure on energy utilization and overall energy loss

In Fig. 3.12(a), the temperature of the exhaust gases cools from $d$ to $a$ as the temperature of the water–steam rises from 4 to 1. The smallest temperature differences between the two streams occur at the "pinch points" $x$–5 and $d$–1, sometimes called "the temperature approach". Normally, the minimum allowable value of the temperature approach is 20 K ($^\circ$C) for reasons of satisfactory control, more commonly 30 K.

Figure 3.13 shows a schematic diagram of a dual pressure steam generator which is used to supply a two-stage steam turbine (HP and LP stages). There are two separate boilers, within the boiler casing, namely, the high pressure (HP) boiler which supplies the HP turbine, the exhaust steam from the HP stage of the turbine is then directed to the LP turbine. Steam from the low pressure (LP) boiler is directed to the LP stage of the turbine. Each boiler comprises the usual economiser, evaporator and superheater sections. Figure 3.13(c) is a repetition of Fig. 3.12(a) but is applied to a dual pressure steam cycle. By comparing Figs 3.12(a) and 3.13(c), it will be seen that the temperature difference between the gas side and steam side in the dual pressure cycle is considerably smaller than in the single pressure cycle. This gives the dual pressure cycle a considerable thermodynamic advantage since the smaller the temperature difference between the two streams for a given heat transfer rate, the smaller the degree of irreversibility external to the steam cycle, hence the greater the amount of work which may be able to be produced. However, the smaller the mean temperature difference, the larger the amount of heat transfer surface that would be required to effect the heat transfer rate.
3.6.5 Example of GT–ST Combined Cycle Plant

For simplicity, let us consider a modern gas turbine burning natural gas whose exhaust gases are used to raise steam, which is then used to drive a suitable turbine which drives an alternator. The relevant data for the gas turbine [6] is given in Table 3.1.

If the fuel gas has a calorific value of 48 MJ/kg, the fuel-to-electricity efficiency of the gas turbine is about 33%.

Figures 3.14.1(a) and 3.14.2(a) show the panoramic views of Doswell and Teesside combined cycle power plants, while Figs. 3.14.1(b) and 3.14.2(b) give the details of their respective steam generators (HRSG).
Fig. 3.13 Dual pressure steam cycle (a) HRSG, (b) H.P. and L.P. turbines, (c) T–q diagram, (d) T–s diagram

Table 3.1 Basic parameters for gas turbine of combined cycle plant

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Gas inlet temperature to turbine</td>
<td>1060 °C</td>
</tr>
<tr>
<td>2. Pressure ratio</td>
<td>10.7</td>
</tr>
<tr>
<td>3. Mass flow of air</td>
<td>495 kg/s</td>
</tr>
<tr>
<td>4. Mass flow of fuel/gas</td>
<td>9.2 kg/s</td>
</tr>
<tr>
<td>5. Gas outlet temperature from turbine</td>
<td>533 °C</td>
</tr>
<tr>
<td>6. Electrical power output</td>
<td>144 MW</td>
</tr>
</tbody>
</table>
**Single pressure cycle (Fig. 3.12)** Assuming $c_p$ of the exhaust gases from the gas turbine to be 1.114 kJ/kg K and that the gases are cooled to 107 °C (to give a sufficient working margin above dew point temperature), the allowable temperature drop of the exhaust gases is (553–107) or 446 K, and the rate of heat transfer from the exhaust gases to the steam generator.

$$Q_1 = w \cdot c_p \cdot (T_1 - T_2) = (495 + 9.2) \times 1.114 \times 446 = 251 \text{ kW}$$

Assuming a temperature approach (at pinch point) as 20 K, the temperature of the emerging steam at point 1 is thus limited to 553–20 = 533°C. The problem then is to estimate the largest pressure and mass flow rate of steam which gives a sufficiently large temperature approach at the pinch point $x - 5$ [Fig. 3.12(a)].

Now

$$w_s (h_1 - h_d) = w_g \cdot c_{pg} \cdot (T_d - T_a) \quad (3.21)$$

$$w_s (h_5 - h_d) = w_g \cdot c_{pg} \cdot (T_x - T_a) \quad (3.22)$$

By trial-and-error estimation from the above two Eq. (3.21) and (3.22) and from steam properties, we find $T_1 = 522$ °C. The feedwater temperature, $T_4 = 38$ °C, which shows that at a pressure as low as 5 bar the temperature approach, $T_x - T_5$, is less than 20 K and the corresponding mass flow rate of steam is 74 kg/s.

**Dual pressure cycle** The HP and LP steam (Fig.3.13) are to be calculated by trial-and-error as before, but with added complexity. The detailed treatment is given by Horlock [11]. Employing some regenerative feedheating, the steam conditions yielded are: 6.5 bar, 200 °C and a flow of about 13.5 kg/s for the LP boiler and 80 bar, 520 °C and a flow of about 66 kg/s for the HP boiler.

Assuming a condenser pressure of 0.068 bar, the specific enthalpy drops in the HP and LP turbines are 670 and 570 kJ/kg, respectively. Therefore, the steam turbine power output

$$W_{st} = (13.5 + 66) \times 670 + 66 \times 670$$

$$= 45.3 + 44.2 = 89.5 \text{ MW}$$

Thus we have approximately equal power output from both LP and HP stages and sufficient for an alternator output of 85 MW. Since the heat input to steam plant from the gas turbine exhaust gases has been shown to be 251 MW, the thermal efficiency of the steam cycle is thus 89.5/251 = 0.357 or 35.7%.

Since the power output of the gas turbine was about 144 MW, the output from the combined plant is about 144 + 85 = 229 MW electrical. If the calorific value of fuel is about 48 MJ/kg and the mass flow of fuel consumption is 9.2 kg/s, the fuel-to-electricity efficiency of the combined plant is 229/\((9.2 \times 48) = 0.496\text{ or }49.6\%.

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Fig. 3.14.1 (b)  Heat recovery steam generator of Doswell 663 MW c.c. power plant
Fig. 3.14.2 (b) Heat recovery steam generator of Teesside, 1875 MW c.c. power plant
3.6.6 Exergy Analysis of Combined Cycles

The exergy of a system is the maximum work obtainable as the system comes to equilibrium with the surroundings (Bejan, 1992). The higher the value of exergy, more is the work obtainable from the system. The First Law of Thermodynamics makes only an energy balance of a system or a control volume. It does not make any distinction of different forms of energy, particularly between work and heat, or heat (internal energy) available at different temperatures. It is the Second Law which asserts that from engineering viewpoint, all forms of energy are not of the same quality. Energies of two systems may be quantitatively equal, but qualitatively they may be different. When steam is adiabatically throttled, its energy does not change, but its quality degrades. Exergy is a measure of energy quality and exergetic (or second law) efficiency is a measure of the perfectness of a thermal system. While energy of a system in any process remains constant, a part of its exergy is always destroyed.

For dry solid fossil fuels the ratio \( \phi \) of chemical exergy \( \varepsilon_o \) to the net calorific value (NCV) \( (\text{NCV})_o \) with a mass ratio of oxygen to carbon \( o/c \) less than 0.667 is given by Kotas, 1985, p.267.

\[
\frac{\varepsilon_o}{(\text{NCV})_o} = \phi = 1.0437 + 0.1882 \left( \frac{h}{c} \right) + 0.0610 \left( \frac{o}{c} \right) + 0.0404 \left( \frac{n}{c} \right)
\]  

(3.23)

where \( c, h, o \) and \( n \) are the mass fractions of carbon, hydrogen, oxygen and nitrogen, respectively.

For solid fuels with the mass ratio \( 2.67 > o/c > 0.667 \),

\[
\phi = \frac{1.0438 + 0.1882 \left( \frac{h}{c} \right) - 0.2509 \left( 1 + 0.7256 \left( \frac{h}{c} \right) \right) + 0.0383 \left( \frac{n}{c} \right)}{1 - 0.3035 \left( \frac{o}{c} \right)}
\]

(3.24)

For liquid fuels the effect of sulphur was included in the correlation

\[
\phi = 1.0401 + 0.1728 \left( \frac{h}{c} \right) + 0.0432 \left( \frac{o}{c} \right) + 0.2169 \left( \frac{s}{c} \right) \left( 1 - 2.0628 \left( \frac{h}{c} \right) \right)
\]

(3.25)

For gaseous fuels of a given composition, the chemical exergy can be calculated from

\[
\varepsilon_o = - \Delta h_o + T_o \Delta s_o + RT_o \left[ x_{o_2} \ln \frac{p_{o_2}}{p_o} - \Sigma x_k \ln \frac{p_k}{p_o} \right]
\]

(3.26)

where the subscript \( k \) refers to the components of the products of composition. Typical values of \( \phi \) for some industrial fuels are given in Table 3.2.
Table 3.2 Typical values of $\phi$ for some industrial fuels

<table>
<thead>
<tr>
<th>Fuel</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Coke</td>
<td>1.05</td>
</tr>
<tr>
<td>2. Coal</td>
<td>1.06 – 1.10</td>
</tr>
<tr>
<td>3. Wood</td>
<td>1.15 – 1.30</td>
</tr>
<tr>
<td>4. Fuel oils and petrol</td>
<td>1.04 – 1.08</td>
</tr>
<tr>
<td>5. Natural gas</td>
<td>1.04 ± 0.5%</td>
</tr>
<tr>
<td>6. Coal gas</td>
<td>1.00 ± 1%</td>
</tr>
<tr>
<td>7. Blast furnace gas</td>
<td>0.98 ± 1%</td>
</tr>
<tr>
<td>8. Hydrogen</td>
<td>0.985</td>
</tr>
<tr>
<td>9. Carbon monoxide</td>
<td>0.973</td>
</tr>
</tbody>
</table>

Now, for a chemically reactive system

$\varepsilon_o = -\Delta g_o = \text{the change in the standard Gibbs function change (kJ/k mol)}$

and $\Delta h = -(\text{NCV})_o = \text{the change in the enthalpy of formation (kJ/k mol)}$. Now, from Eq. (3.23)

$$\phi = \frac{-\Delta G_o}{\Delta H_o}$$ (3.27)

and

$$\Delta H_o = -w_f \times (\text{NCV})_o \text{ (kW)}$$ (3.28)

where $w_f$ is the fuel burning rate (kg/s).

Therefore, $\Delta G_o$ can be estimated.

Again,

$\Delta G_o = \Delta H_o - T_o \Delta S_o$

$\therefore$ $T_o \Delta S_o = \text{rate of exergy loss in combustion or reaction}$

$$= -w_f \times (\text{NCV})_o + \phi [w_f \times (\text{NCV})_o]$$

$$= w_f \times (\text{NCV})_o (\phi - 1)$$ (3.29)

Various amounts of lost work or exergy loss due to irreversibility can be estimated for each component in the plant as given below. The combined cycle can be assumed as given in Fig. 3.15.

**Compressor:** Rate of energy dissipation or irreversibility in the compressor,

$$P_{\text{comp}} = w_a T_o (s_2 - s_1)$$

$$= w_a T_o \left[ c_p a \ln \frac{T_2}{T_1} - R_a \ln \frac{p_2}{p_1} \right]$$ (3.30)

where $\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/(\gamma \eta_c)}$, $\eta_c$ being the isentropic efficiency of the compressor.

**Combustor:** The rate of energy dissipation due to combustion,

$$I_{\text{comb}} = T_o [(S_p)_3 - (S_R)_2]$$ (3.31)
where \((S_R)_2 = (S_A)_2 + (S_P)_2\) and the subscripts \(P, R, A\) and \(F\) represent products, reactants, air and fuel, respectively.

\[
\therefore I_{\text{comb}} = T_o \left\{ \left[ (S_P)_3 - (S_P)_o \right] + \left[ (S_A)_2 - (S_A)_o + (S_P)_o + (S_A)_o \right] \right\}
\]

Now,

\[
\Delta S_o = (S_P)_o - [(S_F)_o + (S_A)_o]
\]

\[
I_{\text{comb}} = T_o \left\{ \left[ (S_P)_3 - (S_P)_o \right] - \left[ (S_A)_2 - (S_A)_o \right] + \Delta S_o \right\}
\]

\[
= T_o \left\{ w_g c_p \frac{T_3}{T_o} \ln \frac{T_3}{T_o} - w_g \frac{R_g}{p_o} \ln \frac{p_3}{p_o} \right\}
\]

\[
- w_a c_p \frac{T_2}{T_o} - w_a \frac{R_a}{p_o} \ln \frac{p_2}{p_o} + \Delta S_o \right\}
\]

(3.32)

where \(T_o \Delta S_o\) is given in Eq. (3.29).

**Gas turbine:** Rate of exergy loss in the gas turbine,

\[
I_{\text{GT}} = w_g T_o (s_4 - s_3)
\]

\[
= w_g T_o \left[ c_p \ln \frac{T_4}{T_3} - R_g \ln \frac{p_4}{p_3} \right]
\]

(3.33)

where \(\frac{T_4}{T_3} = \left( \frac{p_3}{p_4} \right)^{\frac{(y-1) \eta_T}{\gamma}}\) and \(\eta_T\) is the isentropic efficiency of the gas turbine.
**HRSG:** Rate of exergy loss in the heat recovery steam generator,

\[
I_{HRSG} = T_o \left[ w_s (s_a - s_e) + w_g (s_6 - s_d) \right]
\]

\[
= T_o \left[ w_s (s_a - s_e) + w_g \left( c_{pg} \ln \frac{T_6}{T_4} - R_g \ln \frac{p_6}{p_4} \right) \right]
\]

(3.34)

**Exhaust loss:** Rate of exergy loss due to exhaust flue gases.

\[
I_{exhaust} = \int_{T_6}^{T_e} \left( 1 - \frac{T_6}{T} \right) dQ
\]

\[
= w_g c_{pg} \left[ (T_6 - T_o) - T_o \ln \frac{T_6}{T_o} \right]
\]

(3.35)

**Steam turbine:** Rate of exergy loss in the steam turbine,

\[
I_{ST} = w_s T_o (s_b - s_a)
\]

(3.36)

**Condenser:** Rate of exergy loss in the condenser,

\[
I_{cond} = w_s \frac{T_o}{T_2} (s_c - s_b) + \frac{T_o}{T_2} Q_2 + Q_o
\]

(3.37)

where \( Q_2 \) is the rate of heat rejection from the condenser at temperature \( T_2 \), and \( Q_o \) is the rate of heat loss to the environment.

**Feed pump:** Rate of exergy loss in the feedpump,

\[
I_{FP} = w_s T_o (s_d - s_c)
\]

(3.38)

Rate of exergy losses in the feedwater heaters, cooling towers and so on can similarly be estimated. Total rate of exergy losses in the plant,

\[
\Sigma I = I_{comp} + I_{comb} + I_{GT} + I_{HRSG} + I_{exhaust}
\]

\[
+ I_{ST} + I_{cond} + I_{FP} + ...
\]

(3.39)

Total rate of exergy input = \( -\Delta G_o = -\Delta H_o \times \phi \)

Total rate of power generation = \( W_{GT} + W_{ST} \)

\[\therefore \] Total rate of exergy output (power output) + total rate of exergy loss = total rate of exergy input

Exergetic or second law efficiency

\[
= \frac{\text{total rate of exergy output}}{\text{total rate of exergy input}} = \frac{W_{GT}^p + W_{ST}^p}{\Delta G_o}
\]

(3.40)

The advantage of exergy analysis is that it provides the locations where the exergy losses or irreversibilities are more. Ways and means can then be explored, say, by thermodynamically optimizing the operating or geometrical parameters, so as to reduce these losses to the practical minimum values.
One example illustrating the method of using exergy analysis in such a plant has been given later (Example 3.4). It is seen that the maximum exergy loss occurs in the combustion of fuel.

3.6.7 Choice of Gas Turbine

Figure 3.16 shows that as the pressure ratio \( r_p \left( = \frac{p_2}{p_1} \right) \) increases the efficiency of the Brayton cycle increases, till the maximum or Carnot efficiency is reached, for which

\[
(r_p)_{\text{max}} = \left( \frac{T_{\text{max}}}{T_{\text{min}}} \right)^{y/(y-1)}
\]

where \( T_{\text{max}} \) is the maximum practicable inlet temperature fixed by the material and \( T_{\text{min}} \) is the ambient temperature. However, as \( r_p \) increases, the turbine exhaust temperature \( (T_4) \) decreases, which will in turn decrease the output and efficiency of the bottoming steam cycle.

Again, there is an optimum pressure ratio at which the net work output becomes a maximum (Nag, 1981), which is given by

\[
(r_p)_{\text{opt}} = \left( \frac{T_{\text{max}}}{T_{\text{min}}} \right)^{y/2(y-1)}
\]

and

\[
(W_{\text{net}})_{\text{max}} = c_p \left[ (T_{\text{max}})^{1/2} - (T_{\text{min}})^{1/2} \right]^2
\]

Due to economic considerations, present day gas turbines are generally optimized with respect to not efficiency but maximum power density. Fortunately, this optimum coincides fairly accurately with the optimum
efficiency of the combined cycle plant \( r_p = 10.5 \) to 12). As a result, most of today's gas turbines are optimally suited for combined cycle installations.

Gas turbines of a more complicated design, i.e. with intercooling in the compressor and reheating in the turbine, are less suitable for combined cycles. The gas turbine with the highest efficiency (with intercooling, reheating and regeneration) does not necessarily produce the best overall efficiency of the combined cycle plant. The turbine inlet temperature is a far more important factor.

Gas turbines are standardized. One given machine is used even for widely different ambient conditions. The exhaust heat available for the steam cycle is thus a given quantity and the problem lies in its maximum conversion to mechanical energy. The exhaust gases cannot be cooled down below a certain temperature, the sulphuric acid dewpoint, to avoid low temperature corrosion.

3.6.8 Choice of Steam Plant

The simplest arrangement of a CC plant is a single-pressure system without any special equipment added (Fig. 3.17). The Figure shows that the exhaust gases of a 70 MW gas turbine are used to generate 35 kg/s steam at 34 bar, 475°C. The steam then drives a steam turbine with an output of 34 MW. Because of the good river-water cooling system the pressure in the condenser is 0.04 bar, resulting in a gross efficiency of 45%. The rest of fuel energy is removed in the condenser (28.3%), in the stack (25.2%) or is lost elsewhere (1.5%). The main technical data of the plant (Kehlhoffer, 1991) are given below:

**Fig. 3.17 Combined cycle plant with single pressure steam cycle**
The temperature profiles and the heat absorption rates are plotted in Fig. 3.18.

![Temperature profiles](image)

**Fig. 3.18** Temperature profiles of exhaust gas and water-steam along with heat absorption rates

Figures 3.19 and 3.20 show the energy flow and exergy flow diagrams, respectively. The heat removed in the condenser is only about half that of a conventional steam power plant of the same size. The cooling water requirement is also halved.

A simplified flow diagram for a two-pressure steam cycle system for fuels containing sulphur is shown in Fig. 3.21. Two-pressure or dual cycle steam plant reduces the thermal irreversibility in heat transfer from exhaust gas to steam.
Fig. 3.19 Energy flow diagram of the combined cycle plant with GT and ST together converting 45% of the input energy ($Q_1$) into electricity

Fig. 3.20 Exergy flow diagram of a typical combined cycle power plant

In Fig. 3.19, the symbols used are being explained below:

$Q_1$ = energy input
$V_1$ = energy loss in condenser
$V_2$ = energy loss in stack
\( V_3 = \) energy loss due to radiation in HRSG
\( V_4 = \) energy loss in flue gas bypass
\( V_5 = \) energy loss in generator and radiation, gas turbine
\( V_6 = \) energy loss in generator and radiation, steam turbine

The symbols used in Fig. 3.20 are explained below:

\( E = \) exergy input
\( V_1 = \) exergy loss in condenser
\( V_2 = \) exergy loss in stack
\( V_3 = \) exergy loss in HRSG
\( V_4 = \) exergy loss in flue gas bypass
\( V_5 = \) exergy loss in gas turbine
\( V_6 = \) exergy loss in steam turbine
\( E_k = \) exergy supplied in the HRSG

### 3.6.9 Coal-based Combined Cycle Plants

Coal is a lower grade fuel compared to oil and natural gas, but reserves of it are very large and much effort has been devoted to developing clean coal technologies. Successful utilisation of coals for combined cycle power generation has necessitated the development of firing systems whose products of combustion have

**Fig. 3.21** A simplified flow diagram for a combined cycle plant with two-pressure steam cycle
(a) sufficiently low concentrations of particulates to reduce erosion and ensure a satisfactory life of the gas turbine.
(b) sufficiently low concentrations of pollutant gases and particulates in the exhaust to satisfy environmental legislation relating to discharges from power plants.

To reduce the concentration of particulates in products of combustion before entering the gas turbine, hot gas clean-up systems like multi-cyclones, ceramic filters and so on have been developed. For control of emission of oxides of sulphur and nitrogen, different techniques like low NO\textsubscript{X} burners, staged combustion, flue gas scrubbing, etc. are being put into use.

Following are the two dominant coal-based technologies.
1. Pressurized Fluidized Bed Combustion (PFBC) System, which may be either a bubbling fluidized bed or a circulating fluidized bed.
2. Integrated Gasification Combined Cycle (IGCC).

1. **PFBC-based combined cycle** Figure 3.22 shows a schematic of a system using a pressurized fluidized bed combustor (PFBC) to supply hot gas at elevated pressure to a gas turbine via a hot gas clean-up system. Coal and limestone are supplied to the pressurized combustor. The limestone used as the bed material absorbs sulphur (see Chapter 6). Cooling tubes immersed in the fluidized bed are used to generate steam which is supplied to the steam turbine. The combustion products leaving the combustor are passed through a clean-up system before being expanded in the gas turbine. The exhaust gases are then passed through a heat exchanger (economiser) which heats the feedwater before being discharged.

![PFBC combined cycle diagram](image-url)

**Fig. 3.22** A PFBC combined cycle
Some typical data of a PFBC-based combined cycle plant (Pillai, 1991) is

| Steam conditions | : 180 bar, 540 °C |
| Combustor exit temperature | : 850 °C |
| Heat transfer from GT exhaust to steam plant | : 10–15% of the total |
| Ratio of GT power to ST power | : ~ 0.25 |
| Net efficiency | : 42–44% |
| Combustor pressure | : 10–16 bar |

The exit temperature of the exhaust gases may be estimated from (Nag, 1995),

\[ T_c - T_d = \eta_T \frac{1}{T_c} \left[ 1 - \frac{1}{r^{(\gamma - 1)/\gamma}} \right] \tag{3.44} \]

where

- \( T_c \) = gas temperature at inlet to the gas turbine.
- \( T_d \) = gas exit temperature,
- \( \eta_T \) = isentropic efficiency of the gas expansion,
- \( r \) = pressure ratio of the expansion, and
- \( \gamma \) = ratio of specific heats of gas.

Taking \( \gamma = 1.3 \), \( \eta_T = 0.85 \) and \( T_c = 850 \, ^\circ\text{C} = 1123 \, \text{K} \); for \( r = 10 \), \( T_d = 705 \, \text{K} \) (432 °C); and for \( r = 16 \), \( T_d = 646 \, \text{K} \) (373 °C). Thus, as the pressure ratio is raised, there is less energy available in the exhaust gases from the gas turbine as a result of which the output of the steam cycle decreases. The gas turbine efficiency is, however, raised when the pressure ratio is increased.

The temperature in the PFBC is limited to about 850 °C because this is the most favourable temperature of sulphur retention and is below the ash fusion temperature of most coals.

2. **Integrated gasification combined cycle (IGCC)**

Coal is gasified, either partially or wholly, and the synthetic gas produced after clean-up is burnt in the combustion chamber of the gas turbine. It is called an integrated gasification combined cycle (IGCC). The main features of the cycle are shown in Fig. 3.23. Coal and limestone are fed to a pressure vessel, the coal being gasified by oxygen and steam. The ash and limestone form a molten slag which is discharged and the synthetic gas produced is cooled. The use of air instead of oxygen produces a gas of lower calorific value. The exhaust gases from the GT raise steam in the HRSG. Several different forms of gasifier have been developed, e.g. Texaco, Shell, Dow, Lurgi and so on. The Texaco process has been used in the Cool Water demonstration plant in California as given in Table 3.3 (Plumley, 1985).

The thermodynamic performance of an IGCC power plant studied by Nag and De (1998) shows that there is an optimum pressure ratio for the gas cycle at a given temperature ratio \( (T_f/T_0) \) for maximum overall cycle efficiency.
Fig. 3.23 Gasifier combined plant (IGCC)

Table 3.3 Cool Water IGCC Plant

<table>
<thead>
<tr>
<th>Design operating conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Gasifier pressure</td>
</tr>
<tr>
<td>Gasifier temperature</td>
</tr>
<tr>
<td>2. HCV of syngas</td>
</tr>
<tr>
<td>3. Gas turbine power output</td>
</tr>
<tr>
<td>4. Steam turbine inlet pressure</td>
</tr>
<tr>
<td>Steam turbine inlet temperature</td>
</tr>
<tr>
<td>Steam turbine output</td>
</tr>
<tr>
<td>5. Coal supply</td>
</tr>
<tr>
<td>6. Gross efficiency</td>
</tr>
<tr>
<td>7. Net output</td>
</tr>
<tr>
<td>8. Net heat rate</td>
</tr>
<tr>
<td>9. Net efficiency</td>
</tr>
</tbody>
</table>

The system shown in Fig. 3.24 is known as the British Coal "Topping Cycle" and it allows the gas turbine inlet temperature to be raised considerably higher than 850 °C by burning fuel gas produced by partial gasification of the coal. Char from the gasifier is burnt in the PFBC to raise hot gas and generate steam. A gas turbine inlet temperature of 1200 °C yields a significant increase in the efficiency of the combined cycle. The temperature of exhaust gases from the gas turbine can be estimated as before.

For \( T_c = 1200 °C = 1473 K \) and \( r = 16, \)

\[
1473 - T_d = 0.85 \times 1473 \left[ 1 - \left( \frac{1}{16} \right)^{0.25} \right] = 626
\]

or

\( T_d = 847 K \) or \( 574 °C. \)
Fig. 3.24 Outline of PFBC topping cycle (single shaft gas turbine)

Compared with 373 °C as found earlier, for \( T_c = 800 \) °C. The higher gas turbine exit temperature (574 °C) allows more heat recovery from the exhaust gases and hence greater heat supply to the steam cycle.

Figure 3.25 shows a circulating fluidized bed (CFB) being used for burning char and fines from the pressurized gasifier to generate steam. The low pressure turbine exhaust gases as well as the gases exiting the CFBC are used for raising steam in the clean gas boiler (Dawes et al., 1991).

Fig. 3.25 Outline of circulating fluidized bed combustion (CFBC) topping cycle

An interesting development of the bottoming cycle is the one as proposed by Kalina (1994) in which a mixture of ammonia and water is used as the working
fluid. When the binary aqua-ammonia mixture is heated, the ammonia begins to boil first, since its boiling point is lower. As it boils off, the concentration of ammonia in the mixture decreases and the boiling point of the mixture increases. This leads to a better match for the temperature profiles of the exhaust of the topping GT plant and the boiling binary mixture in the bottoming cycle, thus reducing thermal irreversibility. An exergy analysis of the Kalina cycle was made by Nag and Gupta (1998) to find the optimum ammonia concentration at the turbine inlet.

Using the latest technology, future 500 MWe IGCC stations could be designed to operate with an efficiency of 46% (LHV). This is the result of optimal integration of ASEA Brown Boveri (Sweden) type 13E gas turbine with the Shell Coal Gasification Process (USA) and an elevated pressure air separation unit (for oxygen) from Air Products (USA) (Demkolec, 1990).

Based on studies made by Westinghouse (USA), conceptual designs and planning for development of a direct coal-fired advanced combined cycle have been completed, which will burn unbenefticated low-cost utility-grade coal and meet environmental standards, using a pressurized (14 bar) slagging combustor, hot gas clean-up and a modified gas turbine exhausting into a HRSG (Bannister et al., 1992).

Just as combined cycles have already replaced steam cycles when noble fuels (i.e. natural gas or distillate oils) are utilized to generate electricity, they are likely to become predominant also when coal is the fuel because integrated gasification into a combined cycle yields higher operating efficiency as well as the most environmentally benign conversion of coal to electricity.

### 3.6.10 Steam Injection into Gas Turbine (STIG)

Steam injection into gas turbines for power augmentation represents a combined Brayton and Rankine cycle in which steam generated in the HRSG by the exhaust gases from the gas turbine is injected into the combustion chamber of the gas turbine (Fig. 3.26). Conceptually, it is equivalent to a combined cycle except that the steam is expanded together with the gas in the same turbine instead of a separate steam turbine.

![Diagram of Steam Injection into Gas Turbine](image)

**Fig. 3.26** Basic steam injection gas turbine (STIG)
One way to reduce NO$_x$ formation during combustion is by lowering the temperature of the flame, since the speed of reaction producing NO$_x$ is noticeably rapid only at very high temperatures. Injecting water or steam into the combustor can produce the temperature reduction desired. However, it has been observed that there is a drop in temperature of more than 2% with steam injection and almost 5% with water injection (Kehlhoffer, 1991). The status of steam injected gas turbines (STIG) has been discussed by Tuzson (1992). A recent EPRI (USA) study reports that there is no special economic advantage of STIG (Finckh and Pfost, 1992).

3.6.11 Repowering

The conversion of older power plants into combined cycle units—known as repowering—is one interesting way to continue using at least parts of older steam power plants which have become uneconomical. In this procedure, the boilers are normally replaced with high output modern gas turbines and waste heat boilers. Steam turbines of older power stations with relatively low steam inlet pressure and temperature are well adapted for combined cycle operation. These 20–25 years old steam turbines still have a considerable service life left, but their boilers are often ready for scrapping.

Figures 3.27 and 3.28 show the example of a conventional steam turbine plant, before and after such a conversion (repowering). It was possible to reuse the following components:

1. building
2. steam turbine and generator
3. condenser
4. main cooling system
5. main transformer
6. high voltage equipment

Fig. 3.27 Steam Power plant before conversion to a combined cycle plant
1. Steam generator  5. Condenser
2. Steam turbine   6. Feed pump
3. Feedwater tank, deaerator  7. Condensate pump
4. Low pressure preheater

The following components were dismantled:

1. boiler  2. piping and fittings
3. feedwater heaters  4. condensate pumps
5. boiler feed pumps  6. control equipment

The list may vary from one case to another. Feedwater preheating system for a CC plant is different in design from that for a conventional steam power plant because of thermodynamic considerations. In most cases, no extraction points are needed any more on the steam turbine (Fig. 3.28), which increases the power output from the turbine.

![Diagram of a combined-cycle plant with existing steam turbine](image)

**Fig. 3.28** Combined-cycle plant with existing steam turbine

2. Compressor  10. Steam bypass
3. Flue gas bypass  11. Feedwater tank/deaerator
4. Superheater  12. HP Feedwater pump
5. Evaporator  13. Condensate pump
6. Economizer  14. LP evaporator
7. Drum  15. LP feedwater pump
8. Steam turbine  16. LP drum

Table 3.4 shows the significant gain in efficiency that can be attained with repowering. The power output of the plant has actually been tripled. Since the design of the gas turbine is standardized, there are fixed limits on the amount of
steam that can be produced, and these do not necessarily lie close to the design values for the steam turbine. One advantage of supplementary firing in repowering is the possibility of closely adapting the steam condition according to the requirement of the steam turbine.

**Table 3.4** Comparison of steam power plant before and after repowering to a combined cycle plant

<table>
<thead>
<tr>
<th></th>
<th>Before repowering</th>
<th>After repowering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net output from power plant</td>
<td>30.3 MW</td>
<td>102.2 MW</td>
</tr>
<tr>
<td>Power output from ST</td>
<td>32.0 MW</td>
<td>35.0 MW</td>
</tr>
<tr>
<td>Power output from GT</td>
<td>-</td>
<td>68.4 MW</td>
</tr>
<tr>
<td>Station service power</td>
<td>1.7 MW</td>
<td>1.2 MW</td>
</tr>
<tr>
<td>Heat input</td>
<td>107 MW</td>
<td>228 MW</td>
</tr>
<tr>
<td>Net efficiency</td>
<td>28.3%</td>
<td>44.8%</td>
</tr>
</tbody>
</table>

**3.6.12 Combined Cycle Plants for Cogeneration**

Thermodynamic superiority of the combined cycle plant over a conventional power plant is even more pronounced in cogeneration plants than it is in plants used only to generate electricity. Whenever both electrical power and process steam are needed, it is thermodynamically and economically better to produce both products in a single plant.

Figure 3.29 shows a combined cycle cogeneration plant where a back pressure steam turbine has been used. Table 3.5 gives the relevant operating data for such a plant (Kehlhoffer, 1991).

1. Waste heat boiler
2. Gas turbine
3. Steam user
4. Feedwater tank deaerator
5. Steam reducing valve
6. Supplementary firing
7. Flue gas bypass
8. Backpressure steam turbine

![Diagram of the principle of a combined-cycle plant used for cogeneration](image)
The pressure of process steam and the power coefficient, defined as the ratio of electricity generated to the process heat supplied, are important for design. The higher is the pressure of process steam, the less is the electricity produced. If it is very high, the use of a steam turbine becomes questionable since the pressure difference is then too small. In that case, the steam process reduces to a waste heat boiler.

Supplementary firing makes it possible to lower the power coefficient. However, this capability is limited because lowering it too much would reduce the thermodynamic advantage of the combined cycle plant. Combined cycle plants should be employed only where the power coefficient is high.

**Table 3.5 Operating data of a combined cycle cogeneration plant**

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Natural gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas turbine power output</td>
<td>69.1 MW</td>
</tr>
<tr>
<td>Back pressure steam turbine output</td>
<td>44.7 MW</td>
</tr>
<tr>
<td>Station service power.</td>
<td>1.4 MW</td>
</tr>
<tr>
<td>Net power output of the plant</td>
<td>112.4 MW</td>
</tr>
<tr>
<td>Heat input to the gas turbine (LHV)</td>
<td>230.0 MW</td>
</tr>
<tr>
<td>Heat input in supplementary firing (LHV)</td>
<td>79.6 MW</td>
</tr>
<tr>
<td>Process steam flow</td>
<td>65.3 kg/s</td>
</tr>
<tr>
<td>Process steam pressure</td>
<td>3.5 bar</td>
</tr>
<tr>
<td>Thermal energy of process steam</td>
<td>152 MW</td>
</tr>
<tr>
<td>Rate of fuel utilization</td>
<td>85.4%</td>
</tr>
</tbody>
</table>
| Power coefficient \[
\frac{\text{electricity generated}}{\text{process heat supplied}}\] | 0.74        |
| Electrical yield                          | 36.8%       |
| Efficiency of power production            | 79.9%       |

An extraction or pass-but steam turbine offers considerable design and operating flexibility to achieve high power coefficients for combined cycle cogeneration plants for district heating, desalination of sea-water and so on.

### 3.6.13 Environmental Impact

Very efficient desulphurization and clean-up of the coal gas in the gas clean-up system in a combined cycle plant produce a very low environmental impact. Because of the low combustion temperature, only small amounts of NOx are formed. Due to the long residence time and low combustion temperature the amount of CO formed is not significant. The overall efficiency of a combined cycle plant is high and so the amount of CO2 discharged into the atmosphere per unit mass of fuel burnt is less, thus mitigating the green-house effect.
3.6.14 Scope of GT-ST Combined Cycle

Coal gasification combined cycle technology found niche markets in areas where coal is readily available. Several large scale IGCC plants are now being developed worldwide. Three similar PFBC facilities are now operating in Spain, Sweden and the USA (Collins, 1993). The Sweden plant commissioned in 1990 delivers 135 MW to the electric grid and 225 MW to the district heating system.

Of the 113 GW (i.e. 113,000 MW) new generating capacity to be installed in the USA during the 90s, it has been estimated that 51.9 GW will be combined cycle based facilities (Collins, 1993).

A significant number of more than 3500 utilities of 20 GW capacity in the USA completing 30 years service would be candidates for repowering, where the aging steam generator is likely to be replaced by a gas turbine/generator and an HRSG (Collins, 1993). The giant combined cycle power station at Seoinchin, South Korea, commissioned in 1993 and fuelled by natural gas has a total electric generating capacity of 1887 MW and an efficiency exceeding 55% which is the world's top overall efficiency (Power, 1993).

In India, natural gas-based combined cycle power plants under the NTPC are operating at (i) Auraiya (near Kanpur, 652 MW), (ii) Anta (Rajasthan, 430 MW), (iii) Kawas (M.P., 650 MW), (iv) Dadri (near Delhi, 817 MW) and (v) Gandhar (Surat, 650 MW).

The technology of combined cycle power generation:

(a) is well validated, even as a base-load power station,
(b) is more efficient compared to highly developed coal fired steam power plants,
(c) has low levels of pollutant emission,
(d) can be built quickly and relatively cheaply, and
(e) can be profitably employed for repowering an old steam power plant.

It bids fair to become the dominant technology for power production in the twenty-first century.

Example 3.1 A mercury cycle is superposed on the steam cycle operating between the boiler outlet condition at 40 bar, 400°C and the condenser temperature of 40°C. The heat released by mercury condensing at 0.2 bar is used to impart the latent heat of vaporization to the water in the steam cycle. Mercury enters the mercury turbine as saturated vapour at 10 bar. Compute (a) kg of mercury circulated per kg of water, and (b) the efficiency of the combined cycle.

The property values of saturated mercury are given below:

<table>
<thead>
<tr>
<th>p(bar)</th>
<th>t(°C)</th>
<th>h_f (kJ/kg)</th>
<th>h_g (kJ/kg)</th>
<th>s_f (kJ/kg K)</th>
<th>s_g (kJ/kg K)</th>
<th>v_f (m³/kg)</th>
<th>v_g (m³/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>515.5</td>
<td>72.23</td>
<td>363.0</td>
<td>0.1478</td>
<td>0.5167</td>
<td>80.9 × 10⁻⁶</td>
<td>0.0333</td>
</tr>
<tr>
<td>0.2</td>
<td>277.3</td>
<td>38.35</td>
<td>336.55</td>
<td>0.0967</td>
<td>0.6385</td>
<td>77.4 × 10⁻⁶</td>
<td>1.163</td>
</tr>
</tbody>
</table>
**Solution**  With reference to Fig. E3.1 for the steam cycle,

\[ h_1 = 3213.6 \text{ kJ/kg} \]
\[ s_1 = 6.7690 \text{ kJ/kg K} = s_2 = 0.5725 + x_2(8.2570 - 0.5725) \]
\[ x_2 = 0.8064 \]
\[ h_2 = 167.57 + 0.8064 \times 2406.7 = 2074.8 \text{ kJ/kg} \]
\[ h_3 = 167.57 \text{ kJ/kg} \]
\[ h_4 = 167.57 + 40 \times 100 \times 1.008 \times 10^{-3} = 171.6 \text{ kJ/kg} \]
\[ h_5 = 1087.31, \quad h_6 = 2801.4 \text{ kJ/kg} \]

For the mercury cycle,
\[ h_a = 363 \text{ kJ/kg} \]
\[ s_a = 0.5167 \text{ kJ/kg K} = s_b = 0.0967 + x_b (0.6385 - 0.0967) \]
\[ x_b = 0.7751 \]
\[ h_b = 38.35 + 0.7751 (336.55 - 38.35) = 269.48 \text{ kJ/kg} \]
\[ h_c = 38.35 = h_d \]

Let \( m \) = mass of mercury circulated per kg of steam. Then, from energy balance,

\[ m = \frac{h_6 - h_5}{h_b - h_c} = \frac{2801.4 - 1087.31}{269.48 - 38.35} = 7.4159 \text{ kg Hg/kg H}_2\text{O} \]

**Ans.** (a)

\[ Q_1 = m(h_a - h_d) + 1(h_1 - h_6) + 1(h_5 - h_4) \]
\[ = 7.4159(363 - 38.35) + (3213.6 - 2801.4) + (1087.31 - 171.6) \]
\[ = 3733.01 \text{ kJ/kg} \]
\[ Q_2 = h_2 - h_3 = 2074.8 - 167.57 = 1907.23 \text{ kJ/kg} \]

\[ h_{\text{combined cycle}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{1907.23}{3733.01} = 0.489 \quad \text{or,} \quad 48.9\% \quad \text{Ans. (b)} \]

**Example 3.2** In a cogeneration binary cycle, superheated steam enters the turbine with a mass flow rate of 5 kg/s at 40 bar, 440°C and expands isentropically to 1.5 bar. Half of the flow is extracted at 1.5 bar and used for industrial process heating. The remaining steam passes through a heat exchanger which serves as the boiler of the Refrigerant-12 cycle and the condenser for the steam cycle. The condensate leaves the heat exchanger as saturated liquid at 1 bar, where it is combined with the return flow from the industrial process at 60°C and 1 bar, before being pumped isentropically to the steam generator. The Refrigerant-12 cycle is an ideal Rankine cycle with refrigerant entering the turbine at 16 bar, 100°C and saturated liquid leaving the condenser at 9 bar. Determine (a) the rate of heat transfer in the steam generator, (b) the net power output of the binary cycle, (c) the rate of heat transfer to the industrial process.

**Solution** For the steam cycle (Fig. E3.2),

\[ h_1 = 3307.1 \text{ kJ/kg}, s_1 = 6.9041 \text{ kJ/kg k} \]

\[ h_2 = 2570.8 \text{ kJ/kg}, h_3 = 417.46 \text{ kJ/kg} \]

\[ T_6 = 60°C, \text{ compressed liquid} \]

\[ h_6 = h_f (60) + v_f (60) (p - p_{sat,60°C}) \]

\[ = 251.13 + (1.0172 \times 10^{-3}) (1 - 0.1994) \times 100 = 251.2 \text{ kJ/kg} \]

**Fig. E3.2**

Energy balance for adiabatic mixing of the streams 3 and 6 gives

\[ \dot{m}_3 h_3 + \dot{m}_6 h_6 = (\dot{m}_3 + \dot{m}_6) h_4 \]

where

\[ \dot{m}_3 = \dot{m}_6 = 2.5 \text{ kg/s} \]

\[ h_4 = \frac{2.5 \times 417.46 + 2.5 \times 251.2}{5} = 334.33 \text{ kJ/kg} \]
\[ T_4 = 80^\circ C, \ h_5 = h_4 + \nu_4 (p_5 - p_4) \]

or \( h_5 = 334.33 + (1.0291 \times 10^{-3}) (40 - 1) 100 = 338.34 \text{ kJ/kg} \)

For the R-12 cycle,

At 16 bar, 100 \(^\circ\)C, \( h_a = 241.58 \text{ kJ/kg} \)

\[ s_a = 0.7656 \text{ kJ/kg K} \]
\[ h_b = 229.43 \text{ kJ/kg} \]
\[ h_c = 71.93 \text{ kJ/kg} \]
\[ h_d = 71.93 + 0.7914 \times 10^{-3} (16 - 9) 100 = 72.48 \text{ kJ/kg} \]
\[ \dot{Q}_1 = \dot{m}_1 (h_1 - h_5) = 5(3307.1 - 338.34) = 14.844 \text{ kW} \]

**Ans. (a)**

\[ (W_{\text{net}})_{\text{steam}} = W_{T_1} - W_{P_1} = 5[(3307.1 - 2570.8) - (338.34 - 334.33)] = 3661 \text{ kW} \]

\[ \dot{m}_1 (h_2 - h_3) = \dot{m}_{R-12} (h_a - h_d) \]

\[ 2.5(2570.8 - 217.46) = \dot{m}_{R-12} (241.58 - 72.48) \]

\[ \therefore \dot{m}_{R-12} = 31.83 \text{ kg/s} \]

\[ (W_{\text{net}})_{R-12} = \dot{m}_{R-12} [(h_a - h_b) - (h_d - h_c)] \]

\[ = 31.83 [(241.58 - 229.43) - (72.48 - 71.93)] = 368.9 \text{ kW} \]

Total power output = 3661 + 368.9 = 4030 kW

**Ans. (b)**

For the industrial process,

\[ \dot{Q}_{li} = \dot{m}_{\text{process}} (h_2 - h_6) = 2.5 (2570.8 - 251.2) = 5799 \text{ kW} \]

**Ans. (c)**

**Example 3.3**

In a combined gas turbine-steam turbine power plant, the exhaust gas from the open cycle gas turbine is the supply gas to the steam generator of the steam cycle at which additional fuel is burned in the gas. The pressure ratio for the gas turbine is 7.5, the air inlet temperature is 15 \(^\circ\)C and the maximum cycle temperature is 750\(^\circ\)C.

Combustion of additional fuel raises the gas temperature to 750 \(^\circ\)C and the gas leaves the steam generator at 100\(^\circ\)C. The steam is supplied to the turbine at 50 bar, 600 \(^\circ\)C and the condenser pressure is 0.1 bar. The total power output of the plant is 200 MW. The calorific value of the fuel burned is 43.3 MJ/kg. Neglecting the effect of the mass flow rate of fuel on the air flow, determine (a) the flow rates of air and steam required, (b) the power outputs of the gas turbine and steam turbine, (c) the thermal efficiency of the combined plant, (d) the air fuel ratio. Take \( c_p = 1.11 \text{ kJ/kg K} \) and \( \gamma = 1.33 \) for combustion gases, and \( c_p = 1.005 \text{ kJ/kg K} \) and \( \gamma = 1.4 \) for air. Neglect pump work.

**Solution** For the gas turbine cycle (Fig. E3.3)

\[ \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} = (7.5)^{0.4/1.4} \]

\[ T_2 = 288 \times (7.5)^{2/7} = 512.165 \text{ K} \]
\[ T_4 = T_3 \left( \frac{p_4}{p_3} \right)^{\left(\frac{\gamma - 1}{\gamma}\right)} = \frac{1023}{(7.5)^{0.33/1.33}} = 620.52 \text{ K} \]

For the steam cycle, from Mollier chart,

\[ h_a = 3670, \ h_b = 2305, \ h_c = h_d = 192 \text{ kJ/kg} \]

\[ (W_{\text{net}})_{\text{GT}} = m_a \ c_{p_a} \ (T_3 - T_4) - m_a \ c_{p_a} \ (T_2 - T_1) \]

\[ = m_a \ 1.11 \ (1023 - 620.52) - m_a \ 1.005 \ (512.65 - 288) \]

\[ = 221.47 \ m_a \]

\[ (W_T)_{\text{ST}} = \dot{m}_s \ (h_a - h_b) = \dot{m}_s (3670 - 2305) = 1365 \ \dot{m}_s \]

\[ 1365 \ \dot{m}_s + 221.47 \ m_a = 200 \times 10^3 \]  \hspace{1cm} (1)

Now, \[ m_a \ c_{p_a} \ (T_5 - T_b) = \dot{m}_s \ (h_a - h_d) \]

\[ m_a \ 1.11 \ (1023 - 373) = \dot{m}_s \ (3670 - 192) \]

\[ \therefore \quad \frac{\dot{m}_s}{m_a} = \frac{715}{3478} = 0.2056 \] \hspace{1cm} (2)

Substituting in Eq. (1),

\[ 1365 \times 0.2056 \ m_a + 221.47 \ m_a = 200 \times 10^3 \]
\[ m_a = \frac{200 \times 10^3}{500.26} = 398.34 \text{ kg/s} \]
\[ m_s = 0.2056 \times 398.34 = 81.9 \text{ kg/s} \]  
\textit{Ans. (a)}

\[ (W_{\text{net}})_{\text{GT}} = 221.47 \times 398.34 \times 10^{-3} = 88.22 \text{ MW} \]
\[ (W_{\text{T}})_{\text{ST}} = 200 - 88.22 = 111.78 \text{ MW} \]  
\textit{Ans. (b)}

\[ Q_1 = m_a \ c_{p_s} \ (T_3 - T_2 + T_5 - T_4) \]
\[ = 398.34 \times 1.11 \times [1023 - 512.165 + 1023 - 620.52] \]
\[ = 400.19 \text{ MW} \]

\[ h_{\text{thermal}} = \frac{W_{\text{net}}}{Q_1} = \frac{200}{400.19} = 50\% \]  
\textit{Ans. (c)}

\[ Q_1 = m_f \times 43.3 \times 10^3 \text{ kJ/kg} \]

where \( m_f \) is the fuel burning rate.

\[ m_f \times 43.3 \times 10^3 = m_a \times 1.11 \times 913.31 \]

\[ \frac{m_a}{m_f} = \frac{43300}{1013.77} = 42.7 \]  
\textit{Ans. (d)}

\section*{Example 3.4}
The following data refer to a combined cycle power plant:

- Inlet condition of air to the compressor: 1 bar, 25 °C
- Pressure ratio of the compressor: 8
- Maximum gas temperature at inlet to the gas turbine: 900 °C
- Pressure drop in the combustion chamber: 3%
- Efficiency of the compressor: 0.88
- Efficiency of the gas turbine: 0.88
- Calorific value of liquid octane (C\(_8\)H\(_{18}\)) used as fuel: 44.43 MJ/kg
- Specific heat of air: 1.006 kJ/kg K
- Specific heat of gas: 1.148 kJ/kg K
- Specific heat ratio of gas: 1.333
- Specific heat ratio of air: 1.4
- Condition of steam at inlet to the steam turbine: 40 bar, 425 °C
- Condenser pressure: 0.04 bar
- Feed water temperature to the HRSG: 170.4 °C
- Efficiency of the steam turbine: 0.82
- Pressure drop of gas in the HRSG: 5 kPa
- Steam flow rate: 29.235 kg/s

Assume \( \psi = (\Delta G_0)/(\Delta H_0) = 1.0401 + 0.1728 \left( \frac{h}{c} \right) \)

where \( (h/c) \) is the mass ratio of hydrogen to carbon in the fuel.

(a) Determine the total power output and the overall efficiency of the combined cycle plant and estimate the "lost heat" coefficient \( x_L \). (b) Give an energy balance of the plant and estimate the exergetic efficiency.
**Solution**  Gas turbine plant:

\[ p_1 = 1 \text{ bar}, \quad p_2 = 8 \text{ bar}, \quad T_1 = 298 \text{ K}, \quad \eta_c = 0.88 \]

\[ \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/(\gamma \eta_c)} = 8^{0.4/(1.4 \cdot 0.88)} = 8^{0.325} = 1.965 \]

\[ \therefore \quad T_2 = 298 \times 1.965 = 586 \text{ K} = 313 ^\circ \text{C} \]

**Combustor:**

Pressure loss = 0.03 \times 8 = 0.24 \text{ bar}

\[ \therefore \quad p_3 = 8 - 0.24 = 7.76 \text{ bar} \]

Let the flow rate of combustion gas be 1 kg/s and that of fuel \( f \) kg/s.

\[ \therefore \text{ The flow of air} = (1 - f) \text{ kg/s} \]

Therefore,

\[ f \times CV = 1 \cdot c_p \left( T_3 - T_4 \right) - \left( 1 - f \right) c_p \left( T_2 - T_0 \right) \]

\[ f \times 44,430 = 1 \times 1.148 \left( 900 - 25 \right) - \left( 1 - f \right) \left( 1.006 \right) \left( 313 - 25 \right) \]

\[ = 1004.5 - 289.7 \left( 1 - f \right) \]

\[ f = \frac{714.8}{44140.7} = 0.0162 \text{ kg/s} \]

Air fuel ratio \[ \frac{1 - f}{f} = \frac{0.9838}{0.0162} = 60.73 \]
Now, $C_8H_{18} + 12.5O_2 = 8CO_2 + 9H_2O$

Air fuel ratio for stoichiometric combustion $= \frac{12.5 \times 32}{0.232 \times 114} = 15.12$

Excess air $= \frac{60.73 - 15.12}{15.12} = 3.02$ or $302\%$

**Gas turbine:** $p_4 = 1 + 0.05 = 1.05$ bar, $T_3 = 1173$ K

\[
\frac{T_3}{T_4} = \left( \frac{p_3}{p_4} \right)^{(\gamma-1)\eta_f/\gamma} = \left( \frac{7.76}{1.05} \right)^{(0.333 \times 0.88)/(1.333)}
\]

\[
= (7.39)^{0.22} = 1.553
\]

\[
T_4 = \frac{1173}{1.553} = 755 \text{ K } = 482 \text{ °C}
\]

This is the turbine exhaust gas temperature.

**HRSG:**

Let the pinch point temperature difference $(T_5 - T_f)$ be 30 °C,

\[
T_f = (T_{\text{sat}})_{40\text{ bar}} = 250.4 \text{ °C}
\]

\[
\therefore \quad T_5 = 250.4 + 30 = 280 \text{ °C}
\]

Now, from steam tables, $h_a = 3272$ kJ/kg and $h_f = 1087$ kJ/kg. By energy balance,

\[
w_g c_{p_g} (T_4 - T_5) = w_s (h_a - h_f)
\]

\[
1 \times 1.148(482 - 280) = w_s(3272 - 1087.31)
\]

\[
\therefore \quad w_s = 0.106 \text{ kg/s}
\]

The total heat transfer in the HRSG yields the stack temperature $T_6$.

At 170.4 °C, $h_c = 721.1$ kJ/kg

\[
1.148 (482 - T_6) = 0.106(3272 - 721)
\]

\[
\therefore \quad T_6 = 247 \text{ °C}
\]

**Power output:**

\[
h_a = 3272 \text{ kJ/kg}
\]

\[
s_a = 6.853 \text{ kJ/kg} = s_{bs} = 0.4226 + x_{bs} \times 8.052
\]

\[
x_{bs} = 0.7986
\]

\[
h_{bs} = 121.46 + 0.7986 \times 2432.9 = 2064.37
\]

Neglecting pump work, the steam turbine output is

\[
W_{ST} = w_s (h_a - h_{bs}) \eta_f = 29.235(3272 - 2064.37) \times 0.82 = 28,863 \text{ kW}
\]

Mass flow rate of gas in gas turbine

\[
w_g = \frac{29.235}{0.106} = 275.8 \text{ kg/s}
\]
Air flow rate entering the compressor

\[ w_s = (1 - f)275.8 \times 0.9838 \times 275.8 = 271.3 \text{ kg/s} \]

Power output from the gas turbine

\[ W_{GT} = 275.8 \times 1.148(900 - 482) - 271.3 \times 1.006(313 - 25) \]
\[ = 132,346.5 - 78,603.2 = 53,744 \text{ kW} \]

\[ \therefore \text{Total power output} = W_{ST} + W_{GT} = 28,863 + 53,744 = 82,607 \text{ kW} \quad \text{Ans.} \]

Fuel mass flow rate, \( w_f = 0.0162 \times 275.8 = 4.466 \text{ kg/s} \)

The overall efficiency of the combined plant

\[ \eta_o = \frac{82,607}{4,466 \times 44,430} = 0.4163, \text{ or } 41.63\% \quad \text{Ans.} \]

Efficiency of the steam plant

\[ \frac{h_a - h_b}{h_a - h_e} = \frac{(3272 - 2064) 0.82}{3272 - 721} \]
\[ = \frac{990.6}{2551} = 0.388 \]

The efficiency of the GT plant

\[ \frac{53,744}{4,466 \times 44,430} = 0.2709 \]

"Lost heat" coefficient in the exhaust stack

\[ x_L = \frac{275.8 \times 1.148 (247 - 25)}{4,466 \times 44,430} = 0.354 \]

From Eq. (3.8), the overall efficiency is

\[ \eta_o = \eta_1 + \eta_2 - \eta_1 \eta_2 - \eta_2 x_L \]
\[ = 0.271 + 0.388 - 0.271 \times 0.388 - 0.388 \times 0.354 \]
\[ = 0.659 - 0.105 - 0.137 = 0.417 \]

Thus, it is close to the value of 0.4163 obtained earlier.

**Exergy fluxes and irreversibilities:**

Given

\[ \psi = \frac{-\Delta G_o}{-\Delta H_o} = 1.0401 + 0.1728 \left( \frac{h}{c} \right) \]

where \( h/c \) is the mass ratio of hydrogen to carbon in the fuel.

For octane (C\(_8\)H\(_{18}\)), \( \psi = 1.0401 + 0.1729 \left( \frac{18 \times 1}{8 \times 12} \right) = 1.0725 \)

\[ \Delta H_o = w_f \times (CV)_o = 4.466 \times 44,430 = 198,424 \text{ kW} \]
\[ \Delta G_o = \psi \times \Delta H_o = 1.0725 \times 198,424 = 212,810 \text{ kW} \]

\[ \therefore T_o \Delta S_o = \Delta G_o - \Delta H_o = 212,810 - 198,424 = 14,386 \text{ kW} \]
Various amounts of lost work or exergy destruction due to irreversibility:

**Compressor:** Lost work or rate of energy dissipation or irreversibility

\[ I_c = w_a T_o (s_2 - s_1) \]

where \( s_2 - s_1 = c_{p_x} \ln \frac{T_2}{T_1} - R_a \ln \frac{P_2}{P_1} \) where \( R_a = \frac{c_p (\gamma - 1)}{\gamma} \)

\[ = 1.006 \ln \frac{313 + 273}{298} - \frac{0.4 \times 1.006}{1.4} \ln 8 \]

\[ = 0.6795 - 0.5977 = 0.0818 \text{ kJ/kg K} \]

\[ \therefore I_c = 271.3 \times 298 \times 0.0818 = 6613 \text{ kW} \]

**Combustor:** Rate of energy dissipation or lost work in the combustor is

\[ \dot{I}_{\text{comb}} = T_o [(\dot{S}_p)_3 - (\dot{S}_R)_2] \]

where \((\dot{S}_R)_2 = (\dot{S}_A)_2 + (\dot{S}_p)_o\) and the subscripts P, R, A and F represent products, reactants, air and fuel respectively.

or \[ \dot{I}_{\text{comb}} = T_o [(\dot{S}_p)_3 - (\dot{S}_p)_o + (\dot{S}_p)_o - (\dot{S}_A)_2 - (\dot{S}_A)_o + (\dot{S}_A)_o] \]

Now,

\[ \Delta \dot{S}_o = (\dot{S}_p)_o - [(\dot{S}_F)_o + (\dot{S}_A)_o] \]

\[ \therefore \dot{I}_{\text{comb}} = T_o [((\dot{S}_p)_3 - (\dot{S}_p)_o) + ((\dot{S}_A)_2 - (\dot{S}_A)_o) + \Delta \dot{S}_o] \]

\[ = T_o \left[ \left\{ w_g c_{p_g} \ln \frac{T_3}{T_2} - w_g R_g \ln \frac{P_3}{P_o} \right\} - \left\{ w_g c_{p_g} \ln \frac{T_2}{T_o} - w_g R_g \ln \frac{P_2}{P_o} \right\} + \Delta \dot{S}_o \right] \]

\[ = 275.8 \times 298 \left[ 1.148 \ln \frac{1173}{298} - \frac{1.148}{4} \ln 7.76 \right] \]

\[ - 271.4 \times 298 \left[ 1.006 \ln \frac{586}{298} - \frac{1.006}{3.5} \ln 8 \right] + 14.386 \]

\[ = 80,947 - 6672 + 14,386 = 88,661 \text{ kW} \]

**Gas turbine:** Rate of energy dissipation in the gas turbine or lost work,

\[ \dot{I}_{\text{GT}} = w_g T_o (s_4 - s_3) \]

where,

\[ s_4 - s_3 = c_{p_x} \ln \frac{T_4}{T_3} - R_g \ln \frac{P_4}{P_3} \]

\[ = -1.148 \ln (1.553) - \frac{1.148}{4} \ln \frac{1.05}{7.76} \]

\[ = -0.5053 + 0.5740 = 0.0687 \text{ kJ/kg K} \]

\[ \therefore \dot{I}_{\text{GT}} = 275.8 \times 298(0.0687) = 5646 \text{ kW} \]

**HRSG:** Rate of energy dissipation or lost work in the heat recovery steam generator,

\[ I_{\text{HRSG}} = T_o \Delta \dot{S} = T_o [w_s (s_a - s_c) + w_g (s_6 - s_4)] \]
where \[ s_a - s_e = 6.853 - 2.046 = 4.807 \text{ kJ/kg K} \]

\[ s_6 - s_4 = c_p \ln \frac{T_6}{T_4} - R_g \ln \frac{P_6}{P_4} \]

\[ = 1.148 \ln \frac{755}{520} + \frac{1.148}{4} \ln \frac{105}{100} \]

\[ = -0.428 + 0.014 = -0.414 \text{ kJ/kg K} \]

\[ \therefore \quad \hat{I}_{HRSG} = 298 \times 29.235 \times 4.807 - 275.8 \times 298 \times 0.414 \]

\[ = 41,879 - 34,026 = 7853 \text{ kW} \]

**Steam turbine**: Rate of energy dissipation in the steam turbine or lost work

\[ \hat{I}_{ST} = w_s (s_b - s_a)T_o \]

Now, \[ s_a = 6.853 \text{ kJ/kg K}, \quad h_a = 3272 \text{ kJ/kg}, \quad h_{bs} = 2064.37 \text{ kJ/kg} \]

\[ h_a - h_b = \eta_T (h_a - h_{bs}) = 0.82 \times 1207.63 = 990.26 \text{ kJ/kg} \]

\[ \therefore \quad h_b = 2281.74 \text{ kJ/kg} \]

\[ h_b = 121.46 + x_b \times 2432.9 = 2281.74 \text{ kJ/kg} \]

\[ \therefore \quad x_b = 0.89 \]

\[ \therefore \quad s_b = 0.4226 + 0.89 \times 8.052 = 7.589 \text{ kJ/kg K} \]

\[ \therefore \quad \hat{I}_{ST} = 29.235 (7.589 - 6.853) \times 298 = 6412 \text{ kW} \]

**Exhaust loss**: Rate of exergy loss due to exhaust flue gases,

\[ \hat{I}_{exh} = \int_{T_4}^{T_6} \left(1 - \frac{T_0}{T}\right) dQ = w_g c_p \left[ (T_6 - T_0) - T_0 \ln \frac{T_6}{T_0} \right] \]

\[ = 275.8 \times 1.148 \left[ (247 - 25) - 298 \ln \frac{520}{298} \right] = 17,760 \text{ kW} \]

**Exergy balance**:

<table>
<thead>
<tr>
<th>Exergy input (kW)</th>
<th>Power output (kW)</th>
<th>Exergy losses (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\Delta \dot{G}_o$ = 212,810</td>
<td>$\dot{W}^o_{ST}$ = 28,863</td>
<td>Compressor : 6613</td>
</tr>
<tr>
<td>[ \dot{W}^o_{GT} = 53,744 ]</td>
<td>[ \dot{W}^o_{ST} = 28,863 ]</td>
<td>Combustor : 88661</td>
</tr>
<tr>
<td>[ \dot{W}^o_{HRSG} ]</td>
<td><strong>Total output = 82607 kW</strong></td>
<td>Gas turbine : 5646</td>
</tr>
<tr>
<td><strong>Input = 212,810 kW</strong></td>
<td><strong>Total losses = 1,32,945 kW</strong></td>
<td>HRSG : 7853</td>
</tr>
<tr>
<td><strong>Steam turbine : 6412</strong></td>
<td><strong>Exhaust gases : 17760</strong></td>
<td>Gas turbine : 5646</td>
</tr>
</tbody>
</table>

Exergy output + exergy destruction = 82,607 + 132945 = 215,552 kW which is 1.3% greater than the exergy input.

Exergetic or second law efficiency = $\frac{82607}{212810} = 0.388$ or 38.8%
Example 3.5  For a mercury-steam-sulphur dioxide cycle, the heat rejected in the mercury cycle is given to the steam cycle and the heat rejected in the steam cycle is utilized in the SO₂ cycle. If the efficiencies of the mercury, steam and SO₂ cycles are 0.5, 0.4 and 0.25, respectively, find the overall efficiency of the composite cycle.

Solution  For 3 cycles coupled in series the overall efficiency of the combined cycle is given by

\[ \eta = 1 - (1 - \eta_1) (1 - \eta_2) (1 - \eta_3) \]
\[ = 1 - (1 - 0.50) (1 - 0.40) (1 - 0.25) \]
\[ = 1 - 0.5 \times 0.6 \times 0.75 \]
\[ = 0.775 \text{ or } 77.5\% \]

SHORT-ANSWER QUESTIONS

3.1 What are the flaws of steam as the working fluid in a power cycle?
3.2 Explain the characteristics of an ideal working fluid in a power cycle.
3.3 What are the advantages of liquid metals as the working fluid in a topping cycle?
3.4 What are the drawbacks of liquid metals as the working fluid in the low temperature region of power cycle?
3.5 Why was the mercury-steam cycle abandoned commercially?
3.6 What is a tertiary cycle? What are the probable working fluids in such a cycle?
3.7 Show that the overall efficiency of a two-fluid coupled cycle is equal to the sum of the individual efficiencies of the two cycles minus their product.
3.8 What are the merits of multifluid coupled cycles?
3.9 What are the various types of combined cycle plants? What are the inherent advantages of a such a plant?
3.10 Explain the drawbacks of a stationary gas turbine power plant for generation of electricity?
3.11 What are the advantages of a stationary gas turbine for a utility system?
3.12 What do you understand by a peaking unit? How does a gas turbine adapt to the need of a peaking unit?
3.13 When there is heat loss between two cyclic plants coupled in series, how is the overall efficiency of the plant affected?
3.14 Show that when two cyclic plants operate in parallel, the overall efficiency lies between the efficiencies of these plants.
3.15 Show that the overall efficiency of a series-parallel plant is less than that of two cycles coupled in series.
3.16 Derive an expression for the overall efficiency of series-parallel plants with two plants in series having supplementary heat supply and heat loss in between the two plants.
3.17 Discuss the effect of supplementary heat supply on the overall efficiency of a combined cycle plant.

3.18 What do you understand by limited supplementary firing and maximum supplementary firing in a combined cycle plant?

3.19 Discuss the advantages of combined cycle power generation. Why is it so important in the present day energy scenario?

3.20 Explain the dual-pressure steam cycle in a combined cycle plant. What is its thermodynamic advantage?

3.21 What is the need of an exergy analysis for a combined plant? Give briefly an exergy analysis of a simple GT-ST combined plant.

3.22 What do you understand by exergetic efficiency?

3.23 Show that the optimum pressure ratio of a gas turbine plant for maximum specific work is

\[(r_p)_{opt} = \left( \frac{\eta_T}{\eta_C} \frac{T_{max}}{T_{min}} \right)^{\gamma/(\gamma-1)}\]

3.24 Give the energy flow diagram (Sankey diagram) of a combined cycle plant.

3.25 Give the exergy flow diagram (Grassmann diagram) of a combined cycle plant. How does it differ from the energy flow diagram?

3.26 Explain the use of coal in a combined cycle plant.

3.27 What is integrated gasification combined cycle?

3.28 What is a gas-cleanup system? Why is it needed?

3.29 What is a PFBC? Explain its operation.

3.30 What is a PCFB? Explain its operation.

3.31 What is a STIG? Explain its merits and demerits.

3.32 What do you understand by repowering? Explain its importance.

3.33 Give an example with sketches of a conventional steam power plant before and after repowering.

3.34 How can a combined cycle plant be used for cogeneration? What is its thermodynamic advantage?

3.35 What is power coefficient? How does it affect the performance of a cogeneration plant?

3.36 What is the environmental impact of a combined cycle plant?

3.37 Discuss the scope of GT-ST combined cycle plant in the coming decades.

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**PROBLEMS**

3.1 A binary vapour cycle operates on mercury and steam. Saturated mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhausts at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar. (a) Find the overall efficiency of the cycle. (b) If 50,000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine? (c) Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow? (d) If the steam leaving the mercury condenser is
superheated to a temperature of 300 °C in a superheater located in the mercury boiler, and if the internal efficiencies of the mercury and steam turbines are 0.85 and 0.87, respectively, calculate the overall efficiency of the cycle. The properties of saturated mercury are given below:

<table>
<thead>
<tr>
<th>$p$(bar)</th>
<th>$t$(°C)</th>
<th>$h_f$</th>
<th>$h_g$</th>
<th>$s_f$</th>
<th>$s_g$</th>
<th>$v_f$</th>
<th>$v_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>450</td>
<td>62.93</td>
<td>355.98</td>
<td>0.1352</td>
<td>0.5397</td>
<td>79.9 × 10⁻⁶</td>
<td>0.068</td>
</tr>
<tr>
<td>0.04</td>
<td>216.9</td>
<td>29.98</td>
<td>329.85</td>
<td>0.0808</td>
<td>0.6925</td>
<td>76.5 × 10⁻⁶</td>
<td>5.178</td>
</tr>
</tbody>
</table>

[Ans. (a) 53%, (b) 59.35 × 10⁴ kg/h, (c) 28.5 MW, (d) 46.2%]

3.2 Show that the efficiency of Brayton cycle can be expressed as

$$\eta = \frac{(r - 1)}{r} \left( T_f \eta_T \eta_C - r \right)$$

where $\theta = \frac{T_3}{T_1} = \frac{T_{\text{max}}}{T_{\text{min}}}$, $r = (r_p)$ and $r_p$ = pressure ratio.

Also, show that the optimum pressure ratio for maximum work output is given by

$$(r_{p,\text{opt}}) = (\eta_T \eta_c \theta)^{\frac{1}{2(\gamma - 1)}}$$

3.3 In an electric generating station, using a binary vapour cycle with mercury in the upper cycle and steam in the lower, the ratio of mercury flow to steam flow is 10 : 1 on a mass basis. At an evaporation rate of 10,000,000 kg/h for the mercury, its specific enthalpy rises by 356 kJ/kg while passing through the boiler. Superheating steam in the boiler furnace adds 586 kJ to the steam specific enthalpy. Mercury gives up 251.2 kJ/kg during condensation, and steam gives up 2003 kJ/kg in its condenser. The overall boiler efficiency is 85%. The combined turbine mechanical and generator efficiencies are each 95% for the mercury and steam units. The steam auxiliaries need 5% of the energy generated by the units. Find the overall efficiency of the plant.

3.4 A sodium-mercury-steam cycle operates between 1000 °C and 40 °C. Sodium rejects heat at 670 °C to mercury. Mercury boils at 24.6 bar and rejects heat at 0.141 bar. Both sodium and mercury cycles are saturated. Steam is formed at 30 bar and is superheated in the sodium boiler to 350 °C. It rejects heat at 0.08 bar. Assume isentropic expansions, no heat losses, no regeneration and neglect pumping work. Find (a) the amounts of sodium and mercury used per kg of steam, (b) the heat added and rejected in the composite cycle per kg steam, (c) the total work done per kg steam, (d) the efficiency of the composite cycle, (e) the efficiency of the corresponding Carnot cycle, and (f) the work, heat added, and efficiency of a supercritical steam (single fluid) cycle operating at 250 bar and between the same temperature limits.

For mercury, at 24.6 bar, $h_g = 366.78$ kJ/kg, $s_g = 0.48$ kJ/kg K, and at 0.0141 bar, $s_f = 0.09$ and $s_g = 0.64$ kJ/kg K, $h_f = 36.01$ and $h_g = 330.77$ kJ/kg.
For sodium, at 1000 °C, \( h_g = 4982.53 \text{ kJ/kg} \)
At turbine exhaust, \( h = 3914.85 \text{ kJ/kg} \)
At 670 °C, \( h_f = 745.29 \text{ kJ/kg} \)
For a supercritical steam cycle, the specific enthalpy and entropy at the turbine inlet may be computed by linear extrapolation from the steam tables.

3.5 In a combined gas turbine-steam turbine plant, the exhaust gas from the GT being the supply gas to the steam generator at which a further supply of fuel is burned in the gas. The pressure ratio for the GT is 8, the inlet air temperature is 15 °C and the maximum cycle temperature is 800 °C. Combustion in the steam generator raises the inlet temperature of the exhaust gas from the GT to 800 °C and the gas leaves the steam generator at 100 °C. The condition of steam at ST inlet is 60 bar, 600 °C and the condenser pressure is 0.05 bar. Calculate the flow rates of air and steam required for the total power output of 190 MW and the overall efficiency of the combined plant. Assume ideal processes. What would be the overall air fuel ratio? Take \( c_p \) of combustion gases and air as 1.11 and 1.005 kJ/kg K, and \( \gamma \) for gases and air as 1.33 and 1.4, respectively. Neglect the effect of mass flow rate of fuel on the air flow. Take the calorific value of the fuel as 43.3 MJ/kg.

\[\text{Ans. 4.54, 0.54 and 39.6}\]

3.6 A combined power plant consisting of a closed cycle GT unit (Brayton cycle), using air as the working fluid and a ST unit (Rankine cycle) is to be designed such that the heat rejected at the GT unit is to be utilized to produce steam at the generator for the ST. The air leaving the generator is at 200 °C and it is cooled to the compressor inlet temperature by a second cooler which rejects the heat to waste.

Draw the flow and \( T-s \) diagram of the plant and calculate on the basis of ideal cycles, neglecting the feed pump work of the Rankine cycle, (a) the mass flow rate of steam per kg/s of air flow, (b) the total power output per kg/s of air flow, and (c) the overall efficiency of the plant. Take \( c_p \) of air as 1.005 kJ/kg K and \( \gamma = 1.4 \). The particulars of each cycle are as follows:

<table>
<thead>
<tr>
<th></th>
<th>( \text{GT cycle (Brayton)} )</th>
<th>( \text{ST cycle (Rankine)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (bar)</td>
<td>min 1</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>max 5</td>
<td>30</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>min 20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>max 830</td>
<td>300</td>
</tr>
</tbody>
</table>

\[\text{Ans. (a) 0.0762 kg/s, (b) 314.78 kJ/kg, (c) 49%}\]

3.7 A combined cycle power plant has a total power output of 300 MW. The gas turbine operates with a pressure ratio of 10, air inlet temperature of 30 °C and the maximum gas temperature of 1000 °C. There is the provision for supplementary firing in which the combustion of additional fuel raises the gas temperature to 900 °C. The exhaust gas from the GT flows to a HRSG from which the gas leaves at 110 °C. In the bottoming steam plant the steam is supplied to the turbine at 80 bar, 500 °C and the condenser pressure is 0.1 bar. The calorific value of fuel burned is 43.2 MJ/kg. Neglect the effect of
the mass flow rate of fuel on the air flow and take $c_p = 1.11$ kJ/kg K and $\gamma = 1.33$ for combustion gases and $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$ for air. Neglect pump work. Determine (a) the flow rates of air and steam required, (b) the power outputs of the gas turbine and steam turbine, (c) the overall efficiency of the combined plant, and (d) the air-fuel ratio. Draw the flow and T-s diagrams.

[Ans. (a) 442.62 kg/s, 121.056 kg/s, (b) 146.47 MW, 153.53 MW, (c) 53.5%, (d) 34.1]

**BIBLIOGRAPHY**

The primary fuels which are burned to release heat and generate steam in boilers are the fossil fuels in the form of coal, fuel oil and natural gas, which represent the remains of plant and animal life that are preserved in the sedimentary rocks. Besides these, industrial wastes like blast furnace gas, coke oven gas, refinery gas, sugar factory refuse (bagasse), saw mill wood dust, rice husk, etc. are also used as boiler fuels, often to boost one of the primary fossil fuels. When more than one type of fuel is simultaneously burned to meet the total heating requirement, the boiler is said to have a combination firing.

4.1 COAL

Coal is the principal energy source, particularly in India because of its large deposits and availability. Coal originated from vegetable matter which grew millions of years ago. Trees and plants falling into water decayed and later produced peat bogs. Huge geological upheavals buried these bogs under layers of silt. Subterranean heat, soil pressure and movement of earth’s crust distilled off some of the bog’s moisture and hardened it to form brown coal or lignite. Continuing subterranean activity and metamorphosis produced higher grades of coal. According to geological order of formation, coal may be of the following types: (1) Peat, (2) Lignite, (3) Subbituminous, (4) Bituminous, (5) Subanthracite, and (6) Anthracite, with increasing percentages of carbon. After anthracite, graphite is formed. Anthracite contains more than 86% fixed carbon (in amorphous form) and less volatile matter. Volatile matter helps in the ignition of coal. So, it is often difficult to burn anthracite. Bituminous coal is the largest group containing 46–86% of fixed carbon and 20–40% of volatile matter. It can be low-volatile, medium-volatile and high-volatile. The lower the volatility, the higher the heating value. Lignite is the lowest grade of coal containing moisture as high as 30% and high volatile matter. According to ASTM (American Society of Testing and Materials), peat is not regarded as a rank of coal. Peat contains up to 90% moisture and is not attractive as a utility fuel. Rank carries the meaning
of degree of maturation (carbonisation) and is a measure of carbon content in coal. Lignite is considered to be low rank and anthracite to be high rank.

4.2 COAL ANALYSIS

There are two types of coal analysis: proximate and ultimate, both done on a mass per cent basis. Both these types may be based on: (a) as-received basis, useful for combustion calculations, (b) dry or moisture free basis, (c) dry mineral-matter-free or combustible basis.

4.2.1 Proximate Analysis

The proximate analysis indicates the behaviour of coal when it is heated. When 1 g sample of coal is subjected to a temperature of about 105 °C for a period of 1 hour, the loss in weight of the sample gives the moisture content of the coal.

When 1 g sample of coal is placed in a covered platinum crucible and heated to 950 °C and maintained at that temperature for about 7 min, there is a loss in weight due to the elimination of moisture and volatile matter. The latter may now be determined since moisture has been calculated from the previous test. Volatile matter consists of hydrogen and certain hydrogen–carbon compounds which can be removed from the coal simply by heating it.

By subjecting 1 g sample of coal in an uncovered crucible to a temperature of about 720 °C until the coal is completely burned, a constant weight is reached, which indicates that there is only ash remaining in the crucible. Complete combustion of coal is determined by repeated weighing of the sample.

Fixed carbon is the difference between 100% and the sum of the percentages of moisture, ash and volatile matter. However, this difference does not represent all the carbon that was in the coal. Some of the carbon may have been in the form of hydrocarbons which may have been distilled off while determining the volatile matter. It is also possible that some of this fixed carbon may include sulphur, nitrogen and oxygen. So, the proximate analysis of coal gives

\[
\text{FC} + \text{VM} + \text{M} + \text{A} = 100\% \text{ by mass} \quad (4.1)
\]

The amount of VM indicates whether the coal will burn with a short or long flame and whether it will tend to produce smoke. The more volatile the coal, the more it will smoke.

Figure 4.1 shows the trend in moisture, volatile matter and fixed carbon when expressed on a dry ash-free basis. The general trend with increasing rank is an increase in the heating value and fixed carbon and a corresponding decrease in moisture and VM. This trend is so pronounced that a classification system based on the fuel ratio (ratio of fixed carbon to volatile matter) has been used as a rough indicator of a coal’s rank.

Lower rank coals (lower fuel ratio) are characterised by a greater oxygen content, that aids ignition and enhances combustibility and flame stability. High combustibility improves carbon burnout (reduces carbon carryover) and hence
Coal rank compared with proximate analysis

Boiler efficiency and for pulverized coal-fired units, this allows the coal to be ground to a coarser size. Low rank coals (high moisture content) produce a "self-pulverization" of the coal particles during combustion. As the inherent moisture in the pore structure of the coal is heated and expands rapidly, its volume increases (as water flashes to steam at atmospheric pressure, the volume expansion is 1600 to 1), thus fragmenting the coal particles. This exposes more surface area for combustion.

4.2.2 Ultimate Analysis

The ultimate analysis gives the chemical elements that comprise the coal substance, together with ash and moisture. The coal substance consists of organic compounds of carbon, hydrogen, and oxygen derived from the original vegetable matter. The analysis shows the following components on mass basis: carbon (C), hydrogen (H), oxygen (O), nitrogen (N), sulphur (S), moisture (M) and ash (A). Therefore,

\[ C + H + O + N + S + M + A = 100\% \text{ by mass} \]  \hspace{1cm} (4.2)

The dry and ash free analysis on combustible basis is obtained on dividing C, H, O, N and S by the fraction \( \left(1 - \frac{M + A}{100}\right) \).

4.2.3 Coal Properties

There are certain properties of coal which are important in power plant applications. They are swelling index, grindability, weatherability, sulphur content, heating value and ash softening temperature.

Swelling index Some types of coal during and after release of volatile matter become soft and pasty and form agglomerates. These are called caking
coal. In a fixed bed, such as a travelling grate stoker, the coal must not cake as it burns. The consequent agglomeration disturbs greatly the availability of air and so the coal does not completely burn yielding low combustion efficiency. Coal that does not cake is called free-burning coal. It breaks apart during combustion exposing large surface area to the air, thus enhancing the combustion process. Caking coals are used to produce coke by heating in a coke oven in the absence of air, with the volatile matter driven off. Coal devoid of volatile matter is called coke, which is largely needed in steel plants. A qualitative evaluation method, called the swelling index, has been devised to determine the extent of caking of a coal. A free-burning coal has a high value of swelling index, which indicates that it somewhat expands in volume during combustion. When modern pulverized coal burners are used, the swelling property of coal is, however, of less importance.

**Grindability** Grindability is often an important criterion for selecting a coal. This property of coal is measured by the standard grindability index, which is inversely proportional to the power required to grind the coal to a specified particle size for burning. Grindability of a standard coal is defined as 100. If the coal selected for use at a power plant has a grindability index of 50, it would require twice the grinding power of the standard coal to produce a specified particle size.

**Weatherability** It is a measure of how well coal can be stockpiled for long periods of time without crumbling to pieces. Modern power plants normally stockpile 60 to 90 days’ supply of coal in a large pile near the power plant. The coal unloaded from wagons is packed in a long trapezoidal pile. Excessive crumbling or weathering of the coal due to climatic conditions may result in small particles of coal which can be dispersed by wind or rain.

**Sulphur content** Sulphur content in coal is combustible and generates some energy by its oxidation to SO₂. Sulphur dioxide is a major source of atmospheric pollution. Their is an environmental regulation on SO₂ emission. The operating cost of SO₂ removal equipment need be considered while selecting a coal with high sulphur content.

**Heating value** The heating value or calorific value of coal is a property of fundamental importance. It may be determined on as-received, dry, or dry-and-ash-free basis. It is the heat transferred when the products of complete combustion of a sample of coal (or other fuel) are cooled to the initial temperature of air and fuel. It is normally determined in a standard test in a bomb calorimeter, where a coal sample of known mass is burnt with pure oxygen supply completely in a stainless steel bomb or vessel surrounded by a known mass of water, and the rise in water temperature is noted. Two different heating values are cited for coal. The higher heating value (HHV) assumes that the water vapour in the products condenses and thus includes the latent heat of vaporization of the water vapour formed by combustion. The lower heating value (LHV) assumes that the water vapour formed by combustion leaves as vapour itself. Therefore,
LHV = HHV - \textit{m}_w \ h_{fg} \tag{4.3}

where \textit{m}_w is the mass of water vapour formed given by
\begin{equation}
\textit{m}_w = M + 9H + \gamma_A \ W_A \tag{4.4}
\end{equation}

where \(M\) and \(H\) are the mass fractions of moisture and hydrogen in the coal, \(\gamma_A\) is the specific humidity of atmospheric air and \(W_A\) is the actual amount of air supplied per kg of coal. For energy balance and efficiency calculations of steam generators, HHV of fuel is considered in the USA, whereas LHV is the standard used in European practice.

If the ultimate analysis is known, the HHV of anthracite and bituminous coals can be determined approximately by using Dulong and Petit formula as given below:
\begin{equation}
\text{HHV} = 33.83 \ C + 144.45 \left( H - \frac{0}{8} \right) + 9.38 \ S, \text{ in MJ/kg} \tag{4.5}
\end{equation}

where \(C\), \(H\), \(O\) and \(S\) are mass fractions of carbon, hydrogen, oxygen and sulphur in coal. Assuming the latent heat of vaporization \(h_{fg}\) at the partial pressure of water vapour in the combustion products as 2.395 MJ/kg, the lower heating value of coal from Eq. (4.3) is given by
\begin{equation}
\text{LHV} = \text{HHV} - 2.395 \ \textit{m}_w, \text{ in MJ/kg} \tag{4.6}
\end{equation}

For lower-rank fuels, Eq. (4.5) usually underestimates the HHV.

**Ash softening temperature** The ash softening temperature is the temperature at which the ash softens and becomes plastic. This is somewhat below the melting point of ash. The design of the steam generator greatly depends on the ash softening temperature (a.s.t.) of the coal. If the furnace temperature is higher than the a.s.t., all the ash will melt and would come out of the furnace bottom continuously as molten slag. For a furnace that would discharge ash in the solid form, a high ash softening temperature would be required. A stoker furnace must use coal with a high a.s.t., otherwise clinkers would be formed. Clinkers, which are large masses of fused ash, cause troubles in discharge and also make combustion inefficient.

**Spontaneous combustion** Combustion (oxidation) of coal can take place rapidly as in a furnace or slowly on a stockpile. If it takes place slowly, there is a degradation or loss of energy content and hence in the value of fuel. The factors which influence spontaneous combustion and which can lead to a big fire, are the following.

(a) Rank of coal, low rank coals are more susceptible because of their higher porosity.
(b) Amount of surface area exposed to air.
(c) Ambient temperature, with high solar insolation aiding it.
(d) Oxygen content of coal.
(e) Free moisture in coal
(f) Configuration of the coal stockpile: steep conical piles with coarse coal at the edges and fines near the top are more susceptible because they
promote natural convection (chimney effect) and good air flow through the pile to support combustion as it develops.

To prevent spontaneous combustion, it is important to maintain a dry pile and compaction at regular intervals.

### 4.3 FUEL OIL

Petroleum is believed to have been formed during past geological ages from decayed marine life, both vegetable and animal. Dead marine animals and vegetable matter accumulated for millions of years ultimately got transformed into oil, mainly in sedimentary rocks, by pressure and heat. Oil deposits accumulated in the rocks and sands below the earth’s crust. Oil generally has a body of water below and pressurized natural gas above. Fairly thick and dense earth strata (caprock) cover most deposits preventing seepage (Fig. 4.1a). Oil wells drilled through this layer penetrate the deposits. The pressure forces the gas and oil to the surface. After the pressure has diminished, the oil must be pumped.

![Fig. 4.1(a) An oil deposit](image)

Liquid fuels are an excellent energy source. They are easy to handle, easy to store and easy to burn. They have nearly constant heating values. They are primarily a mixture of hydrocarbon compounds, which may also contain nitrogen, oxygen and sulphur. The bulk of the hydrocarbons belong to the paraffin series, $C_nH_{2n+2}$, like methane ($\text{CH}_4$), ethane ($\text{C}_2\text{H}_6$), propane ($\text{C}_3\text{H}_8$) and butane ($\text{C}_4\text{H}_{10}$) which are gaseous, and pentane ($\text{C}_5\text{H}_{12}$), hexane ($\text{C}_6\text{H}_{14}$) and octane ($\text{C}_8\text{H}_{18}$) which are liquid at STP. In addition, there can be isoparaffins, cycloparaffins and aromatic compounds. The proportion of various hydrocarbon groups varies widely with geographic location. The ultimate analysis of oil indicates the mass fractions of carbon, hydrogen, oxygen, nitrogen and sulphur. Regardless of the crude oil source, its composition is fairly uniform within close limits, as given below

- Carbon 83–87%, Hydrogen 11–16%,
- Oxygen + Nitrogen 0–7%, Sulphur 0–4%

There can also be some moisture and sediment.

Crude oil is seldom used as such. In the refining process it is distilled into a number of fractions. The lighter fractions (having lower boiling point) like
gasoline, aviation fuel, kerosene, light diesel oil, heavy diesel oil, lubrication oil and so on are principally transportation and machine fuels. The heavier fractions are used for boiler fuels and chemical production.

The required physical properties of fuel oil are specific gravity, viscosity, pour point, flash point and heating value. The pour point is the lowest temperature at which the oil will flow under standard pressure conditions. The flash point is the minimum temperature at which the oil may be ignited.

4.4 NATURAL AND PETROLEUM GAS

Natural gas was formed millions of years ago from decaying vegetable matter generally along with petroleum. Oil wells drilled into the geologic formation containing the gas and trapped oil release gas predominantly. In some fields, the natural gas is flared or burned at the wellhead because of lack of facilities to transport the gas. In regions close to the source, natural gas has been used as a power plant fuel.

Transportation of natural gas is made through pipelines. In India there is the HBJ (Hazira-Bijaipur-Jagdishpur) pipeline which transports natural gas from Hazira in Gujarat to Jagdishpur in U.P. Natural gas is the cleanest of all fossil fuels. It is free from ash and mixes well with air to undergo complete combustion producing very little smoke. It consists of a mixture of the most volatile paraffins-methane to pentane. It has a high hydrogen content and produces a considerable amount of water vapour when burned. The heat of combustion varies from 33.5 to 40 MJ/m$^3$. The specific gravity is 0.63 relative to air.

Since the major constituent of all natural gases is methane (critical temperature – 83 °C), cryogenic temperatures are required to maintain the gas as a liquid at moderate pressures (e.g. – 100 °C at 36 bar). Liquid natural gas (LNG) is transported by special tankers and stored in spherical pressure vessels to be used when needed, particularly during peak load.

Compressed natural gas (CNG) is now being considered as the alternative fuel for automobiles. Liquid petroleum gas (LPG) refers to hydrocarbons, such as propane, propylene, butane, butylene and so on, which are liquefied under moderate pressures and at normal temperatures. It is used widely as domestic fuel and also to supplement natural gas flow.

4.5 EMULSION FIRING

A suspension of a finely divided fluid in another is called an emulsion. An emulsion of water in heavy oil has been tried for boiler firing. When atomized, the drops of such an emulsion undergo microexplosions of the entrained water as they enter the hot combustion chamber. This causes additional atomization and enhances fuel surface-to-volume ratio. This promotes combustion, reduces carbon loss by way of soot etc. It also reduces excess air requirement and improves combustion efficiency. It can help conserve fuel oil.
4.6 **COAL–OIL AND COAL–WATER MIXTURES**

Liquid fuels using mixtures of fine coal in oil have been known as colloidal fuel, coal–in–oil and more recently, coal–oil mixtures (COM). This technology has been of interest since the late nineteenth century and tried on a limited scale since 1930, with promising results. In the fifties and sixties, oil and natural gas became progressively cheaper so that economic balance was shifted decidedly in favour of oil and gas–fired plants. After 1973, the price of oil and gas rose sufficiently and the interest in the use of COM was revived again. The main aspects of its preparation and burning are: (1) Grinding of coal to the desired size, (2) Mixing of coal in oil, (3) Flow of COM in pipes, (4) Stability of the colloidal fuel in storage, (5) Combustion of the colloidal fuel, and (6) Economics of its manufacture and use. For coal weight concentrations up to 40% the colloidal mixture obeys the fundamental law for viscous fluid flow in laminar range. It may be transported in pipes without clogging and fed through oil burner nozzles for combustion. For stabilizing the mixture, aluminium stearate and spent alkylation acid could be used (Lord et al., 1982). It has been found that combustion of colloidal fuels up to 40% coal concentration with 99% of the coal powder passing through a 230 mesh (61 µm) was better than, or equal to that of coal or oil alone.

**Advantages of COM**

1. It is cheaper than oil.
2. It can be handled like oil in pipes and by pumps.
3. It has a combustion efficiency that equals or exceeds that of oil.
4. It enables better utilization of storage space than does coal.
5. It assists in conservation of petroleum resources.
6. It eliminates the danger of spontaneous combustion in stored coal.
7. It is heavier than water, so fires are extinguished more easily.
8. It has an ash and moisture content less than that of coal.
9. It better utilizes low-rank coals.
10. It can be burned in present oil-burning equipment with efficiencies exceeding 80%.

Coal–water mixtures (CWM) having greater ability than COM to replace oil are often preferred. The CWM contains 70–80% coal mass, the rest water, plus a fraction of a percent of a stabilizer. Fine coal is suspended in the water and with the injection of a suitable stabilizer it is agitated to form a uniform slurry. The slurry can be handled, stored, transported, and fired like oil. There is a reduction in the heating value, since the water evaporating absorbs the latent heat from the energy released by combustion. So, the steam generator may require to be de-rated.

4.7 **INDUSTRIAL WASTES AND BYPRODUCTS**

Industrial wastes which are combustible are receiving increased attention as fuels for steam generators. They, thus, serve the twin objective, one is to dispose of them and the other is to reduce the use of oil.

Gaseous byproducts are very attractive, particularly the refinery gas and the coke oven gas. Refinery gas is generated during the conversion of crude oil to gasoline and other refinery products. It has a high heating value and is often
blended with lower heating value gas byproducts from the refinery prior to combustion. Coke-oven gas is generated during the production of coke from raw coal in a coke oven where the volatile matter is distilled off and the coke-oven gas is separated from the volatile matter by cooling and extraction. It consists of about 50% hydrogen, 33% methane and the remaining 17% of various other gases (by volume). Its heating value varies from 14.2 to 21.3 MJ/m³.

Regenerator gas produced in refinery by catalytic-cracking processes has about 10% CO and high inert gas and solid contents available at a high temperature of more than 540 °C, and can be burned at the site. Blast furnace gas generated during reduction processes of iron oxide to iron in the blast furnace contains about 30% CO, but has a high dust loading. It requires cleaning prior to burning, otherwise, it would cause plugging of burners and fouling of heat transfer surfaces.

The liquid wastes include solvents, waste oil and oil sludges, oil-water emulsions, polymers, resins, chlorinated hydrocarbons, phenols, tars, combustible chemicals, greases and fats. The main disadvantage of such liquids is that they vary considerably in heating value, flash and fire points, viscosity and moisture content.

There are some solid wastes which are available as fuels, the notable ones being wood waste and sugar factory refuse. Wood waste is found in saw mills and in paper, pulp, furniture, plywood and similar industries. Most woods have about the same chemical composition having a proximate analysis: fixed carbon 25%, volatile matter 70–75%, and ash 0.5–5%, on a dry basis. They vary in density and moisture content. Their heating value ranges between 19.3 to 21.0 MJ/kg. To burn wood efficiently, it must be cut down to chip size to permit continuous feeding and should not have a moisture content exceeding 60 percent. Wood burns more cleanly than oil or coal and causes much less pollution. A few wood-burning power plants are operating in the USA.

Sugarcane waste, also called bagasse, is that portion of sugarcane which remains after the sugar juice is extracted. Consisting mainly of cellulose fibres, its proximate analysis is about 12.5% fixed carbon, 84% volatile matter and 3.5% ash, on a dry basis. It contains more than 50% moisture. To burn it, it is cut to short fibres and fines. Its heating value ranges between 8.4 to 9.77 MJ/kg. Sugar mills generate enough bagasse to meet the demand for cogeneration of both process steam and electricity. Numerous bagasse fired steam and electricity generating plants are in operation near sugar mills throughout the world for decades.

Solid wastes generated by industrial and domestic processes as garbage are often used as boiler fuel in incinerators, particularly in cities, for generation of electricity or process steam.

Apart from municipal wastes and refuse-derived fuels (RDF), another waste that has become popular in recent years is used tyres because of its high heating value (about 32 MJ/kg). Some power plants in the USA burn automobile and truck tyres as their sole fuel. Used tyres are also being burned at coal-fired stations as supplemental fuel similar to municipal refuse.
SYNTHETIC FUELS

Synthetic fuels, also called synfuels, are gaseous and liquid fuels produced largely from coal in an economical and environmentally acceptable manner. These fuels cannot only be used for steam generation in power plants but also for domestic, industrial and transportation purposes.

4.8.1 Coal Gasification

In the nineteenth century and the first part of the twentieth century there was widespread use of coal gas produced by destructive distillation of coal for illuminating and cooking purposes. On heating coal in absence of air, coal was carbonized to coke by removing its volatiles and using them as a byproduct gas. This gas was distributed in urban areas as town gas. The coke thus produced was burned in big beds with less than the stoichiometric quantity of air to yield the producer gas:

\[ 2\text{C} + (\text{O}_2 + 3.76\text{N}_2) \rightarrow 2\text{CO} + 3.76\text{N}_2 \]

Coke Air Producer gas

For each mole of oxygen in air there are 3.76 moles of nitrogen. When the bed was heated to a high temperature, the flow of air was replaced by a flow of steam and water gas was produced:

\[ \text{C} + \text{H}_2\text{O} \rightarrow \text{CO} + \text{H}_2 \]

Coke Steam Water gas

Water gas reaction is endothermic and the bed gets cooled. The steam flow is then replaced by air flow and exothermic partial combustion, Eq. (4.7), reheated the bed. Air flow is again replaced by steam flow. The alternate production of water gas and producer gas continued till the coke bed was exhausted.

Modern gasification techniques are based on the producer and water gas reactions. However, they are more efficient and the production processes are continuous.

The basic process involves several steps. The coal is first ground into a sand-like powder and then preheated and dried to reduce caking during conversion. For a coking coal a high-temperature pretreatment is often used to give the coal particles a thin coating of oxygen to prevent sticking. Figure 4.2 illustrates the basic steps in the gasification process.

Coal gasification results in three gas mixtures, classified according to their heating value. They are called low-heating value, medium-heating value, and high-heating value gas. The process illustrated by the lower flow sheet (Fig. 4.2) yields a low-heating value gas (which is a mixture of water gas and producer gas), called synthesis gas. The process illustrated by the upper flow sheet yields a high-heating value gas, called pipeline gas, with properties close to that of natural gas.
Fig. 4.2 Coal gasification processes

1. The feedstock is reacted with air and steam. The air quantity is less than stoichiometric.
   \[ C + O_2 + 3.76 \text{N}_2 \rightarrow CO_2 + 3.76 \text{N}_2 \]  
   (4.9)
   \( CO_2 \) from this reaction reacts further with additional carbon in the rich mixture to give
   \[ C + CO_2 + 3.76 \text{N}_2 \rightarrow 2CO + 3.76 \text{N}_2 \]  
   (4.10)

   In steam,  \[ C + H_2\text{O} \rightarrow CO + H_2 \]  
   (4.11)

   The result is low-heating value syngas containing CO, H\(_2\), N\(_2\) and some CO\(_2\). It may also contain some CH\(_4\).

2. To obtain medium-heating value gas, a shift reaction is used to produce additional hydrogen, and nitrogen is removed.
   \[ CO + H_2\text{O} \rightarrow CO_2 + H_2 \]  
   (4.12)

3. To produce pipeline gas, a catalytic methanation is carried out in which the products of water gas reaction are reacted over a nickel catalyst at a temperature of about 1100°C and a pressure of 6.8 bar.
   \[ CO + 3H_2 \rightarrow CH_4 + H_2\text{O} \]  
   (4.13)

   The product gas is high-quality gas having a heating value of about 38 MJ/m\(^3\) and can directly substitute for natural gas.

   There is another method of classification, called hydrogasification in which fluidized coal is gasified directly with hydrogen-rich steam to a methane-rich gas that requires very little additional shifting. The overall reaction is of the form
   \[ \text{CH}_{0.8} + 0.55H_2\text{O} + 1.15H_2 \rightarrow 0.575\text{CH}_4 + 0.425 \text{CO}_2 \]  
   (4.14)

   The overall efficiency of conversion by this method is higher than the earlier method.
An attractive application of low-heating value gas in electric generation is as a fuel for a combined cycle power plant employing gas turbine and steam turbine as discussed in Chapter 3.

Notable commercial gasification systems are the Lurgi process, Winkler gasifier and Koppers-Totzek process. In the synthane process developed by US Bureau of Mines, dry crushed coal is fed from a pressurized hopper into a pretreater (Fig. 4.3). The coal is reacted with oxygen and steam at 400 °C, 7 MPa to devolatilize it and prevent caking. The coal mixture flows to the gasifier where it is partially gasified in a dense phase at 800 °C, and later at 1000°C, 7 MPa. Unreacted char and ash are removed from the bottom of the vessel. The synthesis gas passes through a cyclone separator which removes dust and tar. After further cleaning and sulphur removal, the gas can be used as a low-heating value gas. It can be further reacted through a catalytic methanation process to produce pipeline quality gas. Some other commercial gasifiers have been discussed in Chapter 5.

**Fig. 4.3 Synthane coal gasification process**

### 4.8.2 Underground Coal Gasification

Deposits of coal that could not be mined competitively by other processes might be economically utilized by underground gasification. Coal is gasified *in situ* and the gas produced is conveyed to the surface and utilized to meet various needs. *In situ* coal gasification became attractive due to the following advantages:
1. It can extract energy from inaccessible reserves of coal that cannot be mined by conventional techniques, e.g. steeply inclined seams.
2. When the mines become very deep, it becomes uneconomical to raise coal from such deep mines. In situ gasification can help raise energy from such abandoned mines.
3. It reduces mining personnel and equipment and needs less coal-handling and transportation facilities.
4. It is much safer and minimizes occupational hazards.
5. Fuel gas produced by in situ gasification is cheaper than other energy forms.
6. When coal seams are thin, a higher recovery of the coal via in situ gasification is possible.
7. From the environmental standpoint, hydrogen sulphide (H₂S) rather than SO₂ is the predominant form of sulphur produced in underground coal gasification. It can be economically treated by hot carbonate scrubbing (Lamb, 1977).

Two deep drilled holes are made, one for air and the other for product gas (Fig. 4.4). After initial ignition, combustion takes place at the bottom of the air hole and the combustion zone proceeds toward the product gas hole. In the combustion zone, carbon dioxide is formed in the reaction:

![Diagram of in-situ coal gasification]

**Fig. 4.4 In-situ coal gasification**

\[ C + O_2 \longrightarrow CO_2 \] 

(4.15)

Ahead of the combustion zone is a reduction zone where carbon monoxide is formed.

\[ CO_2 + C \longrightarrow 2CO \] 

(4.16)
From moisture in the coal, water gas reaction can take place

\[ C + H_2O \rightarrow CO + H_2 \]  \hspace{1cm} (4.17)

Volatile matter is also released from the coal as the process proceeds. The product gases are very similar to the coal gas or water gas, as described earlier. Thus the heating value is low, about 5.5 MJ/m³. The reaction is difficult to control. Considerable development work is required before this method is commercialized.

### 4.8.3 Coal Liquefaction

The growing shortage of world petroleum supplies and the rapidly increasing cost of the oil has revived interest in producing a liquid fuel from coal. Coal liquefaction technology was stimulated in both Germany and Japan by the World War-II. Japan produced aviation gasoline in a large plant in North Korea (it was under Japan during that time) which converted coke made from coal into calcium carbide in electric furnaces, then to acetylene, acetaldehyde, butyraldehyde, octanol, and finally octane. In a plant in Taiwan (also under Japanese occupation), the Japanese used starch from root vegetables which they fermented to butanol, which was then converted to butyraldehyde, octanol and octane. The most important German process was the *Fischer-Tropsch process*, which is still used commercially by the SASOL plant in South Africa. South Africa has no indigenous oil of its own but has considerable coal deposit. By coal liquefaction it meets all its oil requirement and is thus independent of foreign oil.

The conversion of coal into a liquid fuel requires the addition of hydrogen to the coal. Coal has a ratio of hydrogen atoms to carbon atoms of only 0.8 to 1, while in petroleum this ratio is 1.75 to 1. There are three basic modes that have been used to liquefy coal. These are: (1) hydrogenation, (2) catalytic conversion, and (3) hydropyrolysis.

In the *hydrogenation process*, coal and catalyst are suspended as a slurry, which is reacted with hydrogen at high pressure and moderate temperature to form liquid hydrocarbons. In the *catalytic conversion process*, a synthesis gas is produced from the coal as described earlier. The hydrogen and carbon monoxide in the gas are then combined in the presence of a catalyst to form a liquid hydrocarbon fuel. In *hydropyrolysis*, coal is heated beyond 450 °C, the fraction of coal volatilized greatly exceeds the volatile matter in coal. The hydrogen entrained pulverized coal is flash pyrolyzed. Up to 50% of the coal can thus be liquefied.

The Fischer–Tropsch process first produces a mixture of CO and H₂ from coal and steam. This is followed by catalytic reactions at about 150 °C and 150 bar, which yield a range of hydrocarbons from gaseous methane to higher liquid hydrocarbons. These are then separated with methane going as pipeline gas and the rest going to different liquid fuels.
4.9 BIOMASS

Biomass is organic matter produced by plants, both in land and water. It includes forest crops, the crops which are grown in energy farms, and animal manure. While fossil fuels take millions of years to form, the biomass is an alternate fuel, the source of which may be considered renewable, since plant life renews and adds to itself every year. It is the solar energy stored by way of photosynthesis,

\[
6\text{CO}_2 + 12\text{H}_2\text{O} \xrightarrow{\text{Sunlight}} \text{C}_6\text{H}_{12}\text{O}_6 + 6\text{H}_2\text{O} + 6\text{O}_2
\]

(glucose ejected by air or starch) transpiration

The products of photosynthesis (or biomass), also called biofuels, are bulky and contain large amounts of water. So, it is not economical to transport them over long distances. The stored energy in them must be utilized by direct burning in regions close to the source. The biomass can also be converted to a variety of gaseous, liquid or solid fuels—all biomass-derived and therefore all biofuels.

Land crops may be of the following types:

1. Sugar crops, such as sugarcane refuse or bagasse
2. Herbaceous crops, such as non-woody plants
3. Forest crops, such as cultured hybrid poplar, eucalyptus, sycamore, sweetgum, alder and other hard woods.

Animal and human wastes are an indirect land crop from which methane and ethylene can be produced while retaining the fertilizer value of the manure.

Aquatic crops grown in fresh, sea and brackish waters include seaweeds, marine algae and various kelps.

There are three forms of bioconversion routes:

1. Direct combustion, such as wood waste and bagasse
2. Thermochemical conversion
3. Biochemical conversion

Thermochemical conversion can take two routes, viz., gasification and liquefaction. Gasification is done by heating the biomass with limited oxygen to produce low heating value gas or by making the biomass react with steam and oxygen at a high pressure and temperature to produce medium heating value gas. The latter may also be subjected to liquefaction by converting it to methanol and ethanol.

Biochemical conversion can take two routes: anaerobic digestion and fermentation. Anaerobic digestion is the bacterial decomposition of organic matter (biomass) in the absence of air or oxygen to ultimately produce a gaseous mixture (biogas) of methane and carbon dioxide in a roughly 2 : 1 volume ratio. An anaerobe is a micro-organism that can live and grow without air or oxygen. It gets its oxygen by the decomposition of matter containing it.
The CO₂ gas produced by anaerobic digestion can be removed to produce synthetic natural gas (SNG) of pipeline quality, while the remaining sludge retains its nitrogen to produce a good fertilizer.

*Fermentation* is the breakdown of complex molecules in organic compounds with the help of a ferment such as yeast, bacteria, enzymes, etc. Grains and sugar crops are converted by fermentation into ethanol. Ethanol thus produced can be mixed with gasoline to produce gasohol (90% gasoline, 10% ethanol), which can be used as an automobile fuel.

In areas not suitable for food production, large rapidly growing trees are being cultivated for the production of biofuels. Sycamore and eucalyptus are such promising trees that yield up to 16 tonnes/acre per year. It is estimated that an *energy farm* of area about 900 km² could produce 400 MW of electricity. *Aquatic farms* growing algae, tropical grasses, floating kelp, water hyacinth and so on can yield several hundred tonnes/acre in a year. One interesting concept is to use the warm condenser cooling water from power plants to grow large quantities of algae, kelps etc. Research is in progress to increase the photosynthetic efficiency in the conversion of solar energy to biocrops from the present low value of about 5% to around 10%.

Table 4.1 shows the various bioconversion routes for the production of biofuels, and Table 4.2 gives the heating value of some waste bioproducts.

**Table 4.1 Bioconversion process and products**

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Process</th>
<th>Initial product</th>
<th>Final product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aqueous</td>
<td>1. Anaerobic digestion</td>
<td>Biogas—CH₄ : CO₂ = 2 : 1 (by vol)</td>
<td>1. Methane (38 MJ/m³)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(22.28 MJ/m³)</td>
<td>2. Ethanol (19 MJ/l)</td>
</tr>
<tr>
<td></td>
<td>2. Alcoholic fermentation</td>
<td></td>
<td>3. Oils (35–40 MJ/kg)</td>
</tr>
<tr>
<td></td>
<td>3. Chemical reduction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dry Thermochemical</td>
<td>Pyrolysis</td>
<td>Pyrolytic oils</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(23–30 MJ/kg)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gas (8–15 MJ/m³)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Char (19–32 MJ/kg)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hydrogasification</td>
<td>Methane (38 MJ/m³)</td>
<td>Methanol (16.9 MJ/l)</td>
</tr>
<tr>
<td></td>
<td>Low and medium energy gas</td>
<td>(7–15 MJ/m³)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ammonia, Electricity (3.6 MJ/kWh)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ethane (70.5 MJ/m³)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Char (19–32 MJ/kg)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Biophotolysis</td>
<td>Hydrogen (12.7 MJ/m³)</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.2  Heating value of some waste bioproducts

<table>
<thead>
<tr>
<th>Waste bioproducts</th>
<th>Heating value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood chips</td>
<td>18.6–20.9 MJ/kg dry weight</td>
</tr>
<tr>
<td>Sugarcane bagasse</td>
<td>9.5 MJ/kg</td>
</tr>
<tr>
<td>Cereal straw</td>
<td>16–17 MJ/kg</td>
</tr>
<tr>
<td>Organic refuse</td>
<td>13.2 MJ/kg</td>
</tr>
</tbody>
</table>

4.10 THERMODYNAMIC VIEW

The steam generator system comprises the fuel preparation and firing system, the steam generator itself and the air heaters. Figure 4.5 illustrates how a thermodynamic control volume can be drawn around this equipment. Heat released by the burning of fuel is transferred to the two steam circuits for main steam and reheat steam. Any energy remaining in the flue gas at the exit of the air heater is lost.

An important measure of performance for a power plant is its net plant heat rate (NPHR) defined as the amount of fuel energy or boiler heat input required to generate one kWh and deliver it to the transmission lines. For a coal-fueled steam electric generating unit, the net plant heat rate is given by

![Thermodynamic control volume drawn around equipment](image)

**Fig. 4.5** Thermodynamic control volume drawn around equipment
NPHR = \frac{Q_B}{(W_{net})_{plant}} = \frac{\omega_f \times HHV}{W_T - W_P - AP} \tag{4.19}

where \(Q_B\) = heat input to boiler from fuel (kW), \(\omega_f\) = fuel burning rate (kg/s), \(W_T\) = turbine output (kW), \(W_P\) = pump input (kW), \(AP\) = auxiliary power (kW), HHV = higher heating value (kJ/kg).

The boiler efficiency, \(\eta_B = \frac{\omega_s (h_1 - h_4)}{\omega_f \times HHV}\)

The turbine heat rate (NTHR) = \(\frac{\omega_s (h_1 - h_4)}{W_T - W_P} = \frac{1}{\eta_{cycle}}\)

\[\omega_f \times HHV = \frac{NTHR \times NTO}{\eta_B}\]

\[\text{Fig. 4.6} \quad \text{Example of steam generator surface arrangement}\]

where NTO = net turbine output = \(W_T - W_P\) and \((h_1 - h_4)\) enthalpy rise of water in the boiler.
\[
\text{NPHR} = \frac{\text{NTHR} \times \text{NTO}}{\eta_B \left[ 1 - \frac{\text{AP}}{\text{NTO}} \right]}
\]

Equation 4.20 may be used to estimate the net plant heat rate (kJ/kWh). The overall efficiency \(\eta_0\) of the plant is then

\[
\eta_0 = \frac{3600}{\text{NPHR}}
\]

For a typical net plant heat rate of approximately 10.542 MJ/kWh, the overall plant efficiency would be 34.13 per cent.

Heat transferred within the steam generator must meet three discrete turbine cycle requirements: heat must convert feedwater to steam, superheat this steam and reheat the steam returning from the h.p. turbine. Therefore, the design of a steam generator must focus on arranging appropriate heat transfer surfaces throughout the unit to ensure that the correct quantity of heat is transferred for each requirement. Waterwall surface within the furnace collects heat to boil the feedwater. Economizer surface located at the exit of the convective pass is used to preheat the feedwater prior to feeding it into the waterwalls. Superheater surface is normally located above the furnace and in the backpass. Suspended reheat surface is located in a similar manner; radiant reheat may also be located on front or side walls of the upper furnace. The actual location of surface varies depending on boiler manufacturer, type of fuel and steam pressure. Figure 4.6 illustrates a potential surface arrangement.

**4.11 COMBUSTION REACTIONS**

Combustion is the high temperature oxidation of the combustible elements of a fuel with heat release. The combustible elements in coal and fuel oil are carbon, hydrogen and sulphur. The basic chemical equations for complete combustion are

\[
\begin{align*}
\text{C} + \text{O}_2 & \rightarrow \text{CO}_2 \\
2\text{H}_2 + \text{O}_2 & \rightarrow 2\text{H}_2\text{O} \\
\text{S} + \text{O}_2 & \rightarrow \text{SO}_2
\end{align*}
\]

When insufficient oxygen is present, the carbon will be burned incompletely with the formation of carbon monoxide.

\[
2\text{C} + \text{O}_2 \rightarrow 2\text{CO}
\]

In order to burn a fuel completely, four basic conditions must be fulfilled:

1. Supply *enough* air for complete combustion of fuel.
2. Secure enough *turbulence* for thorough mixing of fuel and air.
3. Maintain a furnace *temperature* high enough to ignite the incoming fuel air mixture.
4. Provide a furnace volume large enough to allow *time* for combustion to be completed.

Apart from adequate air supply, the three T's, *viz.*, *time*, temperature and turbulence have to be kept in mind while designing a furnace. Combustion may
be said to hinge upon the word MATT—M is for mixture (turbulence), so that
the fuel molecules meet the oxygen molecules, A is for the proper air–fuel ratio
in order to support combustion, one T is for temperature, and the other T is for
time. Since the complete mixing of the fuel and air is virtually impossible,
excess air must be supplied to ensure complete combustion. The greater is the
rate of mixing or turbulence, the lower would be the excess air required.

### 4.11.1 Stoichiometric Air

The ultimate analysis of the fuel is given by

\[ C + H + O + N + S + M + A = 1.0 \]

Oxygen needed for the oxidation processes can be calculated as follows:

\[
\begin{align*}
C & \quad + \quad O_2 & = & \quad CO_2 \\
12 \text{ kg} & \quad + \quad 32 \text{ kg} & = & \quad 44 \text{ kg} \\
1 \text{ kg} & \quad + \quad 2.67 \text{ kg} & = & \quad 3.67 \text{ kg} \\
C \text{ kg} & \quad + \quad 2.67C \text{ kg} & = & \quad 3.67C \text{ kg} \\
2H_2 & \quad + \quad O_2 & = & \quad 2H_2O \\
4 \text{ kg} & \quad + \quad 32 \text{ kg} & = & \quad 36 \text{ kg} \\
1 \text{ kg} & \quad + \quad 8 \text{ kg} & = & \quad 9 \text{ kg} \\
H \text{ kg} & \quad + \quad 8H \text{ kg} & = & \quad 9H \text{ kg} \\
S & \quad + \quad O_2 & = & \quad SO_2 \\
32 \text{ kg} & \quad + \quad 32 \text{ kg} & = & \quad 64 \text{ kg} \\
1 \text{ kg} & \quad + \quad 2 \text{ kg} & = & \quad 2 \text{ kg} \\
S \text{ kg} & \quad + \quad S \text{ kg} & = & \quad 2S \text{ kg}
\end{align*}
\]

Oxygen required for complete combustion of 1 kg fuel is

\[ W_{O_2} = 2.67C + 8H + S - O \]

where O is the oxygen in the fuel.

Air contains 23.2% oxygen by mass. Therefore, theoretically air required for
complete combustion of 1 kg fuel is

\[
W_T = \frac{W_{O_2}}{0.232} = \frac{2.67}{0.232} C + \frac{8}{0.232} \left( H - \frac{0}{8} \right) + \frac{1}{0.232} S
\]

or,

\[ W_T = 11.5C + 34.5 \left( H - \frac{0}{8} \right) + 4.3S \quad (4.22) \]

where C, H, O and S are the mass fractions of carbon, hydrogen, oxygen and
sulphur in the fuel as given by the ultimate analysis.

As stated earlier, complete combustion of fuel cannot be achieved if only the
theoretical or stoichiometric air is supplied. Excess air is always needed for
complete combustion. It is expressed as a percentage or by the use of a dilution
coefficient. The per cent excess air supplied is
% excess air = \frac{W_A - W_T}{W_T} \times 100 \quad (4.23)

where \( W_A \) is the actual amount of air supplied for complete combustion of 1 kg fuel. The dilution coefficient, \( d \), is given by,

\[ d = \frac{W_A}{W_T} \quad (4.24) \]

The percentage of excess air varies between 15 and 30% for most large utility boilers.

In the combustion of methane,

\[ \text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} \quad \text{reactants} \rightarrow \text{products} \quad (4.25) \]

Atmospheric air contains 21% oxygen, 78% nitrogen, and 1% argon by volume. In combustion calculations, however, the argon is usually neglected, and air is assumed to consist of 21% oxygen and 79% nitrogen by volume (or molar basis). On a mass basis, air contains 23.2% oxygen and 76.8% nitrogen.

For each mole of oxygen taking part in a combustion reaction, there are 0.79/0.21 or 3.76 moles of nitrogen. So, for the combustion of methane, the reaction can be written as

\[ \text{CH}_4 + 2\text{O}_2 + 2(3.76)\text{N}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} + 7.52\text{N}_2 \quad (4.26) \]

With 150% theoretical air or 50% excess air, the methane combustion reaction can be written as

\[ \text{CH}_4 + 2(1.5)\text{O}_2 + 2(3.76) (1.5)\text{N}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} + \text{O}_2 + 11.28\text{N}_2 \quad (4.27) \]

With less than needed excess air there may be a small amount of CO present in the products, called the flue gases, depending on mixing and turbulence during combustion, e.g. with 15% excess air,

\[ \text{CH}_4 + 2(1.15)\text{O}_2 + 2(1.15) (3.76)\text{N}_2 \rightarrow 0.95\text{CO}_2 + 0.05\text{CO} + 2\text{H}_2\text{O} + 0.325\text{O}_2 + 8.65\text{N}_2 \quad (4.28) \]

### 4.11.2 Actual Air–Fuel Ratio

The actual amount of air supplied per kg fuel can be ascertained from the measured volumetric composition of combustion products. There are several methods to perform these measurements: (a) Orsat analyser, (b) Haldane apparatus, (c) Infra-red gas analyzer, and (d) Gas chromatograph. Of these, Orsat gas analyzer is very simple, portable and most frequently used. It measures the volume or mole fractions of \( \text{CO}_2 \), \( \text{CO} \) and \( \text{O}_2 \) in the dry flue gas.

Samples of flue gas are first collected by suction in sampling bottles. An Orsat analyzer contains three pipettes containing chemical solutions (Fig. 4.7). The reagents normally used are a KOH solution to absorb the \( \text{CO}_2 \) gas, pyrogallol solution to absorb the \( \text{O}_2 \) gas, and a cuprous chloride mixture (\( \text{CuCl}_2 \)) to absorb the CO gas. By opening the stop valve and lowering the levelling bottle which contains water, 100 cm\(^3\) of flue gas sample is collected over water
in the graduated measuring burette. The valve of pipette 1 is opened and the levelling bottle is raised, so that all the gas is passed into the pipette 1. The CO$_2$ content in the flue gas sample is absorbed in the KOH solution. The levelling bottle is lowered and the volume of gas is measured, say, it is 89 cm$^3$. That means 11% of the dry gas is CO$_2$ by volume. In the same manner, volumetric percentages of O$_2$ and CO in the flue gas are measured by opening successively the pipettes 2 and 3. The remaining unabsorbed gas is nitrogen. Since the sample is collected over water, any water vapour in the flue gas would have condensed during the collection process. The SO$_2$ gas will react with water in the container. So the flue gas analysis thus measured is on the dry basis:

$$\text{CO}_2 + \text{CO} + \text{O}_2 + \text{N}_2 = 100\%, \quad \text{by volume} \quad (4.29)$$

Let us assume that the dry flue gas (dfg) analysis of a gas sample is: 12% CO$_2$, 3% CO, 5% O$_2$ and 80% N$_2$. by volume. Therefore, 1 mole of dfg contains 0.12 mol of CO$_2$, 0.03 mol of CO, 0.05 mol of O$_2$ and 0.80 mol of N$_2$. The molecular weight of the dfg then becomes equal to $(0.12 \times 44 + 0.03 \times 28 + 0.05 \times 32 + 0.80 \times 28)$, or 30.12 kg/kg mol. Thus, in general, the molecular weight of dfg is given by

$$M_{\text{dfg}} = 44\text{CO}_2 + 28\text{CO} + 32\text{O}_2 + 28\text{N}_2 \quad (4.30)$$

where CO$_2$, CO, O$_2$ and N$_2$ are the volumetric percentages in dfg as measured. Therefore,
\[
\% \text{ CO}_2 \text{ by mass} = \frac{44 \text{ CO}_2}{44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2} \times 100
\]
\[
\% \text{ O}_2 \text{ by mass} = \frac{32 \text{ O}_2}{44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2} \times 100 \quad (4.31)
\]
\[
\% \text{ N}_2 \text{ by mass} = \frac{28 \text{ N}_2}{44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2} \times 100
\]
\[
\% \text{ CO} \text{ by mass} = \frac{28 \text{ CO}}{44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2} \times 100
\]

This is the gravimetric composition of the dry flue gas. From Eq. (4.21), it is seen that 44 kg of \text{ CO}_2 gas has 12 kg carbon in it. So, (44\text{ CO}_2) kg of \text{ CO}_2 has (12 \text{ CO}) kg of carbon. Similarly, from Eq. (4.20), (28 \text{ CO}) kg of \text{ CO} has (12 \text{ CO}) kg of carbon. So, 1 kg mol of dfg has 12 (\text{ CO}_2 + \text{ CO}) kg of carbon.

\[
\text{Mass of carbon per kg of dfg} = \frac{12(\text{ CO}_2 + \text{ CO})}{44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2} \quad (4.32)
\]

Let \( C_{ab} \) be the mass fraction of carbon \( C \) in the fuel which has been oxidized either to \text{ CO}_2 or \text{ CO}. Then \((C - C_{ab})\) is the mass fraction of unburnt carbon in the refuse. For 1 kg fuel burnt, there should be \( C_{ab} \) kg of carbon in the dfg. Therefore,

\[
\text{Mass of dfg produced per kg fuel} = \frac{C_{ab}(44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2)}{12(\text{ CO}_2 + \text{ CO})} \quad (4.33)
\]

From Eqs ((4.31) and (4.33).

Mass of \text{ N}_2 in dfg per kg fuel
\[
= \frac{C_{ab}(44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2)}{12(\text{ CO}_2 + \text{ CO})} \times \frac{28 \text{ N}_2}{44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2}
\]
\[
= \frac{28 \text{ N}_2 C_{ab}}{12(\text{ CO}_2 + \text{ CO})} \quad (4.34)
\]

This nitrogen in dfg comes from fuel as well as air. So, nitrogen coming with air per kg fuel = \( \frac{28 \text{ N}_2 C_{ab}}{12(\text{ CO}_2 + \text{ CO})} - N \), where \( N \) is the mass fraction of nitrogen in fuel. The actual amount of air supplied per kg fuel is
\[
W_A = \frac{28 \text{ N}_2 C_{ab}}{12(\text{ CO}_2 + \text{ CO})} \times \frac{1}{0.768} - \frac{1}{0.768} N
\]
Since $N$ in fuel is small,

$$W_A = \frac{3.04 N_2 C_{ab}}{\text{CO}_2 + \text{CO}}$$  \hspace{1cm} (4.35)

This is the actual air-fuel ratio used for combustion of the fuel, where $C_{ab}$ is the fraction of carbon in fuel which has been burnt to $\text{CO}_2$ and $\text{CO}$. If $W_A$ is measured by an air flow meter, the degree of burnout of carbon, $C_{ab}$, can be estimated from the relation

$$C_{ab} = \frac{(\text{CO}_2 + \text{CO})W_A}{3.04 N_2}$$  \hspace{1cm} (4.36)

### 4.11.3 Combustion Equation

Let us consider a coal having the following ultimate analysis:

- C – 60%, H – 4%, S – 3.2%, O – 4.8%, N – 2%, M – 5%, and A – 21%.

The exhaust gas has the following volumetric analysis:

$$\text{CO}_2 + \text{SO}_2 = 12\%, \text{CO} = 2\%, \text{O}_2 = 4\% \text{ and N}_2 = 82\%$$

Let $a$ moles of oxygen be supplied for 100 kg fuel. Then, the combustion equation can be written as

$$\frac{60}{12} \text{C} + \frac{4}{2} \text{H}_2 + \frac{3.2}{32} \text{S} + \frac{4.8}{32} \text{O}_2 + a \text{O}_2 + 3.76 a \text{N}_2 \rightarrow b \text{CO}_2 + d \text{CO} + e \text{SO}_2 + f \text{O}_2 + g \text{N}_2 + h \text{H}_2\text{O}$$

By equating the coefficients,

Carbon: $b + d = \frac{60}{12} = 5$ \hspace{1cm} Hydrogen: $h = 2$ \hspace{1cm} Sulphur: $e = \frac{3.2}{32} = 0.1$

Oxygen: $b + \frac{d}{2} + e + f + \frac{h}{2} = \frac{4.8}{32} + a$ \hspace{1cm} Nitrogen: $g = 3.76 a$

From the dfg analysis,

$$\frac{b + e}{b + d + e + f + g} = 0.12 \hspace{1cm} \frac{d}{b + d + e + f + g} = 0.02$$

$$\frac{f}{b + d + e + f + g} = 0.04 \hspace{1cm} \frac{g}{b + d + e + f + g} = 0.82$$

By solving these equations, the coefficients $a$, $b$, $d$, $e$, $f$, $g$ and $h$ are determined and given to be $d = 0.73$, $b = 4.27$, $f = 1.46$, $a = 7.045$ and $g = 26.49$.

Actual air supplied per kg coal = $\frac{32 a}{0.232 \times 100} = 9.72$ kg

Let us now consider the combustion of propane gas ($\text{C}_3\text{H}_8$) with 80% theoretical air.

$$\text{C}_3\text{H}_8 + 5\text{O}_2 + 5(3.76)\text{N}_2 \rightarrow 3\text{CO}_2 + 4\text{H}_2\text{O} + 18.8\text{N}_2$$
With 80% theoretical air, the combustion equation becomes

\[ C_3H_8 + 5(0.8)O_2 + 5(3.76)0.8N_2 \rightarrow aCO + bCO_2 + 4H_2O + 15.04N_2 \]

Carbon balance: \(3 = a + b\)
Oxygen balance: \(8 = a + 2b + 4\)
By solving: \(a = 2, \ b = 1\)
The combustion equation can thus be written as:

\[ C_3H_8 + 4O_2 + 15.04N_2 \rightarrow 2CO + CO_2 + 4H_2O + 15.04N_2 \]

### 4.11.4 Dew Point Temperature

From the combustion equation, the mole fraction of water vapour \((x_{H_2O})\) formed can be estimated:

\[ x_{H_2O} = \frac{n_{H_2O}}{n_{total}} = \frac{n_{H_2O}}{n_{CO_2} + n_{CO} + n_{O_2} + n_{SO_2} + n_{N_2} + n_{H_2O}} \]  

(4.37)

where \(n\) represents the number of moles.

The partial pressure of water vapour in the mixture of gases constituting the flue gas is given by

\[ P_{H_2O} = x_{H_2O}P = x_{H_2O} \times 1 \text{ atm} = x_{H_2O} \text{ atm} \]  

(4.38)

where \(P\) is the total pressure of the exhaust gas mixture which is 1 atm in all utility boilers. The saturation temperature at the partial pressure of water vapour is called the *dew point temperature* (d.p.t.). The flue gases are cooled in heat exchangers like economiser and air preheater so as to minimize the exhaust losses through chimney. These gases, however, should never be cooled below the dew point temperature (Fig. 4.8), as read from the steam tables. If cooled below the d.p.t., the water vapour condenses into liquid droplets which react with \(SO_2\) or \(SO_3\) to form acid. This acid corrodes the metal surfaces of the ducts through which the flue gas flows.

![Fig. 4.8 Dew point temperature of the exhaust flue gases](image)

### 4.11.5 Heating Value of a Fuel

Heats of reaction of coal constituents are given in Table 4.3.
### Table 4.3 Heats of reaction of coal constituents

<table>
<thead>
<tr>
<th>Formula and state</th>
<th>Product of combustion and state</th>
<th>Heat of reaction (kJ/kg mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon (coke)</td>
<td>CO(_2) (g)</td>
<td>-407,000</td>
</tr>
<tr>
<td>Carbon</td>
<td>CO(_2) (g)</td>
<td>-397,000</td>
</tr>
<tr>
<td>Carbon monoxide</td>
<td>CO(_2) (g)</td>
<td>-283,000</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>H(_2)O(l)</td>
<td>-286,000</td>
</tr>
<tr>
<td>Sulphur</td>
<td>SO(_2) (g)</td>
<td>-291,000</td>
</tr>
</tbody>
</table>

For 1 kg of coal containing \(C\) kg carbon, the heat released by the carbon combustion (taking carbon to have heat of reaction of coke) at standard conditions is

Heat released by carbon combustion

\[
= C \text{ kg} \times 407,000 \left(\frac{\text{kJ}}{\text{kg mol}}\right) \times \frac{\text{kg mol}}{12 \text{ kg}} = 33917 \left(\frac{\text{kJ}}{\text{kg}}\right)
\]

Similarly, heat released by sulphur = \[
= \frac{291000S}{32} = 9094S \text{ kJ/kg}
\]

The available hydrogen is that hydrogen which is available for combustion, and is the total hydrogen less than that required to combine with the oxygen in the coal, \(H - \frac{0}{8}\).

Heat released by hydrogen combustion

\[
= \left(H - \frac{0}{8}\right) \frac{286000}{2} = 143,000 \left(H - \frac{0}{8}\right) \text{ kJ/kg}
\]

Therefore, the total heat released by complete combustion of 1 kg coal is

\[
\text{HHV} = 33.917C + 143 \left(H - \frac{0}{8}\right) + 9.094S \text{ MJ/kg} \quad (4.39)
\]

The equation is very close to Dulong’s formula, as given by Eq. (4.5).

### 4.11.6 Control of Excess Air

Proper control of the right amount of excess air maintains optimum combustion efficiency. Amounts of CO\(_2\) and O\(_2\) in combustion gases are indexes of excess air. The desirable CO\(_2\) level depends on the fuel and the optimum excess air for the furnace (Fig. 4.9). Desirable O\(_2\) values depend much less on the type of fuel (Fig. 4.10). This makes O\(_2\) measurement the preferred method for combustion control. If the measured O\(_2\) content is more than that desired, the air supply is to be reduced. If the O\(_2\) measured is less than that desired, air supply is to be increased.
Fig. 4.9  CO$_2$ variation in flue gas with fuels and excess air

Fig. 4.10  O$_2$ variation in flue gas with excess air

Boiler losses are estimated as outlined in Section 4.13 for different steam outputs. The excess air is then adjusted by controlling air supply to show the optimum value of CO$_2$ or O$_2$. The optimum value of excess air for best combustion efficiency is then ascertained (Fig. 4.11). The excess air can also be determined using the following relation (Skrotzki and Vopat, 1960).

Fig. 4.11  Optimum excess air for maximum combustion efficiency
\[
\text{% excess air} = \frac{O_2 - 0.5CO}{0.264 N_2 - (O_2 - 0.5CO)} \times 100 \tag{4.40}
\]

where \(O_2\), \(CO\) and \(N_2\) are the volumetric percentages in the dry flue gas. An approximate formula for the excess air is

\[
\text{% excess air} = 100 \left[ \frac{(CO_2)_0}{(CO_2 + CO)} - 1 \right] = 100 \left[ \frac{O_2 - 0.5CO}{21 - O_2} \right]
\]

where \((CO_2)_0 = \% CO_2\) in the stoichiometric dry products, \(CO_2, CO, O_2 = \%\) in the actual products. Instruments are used to regulate the air flow, gas flow and steam flow. It aids combustion control.

### 4.12 MASS BALANCE OF A STEAM GENERATOR

Analyses of fuel, refuse from ash pit, and flue gas, and the dry bulb and wet bulb temperatures of entering air are recorded. On the basis of 1 kg coal, Fig. 4.12 gives the material balance for a boiler furnace, where \(W_A\) is the amount of air supplied.

\[
\begin{align*}
1 \text{ kg coal} &= \text{C + H + O + S + N + M + A} \\
W_A &\rightarrow \text{Boiler furnace} \rightarrow \text{W}_\text{dfig} = \text{CO}_2 + \text{CO} + O_2 + N_2 + \text{SO}_2 \\
&\rightarrow \text{W}_\text{H}_2\text{O} = 9\text{H} + \text{M} + \gamma_A W_A \\
&\rightarrow \text{Refuse} = \text{A} + \text{C} - C_{ab}
\end{align*}
\]

**Fig. 4.12 Material balance for a boiler furnace**

\[
W_A + \text{C + H + O + S + N + M + A} = W_{\text{dfig}} + 9 \text{H} + \text{M} + \text{A} + \text{C} - C_{ab}
\]

\[
W_A = W_{\text{dfig}} + 8 \left( \text{H} - \frac{0}{8} \right) - C_{ab} - \text{N} - \text{S} \tag{4.41}
\]

Mass of dfig produced per kg coal,

\[
W_{\text{dfig}} = \frac{C_{ab}(44\text{CO}_2 + 28\text{CO} + 28\text{N}_2 + 32\text{O}_2)}{12(\text{CO}_2 + \text{CO})}
\]

\[
= \frac{C_{ab}[11\text{CO}_2 + 7\text{CO} + 8\text{O}_2 + 7(100 - \text{CO}_2 - \text{CO} - \text{O}_2)]}{3(\text{CO}_2 + \text{CO})}
\]

\[
= \frac{C_{ab}(4\text{CO}_2 + \text{O}_2 + 700)}{3(\text{CO}_2 + \text{CO})} \tag{4.42}
\]
Volume of flue gases (wet) produced per kg coal

\[ V_{fg} = \left[ \frac{W_{dfg}}{M_{dfg}} + \frac{W_{H_2O}}{18} \right] \times 22.4 \times \frac{t_g}{273} \times \frac{101.325}{P_g} \text{ m}^3 \]  \hspace{1cm} (4.43)

where the pressure of gas \( P_g \) is in kPa and \( M_{dfg} \) is the molecular weight of \( dfg \).

The dry refuse analysis by mass gives \( A_R + C_R = 1.00 \), where subscript \( R \) represents the refuse.

In 1 kg coal, \( A = W_R \times A_R \), where \( W_R \) is the amount of refuse per kg coal and \( A_R \) is the mass fraction of ash in the refuse.

\[ W_R = \frac{A}{A_R} \]

Mass of unburnt carbon in refuse per kg coal = \( C_R W_R = C_R \frac{A}{A_R} \)

Carbon burnout in dry gas

\[ C_{ab} = C - C_R \frac{A}{A_R} \]  \hspace{1cm} (4.44)

### 4.13 ENERGY BALANCE OF A STEAM GENERATOR

The fuel supplied to a furnace when completely burned releases its heating value. This energy converts the feedwater pumped to the boiler into steam (Fig. 4.13).

**Fig. 4.13  Energy balance of a steam generator**

**Basis: 1 kg fuel**

1. **Useful energy**

\[ Q_1 = \frac{w_s}{w_f} \left[ (h_2 - h_1) \right] + (1 - m_1) (h_4 - h_3) \text{ kJ/kg} \]  \hspace{1cm} (4.45)
where \( w_s \) = steam generation rate, kg/s
\( w_f \) = fuel burning rate, kg/s
\( m_1 \) = mass fraction of inlet steam bled from the h.p. turbine before reheating

2. Energy loss due to dry exhaust gas

\[
Q_2 = W_{dfg} \cdot c_{p,fg} (t_g - t_a), \text{kJ/kg} \quad (4.46)
\]
where \( W_{dfg} \) = mass of \( dfg \) produced per kg fuel,
\( c_p \) = average specific heat of \( dfg \), kJ/kg K
\( t_g \) = exhaust gas temperature, \( ^\circ \text{C} \)
\( t_a \) = ambient temperature, \( ^\circ \text{C} \)

3. Energy loss due to unburnt carbon

\[
Q_3 = \text{mass of unburnt carbon} \times \text{its heating value}
\]
\[
= (C - C_{ab}) \cdot \frac{407000}{12} = 33917 \cdot (C - C_{ab}) \text{kJ/kg} \quad (4.47)
\]

4. Energy loss due to incomplete combustion

Loss of energy per kg of C oxidized to CO (from Table 4.3)
\[
= \frac{283,000}{12} \text{kJ/kg carbon} = 2358.3 \text{kJ/kg carbon}
\]

Loss of energy per kg fuel

\[
Q_4 = 2358.3 \cdot \frac{\text{kJ}}{\text{kg carbon}} \times \frac{\text{kg dfg}}{\text{kg fuel}} \times \frac{28 \text{ CO}}{44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2} \times \frac{\text{kg CO}}{\text{kg dfg}} \times \frac{12 \text{ kg carbon}}{28 \text{ kg CO}}
\]
\[
= 10100 \cdot W_{dfg} \cdot \frac{28 \text{ CO}}{44 \text{ CO}_2 + 28 \text{ CO} + 32 \text{ O}_2 + 28 \text{ N}_2} \cdot \frac{\text{kJ}}{\text{kg}} \quad (4.48)
\]

5. Energy loss due to moisture in fuel

\[
Q_5 = M \cdot \left[4.187(100 - t_f) + 2256.8 + 2.09 (t_g - 100)\right] \frac{\text{kJ}}{\text{kg}} \quad (4.49)
\]
where \( t_f \) = temperature of fuel entering the furnace.

6. Energy loss due to hydrogen in fuel

\[
Q_6 = 9H \cdot \left[4.187(100 - t_f) + 2256.8 + 2.09 (t_g - 100)\right] \frac{\text{kJ}}{\text{kg}} \quad (4.50)
\]

7. Energy loss due to moisture coming with air supplied

\[
Q_7 = \gamma_A W_A \times c_p (t_g - t_a), \text{kJ/kg} \quad (4.51)
\]
where \( \gamma_A = \) specific humidity of air, (kg moisture)/(kg dry air)
8. Energy loss due to ash and slag

\[ Q_b = [(C - C_{ab}) + A] \bar{c}_p (t_{fu} - t_d) \text{ kJ/kg} \]  

where

- \( \bar{c}_p \) = the average specific heat of ash, kJ/kg K
- \( t_{fu} \) = the temperature of the furnace, °C

9. Energy loss due to convection and radiation from the boiler surface.

\[ Q_9 = (h_c + h_r) A (t_w - t_a)/w_f \]  

where

- \( h_c \) = convective heat transfer coefficient, W/m² K
- \( h_r \) = radiative heat transfer coefficient, W/m² K
- \( A \) = total surface area exposed to the ambient air, m²
- \( t_w \) = temperature of the wall surface of the boiler, °C
- \( t_a \) = ambient temperature, °C

Energy released by complete combustion of 1 kg fuel = HHV.

Energy utilized in the heating of the working fluid,

\[ Q_1 = \text{HHV} - \Sigma \text{energy losses} = \text{HHV} - (Q_2 + Q_3 + ... + Q_9) \]

Therefore, efficiency of the steam generator,

\[ \eta_{\text{st. gen}} = \frac{\text{energy utilized}}{\text{energy released}} = \frac{\text{HHV} - \Sigma \text{energy losses}}{\text{HHV}} \]

\[ = \frac{w_8 [(h_2 - h_1) + (1 - m_1) (h_4 - h_3)]}{w_f \times \text{HHV}} \]  

All energy losses at a particular load can be measured, and from Eq. (4.54) the efficiency of the steam generator can be estimated. Then, with the help of Eq. (4.55), the fuel consumption rate of the boiler can be determined.

### 4.14 DRAUGHT (OR DRAFT) SYSTEM

Large amounts of air are needed for combustion of the fuel. The gaseous combustion products in huge quantity have also to be removed continuously from the boiler furnace. To produce the required flow of either air or combustion gas, a pressure differential is needed. The term “draught” or “draft” is used to define the static pressure in the furnace, in the various ducts, and the stack. The function of the draught system is basically two-fold:

1. To supply to the furnace the required quantity of air for complete combustion of fuel.
2. To remove the gaseous products of combustion from the furnace and throw these through chimney or stack to the atmosphere.

There are two ways of producing draught: (a) natural draught and (b) mechanical draught.
4.14.1 Natural Draught

The natural draught is produced by a chimney or a stack. It is caused by the density difference between the atmospheric air and the hot gas in the stack. For a chimney of height $H$ metres (Fig. 4.14) the draught or pressure difference ($N/m^2$) produced is given by

$$\Delta p = gH (\rho_a - \bar{\rho}_g) \quad (4.56)$$

**Fig. 4.14** Draught produced by a chimney

where $\rho_a = \text{density of atmospheric air, kg/m}^3$

$\rho_g = \text{average gas density in the chimney, kg/m}^3$

$g = \text{acceleration due to gravity, 9.81 m/s}^2$

Let us assume that the volume of the products of combustion is equal to the volume of air supplied for combustion, both volumes being measured at the same temperature. Thus, the volume of 1 kg of flue gas at NTP

$$V_a = \frac{0.287 \times 273}{101.325} = 0.7733 \text{ m}^3/\text{kg}$$

The volume of $m$ kg of air per kg fuel at temperature $T_a$

$$V_a = 0.7733 \times \frac{T_a}{273}$$

$$\rho_a = \frac{1}{0.7733} \times \frac{273}{T_a} = 1.293 \left(\frac{273}{T_a}\right) \text{ kg/m}^3$$

The mass of flue gas will be $(m + 1)$ kg and its temperature is $T_g$.  

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Density of flue gas, $\rho_g = \frac{m + 1}{0.7733 \times m \times T_g / 273}$

$$= 1.293 \left( \frac{273}{T_g} \right) \left( \frac{m + 1}{m} \right) = 353 \left( \frac{m + 1}{m} \right)$$

\[ \therefore \text{ Draught produced (from Eq. 4.56)} \]

$$\Delta p = 1.293 \times 273 \left[ \frac{1}{T_a} - \frac{m + 1}{m} \frac{1}{T_g} \right] gH$$

$$= 353 \ gH \left[ \frac{1}{T_a} - \frac{m + 1}{m} \frac{1}{T_g} \right] \tag{4.57}$$

If the draught is measured in terms of water column and the value is $h$ mm,

$$\Delta p = 10^3 \ g h \times 10^{-3} = 353 \ gH \left[ \frac{1}{T_a} - \frac{m + 1}{m} \frac{1}{T_g} \right]$$

\[ \therefore \]

$$h = 353 \ H \left[ \frac{1}{T_a} - \frac{m + 1}{m} \frac{1}{T_g} \right] \tag{4.58}$$

where $h$ is in mm and $H$ is in m.

For given $H$ and $T_a$, the natural draught depends on the average gas temperature $T_g$. Higher is the $T_g$, higher is the draught produced. But a high $T_g$ means a large exhaust loss through chimney resulting in a lower boiler efficiency. With an optimum $T_g$, the amount of draught produced by density difference is thus limited.

Tall and conspicuous from a distance, chimneys or stacks are used in all power plants. In early steam generators, all the needed draught used to be produced by stacks alone to overcome all the pressure losses at the required gas flow rates. In modern boilers, the fuel burning rate is high, and so the rate of air supply as well as the rate of flue gas removal is high. There are also various heat exchangers like superheaters, re heater, economizer, and air preheater on the way to cause large pressure losses for which stacks alone are insufficient, and fans are added for producing mechanical draught. Stacks have thus two functions:

1. To assist the fans in overcoming pressure losses.
2. To help disperse the gas effluent into the atmosphere at a sufficient height to cause minimum atmospheric pollution.

Stacks introduce pressure losses of their own. These pressure losses are caused by wall friction and the pressure equivalent to the kinetic energy of the gases leaving the stack. Therefore, the actual draught produced is less than the theoretical value $\Delta p$ given by Eq. (4.58), so that
\[ \Delta p_{\text{act}} = \Delta p - \rho_s \frac{V^2}{2} \left(1 + \frac{fH}{D}\right) \]  

(4.59)

where  
- \( f \) = friction factor, dimensionless  
- \( H \) = stack height, m  
- \( D \) = stack inside diameter, m  
- \( V \) = stack exit velocity, m/s  
- \( \rho_s \) = average gas density in the stack, kg/m³

Dispersion of the flue gases into the atmosphere is defined as the movement of the flue gases horizontally as well as vertically and their dilution by the atmosphere. Existing wind causes the horizontal motion of the flue gases. The vertical motion of flue gases results from the upward motion of high-velocity warm stack-exit gases to much higher elevations. The exit velocity of flue gases at stack exit results in a plume rise \( \Delta H \) above the actual stack (Fig. 4.15). The gases bend in the direction of wind flow. The plume height \( \Delta H \) is the height of a virtual point source above the stack obtained by extending the lines of dispersion backward. The effective stack height \( H_e \) is, thus, given by

\[ H_e = H + \Delta H \]  

(4.60)

**Fig. 4.15** Dispersion of flue gases at stack exit

Higher is the velocity of flue gases at stack exit, higher will be the plume rise \( \Delta H \) and greater will be the dilution of exiting gases. There are several analytical methods of estimating \( \Delta H \). Most of these utilize a momentum term that accounts for the vertical momentum of the gas caused by the stack exit velocity and a buoyancy term that accounts for the difference between stack gas and atmospheric densities, pertinent to the prevailing situation of mixed convection in the stack.
**Correlation of Carson and Moses**

\[
\Delta H = 2.62 \frac{Q_e^{0.5}}{V_w} - 0.029 \frac{V_s D}{V_w}
\]  

(4.61)

where,
- \( V_s \) = stack gas exit velocity, m/s
- \( D \) = stack diameter, m
- \( V_w \) = wind velocity at stack exit, m/s
- \( Q_e \) = heat emission, J/s = \( m c_p (T_s - T_a) \)
- \( m \) = gas mass flow rate, kg/s
- \( c_p \) = specific heat of gas = 1005 J/kg K for dry air at low temperature
- \( T_s \) = gas temperature at stack exit, K
- \( T_a \) = air temperature at stack exit, K

**Correlation of Briggs**

\[
\Delta H = \frac{114CF^{1/3}}{V_w}
\]  

(4.62)

where \( C \) = dimensionless temperature gradient parameter

\[
= 1.58 - 41.4 \frac{\Delta \theta}{\Delta Z}
\]

\( \frac{\Delta \theta}{\Delta z} \) = air temperature gradient, K/m which is zero for neutral atmospheric stability conditions

\[
F = \text{buoyancy flux} = \frac{gV_s D^2 (T_s - T_a)}{4T_a}, \text{ m}^4/\text{s}^2
\]

\( g \) = acceleration due to gravity = 9.81 m/s²

### 4.14.2 Condition for Maximum Discharge through the Chimney

Mass of hot gases flowing through the chimney

\[
\dot{m}_g = \rho_g A V_g
\]

Since the density of hot gases is inversely proportional to its temperature, \( \rho_g = C_1/T_g \), where \( C_1 \) is constant and the velocity of gases \( V_g = C \sqrt{2gH_g} \), we have

\[
\dot{m}_g = \frac{C_1}{T_g} \times A \times C \sqrt{2gH_g}
\]

where \( H_g \) = hot gas column in m.

\[
\therefore \quad \dot{m}_g = AC_2 \sqrt{2gH_g \left[ \frac{m}{m+1} \frac{T_g}{T_a} - 1 \right]} \times \frac{1}{T_g}
\]
For a given height of the chimney, \( H \), we can write

\[
m_{g} = C_3 \left[ \frac{m}{m + 1} \frac{1}{T_a T_g} - \frac{1}{T_g^2} \right]^{3/2}
\]

Thus the maximum discharge is a function of \( T_g \)

\[
\frac{d m_g}{dT_g} = 0 = -\frac{m}{m + 1} \frac{1}{T_a T_g^2} + \frac{2}{T_g^3}
\]

\[
T_g = 2T_a \frac{m + 1}{m} \text{ and } \frac{T_g}{T_a} = 2 \left( \frac{m + 1}{m} \right)
\]

(4.63)

Now,

\[
\Delta p = \rho_g g H_g = 353 \rho g \left( \frac{1}{T_a} - \frac{m + 1}{m} \frac{1}{T_g} \right)
\]

Since \( \rho_g = \frac{353}{T_g} \frac{m + 1}{m} \), on substitution

\[
H_g = H \left( \frac{m T_g}{m + 1 T_a} - 1 \right)
\]

\[
= H \left( \frac{m}{m + 1} \cdot 2 \frac{m + 1}{m} - 1 \right) = H
\]

Thus the height of the hot gas column is equal to the chimney height for maximum discharge.

The draught in mm of water column

\[
h = \frac{353H}{T_a} \left[ 1 - \frac{m + 1}{m} \times \frac{T_a}{T_g} \right] = \frac{353H}{T_a} \left[ 1 - \frac{m + 1}{m} \cdot \frac{m}{2(m + 1)} \right]
\]

\[
= \frac{353H}{2T_a} = \frac{176.5H}{T_a}
\]

(4.64)

4.14.3 Mechanical Draught

Mechanical draught is produced by fans. There are two types of fans in use today: forced draught (FD) and induced draught (ID) fans. When either one is used alone, it should overcome the total air and gas pressure losses within the steam generator.

Forced draught fans are installed at inlet to the air preheater. They handle cold air. So they have less maintenance problems, consume less power (since cold air has low specific volume, and work input per unit mass flow rate is given by \( \dot{v}dp \)) and therefore, their capital and operating costs are lower. If \( \omega_f \) is the fuel burning rate, \( v \) is the specific volume of inlet air and \( \Delta p_{FD} \) is the pressure head developed by the fan to overcome all the pressure losses, then the power required to drive the FD fan is given by
Power input = \[
\frac{W_t \times \frac{A}{F} \times v \times \Delta p}{\eta_{FD}}
\]  
(4.65)

where \(A/F\) is the air—fuel ratio and \(\eta_{FD}\) is the efficiency of the forced draught fan. For good reliability two forced draught fans operating in parallel are normally used, each capable of undertaking at least 60% of full load air flow when the other is out of service.

The forced draught fan if used alone, as in many large steam generators and almost all marine applications, maintains the entire system up to the stack entrance under positive gauge pressure. The furnace is then said to be pressurized. Utmost care is taken so that noxious gases do not leak out to the atmosphere. It thus needs a gas-tight furnace construction and properly designed ignition openings and inspection doors. The stack in such a situation is shorter and meant only for disposal of flue gases.

Induced draught fans are normally located at the foot of the stack. They handle hot combustion gases. Their power requirements are, therefore, greater than forced draught fans. In addition, they must cope with corrosive combustion products and fly ash. Induced draught fans are seldom used alone. They discharge essentially at atmospheric pressure and place the system upstream under negative gauge pressure. If \(v_g\) is the specific volume of flue gases handled by the ID fan and \(\Delta p_{ID}\) is the pressure head developed, then the power needed to drive the ID fan is given by

\[
\text{Power input} = \frac{w_t \left(1 + \frac{A}{F}\right) \times v_g \times \Delta p_{ID}}{\eta_{ID}}
\]  
(4.66)

When both forced and induced draught fans are used in a steam generator, the FD fans push atmospheric air through the air preheater, dampers, various air ducts, and burners into the furnace, and the ID fan sucks out the flue gases through the heat transfer surfaces in the superheaters, reheaters, economiser, gas-side air preheater and dust collectors and discharge into the stack (Fig. 4.16). The stack because of its height, adds a natural driving pressure of its own. In such a case the furnace is said to operate with balanced draught, meaning that the pressure in it is approximately atmospheric. Actually, it is maintained at a slightly negative gauge pressure to ensure that any leakage would be inward. Modern boilers are mostly designed with balanced draught firing.

A typical large power plant (600 MWe) may require two 1320 kW forced draught fans and two 3000 kW induced draught fans.

### 4.14.3 Pressure and Flow Measurement

The total pressure at a point in the system is the sum of static and dynamic pressures:

\[
p_t = p_s + p_d
\]  
(4.67)
Fig. 4.16  Balanced draught system in a modern boiler

where

\[ p_t = \text{total pressure, N/m}^2 \]
\[ p_s = \text{static pressure, N/M}^2 \]
\[ p_d = \text{dynamic pressure} = \rho_g \frac{\bar{V}^2}{2}, \text{N/m}^2 \]
\[ \rho_g = \text{gas density, kg/m}^3 \]
\[ \bar{V} = \text{velocity of gas, m/s} \]

Although the SI units of pressure are pascals (N/m²), it is still customary to state the static and dynamic pressures in mm of water since measurements are commonly made using a simple, standard U-tube with water as the manometer fluid.

One metre head of water produces a pressure of

\[ p = \rho g H = 1000 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times 1 \text{ m} = 9810 \text{ N/m}^2 \]

In terms of mm of water, the dynamic pressure head, \( p_d \), which would be measured by a pitot tube connected to a water filled manometer, becomes

\[ p_d = \frac{\rho g V^2}{9810 \times 2} \times 1000 = \frac{\rho g V^2}{19.62}, \text{ mm H}_2\text{O} \]

where \( \rho_g \) is in kg/m³ and \( V \) is in m/s. Then the volume flow rate of gas in a duct is given by

\[ \bar{V}_o = A \left[ 19.62 \frac{p_d}{\rho_g} \right]^{1/2} \text{ m}^3/\text{s} \]

where \( A \) is the duct cross-sectional area in m².
\[ V_o = 4.43A \sqrt{\left( \frac{p_d}{\rho_g} \right)} \text{ m}^3/\text{s} \]  
(4.68)

where \( p_d \) is in mm of water and \( \rho_g \) is in kg/m\(^3\).

The flue gas density depends on the moisture content and temperature. For 5% moisture and at 560 K, flue gas density is approximately 0.649 kg/m\(^3\). Combustion air density at 560 K, is 0.636 kg/m\(^3\).

### 4.15 FANS

FD and ID fans operate continuously for long periods, up to 1 or 1 \( \frac{1}{2} \) years. So, these must be well designed, ruggedly constructed, well balanced, and highly efficient over a wide range of outputs. Typical fans have capacities of 700 m\(^3\)/s of volume flow producing 152 mm water static pressures (about 0.15 bar).

There are two types of fans, viz., centrifugal and axial. In the centrifugal fan, the gases are accelerated radially through curved or flat impeller blades from rotor to a spiral or volute casing. In the axial fan, gases are accelerated parallel to the rotor axis. This is similar to a table fan, but here the fan is housed in a casing to develop static pressure. Axial fans have higher capital costs.

Centrifugal fans can have forward-curved, flat or backward curved impeller blades (Fig. 4.17). The velocity triangles at exit from the tip of the blades are shown, where the absolute velocity of gas \( V \) is the same in all the three cases. It is seen that for the same \( V \) the blade tip velocity \( V_b \) is the highest for the backward-curved blades (c) and the lowest for the forward-curved blades (a). Since \( V_b = (\pi DN)/60 \), for the same tip diameter \( D \), the rpm \( N \) is the highest for the backward-curved and the lowest for the forward-curved blades. The FD fans should have high \( V_b \) so as to rotate at high speeds and handle large volume flow of air. Therefore, centrifugal fans with backward-curved blading are normally used for FD fans. The ID fans handle dust-laden flue gases and so the blades are subject to erosion by the fly ash. The erosion rate of blades is lower if the blade tip speed \( V_b \) is less and the fan rotates at lower speeds. Therefore, centrifugal fans having forward-curved or flat blading are used for ID fans. Low-speed fans with flat blades are used for particularly dirty or corrosive gases.

There are two common methods of controlling the output of fans: damper control and variable-speed control.

In damper control, a damper (butterfly valve) is used to control the flow of gas by regulating the flow area according to the load. It has the advantage of low capital cost since it needs a simple constant-speed induction ac motor. It, however, offers additional flow resistance which is overcome by increasing the power input. Dampers are usually put on the outlet side of the fan, although dampers at inlet to the fan are sometimes used. Inlet dampers consume less power than outlet dampers but are only effective for moderate load changes near full load. When used with FD fans, they are normally used in combination with outlet dampers. The use of a two-speed ac drive motor in conjunction with damper control can also reduce power consumption, since two-speed motors are less expensive than variable-speed ac drives.
Fig. 4.17 Centrifugal blading: (a) forward-curved, (b) flat, and (c) backward-curved. Vector diagrams show blade tip velocity $V_b$, gas velocity relative to blade $V_r$, and absolute velocity of gas leaving blade $V$. In all cases, $V$ is the same.

Variable-speed control has the advantage of less power consumption and is the most efficient method for fan control. The effect of speed on fan performance is:

- Volume flow $\propto N$
- Pressure head $\propto N^2$
- Power input $\propto N^3$

where $N$ is the rpm of the fan. Thus, reducing speed, say by 70%, reduces the capacity to 70%, the pressure to about 50%, and the power input to about 35%.

The types of drives are:

1. variable-speed steam turbine
2. hydraulic coupling
3. variable-speed dc motor
4. multiple speed ac motor
5. electronically adjustable motor drive

The major disadvantage of variable speed control is the higher capital costs than damper control.
Two other types of fans are used in power plants: primary air fans and gas recirculation fans. Primary air fans supply air to dry and transport pulverized coal to the furnace. Gas recirculation fans are used to recirculate gas from a point between the economizer and air preheater back to the bottom of the furnace as part of a steam temperature control system.

Fans are a major source of noise in power plants. To reduce this noise, they are often housed in thick masonry acoustical enclosures or equipped with silencers, or both.

### 4.16 HEAT OF COMBUSTION

The first law of thermodynamics was treated so far ignoring such energy terms as chemical, electrical, and magnetic, in the energy balance for a system. When dealing with combustion systems, however, the chemical energy term cannot be ignored.

#### 4.16.1 Open System

For a chemically reactive system (Fig. 4.18), the steady state steady flow energy equation can be written as

\[
H_R + Q_{CV} = H_P + W_{CV}
\]

ignoring the changes in kinetic and potential energies. Here, \(H_R\) and \(H_P\) are the enthalpies of the reactants and products, respectively, evaluated for their constituents at their respective pressures and temperatures; \(Q_{CV}\) is the net heat added to the system from the surroundings (which is usually negative, since in combustion heat is usually rejected to the surroundings); and \(W_{CV}\) is the net work done by the system.

Both the reactants and products are usually composed of several constituents. Therefore, Eq. (4.69) is written in the form

\[
\sum_R (mh) + Q_{CV} = \sum_P (mh) + W_{CV}
\]

where \(m\) is the mass and \(h\) is the specific enthalpy of each constituent. Let us consider the combustion of ethane.

\[
C_2H_6 + 3.5O_2 + (3.5) (3.76) N_2 \rightarrow 2CO_2 + 3H_2O + (3.5) (3.76) N_2
\]
The enthalpies of the various reactants and products are those that start at the same datum of composition, temperature and pressure, which are arbitrarily chosen as the elemental substances at 25 °C and 1 atm, respectively. For example, C₂H₆ is formed from elemental carbon C and hydrogen H₂ (Fig. 4.19). When the reactants carbon and hydrogen enter the reactor at 25 °C, 1 atm and the product C₂H₆ is cooled back to 25 °C, 1 atm, 2817.3 kJ/kg C₂H₆ of heat is rejected. The reaction may be written as

\[ 2C + 3H₂ \rightarrow C₂H₆ + 2817.3 \text{ kJ/kg C₂H₆} \]

Similarly, for CO₂,

\[ C + O₂ \rightarrow CO₂ + 8946.8 \text{ kJ/kg CO₂} \]

![Diagram showing the formation of ethane](image)

\[ Q_{CV} = -2817.3 \text{ kJ/kg C₂H₆} = (h_f^0)_{C₂H₆} \]

**Fig. 4.19** Formation of ethane

---

**Table 4.4** Enthalpy of formation \( h_f^0 \) at 25 °C, 1 atm

<table>
<thead>
<tr>
<th>Substance</th>
<th>Formula</th>
<th>Molecular weight</th>
<th>State</th>
<th>( h_f^0 ) (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>C</td>
<td>12.011</td>
<td>solid</td>
<td>0</td>
</tr>
<tr>
<td>Oxygen</td>
<td>O₂</td>
<td>32.000</td>
<td>gas</td>
<td>0</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>H₂</td>
<td>2.016</td>
<td>gas</td>
<td>0</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>N₂</td>
<td>28.016</td>
<td>gas</td>
<td>0</td>
</tr>
<tr>
<td>Sulphur</td>
<td>S</td>
<td>32.060</td>
<td>solid</td>
<td>0</td>
</tr>
<tr>
<td>Carbon monoxide</td>
<td>CO</td>
<td>28.011</td>
<td>gas</td>
<td>-3948.3</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>CO₂</td>
<td>44.011</td>
<td>gas</td>
<td>-8946.8</td>
</tr>
<tr>
<td>Water</td>
<td>H₂O</td>
<td>18.016</td>
<td>liquid</td>
<td>-15875.5</td>
</tr>
<tr>
<td>Methane</td>
<td>CH₄</td>
<td>16.043</td>
<td>gas</td>
<td>-4669.8</td>
</tr>
<tr>
<td>Ethane</td>
<td>C₂H₆</td>
<td>30.070</td>
<td>gas</td>
<td>-2817.3</td>
</tr>
<tr>
<td>Propane</td>
<td>C₃H₈</td>
<td>44.097</td>
<td>gas</td>
<td>-2356.3</td>
</tr>
<tr>
<td>Butane</td>
<td>C₄H₁₀</td>
<td>58.124</td>
<td>gas</td>
<td>-2171.6</td>
</tr>
<tr>
<td>Octane</td>
<td>C₈H₁₈</td>
<td>114.230</td>
<td>liquid</td>
<td>-2189.5</td>
</tr>
<tr>
<td>Nitric oxide</td>
<td>NO</td>
<td>30.008</td>
<td>gas</td>
<td>-3020.8</td>
</tr>
<tr>
<td>Nitrogen dioxide</td>
<td>NO₂</td>
<td>46.008</td>
<td>gas</td>
<td>-733.3</td>
</tr>
<tr>
<td>Sulphur dioxide</td>
<td>SO₂</td>
<td>64.060</td>
<td>gas</td>
<td>-4632.8</td>
</tr>
</tbody>
</table>
The quantities 2817.3 and 8946.8 leave the system and hence are negative. They are called enthalpies of formation ($h^0_f$). Table 4.4 gives values of $h^0_f$ for various substances at 25 °C, 1 atm. The enthalpy of all elements at the standard reference state (25 °C, 1 atm) is assigned the value of zero. It may be noted that if the combustion equation involves a liquid, such as octane or water, there will be two enthalpies of formation, depending upon whether they start or end in a liquid or vapour state.

Because chemical reactions, such as Eq. (4.70) are balanced in terms of moles and not masses, and because $m = n\mu$, where $n =$ number of moles and $\mu =$ molecular weight, Eq. (4.70) may be written as

$$\sum_R n_i h_i + Q_{CV} = \sum_R n_e h_e + W_{CV}$$

(4.72)

where $\tilde{h}$ is in kJ/kg mol. Table 4.5 gives the values of the enthalpy of formation ($h^0_f$) of a number of substances in kJ/kg mol at 25 °C, 1 atm.

In most cases, however, the reactants and products are not at 25 °C, 1 atm. Therefore, the change of enthalpy (in a constant pressure of SSSF process) between 25 °C, 1 atm and the given state must be known. Thus the enthalpy at any temperature and pressure, $\tilde{h}_{T, p}$ is

$$\tilde{h}_{T, p} = (\tilde{h}^0_f)_{298K, 1 \text{ atm}} + (\Delta \tilde{h})_{298 K, 1 \text{ atm}} \rightarrow T, p$$

For convenience, the subscripts are usually dropped, and

$$\tilde{h}_{T, p} = h^0_f + \Delta \tilde{h}$$

(4.73)

**Table 4.5** Enthalpy of formation, Gibbs function of formation, and absolute entropy of various substances at 25 °C, 1 atm pressure

<table>
<thead>
<tr>
<th>Substance</th>
<th>Molecular Weight, $M$</th>
<th>$h^0_f$ kJ/kg mol</th>
<th>$g^0_f$ kJ/kg mol</th>
<th>$s_0$ kJ/kg mol K</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO(g)</td>
<td>28.001</td>
<td>-119529</td>
<td>-137150</td>
<td>197.653</td>
</tr>
<tr>
<td>CO$_2$(g)</td>
<td>44.001</td>
<td>-393522</td>
<td>-394374</td>
<td>213.795</td>
</tr>
<tr>
<td>H$_2$O(g)</td>
<td>18.015</td>
<td>-241827</td>
<td>-228583</td>
<td>188.883</td>
</tr>
<tr>
<td>H$_2$O(1)</td>
<td>18.015</td>
<td>-285838</td>
<td>-237178</td>
<td>70.049</td>
</tr>
<tr>
<td>CH$_4$(g)</td>
<td>16.043</td>
<td>-74873</td>
<td>-50751</td>
<td>186.256</td>
</tr>
<tr>
<td>C$_2$H$_2$(g)</td>
<td>26.038</td>
<td>+226731</td>
<td>+209234</td>
<td>200.958</td>
</tr>
<tr>
<td>C$_2$H$_4$(g)</td>
<td>28.054</td>
<td>+52283</td>
<td>+68207</td>
<td>219.548</td>
</tr>
<tr>
<td>C$_2$H$_6$(g)</td>
<td>30.070</td>
<td>-84667</td>
<td>-32777</td>
<td>229.602</td>
</tr>
<tr>
<td>C$_3$H$_8$(g)</td>
<td>44.097</td>
<td>-103847</td>
<td>-23316</td>
<td>270.019</td>
</tr>
<tr>
<td>C$_4$H$_10$(g)</td>
<td>58.124</td>
<td>-126148</td>
<td>+16914</td>
<td>310.227</td>
</tr>
<tr>
<td>C$<em>8$H$</em>{18}$(g)</td>
<td>114.23</td>
<td>-208447</td>
<td>+16859</td>
<td>466.835</td>
</tr>
<tr>
<td>C$<em>8$H$</em>{18}$(l)</td>
<td>114.23</td>
<td>-249952</td>
<td>+6940</td>
<td>360.896</td>
</tr>
</tbody>
</table>

where $\Delta \tilde{h}$ represents the difference in enthalpy between any given state and the enthalpy at 298 K, 1 atm.
Table A.4 in the appendix gives the values of $\Delta \bar{h} = \bar{h}^0 - \bar{h}_{298}$ in kJ/kg mol for various substances at different temperatures.

### 4.16.2 Closed System

Combustion equations for fuels burning in a closed system, such as a cylinder or bomb, may be obtained by writing the energy balance for the closed system. Thus,

$$U_R + Q = U_p + W \tag{4.74}$$

where $W$ is the non-flow work. For gases,

$$H = U + pV = U + n \bar{R} T$$

where $\bar{R}$ is the universal gas constant.

### 4.17 HEATING VALUES: ENTHALPY OF COMBUSTION

The enthalpy of combustion is defined as the difference between the enthalpy of products and the enthalpy of the reactants when complete combustion occurs at a given temperature and pressure. Therefore,

$$\bar{h}_{RP} = H_p - H_R$$

or

$$\bar{h}_{RP} = \sum_R n_e (\bar{h}_e^0 + \Delta \bar{h})_e - \sum_R n_i (\bar{h}_i^0 + \Delta \bar{h})_i \tag{4.75}$$

where $\bar{h}_{RP}$ is the enthalpy of combustion of the fuel. The values of the enthalpy of combustion of different hydrocarbon fuels at 25 °C, 1 atm are given in Table 4.6.

The internal energy of combustion, $\bar{u}_{RP}$ is defined in a similar way.

$$\bar{u}_{RP} = U_p - U_R$$

$$= \sum_R n_e \left( \bar{h}_e^0 + \Delta \bar{h} - p \bar{V} \right)_e - \sum_R n_i \left( \bar{h}_i^0 + \Delta \bar{h} - p \bar{V} \right)_i \tag{4.76}$$

If all the gaseous constituents are considered ideal gases and the volume of liquid and solid considered is assumed to be negligible compared to gaseous volume,

$$\bar{u}_{RP} = h_{RP} - \bar{R} T \left( n_{\text{gaseous products}} - n_{\text{gaseous reactants}} \right) \tag{4.77}$$

In the case of a constant pressure or steady flow process, the negative of the enthalpy of combustion is frequently called the heating value at constant pressure, which represents the heat transferred from the chamber during combustion at constant pressure.

The internal energy of combustion, $\bar{u}_{RP}$ is defined in a similar way.

$$\bar{u}_{RP} = U_p - U_R$$

$$= \sum_R n_e \left( \bar{h}_e^0 + \Delta \bar{h} - p \bar{V} \right)_e - \sum_R n_i \left( \bar{h}_i^0 + \Delta \bar{h} - p \bar{V} \right)_i \tag{4.76}$$
<table>
<thead>
<tr>
<th>Hydrocarbon</th>
<th>Formula</th>
<th>Liquid ( H_2O ) in Products (Negative of Higher Heating Value)</th>
<th>Gaseous Hydrocarbon kJ/kg fuel</th>
<th>Liquid Hydrocarbon kJ/kg fuel</th>
<th>Liquid ( H_2O ) in Products (Negative of Lower Heating Value)</th>
<th>Gaseous Hydrocarbon kJ/kg fuel</th>
<th>Gaseous Hydrocarbon kJ/kg fuel</th>
</tr>
</thead>
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<td><strong>Paraffin family</strong></td>
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<td>-50010</td>
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<td>-51875</td>
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<td>-47484</td>
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<td>-46353</td>
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<td>Octane</td>
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<td>-44788</td>
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<td>Decane</td>
<td>( \text{C}<em>{10}\text{H}</em>{22} )</td>
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<td>-48000</td>
<td>-44239</td>
<td>-44598</td>
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<tr>
<td>Dodecane</td>
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<td><strong>Olefin family</strong></td>
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<td>-50296</td>
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<td>-48453</td>
<td>-45316</td>
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<tr>
<td>Pentene</td>
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<td>Hexene</td>
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<td>Heptene</td>
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<td>-44662</td>
<td>-44662</td>
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</tr>
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<td>Octene</td>
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<td>-47693</td>
<td>-44556</td>
<td>-44556</td>
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<td></td>
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<td>Nonene</td>
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<td>-47612</td>
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<td>-44475</td>
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<td></td>
</tr>
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<td>Decene</td>
<td>( \text{C}<em>{10}\text{H}</em>{20} )</td>
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<td><strong>Alkylbenzene family</strong></td>
<td></td>
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<tr>
<td>Benzene</td>
<td>( \text{C}_6\text{H}_6 )</td>
<td>-41831</td>
<td>-42266</td>
<td>-40141</td>
<td>-40576</td>
<td></td>
<td></td>
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<tr>
<td>Methylbenzene</td>
<td>( \text{C}_7\text{H}_8 )</td>
<td>-42473</td>
<td>-42847</td>
<td>-40527</td>
<td>-40937</td>
<td></td>
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<tr>
<td>Ethylbenzene</td>
<td>( \text{C}<em>8\text{H}</em>{10} )</td>
<td>-42997</td>
<td>-43395</td>
<td>-40924</td>
<td>-41322</td>
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</tr>
<tr>
<td>Propylbenzene</td>
<td>( \text{C}<em>9\text{H}</em>{12} )</td>
<td>-43416</td>
<td>-43800</td>
<td>-41219</td>
<td>-41603</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Butylbenzene</td>
<td>( \text{C}<em>{10}\text{H}</em>{14} )</td>
<td>-43748</td>
<td>-44123</td>
<td>-41453</td>
<td>-41828</td>
<td></td>
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</tr>
</tbody>
</table>

If all the gaseous constituents are considered ideal gases and the volume of liquid and solid considered is assumed to be negligible compared to gaseous volume, 

\[
\bar{u}_{RP} = h_{RP} - \bar{R} T (n_{\text{gaseous products}} - n_{\text{gaseous reactants}})
\]  

(4.77)

In the case of a constant pressure or steady flow process, the negative of the enthalpy of combustion is frequently called the heating value at constant
pressure, which represents the heat transferred from the chamber during combustion at constant pressure.

Similarly, the negative of the internal energy of combustion is sometimes designated as the heating value at constant volume in the case of combustion because it represents the amount of heat transfer in the constant volume process.

The higher heating value (HHV) or higher calorific value (HCV) is the heat transferred when H₂O in the products is in the liquid state (see Section 4.2.3). The lower heating value (LHV) or lower calorific value (LCV) is the heat transferred in the reaction when H₂O in the products is in the vapour state.

4.18 THEORETICAL FLAME TEMPERATURE

If a combustion process occurs adiabatically in the absence of work transfer or changes in KE and PE, then the steady flow energy equation becomes

\[ H_R = H_P \]  \quad \text{or} \quad \sum n_i \bar{h}_i = \sum n_c \bar{h}_c \]

or

\[ \sum n_i \left( \bar{h}_i^0 + \Delta \bar{h}_i \right) = \sum n_c \left( \bar{h}_c^0 + \Delta \bar{h}_c \right) \]  \quad (4.78)

For such a process, the temperature of the products is called the adiabatic or theoretical flame temperature, which is the maximum temperature achieved for the given reactants. The maximum theoretical flame temperature of a fuel corresponds to complete combustion with just the theoretically required quantity of pure oxygen. Similarly, the maximum flame temperature using air corresponds to complete combustion with the stoichiometric quantity of air. This temperature is lower than the maximum flame temperature in pure oxygen because of the dilution effect of nitrogen. In actual combustion chambers, flame temperature is further reduced due to incomplete combustion, excess air requirements, heat losses to the surrounding walls, and by heat loss to ash particles suspended in the combustion products.

The adiabatic flame temperature can be controlled by the amount of excess air supplied. The maximum permissible temperature in a gas turbine is fixed from metallurgical considerations (i.e., the material used for gas turbine blades). So, close control of the temperature of the products is achieved by controlling the excess air. For a given reaction, the adiabatic flame temperature can be theoretically computed by trial and error. The energy of the reactants \( H_R \) being known, a suitable temperature is chosen for the products so that the energy of the products. \( H_P \) at that temperature becomes equal to the energy of the reactants \( H_R \). This helps in the selection of material for the combustor.

Figure 4.20 shows the variation of adiabatic flame temperature with excess air. Combustion temperatures so calculated are higher than actual because dissociation of some of the products takes place at high temperatures. For example, some CO₂ may dissociate to form CO and \( \frac{1}{2} \) O₂, which is an endothermic reaction that lowers the temperature. Chemical equilibrium occurs at a certain temperature when the reaction rate is the same in both directions, i.e., when
**Fig. 4.20** Variation of adiabatic flame temperature with excess air

\[ \text{CO} + \frac{1}{2} \text{O}_2 \rightleftharpoons \text{CO}_2 \quad \text{and} \quad \text{H}_2 + \frac{1}{2} \text{O}_2 \rightleftharpoons \text{H}_2\text{O} \]

The higher is the temperature, more is the dissociation. However, the effect of dissociation is lower for lean mixtures because the excess oxygen tends to drive the reactions toward completion, and for higher furnace pressure. Thus, the effect of dissociation is not significant in steam generator furnace combustion, which occurs with excess air at about 1500 °C.

It may be noted that all combustion calculations can be done if there is heat transfer to or from the environment, work done by or on the system, or even changes in kinetic or potential energy between products and reactants by including the appropriate terms in the steady state steady flow energy equation.

### 4.19 FREE ENERGY OF FORMATION

The thermodynamic potential which controls equilibrium in a system depends on the particular constraints imposed on the system. If the constraints are constant temperature and pressure, for any process

\[ \text{d}G_{TP} < 0 \quad (4.79) \]

where \( G \) is the Gibbs function defined as \( G = H - TS \), which is also called the free energy. For a process at constant \( T \) and \( P \), \( G \) always decreases and becomes a minimum at the state of equilibrium. It is the free energy, \( G \), which determines whether a chemical reaction will occur spontaneously. The driving force in a chemical reaction is the difference in free energy between the initial and final states, \( \Delta G \), where at constant temperature

\[ \Delta G = \Delta H - T\Delta S \quad (4.80) \]

The free energy of the chemical elements at 1 atm. 25°C is arbitrarily assumed to be zero. The *standard free energy of formation of a compound*, \( g_f^0 \), is the free energy change when one mole of the compound is formed directly from its constituent elements, both the reactants and the products being at 298 K and partial pressures of 1 atm. If the reaction takes place at some higher temperature, the free energy of formation \( g_f^0 \) will have a different value. In actual
situations, the partial pressures of the reactants and products will not be 1 atm. The effect of such pressure changes on the free energy of a substance can be evaluated from the relation at constant temperature

\[ \text{dg} = v \, dp \]

where \( v \) is the molal specific volume. For an ideal gas in an isothermal process from the standard pressure \( p_0 \) to the operating pressure \( p_1 \),

\[ \Delta \bar{g} = \int_{p_0}^{p_1} v dp = \bar{R} T \ln \frac{p_1}{p_0} \]  \hspace{1cm} (4.81)

where \( \bar{R} \) is the universal gas constant. The free energy change

\[ \Delta G = \bar{g}_f^0 + \Delta \bar{g} = \bar{g}_f^0 + \bar{R} T \ln \frac{p_1}{p_0} \]  \hspace{1cm} (4.82)

If \( p \) is expressed in atmospheres and \( p_0 = 1 \) atm.

\[ \Delta G = \bar{g}_f^0 + \bar{R} T \ln p_1 \]  \hspace{1cm} (4.83)

If the substance is not an ideal gas, we define a quantity \( f \), called the fugacity such that

\[ \lim_{p \to 0} \frac{f}{p} = 1 \]  \hspace{1cm} (4.84)

and

\[ \Delta G = \bar{g}_f^0 + \bar{R} T \ln \frac{f}{f_0} \]  \hspace{1cm} (4.85)

### 4.20 EQUILIBRIUM CONSTANT \( K_p \)

Let us consider a reaction

\[ \text{C}_2\text{H}_6(\text{g}) + 3.5 \text{O}_2(\text{g}) \longrightarrow 2\text{CO}_2(\text{g}) + 3\text{H}_2\text{O}(\text{g}) \]

taking place at temperature \( T \). The initial state is defined by a partial pressure of ethane, \( p_{\text{C}_2\text{H}_6} \), and the partial pressure of oxygen, \( p_{\text{O}_2} \). In the final state, \( \text{CO}_2 \) and \( \text{H}_2\text{O} \) have partial pressures of \( p_{\text{CO}_2} \) and \( p_{\text{H}_2\text{O}} \). Using the free energies of formation, the free energy change for the system \( \Delta g \) can be expressed as

\[ \Delta G = 2(\Delta g_{\text{CO}_2}) + 3(\Delta g_{\text{H}_2\text{O}}) - [(\Delta g_{\text{C}_2\text{H}_6}) + 3.5 (\Delta g_{\text{O}_2})] \]  \hspace{1cm} (4.86)

where from Eq. (4.83)

\[
2(\Delta g_{\text{CO}_2}) = 2 (\bar{g}_f^0)_{\text{CO}_2} + \bar{R} \, T \ln p^3_{\text{CO}_2} \\
3 (\Delta g_{\text{H}_2\text{O}}) = 3 (\bar{g}_f^0)_{\text{H}_2\text{O}} + \bar{R} \, T \ln p^3_{\text{H}_2\text{O}} \\
(\Delta g_{\text{C}_2\text{H}_6}) = (\bar{g}_f^0)_{\text{C}_2\text{H}_6} + \bar{R} \, T \ln p_{\text{C}_2\text{H}_6} \\
3.5 (\Delta g_{\text{O}_2}) = 3.5 (\bar{g}_f^0)_{\text{O}_2} + \bar{R} \, T \ln p^{3.5}_{\text{O}_2}
\]

Substituting in Eq. (4.86), the free energy change becomes
\[
\Delta G = 2(g^0)_{\text{CO}_2} + 3(g^0)_{\text{H}_2\text{O}} - [(g^0)_{\text{C}_2\text{H}_6} + 3.5 (g^0)_{\text{O}_2}]
+ \overline{R} \ T (\ln p^2_{\text{CO}_2} + \ln p^3_{\text{H}_2\text{O}} - \ln p_{\text{C}_2\text{H}_6} - \ln p^{3.5}_{\text{O}_2})
\] (4.87)

The first four terms of the above equation represent \(\Delta G^0\), the change in the free energy for the reaction when the products and the reactants all have partial pressures of 1 atm, while the remaining terms represent the effect of partial pressures on the free energy change. Making the standard free energy change \(\Delta g^0_T\) as

\[
\Delta G^0 = n_3g_3 + n_4g_4 - n_1g_1 - n_2g_2
\]

where \(n\)'s are the stoichiometric coefficients and \(g\)'s are the free energy of formation, Eq. (4.87) can be written as

\[
\Delta G = \Delta G^0 + \overline{R} \ T \ln \frac{p^2_{\text{CO}_2} p^3_{\text{H}_2\text{O}}}{p_{\text{C}_2\text{H}_6} p^{3.5}_{\text{O}_2}}
\] (4.88)

In generalized notations,

\[
\Delta G = \Delta G^0 + \overline{R} \ T \ln \frac{p^v_3 p^v_4}{p^v_1 p^v_2}
\] (4.89)

where \(p_3\) and \(p_4\) are the partial pressures of the products, and \(p_1\) and \(p_2\) are those of the reactants for the reaction

\[
n_1A_1 + n_2A_2 \rightarrow n_3A_3 + n_4A_4
\]

For exothermic readings, \(\Delta G^0\) is negative and for endothermic reactions it is positive. In Eq. (4.89), \(\Delta G^0\) is normally much higher than the second term representing the effect of partial pressures. In a chemical reaction, as the reactants are consumed and the products are formed, the partial pressures of the reactants decrease and those of the products increase, which make \(\Delta G\) less negative and thus, decrease the net driving force for the reaction. In the reverse reaction, \(A_3\) may react with \(A_4\) to form \(A_1\) and \(A_2\). The free energy change for this reverse reaction

\[
n_3A_3 + n_4A_4 \rightarrow n_1A_1 + n_2A_2
\]

\[
\Delta G = (\Delta G^0)_{\text{reverse}} + \overline{R} \ T \ln \frac{p^v_1 p^v_2}{p^v_3 p^v_4}
\] (4.90)

As the forward reaction proceeds the driving force for the reverse reaction increases. Because of the competing reactions, the reactive system tends to reach an equilibrium condition in which the two driving forces are equal. When this occurs, we have

\[
n_1g_1 + n_2g_2 = n_3g_3 + n_4g_4
\]

so that \(\Delta G = 0\) at equilibrium and from Eq. (4.87),

\[
\Delta G^0 = - \overline{R} \ T \ln \frac{p^v_3 p^v_4}{p^v_1 p^v_2}
\] (4.91)
In an isothermal process, $\Delta G^0$ and $T$ are constant, and so the logarithm of the quotient on the r.h.s. of Eq. (4.91) is constant for equilibrium at a given temperature. This constant is called the equilibrium constant $K_p$ expressed in terms of partial pressures. Therefore, Eq. (4.91) can be expressed as

$$\Delta G^0 = - R T \ln K_p$$  \hspace{1cm} (4.92)

where

$$K_p = \frac{p_3^{v_3} p_4^{v_4}}{p_1^{v_1}}$$  \hspace{1cm} (4.93)

When any of the reactants is not an ideal gas, the appropriate partial pressure must be replaced by a fugacity ratio. If $A_2$ is not an ideal gas, then

$$K_p = \frac{p_3^{v_3} p_4^{v_4}}{p_1^{v_1} (f_2/f_02)^{v_2}}$$  \hspace{1cm} (4.94)

If one of the compounds is a solid, the fugacity ratio for that compound is unity. If $A_2$ is a solid,

$$K_p = \frac{p_3^{v_3} p_4^{v_4}}{p_1^{v_1}}$$  \hspace{1cm} (4.95)

From Eq. (4.90), the equilibrium constant at a given temperature $T$ is given by

$$K_p = \exp \left( - \frac{\Delta G^0}{RT} \right)$$  \hspace{1cm} (4.96)

With data of $\Delta G^0$ as a function of $T$ being available, the values of log $K_p$ for typical reactions relevant to combustion are computed and plotted in Fig. 4.21.

In the combustion of coal, there are several competing reactions which determine the degree of carbon combustion:

$$\text{C(S) + O}_2(\text{g}) \rightleftharpoons \text{CO}_2(\text{g})$$  \hspace{1cm} (4.97a)

$$\text{C(S) + } \frac{1}{2} \text{O}_2(\text{g}) \rightleftharpoons \text{CO( g)}$$  \hspace{1cm} (4.97b)

$$\text{C(S) + CO}_2(\text{g}) \rightleftharpoons 2\text{CO( g)}$$  \hspace{1cm} (4.97c)

$$\text{CO(g) + } \frac{1}{2} \text{O}_2(\text{g}) \rightleftharpoons \text{CO}_2(\text{g})$$  \hspace{1cm} (4.97d)

The hydrogen and sulphur present are burned to $\text{H}_2\text{O}$ and $\text{SO}_2$. The equilibrium constant for the reaction (4.97a) is given by

$$K_p = \frac{p_{\text{CO}_2}}{p_{\text{O}_2}}$$  \hspace{1cm} (4.98)

and for the reaction (4.97c), it is

$$K_p = \frac{(p_{\text{CO}})^2}{p_{\text{CO}_2}}$$  \hspace{1cm} (4.99)
Fig. 4.21  *Equilibrium constants for selected chemical reactions*

In modern coal burning plants, the gases leaving the flame are above 450 °C and the equilibrium constant $K_p$ is very large for reactions (4.97a), (4.97b) and (4.97d). It is, therefore, normally assumed that these reactions are completed. Reaction (4.95c) is a slower reaction with the relative amounts of CO₂ and CO given by Eq. (4.98). The temperature dependent value of the equilibrium constant is obtained from Fig. 4.21.

4.21 EFFECT OF DISSOCIATION

It was mentioned in Section 4.16 that the effect of dissociation is to reduce the flame or products temperature since heat is absorbed (endothermic process) in the dissociation. At high temperatures of combustion, dissociation of the products becomes significant. For example, in the combustion of carbon monoxide:

$$\text{CO} + \frac{1}{2} \text{O}_2 \rightleftharpoons \text{CO}_2$$

The reaction proceeds in both directions, and a fraction, $a$, of CO₂ produced is dissociated into CO and O₂:

$$a \text{ CO}_2 \rightarrow a \left( \text{CO} + \frac{1}{2} \text{O}_2 \right)$$

In the equilibrium condition, the products consist of

$$(1 - a) \text{ CO}_2 + a \text{ CO} + \frac{1}{2} a \text{ O}_2$$
The degree of dissociation, \( a \), can be determined from the total pressure of the products mixture, \( p \), and the equilibrium constant \( K_p \) (which is a function of temperature). For the above reaction,

\[
K_p = \frac{p_{CO_2}}{p_{CO} (p_{O_2})^{1/2}} \tag{4.100}
\]

Under appropriate conditions carbon can reduce water vapour to produce CO and \( H_2 \), which is an important reaction for coal gasification. However, in the presence of oxygen, CO and \( H_2 \) produced will again get oxidized, and the reduction of steam need not be considered in normal combustion calculations.

In case of incomplete combustion, the amount of heat released depends upon the relative amounts of \( CO_2 \) and CO formed, which in turn depend on the chemical equilibrium achieved as given by

\[
K_p = \frac{p_{CO}^2}{p_{CO_2}}
\]

Let us consider the combustion process as given by the overall equation,

\[
1C + 1H + bS + \left( \frac{a}{4} + b + d \right) (O_2 + 3.76N_2) \xrightarrow{\text{\ visually \}} (1-x-y)C
\]

\[+ yCO_2 + xCO + \frac{a}{2} \text{H}_2\text{O} + b\text{SO}_2 + \left( d + \frac{a}{4} + b \right) (3.76 \text{ N}_2) \tag{4.101}\]

By oxygen balance,

\[
\frac{a}{4} + b + d = y + \frac{x}{2} + \frac{a}{4} + b
\]

\[y + \frac{x}{2} = d \]

or,

\[x = 2 (d - y) \]

The partial pressure of CO is then given by

\[
p_{CO} = \frac{x}{x + y + \frac{a}{2} + b + \left( d + \frac{a}{4} + b \right) 3.76} = \frac{x}{\alpha}
\]

\[\therefore \]

\[
p_{CO} = \frac{2(d - y)}{\alpha}
\]

where \( p_x \) represents the partial pressure of a component \( x \).

Hence:

\[
p_{CO_2} = \frac{1 - \alpha}{1 - \alpha + \alpha + \frac{1}{2} \alpha} p = \frac{1 - \alpha}{1 + \frac{1}{2} \alpha} p \tag{4.102}
\]

\[
p_{CO} = \frac{\alpha p}{1 + \frac{1}{2} \alpha} \quad \text{and} \quad p_{O_2} = \frac{\frac{1}{2} \alpha p}{1 + \frac{1}{2} \alpha}
\]

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\[ K_p = \frac{1 - \alpha}{\alpha} \left( \frac{\alpha + 2}{\alpha p} \right)^{1/2} \]  

(4.103)

**Example 4.1** In an oil fired boiler the fuel had an analysis by mass: carbon 84\%, hydrogen 10\% sulphur 3.2\%, oxygen 1.6\%, remainder incombustible. The analysis of dry flue gas by volume gave: combined \( \text{CO}_2 + \text{SO}_2 \) 15.72\%, \( \text{O}_2 \) 1\%, there being no \( \text{CO} \) or \( \text{SO}_3 \). Calculate per kg of fuel (a) mass of air supplied, (b) percentage excess air supplied, (c) mass of dry flue gas formed, and (c) mass of water vapour formed.

**Solution** Let us consider 100 kg of fuel oil. Let \( x \) moles of oxygen are supplied for combustion. The combustion equation can be written as:

\[
\frac{84}{12} \text{C} + \frac{10}{2} \text{H}_2 + \frac{3.2}{32} \text{S} + \frac{1.6}{32} \text{O}_2 + x\text{O}_2 + 3.76 x\text{N}_2
\]

\[ \rightarrow b\text{CO}_2 + d\text{SO}_2 + e\text{O}_2 + f\text{N}_2 + g\text{H}_2\text{O} \]

Equating coefficients:

Carbon
\[
\frac{84}{12} = b \quad \therefore b = 7
\]

Hydrogen
\[
\frac{10}{2} = g \quad \therefore g = 5
\]

Oxygen
\[
\frac{1.6}{32} + x = b + d + e + \frac{g}{2}
\]

Sulphur
\[
\frac{3.2}{32} = d \quad \therefore d = 0.1
\]

Nitrogen
\[
3.76x = f
\]

Volumetric analysis:
\[
\frac{b + d}{b + d + e + f} = 0.1572, \quad \frac{e}{b + d + e + f} = 0.01
\]

On dividing,
\[
\frac{b + d}{e} = 15.72
\]

or
\[
e = 7.1/15.72 = 0.452
\]

Substituting in \( \text{O}_2 \) balance,
\[
0.05 + x = 7 + 0.1 + 0.452 + 2.5 \quad \text{or} \quad x = 10.002
\]

Mass of oxygen supplied = 10.002 \( \times 32 \) = 320.1 kg

\[ \therefore \text{Mass of air supplied for 100 kg fuel} = \frac{320.1}{0.232} = 1380 \text{ kg} \]

\[ \therefore \text{Mass of air supplied per kg fuel, } W_A = 13.80 \text{ kg} \]  

*Ans. (a)*
Theoretical air required per kg fuel

\[
W_{Th} = 11.5C + 34.5 \left( H - \frac{0}{8} \right) + 4.3S \\
= 11.5 \times 0.84 + 34.5 \left( 0.1 - \frac{0.016}{8} \right) + 4.3 \times 0.032 \\
= 9.66 + 3.38 + 0.14 = 13.18 \text{ kg}
\]

Percentage excess air = \[
\frac{W_A - W_{Th}}{W_{Th}} \times 100 = \frac{13.80 - 13.18}{13.18} \times 100 \\
= 4.7\%
\] \hspace{1cm} \text{Ans.(b)}

Mass of dry flue gas formed for 100 kg fuel

\[
= bCO_2 + dSO_2 + eO_2 + fN_2 \\
= 7 \times 44 + 0.1 \times 64 + 0.452 \times 32 + 3.76 \times 10.002 \times 28 \\
= 308 + 6.4 + 14.46 + 1053.01 = 1381.87 \text{ kg}
\]

Mass of dfg formed per kg fuel = 13.82 kg \hspace{1cm} \text{Ans.(c)}

Mass of water vapour formed per kg fuel

\[
gH_2O = \frac{5 \times 18}{100} = 0.90 \text{ kg}
\] \hspace{1cm} \text{Ans.(d)}

Example 4.2 Propane gas is reacted with air in such a ratio that an analysis of the dry products of combustion gives CO\(_2\) 11.5\%, O\(_2\) 2.7\%, and CO 0.7\%. What is the percentage excess air used?

Solution Let \(x\) moles of propane are reacted by a moles of oxygen supplied. The combustion equation is thus,

\[
xC_3H_8 + aO_2 + 3.76aN_2 \rightarrow 11.5CO_2 + 0.7CO + 2.7O_2 + 85.1N_2 + bH_2O
\]

Equating moles for

Nitrogen: \(3.76a = 85.1\) \hspace{1cm} \text{or} \hspace{1cm} a = 22.65

Carbon: \(3x = 11.5 + 0.7 = 12.2\) \hspace{1cm} \text{or} \hspace{1cm} x = 4.07

Oxygen: \(22.65 = 11.5 + \frac{0.7}{2} + 2.7 + \frac{b}{2}\) \hspace{1cm} \text{or} \hspace{1cm} b = 16.2

Hydrogen: \(8(4.07) = 2(16.2)\)

Therefore, the combustion equation becomes

\[
4.07C_3H_8 + 22.65O_2 + 85.1N_2 \rightarrow 11.5CO_2 + 0.7CO \\
+ 2.7O_2 + 85.1N_2 + 16.2H_2O
\]
For 1 mole of C$_3$H$_8$,

\[ C_3H_8 + 5.57O_2 + 20.91N_2 \longrightarrow 2.83CO_2 + 0.17CO + 0.66O_2 + 20.91N_2 + 3.98 H_2O \]

Stoichiometric combustion equation is

\[ C_3H_8 + 5O_2 + 5(3.76)N_2 \longrightarrow 3CO_2 + 4H_2O + 18.8N_2 \]

Theoretically, 5 moles of oxygen are required. But in the actual case 5.57 moles of O$_2$ are supplied. Therefore, the excess air supplied is

\[
\frac{5.57 - 5.00}{5.00} \times 100 \quad \text{or} \quad 11.4\% 
\]

\textit{Ans.}

**Example 4.3** An unknown hydrocarbon fuel, C$_x$H$_y$, was allowed to react with air. An Orsat analysis was made of a representative sample of the product gases with the following result: CO$_2$ 12.1%, O$_2$ 3.8% and CO 0.9%. Determine (a) the chemical equation for the actual reaction, (b) the composition of the fuel, (c) the air fuel ratio during the test, and (d) the excess or deficiency of air used.

**Solution** The combustion equation can be written as

\[ C_xH_y + aO_2 + 3.76aN_2 \rightarrow 12.1CO_2 + 3.8O_2 + 0.9CO + 83.2N_2 + bH_2O \]

Equating moles for

Nitrogen: \[ 3.76a = 83.2 \quad \text{or} \quad a = 22.1 \]

Oxygen: \[ 2 \times 12.1 = 2 \times 12.1 + 2 \times 3.8 + 0.9 + b \quad b = 11.5 \]

Carbon: \[ x = 12.1 + 0.9 = 13.0 \]

Hydrogen: \[ y = 2b = 2 \times 11.5 = 23 \]

The combustion equation becomes

\[ C_{13}H_{23} + 22.1O_2 + 83.2N_2 \rightarrow 12.1CO_2 + 3.8O_2 + 0.9CO + 83.2N_2 + 11.5H_2O \]

\textit{Ans.}(a)

The composition of the fuel is C$_{13}$H$_{23}$. \textit{Ans.}(b)

Stoichiometric equation is

\[ C_{13}H_{23} + 18.75O_2 + 3.76 (18.75)N_2 \rightarrow 13CO_2 + 11.5H_2O + (18.75)3.76N_2 \]

\[ \% \text{ excess air} = \frac{22.1 - 18.75}{18.75} \times 100 = 18\% \]

\textit{Ans.}(c)

Actual air supplied per mole of C$_{13}$H$_{23}$

\[ = \frac{22.1 \times 32}{0.232} = 3048.276 \text{ kg} \]
Air–fuel ratio = \frac{3048.276}{179} = 17.03 \hspace{1cm} \text{Ans.}(d)

Example 4.4 A steam generator operates under the following conditions:
Coal analysis: carbon 61.0, hydrogen 4.0, oxygen 3.0, nitrogen 2.0, sulphur 1.0 moisture 4.0 and ash 25.0
HHV of coal = 24.3 MJ/kg.
Dry flue gas analysis: CO$_2$ 12.0\%, CO 1.5\%, O$_2$ 7.0\% and N$_2$ 79.5\%
Unburnt carbon = nil
Exhaust gas temperature = 170 °C
Energy loss other than dry exhaust loss and incomplete combustion = 3\% of HHV
Steam generation rate = 150 t/h
Steam condition at boiler outlet = 100 bar, 500 °C
Feedwater inlet temperature = 160 °C

Determine (a) the amount of dry flue gas produced per kg fuel, (b) the dry exhaust loss and incomplete combustion loss per kg fuel, (c) the boiler efficiency, (d) the fuel burning rate, (e) the percentage of excess air used, and (f) the percentage of energy absorbed in the superheater. Given: heat of reaction of C + \frac{1}{2} O$_2$ \rightarrow CO$_2$ is 33.083 MJ/kg carbon and that of C + \frac{1}{2} O$_2$ \rightarrow CO is 9.5 MJ/kg carbon, $\bar{c}_p$ of dfg is 1.05 kJ/kg K, and the ambient air temperature is 30 °C.

Solution Mass of dry flue gas produced per kg fuel
\[ = \frac{C_{ab} (44CO_2 + 28CO + 32O_2 + 28N_2)}{12(CO_2 + CO)} \]
\[ = \frac{0.61(44 \times 0.12 + 28 \times 0.015 + 32 \times 0.07 + 28 \times 0.795)}{12(0.12 + 0.015)} \]
\[ = \frac{0.61 \times 30.2}{1.62} = 11.37 \text{ kg} \hspace{1cm} \text{Ans.}(a) \]

Energy loss due to dry exhaust gases
\[ = w_{dfg} \bar{c}_p (t_g - t_a) = 11.37 \times 1.05 \times (170 - 30) \]
\[ = 1671.39 \text{ kJ/kg fuel} \hspace{1cm} \text{Ans.}(b) \]

Energy loss due to incomplete combustion
\[ = w_{dfg} \frac{28CO}{44CO_2 + 28CO + 32O_2 + 28N_2} \times (33083 - 9500) \frac{12}{28} \]
\[ = 11.37 \frac{28 \times 0.015}{44 \times 0.12 + 28 \times 0.015 + 32 \times 0.07 + 28 \times 0.795} \times 10107 \]
\[
\frac{11.37 \times 0.42 \times 10107}{30.2} = 1598.18 \text{ kJ/kg fuel} \quad \text{Ans.(c)}
\]

Energy loss other than dry exhaust loss and incomplete combustion loss = 0.03 \times 24.3 = 0.729 \text{ MJ/kg fuel}

Total energy loss = 1.671 + 1.598 + 0.729 = 3.998 \text{ MJ/kg fuel}

Boiler efficiency = \frac{\text{HHV} - \Sigma \text{losses}}{\text{HHV}} \times 100 = \frac{24.3 - 3.998}{24.3} \times 100

= 83.54\% \quad \text{Ans.(c)}

Again,
\[\eta_{\text{boiler}} = \frac{w_s (h_1 - h_f)}{w_f \times \text{HHV}}\]

At 100 bar, 500 °C, \( h_1 = 3373.7 \text{ kJ/kg} \)

At 160 °C, \( h_f = 675.55 \text{ kJ/kg} \)

On substitution, 0.8354 = \frac{150,000(3373.7 - 675.55)}{w_f \times 24300}

\therefore \quad w_f = 19937 \text{ kg/h} = 19.937 \text{ t/h} = 5.538 \text{ kg/s} \quad \text{Ans.(d)}

Theoretical air required per kg fuel,
\[W_{\text{Th}} = 11.5C + 34.5 \left( H - \frac{O}{8} \right) + 4.3S\]
\[= 11.5 \times 0.61 + 34.5 \left( 0.04 - \frac{0.03}{8} \right) + 4.3 \times 0.01\]
\[= 7.015 + 1.25 + 0.043 = 8.31 \text{ kg} \]

Actual air supplied per kg fuel
\[W_A = \frac{3.04N_2C_{ab}}{CO_2 + CO} \frac{1}{0.768} \text{ N}\]
\[= \frac{3.04 \times 0.795 \times 0.61}{0.12 + 0.015} \times \frac{1}{0.768} \times 0.02 = 10.89 \text{ kg} \]

Percentage excess air used = \frac{W_A - W_{\text{Th}}}{W_{\text{Th}}} \times 100 = \frac{10.89 - 8.31}{8.31} \times 100

= 31.05\% \quad \text{Ans.(e)}

At 100 bar, \( h_g = 2724.7 \text{ kJ/kg} \)

Percentage of energy absorbed in the superheater
\[\frac{h_1 - h_g}{h_1 - h_f} \times 100 = \frac{3373.7 - 2724.7}{3373.7 - 675.55} \times 100 = 24.05\% \quad \text{Ans.(f)}\]
Example 4.5  The ultimate analysis of a fuel oil is given to be: carbon 83.7%, hydrogen 12.7, sulphur 0.7, nitrogen 1.7 and oxygen 1.2. The combustion air has a dry bulb temperature of 27 °C and a wet bulb temperature of 21°C. With 30% excess air and assuming complete combustion, find (a) the total volume of combustion products at 200 °C and 1.013 bar, and (b) the dry flue gas analysis based on CO₂, O₂ and N₂.

Solution  Theoretical air required per kg fuel,

\[ W_{\text{th}} = 11.5 \times 0.837 + 34.5 \left(0.127 - \frac{0.012}{8}\right) + 4.3 \times 0.007 \]

\[ = 13.98 \text{ kg} \]

\[ W_A = 1.3 \times 13.98 = 18.174 \text{ kg/kg fuel} \]

From the psychrometric chart, at DBT = 27 °C and WBT = 21°C, specific humidity = 0.0132 kg moisture/kg dry air.

Water vapour entering with air per kg fuel

\[ = 18.174 \times 0.0132 = 0.24 \text{ kg vap/kg fuel} \]

Total water vapour formed per kg fuel = 9H + M + \( \gamma_A W_A \)

\[ = 9 \times 0.127 + 0 + 0.24 \]

\[ = 1.383 \text{ kg} \]

Mass of combustion gases per kg fuel:

\[ \text{CO}_2 = \frac{44}{12} \times 0.837 = 3.07; \text{CO} = 0 \text{ (complete combustion)} \]

\[ \text{O}_2 = 0.232 \times 0.30 \times 13.98 = 0.97 \]

\[ \text{N}_2 = 0.768 (1 + 0.3) \times 13.98 + 0.017 = 13.99 \]

\[ \text{SO}_2 = \frac{64}{32} \times 0.007 = 0.02; \quad H_2O = 1.383 \]

Moles of combustion gases formed per kg fuel

\[ = \frac{3.07}{44} + \frac{0.97}{32} + \frac{13.99}{28} + \frac{0.02}{64} + \frac{1.383}{18} \]

\[ = 0.0698 + 0.0303 + 0.4992 + 0.0003 + 0.0769 = 0.6765 \]

Volume of flue gases at 200 °C, 1.013 bar per kg fuel

\[ V_G = 0.6765 \times 22.4 \times \frac{273 + 200}{273} \times \frac{1.013}{1.013} = 26.26 \text{ m}^3 \quad \text{Ans.(a)} \]

Composition of dry flue gas by volume:

\[ \text{CO}_2 = \frac{0.0698}{0.0698 + 0.0303 + 0.4992} = 0.1164 \text{ or } 11.64\% \]
\[
O_2 = \frac{0.0303}{0.5993} = 0.0505 \text{ or } 5.05\%
\]
\[
N_2 = \frac{0.4992}{0.5993} = 0.8331 \text{ or } 83.31\% \quad \text{Ans. (b)}
\]

**Example 4.6** The volumetric analysis of a fuel gas is given as: \(C_2H_6\) 22.6%, \(CH_4\) 73.6%, \(CO_2\) 2.4% and \(N_2\) 1.4%. Assuming combustion air to be dry and in 25% excess, find (a) the molecular weight of the combustion products, (b) the total gas volume for complete combustion at 260 °C, 1.013 bar, and (c) the dry flue gas analysis based on \(CO_2\), \(O_2\) and \(N_2\).

**Solution** Assuming 100 moles of fuel gas, the combustion equation can be written as
\[
22.6C_2H_6 + 73.6CH_4 + 2.4CO_2 + 1.4N_2 + 226.3 \left( O_2 + 3.76N_2 \right) \times 1.25
\]
\[= 121.2CO_2 + 215.0H_2O + 56.6O_2 + 1065.4N_2 \]
Molecular weight of fuel gas
\[= 0.226 \times 30 + 0.736 \times 16 + 0.024 \times 44 + 0.014 \times 28 = 20 \text{ kg/kg mol} \]
Moles of air supplied \[= \frac{226.3(32 + 3.76 \times 28) \times 1.25}{28.96} = 1340 \]
Total reaction moles = 100 moles of fuel gas + 1340 moles of air = 1440 moles of combustion gas
Molecular weight of combustion gas
\[= \frac{121.2 \times 44 + 215 \times 18 + 56.6 \times 32 + 1065.4 \times 28}{1440} \]
\[= \frac{40845.2}{1440} = 28.36 \text{ kg/kg mol} \quad \text{Ans. (a)} \]
Total number of moles of combustion gas per kg fuel gas
\[= \frac{1440 \text{ kg mol}}{100 \text{ kg mol fuel gas}} \times \frac{\text{kg mol of fuel gas}}{20 \text{ kg}} = 0.72 \]
Volume of combustion products per kg fuel gas
\[V_G = 0.72 \times 22.4 \frac{273 + 260}{273} = 31.49 \text{ m}^3/\text{kg fuel} \quad \text{Ans. (b)} \]
Gas analysis by volume:
\[CO_2 = \frac{121.2}{121.2 + 56.6 + 1065.4} = 0.098 \text{ or } 9.8\% \]
\[O_2 = \frac{56.6}{1243.2} = 0.046 \text{ or } 4.6\% \]
\[ N_2 = \frac{1065.4}{1243.2} = 0.856 \text{ or } 85.6\% \quad \text{Ans.}(c) \]

**Example 4.7** Test data show the following gas analyses before and after the air preheater.

<table>
<thead>
<tr>
<th></th>
<th>Entering</th>
<th>Leaving</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>9.7</td>
<td>9.2</td>
</tr>
<tr>
<td>O₂</td>
<td>4.0</td>
<td>4.9</td>
</tr>
<tr>
<td>N₂</td>
<td>86.3</td>
<td>85.9</td>
</tr>
</tbody>
</table>

The coal used shows a carbon percentage of 72% by mass. Estimate the air leakage into the air preheater per kg of coal fired.

**Solution** Before air preheater:

\[
W_A = \frac{3.04 \times N_2 \times C_{ab}}{CO_2 + CO} = \frac{3.04 \times 0.859 \times 0.72}{0.097}
\]

\[= 19.42 \text{ kg air/kg fuel}\]

After air preheater:

\[
W_A' = \frac{3.04 \times 0.859 \times 0.72}{0.092} = 20.42 \text{ kg air/kg fuel}
\]

\[\therefore \text{ Air leakage } = W_A - W_A' = 1 \text{ kg air/kg fuel}\]

**Example 4.8** A textile factory has a battery of 6 Lancashire boilers, each supplying 6 t/h of steam at 16 bar, 250 °C from feedwater at 30 °C. The boilers burn fuel oil of calorific value 43.96 MJ/kg with an overall efficiency of 75%. For efficient combustion 16 kg of air per kg fuel is required for which a draught of 20 mm of water gauge is required at the base of the chimney. The flue gases leave the boilers at 320 °C. The average temperature of the gases in the stack may be taken to be 300 °C. The atmosphere is at 30 °C.

Assuming the velocity of gases at stack exit to be negligible, determine the height of the stack and the diameter at its base.

**Solution** The draught produced is given by

\[
Dp = gH (\rho_a - \rho_g) = g\frac{p_a}{R_a} \left( \frac{1}{T_a} - \frac{1}{T_g} \right)
\]

\[20 \times 10^{-3} \text{ m} \times g \times 10^3 \frac{\text{kg}}{m^3} \text{ s}^2 = g \times H \times \frac{1.013 \times 100}{0.287} \left( \frac{1}{303} - \frac{1}{573} \right)
\]

\[\therefore H = \frac{20 \times 0.287 \times 303 \times 573}{1013 \times 270} \text{ m} = 36.44 \text{ m} \quad \text{Ans.}\]
Now \[ \eta_{\text{boiler}} = \frac{w_f (h_f - h_{\text{feed}})}{w_f \times CV} \]

\[ 0.75 = \frac{6000 \times 6 (2919.2 - 125.8)}{w_f \times 43,960} \]

\[ w_f = \frac{36000 \times 2793.4}{0.75 \times 43,960} = 3050.12 \text{ kg/h} = 0.847 \text{ kg/s} \]

\[ w_a = 16 \times 0.847 = 13.55 \text{ kg/s} \]

\[ w_{fg} = 17 \times 0.847 = 14.4 \text{ kg/s} \]

Again,

\[ w_{fg} = \rho_{fg} AV_g = \frac{\rho_{fg} \pi}{RT_g} \frac{\pi}{4} D^2 \left(2gH\right)^{1/2} \]

\[ 14.4 = \frac{101.3 \times \pi}{0.287 \times 593} \frac{\pi}{4} D^2 \left(2 \times 9.81 \times 36.44\right)^{1/2} \]

\[ D^2 = \frac{14.4 \times 0.287 \times 593 \times 4}{101.3 \times \pi \times 26.74} = 1.152 \]

or \[ D = 1.073 \text{ m} \quad \text{Ans.} \]

**Example 4.9** Compute the required motor capacity needed for the FD fan under the following conditions:

- Coal rate: 10 tonnes/h
- Coal analysis: C 78.0, H 3.0, O 3.0, S 1.0, M 7.0, A 8.0
- Excess air: 30%
- Plenum chamber pressure: 180 mm water gauge
- Mechanical efficiency of the fan: 60%
- Room temperature: 30 °C

**Solution**

\[ W_{Th} = 11.5C + 34.5 \left(H - \frac{O}{8}\right) + 4.3S \]

\[ = 11.5 \times 0.78 + 34.5 \left(0.03 - \frac{0.03}{8}\right) + 4.3 \times 0.01 \]

\[ = 9.907 \text{ kg air/kg fuel} \]

\[ W_A = 9.907 \times 1.3 = 12.9 \text{ kg air/kg fuel} \]

\[ v_{\text{air}} = \frac{RT}{p} = \frac{0.287 \times 303}{101.325} = 0.858 \text{ m}^3/\text{kg} \]

FD fan motor capacity = \[ \frac{W_A \times w_f \times v_{\text{air}} \Delta p}{\eta_{\text{fan}}} \]
\[= 12.9 \times 10000 \times 0.858 \times 180 \times 10^{-3} \times 9.81 \times 10^3 \]
\[= \frac{3600 \times 0.6}{9048 \text{ W} = 90.48 \text{ kW}} \quad \text{Ans.} \]

**Example 4.10** For the above boiler (Example 4.9), if the draught produced by the ID fan is 250 mm water gauge, the gas temperature is 180 °C and the fan efficiency is 52%, find the motor capacity of the ID fan.

**Solution**

\[w_{fg} = w_r + w_a = 10 \times 10^3 + 12.9 \times 10 \times 10^3 \]
\[= 139 \times 10^3 \text{ kg/h} = 38.61 \text{ kg/s} \]
\[v_{fg} = v_a \frac{t_g + 273}{t_a + 273} = 0.858 \frac{453}{303} = 1.283 \text{ m}^3/\text{kg} \]

ID fan motor capacity

\[= \frac{38.61 \times 1.283 \times 250 \times 10^{-3} \times 9.81 \times 10^3}{0.52} \text{ W} \]
\[= 233632 \text{ W} = 233.63 \text{ kW} \]

**Example 4.11** During a boiler trial a partial analysis of dry flue gas showed 13.2% CO2 and 3.2% O2 by volume. Some CO was probably present but its percentage was not measured. The analysis of the coal burnt was 88% carbon, 4.4% hydrogen and 7.6% ash. The moisture in the fuel was nil. Assuming that all the carbon and hydrogen have been burnt, estimate (a) the complete volumetric composition of the dry flue gas, (b) the actual amount of air supplied per kg coal, (c) mass of water vapour formed per kg coal, and (d) the dew point temperature of the flue gas.

**Solution** For 100 kg of coal, let \(a\) moles of oxygen are supplied.

\[
\frac{88}{12} \text{C} + \frac{4.4}{2} \text{H}_2 + a\text{O}_2 + 3.76a\text{N}_2 \rightarrow b\text{CO}_2 + d\text{CO} + e\text{O}_2 + f\text{N}_2 + g\text{H}_2\text{O} \]

Carbon balance: \[\frac{88}{12} = 7.333 = b + d \quad (1)\]

Hydrogen balance: \[2.2 = g \]

Oxygen balance: \[a = b + \frac{d}{2} + e + \frac{g}{2} = b + \frac{d}{2} + e + 1.1 \quad (2)\]

Nitrogen balance: \[3.76a = f \quad (3)\]

From dfg analysis, \[0.132 = \frac{b}{b+d+e+f} \quad (4)\]
\[ 0.032 = \frac{e}{b + d + e + f} \quad (5) \]

Dividing (4) by (5), \( \frac{b}{e} = 4.125 \) \quad (6)

From Eqs (1), (2) and (6),

\[ a = 7.333 - d + \frac{d}{2} + \frac{7.333 - d}{4.125} + 1.1 = 10.21 - 0.742d \]

\[ \therefore \quad f = 38.39 - 2.79d \]

From Eq. (4),

\[ 0.132 = \frac{\frac{7.333 - d}{7.333 - d + d + \frac{7.333 - d}{4.125}} + 38.39 - 2.79d}{47.5 - 3.032d} = \frac{7.333 - d}{47.5 - 3.032d} \]

\[ \therefore \quad d = 1.772 \]

\[ \therefore \quad a = 10.21 - 0.742 \times 1.772 = 8.895 \]

\[ b = 7.333 - 1.772 = 5.561 \]

\[ e = \frac{5.561}{4.125} = 1.348 \]

\[ f = 3.76 \times 8.895 = 33.45 \]

Mass of air supplied for 100 kg coal = \( \frac{8.895 \times 32}{0.232} = 1227 \) kg

\[ \therefore \quad \text{Mass of air supplied per kg coal} = 12.27 \text{ kg} \]

\[ \text{Total number of moles of dfg} = b + d + e + f = 42.131 \]

\[ \% \text{ CO}_2 \text{ by volume} = \frac{5.561}{42.131} \times 100 = 13.20\% \]

\[ \% \text{ O}_2 \text{ by volume} = \frac{1.348}{42.131} \times 100 = 3.20\% \]

\[ \% \text{ CO} \text{ by volume} = \frac{1.772}{42.131} \times 100 = 4.20\% \]

\[ \% \text{ N}_2 \text{ by volume} = \frac{33.45}{42.131} \times 100 = 79.40\% \]

\[ \text{Mass of water vapour formed per kg coal} = \frac{2.2 \times 18}{100} = 0.396 \text{ kg} \]

\[ \text{Mole fraction of water vapour in flue gas} \]

\[ \text{Ans.}(b) \]

\[ \text{Ans.}(a) \]
\[ s = \frac{g}{b + d + e + f + g} = \frac{2.2}{42.131 + 2.2} = 0.0496 \]

\[ \therefore \text{Partial pressure of water vapour} = x_{H_2O} P_{\text{total}} = 0.0496 \times 101.325 = 5.026 \text{ kPa} \]

\[ \therefore \text{Dew point temperature (from steam tables)} = 32.9 \degree C \quad \text{Ans.} (d) \]

**Example 4.12** A 200 m high 4 m dia stack emits 1000 kg/s of 100 \degree C gases into 5 \degree C air. The prevailing wind velocity is 50 km/h. The atmosphere is in a condition of neutral stability. Calculate the height of the gas plume.

**Solution** Using the correlation of Carson and Moses,

\[ \Delta H = 2.62 \left( \frac{Q_e}{V_w} \right)^{1/2} - 0.029 \frac{V_s D}{V_w} \]

where,

- \( V_s \) = stack gas exit velocity, m/s
- \( D \) = stack dia, m
- \( V_w \) = wind velocity, m/s
- \( Q_e \) = heat emission from plume, watts = \( \dot{m} c_p (T_s - T_a) \)
- \( \dot{m} \) = gas flow rate, kg/s
- \( T_s \) = stack exit gas temperature, K
- \( T_a \) = ambient air temperature, K
- \( Q_e = 1000 \times 1.005 (100 - 5) \]
  \[ = 95475 \text{ kW} = 95.475 \times 10^6 \text{ W} \]
- \( V_w = 50000/(60 \times 60) = 13.89 \text{ m/s} \)
- \( V_s = \dot{m}/\rho A \), where \( \rho = (101.325)/(0.287 \times 373) \)
  \[ = 0.9465 \text{ kg/m}^3 \]
- \( V_s = \frac{1000}{0.9465 \times \frac{\pi}{4} \times 4^2} = 84.08 \text{ m/s} \)

\[ \Delta H = 2.62 \left( \frac{95.475}{13.89} \right)^{1/2} \times 10^3 - 0.029 \frac{84 \times 4}{13.89} \]

\[ = 1843 - 0.7015 = 1842.3 \text{ m} \quad \text{Ans.} \]

**Example 4.13** A gas turbine is supplied with a fuel of calorific value of 20 MJ/kg and gravimetric analysis 65% C, 25% H₂, 10% O₂. The compressor takes in air at 1 bar, 27 \degree C and compresses it isentropically to a pressure of 4 bar. The exhaust gases heat the air leaving the compressor, before it enters the combustion chamber.
Assuming that the regenerator effectiveness is 78%, the expansion in the turbine is isentropic, and the analysis of the dry exhaust gas is 6% CO₂, 1.5% CO, determine the maximum temperature in the cycle, thermal efficiency of the plant, and specific fuel consumption. For air, \( c_p = 1.005 \text{ kJ/kg K} \), \( g = 1.4 \) and for combustion products, \( c_p = 1.15 \text{ kJ/kg K} \), \( g = 1.33 \).

**Solution** Let us consider 1 kg fuel and let \( x \) kg mol of air is supplied. Then the combustion equation can be written as

\[
\frac{0.65}{12} \text{C} + \frac{0.25}{2} \text{H}_2 + \frac{0.10}{32} \text{O}_2 + 0.21x \text{O}_2 + 0.79x \text{N}_2 \rightarrow a\text{CO}_2 + b\text{CO} + c\text{CO}_2 + d\text{N}_2 + e\text{H}_2\text{O}
\]

A carbon balance gives, \( \frac{0.65}{12} = a + b = 0.0542 \)

A hydrogen balance gives, \( \frac{0.25}{2} = e = 0.125 \)

An oxygen balance gives, \( \frac{0.10}{32} + 0.21x = a + \frac{b}{2} + c + \frac{e}{2} \)

A nitrogen balance gives \( 0.79x = d \)

Also,

\[
\% \text{CO}_2 = \frac{a}{a + b + c + d} = 0.06
\]

\[
\% \text{CO} = \frac{b}{a + b + c + d} = 0.015
\]

\[
\therefore \quad \frac{b}{a} = \frac{0.015}{0.06} = \frac{1}{4} = 0.25
\]

\[\therefore \quad a + b = a + 0.25a = 1.25a = 0.0542 \]

or

\[a = 0.0434 \quad \text{and} \quad b = 0.0108\]

\[
\frac{0.10}{32} + 0.21x + d = a + \frac{b}{2} + c + \frac{e}{2} + d
\]

Also, \( 0.0031 + 0.21x + 0.79x = (a + b + c + d) - \frac{1}{2}b + \frac{e}{2} \)

\[
= 0.7233 - \frac{0.0108}{2} + \frac{0.125}{2}
\]

\[
= 0.7804
\]

\[\therefore \quad x = 0.7773 \text{ k mol.} \]

**Hence, air supplied** \( 0.7773 \times 28.96 = 22.54 \text{ kg/kg fuel} \)
\[
\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\gamma} = 4^{0.4/1.4} = 1.486
\]

\[T_2 = 446 \text{ K}\]

An energy balance on the combustion chamber gives

\[w_a c_p (T_2 - T_0) + w_f (C V) = (w_a + w_f) c_p (T_3 - T_0)\]

\[22.54 (1.005) (446 - 288) + 1.0 (20,000) = (22.54 + 1) (1.15) (T_3 - 288)\]

\[T_0 = \text{datum temperature} = 15 \degree \text{C assumed}\]

\[T_3 = 1159 \text{ K} = \text{maximum temperature of the cycle}\]

\[\frac{T_3}{T_4} = (4)^{0.33/1.33}\]

\[\therefore T_4 = 820 \text{ K}\]

Regenerator effectiveness, \(e\) is

\[e = \frac{T_5 - T_2}{T_4 - T_2} = \frac{T_5 - 446}{820 - 446} = 0.78\]

\[\therefore T_5 = 738 \text{ K}\]

\[W_c = w_a c_p (T_2 - T_1) = 22.54 \times 1.005 (446 - 300) = 3307.3 \text{ kW}\]

\[W_T = 23.54 \times 1.15 (1159 - 820) = 9177.1 \text{ kW}\]

\[Q_1 = 23.54 (1.15) (11.59 - 738) = 11396.9 \text{ kW}\]

\[\therefore \eta_{\text{cycle}} = \frac{9177.1 - 3307.3}{11396.9} = 0.515, \quad \text{or} \quad 51.5\% \quad \text{Ans.}\]

Specific fuel consumption

\[
\frac{3600}{9177.1 - 3307.3} = \frac{3600}{5869.8}
\]

\[= 0.613 \text{ kg/kWh} \quad \text{Ans.}\]

Example 4.14 A boiler uses 2000 kg/h of coal. The temperature of air supplied is 300 K, and the average temperature of the flue gas leaving the chimney is 650 K. The 33-m high steel chimney produces a draught of 20 mm of water column. Determine the (a) quantity of air supplied per kg of coal, (b) the draught in terms of column of hot gases, and (c) the base diameter of the chimney, assuming that 10% of the theoretical draught is used for creating the flow velocity of gases through the chimney.

Solution

\[h = 353 H \left[ \frac{1}{T_a} - \frac{m+1}{m} \frac{1}{T_g} \right]\]
\[ 20 = 353 \times 33 \left[ \frac{1}{300} - \frac{m+1}{m} \frac{1}{650} \right] \]

\[ \therefore \quad m = 19.727 \text{ kg air/kg coal} \quad \text{Ans. (a)} \]

(b) Let \( H_g \) be the height of the hot gas column producing the draught. Then

\[ \Delta p = \rho_g g H_g = 353 \ gH_g \left[ \frac{4}{T_a} - \frac{m+1}{m} \frac{1}{T_g} \right] \]

where

\[ \rho_g = 353 \frac{m+1}{m} \frac{1}{T_g} = 353 \times \frac{20.727}{19.727} \times \frac{1}{650} = 0.5706 \ \text{kg/m}^3 \]

\[ H_g = H \left[ \frac{m \frac{T_g}{m+1} \frac{T_a}{T_g} - 1}{m+1} \right] = 33 \left[ \frac{19.73}{20.73} \times \frac{650}{300} - 1 \right] = 35 \text{ m} \]

\text{Ans. (b)}

(c) Mass flow of flue gas = mass of coal + mass of air

\[ = 2000 \ (1 + 19.727)/3600 \]

\[ = 11.515 \ \text{kg/s} \]

\[ \therefore \quad \text{Volume flow of flue gas} = \frac{11.515}{0.5706} = 20.18 \text{ m}^3/\text{s} \]

Flue gas velocity, \( V_g = \sqrt{2gH_g} = \sqrt{2 \times 9.18 \times 0.1 \times 35} \]

\[ = 8.29 \text{ m/s} \]

\[ \frac{\pi}{4} \ d^2 \times V_g = 20.18 \]

\[ \frac{\pi}{4} \ d^2 = \frac{20.18}{8.29} \]

\[ \therefore \quad d = \text{base diameter of chimney} \]

\[ = 1.76 \text{ m} \quad \text{Ans. (c)} \]

Example 4.15 A forced draught fan supplies air at 10 m/s against a draught of 20 mm of water across the fuel bed. Estimate the power required to run the fan if 2500 kg/h of coal is consumed and 16 kg of air is supplied per kg of coal burned. The temperature of the flue gas and the ambient air may be taken as 600 K and 300 K respectively.

If the forced draught fan is replaced by an induced draught fan, what will be the power required to drive the fan?

\textbf{Solution}

Density of air, \( \rho_a = \frac{\rho}{RT} = \frac{101.3}{0.287 \times 300} = 1.176 \ \text{kg/m}^3 \)
Velocity head \( \rho V^2/2 = 1.176 \times \frac{100}{2} = 58.83 \text{ N/m}^2 \)

Static pressure head = 20 mm of water
\[ = 0.02 \times 10^3 \times 9.81 = 196.2 \text{ N/m}^2 \]

Total pressure head to be produced by the fan
\[ \Delta p = 196.2 + 58.83 = 255.03 \text{ N/m}^2 \]

Volume of air to be handled
\[ \dot{V} = \frac{2500 \times 16 \times 1}{3600 \times 1.176} = 9.45 \text{ m}^3/\text{s} \]

\[ \therefore \text{ Power required by the FD fan} = \frac{\dot{V} \Delta p}{1000} \]
\[ = \frac{9.45 \times 255.03}{1000} = 2.41 \text{ kW} \quad \text{Ans.} \]

If an ID fan replaces the FD fan, since the ID fan handles flue gases,

Density of flue gas, \( \rho_g = \frac{m+1}{m} \times \frac{353}{T_g} \)
\[ = \frac{17}{16} \times \frac{353}{600} = 0.625 \text{ kg/m}^3 \]

Velocity head = \( 0.625 \times \frac{100}{2} = 31.25 \text{ N/m}^2 \)

Draught to be produced
\[ \Delta p = 196.2 + 31.25 = 227.45 \text{ N/m}^2 \]

Volume flow rate of flue gas, \( \dot{V}_g \)
\[ = \frac{2500 \times 17}{3600} \times \frac{1}{0.625} = 18.88 \text{ m}^3/\text{s} \]

Power required by the ID fan
\[ = 18.88 \times 227.45 \times 10^{-3} = 4.29 \text{ kW} \quad \text{Ans.} \]

### SHORT-ANSWER QUESTIONS

4.1 What are the fuels used for generation of steam?
4.2 How was coal formed?
4.3 What is rank of coal? What are the different ranks of coal?
4.4 What are the types of coal analysis?
4.5 What is proximate analysis of coal?
4.6 What do you understand by ultimate analysis of coal?
4.7 What are the important coal properties relevant to boilers?
4.8 What is the difference between a caking coal and a free-burning coal?
4.9 What is swelling index?
4.10 Define grindability index of a coal.
4.11 What do you understand by weatherability of coal?
4.12 What is a bomb calorimeter?
4.13 How is lower heating value of coal different from the higher heating value?
4.14 What is ash softening temperature? What is its effect on boiler performance?
4.15 How is fuel oil formed?
4.16 How is oil brought out from the oil-well?
4.17 What are the main constituents of fuel oil?
4.18 What is the ultimate analysis of fuel oil?
4.19 What are the required properties of fuel oil for boiler firing?
4.20 What is pour point?
4.21 What is flash point?
4.22 How is the natural gas formed?
4.23 What is the approximate heating value of natural gas?
4.24 What are the major constituents of natural gas?
4.25 What is compressed natural gas? What is its probable application?
4.26 What is LPG? What are its constituents? Where is it used?
4.27 What is an emulsion?
4.28 How does an emulsion of water in heavy oil promote combustion?
4.29 What is a coal-oil mixture (COM)?
4.30 How is COM prepared?
4.31 What are the merits of COM as a boiler fuel?
4.32 What is the stabilizer used for COM?
4.33 Explain the merits and demerits of the coal-water mixture as a boiler fuel.
4.34 What are the industrial wastes and byproducts used as boiler fuels?
4.35 Evaluate wood waste as a boiler fuel.
4.36 What is an incinerator? Where is it used?
4.37 What are synfuels?
4.38 How is coal gasified?
4.39 What is a town gas?
4.40 What is water gas reaction?
4.41 How are gases classified according to heating value?
4.42 What is synthesis gas?
4.43 What is pipeline gas?
4.44 What is hydrogasification?
4.45 What is underground gasification of coal?
4.46 What are the attractive features of underground gasification of coal?
4.47 How is coal liquefied? What are the different products?
4.48 What is Fischer—Tropsch process?
4.49 What is Sasol plant in South Africa famous for?
4.50 What are the basic modes of liquefying coal?
4.51 What is hydrogenation process?
4.52 What do you understand by catalytic conversion process?
4.53 What is hydropyrolysis?
4.54 What is biomass? How is it formed?
4.55 What are biofuels? How can these be produced?
4.56 What are the routes of bioconversion?
4.57 What is thermochemical conversion?
4.58 Explain what you understand by biochemical conversion?
4.59 What is anaerobic digestion?
4.60 What is fermentation?
4.61 How is gasohol produced?
4.62 What is an energy farm?
4.63 Explain the scope of an aquatic farm.
4.64 What is combustion?
4.65 What are the four basic conditions to be fulfilled to burn a fuel efficiently?
4.66 What is MATT?
4.67 Why is excess air always required to be supplied for combustion? What is dilution coefficient?
4.68 What are the different methods of measuring the amount of excess air supplied?
4.69 Why is the flue gas analysis said to be measured on the dry basis?
4.70 How can the degree of burnout of carbon in a fuel be determined?
4.71 What do you understand by the dew point temperature of flue gases? Why is it important in boiler design?
4.72 What is Dulong’s formula? How is it arrived at?
4.73 How does the proper control of the right amount of excess air maintain optimum combustion efficiency?
4.74 Why is the oxygen measurement considered the preferred method for combustion control?
4.75 Give the mass balance of a steam generator.
4.76 Give the dry refuse analysis by mass.
4.77 Enlist the various energy losses taking place in a steam generator.
4.78 Define the efficiency of a steam generator.
4.79 What is draught? What are the functions of the draught system?
4.80 How is draught produced in a steam generator?
4.81 What is natural draught? How is it caused?
4.82 What is the function of stack in a boiler?
4.83 What do you understand by (a) dispersion, (b) plume height with reference to a stack?
4.84 What is mechanical (or artificial) draught?
4.85 What is the function of the FD fan? Where is it located?
4.86 What is the function of the ID fan? Where is it located?
4.87 Explain what you understand by balanced draught.
4.88 What is a pressurized furnace? What are its characteristic features?
4.89 How are pressure and volume flow of flue gases measured?
4.90 What are the different types of fans used for producing draught?
4.91 Why are centrifugal fans with backward-curved blading normally used for FD fans?
4.92 Why are centrifugal fans with forward-curved blading normally used for ID fans?
4.93 Explain the two common methods of controlling the output of fans.
4.94 Explain the characteristic features of damper control.
4.95 What are the advantages of the variable speed control of fans? What is the effect of speed on fan performance? What are the types of drives for the fan?
4.96 What is (a) a primary air fan, (b) a gas recirculation fan?
4.97 How is the noise of fans reduced?
4.98 Define (a) enthalpy of formation, (b) internal energy of combustion, and (c) enthalpy of combustion.
4.99 What do you understand by (a) heating value at constant pressure, and (b) heating value at constant volume?
4.100 Explain what you understand by adiabatic flame temperature. What is its importance in the design of a combustor? How is it controlled?
4.101 Explain the effect of dissociation on the combustion temperature.
4.102 What is free energy of formation? What is the standard free energy change?
4.103 Define the equilibrium constant of a chemical reaction. How is it related to the free energy change?

**PROBLEMS**

4.1 A bituminous coal has the following composition: carbon 71.6, hydrogen 4.8, oxygen 6.3, nitrogen 1.3, sulphur 3.4, moisture 3.5 and ash 9.1. Calculate for complete combustion per kg of coal (a) theoretical weight of air per kg of coal, (b) actual weight of air required with 60% excess air, (c) actual volume of air at 1.013 bar, 25 °C, and (d) HHV of coal. Write the complete combustion equation and estimate the LHV of coal at the dew point temperature.

4.2 The analysis of fuel oil is as follows; carbon 83.9, hydrogen 13.9, sulphur 2.2, oxygen 0.0, nitrogen 0.0. Calculate for complete combustion with 20% excess air (a) the flue gas volume in m³/kg of oil at 200 °C and 1.013 bar, and (b) dry gas volumetric analysis.

4.3 A fuel gas consists of 85% methane and 15% ethane by volume. What should be the CO₂ meter reading if the above fuel is burnt with 10% excess air?

\[ \text{Ans. 10.9%} \]

4.4 The wet products of combustion from an oil-fired steam generator furnace pass through a tubular air preheater. The entering temperature is 320 °C. the exit temperature is 130 °C and the mean \( c_p \) is 1.09 kJ/kg K. The oil has a percentage composition by weight: carbon 87.6, hydrogen 9.0, oxygen 2.0, moisture 1.4. The dry volumetric analysis of the products: CO₂ 14.6% and O₂ 2.92%, the remainder being nitrogen. The temperature rise of preheated
air is 180°C. Calculate (a) the air-fuel ratio, and (b) the heat exchanger effectiveness.  

[Ans. (a) 15, (b) 0.82]

4.5 The following data refer to a certain steam generator with pulverized coal system:
Evaporation rate: 150 t/h
Design pressure: 150 bar
Superheater outlet temperature: 500 °C
Economiser inlet temperature of feedwater: 180 °C
Overall boiler efficiency: 90%
Calorific value of coal: 23.1 MJ/kg
Heat release rate in furnace: 186.5 kW/m³
Width: Length: height of furnace = 1 : 2 : 3
Determine (a) the dimensions of the furnace, (b) the fuel burning rate, (c) the percentage of total heat that each of the surface (economiser, evaporator and superheater) contributes.

4.6 The coal supplied to a furnace had the following composition: carbon 85.5, hydrogen 5.0, oxygen 4.0 and incombustibles 5.5. During a test it was found that the ashes discharged from the furnace contained 0.015 kg of unburnt carbon per kg of coal supplied to the furnace. The analysis of the dry flue gas showed CO₂ 14.5% and CO 1.3% by volume. Calculate the mass of air supplied and the complete volumetric composition of the flue gas.

[Ans. 13 kg/kg coal, CO₂ 13.76, CO 1.24, O₂ 3.0, N₂ 76.65 and H₂O 5.35% by volume]

4.7 The percentage analysis by mass of the coal used in a boiler was: Carbon 83, hydrogen 6, oxygen 5 and ash 6. The dry flue gas contained CO₂ 10.50, CO 1.30, O₂ 7.67 and N₂ 80.53, % by volume. The temperatures of the air and flue gas were 15 °C and 215 °C respectively. Find (a) the total mass of air supplied per kg coal, (b) the partial pressure of the steam in the hot flue gas, if the pressure of the flue gas is 1 bar, (c) the heat carried away by the moist flue gas per kg of coal fired including that due to the CO present.

[Ans. (a) 17.13 kg/kg coal, (b) 0.05 bar, (c) (1624 × 4.187 ) kJ/kg coal]

4.8 A coal analysis by mass is 73% C, 12% ash, 15% H₂O. When burned in a boiler the residue contains 18% C by mass. The analysis of the dry flue gas, by volume, is CO₂ 11.8%, CO 1.3%, O₂ 5.5%. Calculate the % C in the coal which undergoes combustion and the actual air used.

[Ans. 97%, 13.47]

4.9 The analysis of a coal by mass is 82% C, 6% H, 6% ash, 2% O and 4% H₂O. Calculate the stoichiometric air-fuel ratio. The actual air supplied is 18 kg/kg fuel. Given that 80% of the carbon is completely burnt and all the hydrogen, calculate the volumetric analysis of the dry products.

[Ans. 11.42 kg/kg; 9.1% CO₂, 2.3% CO, 7.2% O₂, 81.4% N₂]

4.10 A boiler is supplied with duel-fuel burners, using natural gas and fuel oil. Analysis of natural gas, by volume, is 94% CH₄, 2% C₂H₆, and 4% CO₂. Analysis of fuel oil, by mass, is 87% C, 13% H₂. The analysis of dry
4.11 A circular tank 10 m long and 1.5 m diameter is used for oil storage. Calculate the number of days’ oil supply the tank can hold for continuous operation at the following conditions: Steam flow, 2000 kg/h; Steam condition, 15 bar, dry saturated; Feedwater temperature, 110°C; Boiler efficiency, 75%; Specific gravity of fuel oil, 0.82; Calorific value of fuel oil, 42 MJ/kg.

[Ans. 4.08 days]

4.12 The following readings, representing percentage volume of the dry flue gas, were noted on a boiler instrument panel: O_2 meter 1.97% and CO_2 meter 16.52% However, the CO_2 analyzer also absorbs SO_2. Therefore, the CO_2 meter reading includes a percentage of SO_2 as well as a percentage of CO_2. If the fuel used has an analysis by mass: carbon, 84%; hydrogen 6%; sulphur 4%, oxygen 4% and ash 2%, and assuming that no CO or SO_3 was formed, determine the correct complete dry flue gas analysis by volume and state the percentage excess air that was supplied.

[Ans. CO_2 15.80, SO_2 0.28, O_2 1.92 and N_2 82.0%, by vol; 9.2%]

4.13 Coal with composition by weight: carbon 75%, hydrogen 5%, oxygen 5%, moisture 8% and ash 7%, is burnt with excess air. The Orsat analysis of the resulting flue gas shows CO_2 9.09%, O_2 10.55%, CO nil and the balance nitrogen. Determine the weight of air used per kg of coal and the percentage of carbon which is not burnt.

[Ans. 19 kg air/kg coal, 0.048 kg unburnt carbon/kg coal]

4.14 The following stack gas was obtained from a furnace burning bituminous coal: CO_2 = 12.6, CO = 0.2, N_2 = 80.7, O_2 = 6.5
The ashpit refuse contains 8.5% combustible matter, and the coal used has the following analysis:

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>C</td>
<td>73.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>5.0</td>
<td>S</td>
<td>3.0</td>
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<tr>
<td>O</td>
<td>6.0</td>
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</tbody>
</table>

Calculate (a) the mass of dry gas produced per kg coal, and (b) the mass of flue gas produced per kg coal, assuming dry combustion air.

4.15 Test data from a boiler are as follows: Coal analysis: C 75.0, H 5.0, O 7.0, N 2.0, S 1.0, M 1.0 and A 9.0
Dry gas analysis: CO_2 12.3, O_2 6.6, N_2 80.8 and CO 0.3
Refuse analysis: ash 92.0, carbon 8.0
Air has 1.7% moisture
Calculate per kg coal (a) actual air used, (b) air theoretically needed with given moisture and (c) percentage of excess air.

4.16 Data from a boiler test are as follows:
Coal analysis: C 57.6, H 3.73, O 5.8, N 1.07, S 3.29, M 12.00, A 16.50
HHV of coal as fired: 25 MJ/kg
Refuse analysis: C 20.0, A 80.0
Dry flue gas analysis: CO₂ 13.0, O₂ 7.0, CO 0.0, N₂ 80.0

Flue gas temperature, 180 °C
Air data: dry bulb, 20 °C, wet bulb, 15 °C
Make an energy balance of the boiler assuming an “unaccounted and radiation loss” as 3.7%.

4.17 A steam generator operates under the following conditions: Coal analysis: carbon 60.0, hydrogen 4.0, nitrogen 2.0, sulphur 1.5, oxygen 3.0, moisture 4.5 and ash 25.0. Dry flue gas analysis: CO₂ 12.0, CO 1.5, O₂ 7.0 and N₂ 79.5%.

Steam condition at boiler outlet: 100 bar, 500 °C
Feedwater inlet temperature: 170 °C
Steam generation rate: 160 tonnes per hour
Steam generator efficiency: 85%

HHV of coal: 21 MJ/kg

Room temperature: 30 °C, C_{fg} = 1.08 kJ/kgK

Determine (a) the excess air coefficient, (b) the fuel burning rate, (c) the amount of dry flue gas produced per kg coal, and (d) the dry exhaust loss per kg coal. If the pressure losses before and after the furnace are 500 mm and 800 mm of w.g. respectively, and the exhaust gas temperature is 160 °C, find (e) the total power consumed by the FD and the ID fans having efficiencies 70% and 60% respectively.

[Ans. (a) 0.3075 or 30.75% excess air, (b) 6.622 kg/s or 23.84 t/h, (c) 11.185 kg/kg coal, (d) 1614 kJ/kg coal, (e) 427.45 kW by FD fan and 1246.4 kW by ID fan]

4.18 Liquid octane (C₈H₁₈) enters the combustion chamber of a gas turbine at 25 °C and air enters from the compressor at 227 °C. It is determined that 98% of the carbon in the fuel burns to form CO₂ and the remaining 2% burns to form CO. What amount of excess air will be required if the temperature of the products is to be limited to 827 °C? Given: (1) values hₚ^0 of C₈H₁₈ (l), CO₂, CO and H₂O (g) are -249952, -393522, -110529 and -241817 kJ/kg mol, respectively. (2) Values of (hₕ^0 - hₙ^0) for O₂ and N₂ are 6088 and 5912 kJ/kg mol, respectively. (3) Values of (hₕ^0 - hₙ^0) for CO₂, CO, H₂O (g), O₂ and N₂ are 33405, 21686, 25978, 22707 and 21406 kJ/kg mol, respectively.

[Ans. Eqn. 100C₈H₁₈ + aO₂ + 3.76 aN₂ ® 784 CO₂ + 16CO + bO₂ + 900H₂O + 3.76aN₂, Actual A/F = 66, Theoretical A/F = 15.05, % Excess air = 338%]

4.19 The products of combustion of an unknown hydrocarbon fuel CₓHₙ have the following composition as measured by an Orsat apparatus: CO₂ 8.0%, CO 0.9%, O₂ 8.8% and N₂ 82.3%. Determine (a) the values of x and y, (b) the air fuel ratio, (c) the composition of the fuel, and (d) the percentage excess air used.

[Ans. (a) C₈.₉ H₁₈.₅, (b) 24, (c) C 85.23%, H14.77%, (d) 61.6%]

4.20 For the chemical reaction,

\[ \text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O} \]
the equilibrium value of the degree of reaction at 1200 K is 0.55. Determine the equilibrium constant and the Gibbs function change.

\[ \text{Ans. } 1.495, -4005 \text{ J/g mol} \]

4.21 A fuel oil composed only of carbon, hydrogen, and sulphur is used in a steam generator. The dry flue gas analysis gives CO\(_2\) 11.7%, CO 0.44%, O\(_2\) 4.002%, SO\(_2\) 0.176% and N\(_2\) 83.682%. Find (a) the fuel mass composition, (b) the air-fuel ratio by mass, (c) the excess air used, in per cent, and (d) the dew point temperature of the flue gases if their pressure is 2 bar.

4.22 A desulphurized coal has the ultimate analysis on dry combustible basis: C 81.1%, H 4.4%, O 2.7%, and N 1.8%, by mass. It burns in 120% air. The reactants were at 25 °C. Find the adiabatic flame temperature.

\[ \text{Ans. 2186 K} \]

4.23 A power plant is situated at sea level. 1650 kg/s of flue gases enter a 5 m dia stack at 140 °C and leave at 110 °C. The outside air temperature is 10 °C. The stack is designed for a driving pressure of 0.007 bar. Using the Carson and Moses correlation, calculate the flue gas plume height if the prevailing winds are at 80 km/h.

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**BIBLIOGRAPHY**

Combustion Mechanism, Combustion Equipment and Firing Methods

The rate of a chemical reaction, as in the combustion process, is greatly influenced by the temperature, concentration, preparation, and distribution of the reactants and mechanical turbulence. All these factors tend to increase contact between molecules of the reactants.

Higher temperature, for example, increases the velocity of molecular movement, permitting more frequent contacts between molecules. The concentration and distribution of the reactants in a given volume are directly related to the contact between interacting molecules. It assumes greater importance as the reaction nears completion. Mechanical turbulence induced by a properly designed burner system permits greater opportunity for the molecular contact.

In the process of generating steam, the furnaces or burner systems provide controlled, efficient conversion of the chemical energy of fuel to heat energy which, in turn, is transferred to the heat-absorbing surfaces of the steam generator. To do this, the firing systems introduce the fuel and air for combustion, mix these reactants, ignite the combustible mixture, and distribute the flame envelope and the products of combustion.

Furnaces can be broadly divided into two types:

1. Grate-fired furnaces
2. Chamber-type or flame furnaces

![Grate-fired furnace](image)

**Fig. 5.1** Grate-fired furnace
Grate-fired furnaces are suitable for burning solid fuels like coal, wood chips, bagasse, urban waste and so on. The schematic of a grate-fired furnace in which fuel is burned in a bed is shown in Fig. 5.1. Combustion of volatiles, fine carbon particles, and carbon monoxide occurs in the space above the fuel bed. In a chamber type or flame furnace (Fig. 5.2), which is used for firing pulverized coal, fuel oil and gas, a mixture of fuel and air is delivered into the furnace chamber. The fuel ignites and burns as it flows in the furnace space.

![Diagram of a chamber-type or flame furnace](image)

**Fig. 5.2** Chamber-type or flame furnace

### 5.1 KINETICS OF COMBUSTION REACTIONS

The combustion of fuel in a furnace is determined by a number of physical and chemical factors. The physical factors include the process of mixing of fuel and air, the size of fuel particles and the surface area exposed for reaction. The chemical factors are related to the temperature and concentration of the reactants. The complex fields of velocities, temperatures and concentrations together determine the kinetics of chemical reactions.

Reactions involved in combustion of fuel proceed with evolution of heat, i.e. they are exothermic. These include the burning of carbon, hydrogen and sulphur in atmospheric air. At high temperatures, some reactions may occur with heat absorption, i.e. they are endothermic,

\[
\begin{align*}
N_2 + O_2 &= 2NO & -180 \text{ kJ/mol} \\
CO_2 + C &= 2CO & -7.25 \text{ MJ/kg}
\end{align*}
\]

These chemical reactions can proceed in either a forward or reverse direction and are called *reversible*. During combustion of fuel in furnaces, the rate of a direct process, say \( C + O_2 \rightarrow CO_2 \), is extremely higher than that of the reverse process, i.e. \( CO_2 \rightarrow C + O_2 \). The equilibrium of these reactions is shifted towards the formation of the final products and therefore, these processes are irreversible.
The intensity of combustion is characterized by the rate of reaction involved. The homogeneous single phase reaction rate is measured in terms of the variation of the concentration of one of the reacting substances, \( \Delta c_A \), mol/m\(^3\) (or g/m\(^3\)), in a given time interval \( \Delta \tau \), or

\[
r = \frac{\Delta c_A}{\Delta \tau} \quad \text{(mol/m}^3\text{s)}
\]  

(5.1)

The rate of a heterogeneous reaction, \( r_s \), is expressed as the quantity of substance \( m \), in kg, that has reacted on the solid particle surface, \( S \), m\(^2\), during a time interval \( \Delta \tau \), s

\[
r_s = \frac{m}{S \Delta \tau} \quad \text{(kg/m}^2\text{s)}
\]  

(5.2)

Reaction rates obey the law of mass action, according to which the rate of a reaction in a homogeneous medium at a constant temperature is proportional at any instant to the product of the concentrations of the reactants:

\[
r = k c_A^m c_B^n
\]  

(5.3)

where \( c_A \) and \( c_B \) are the concentrations of the reactants \( A \) and \( B \) in the gas mixture, \( m \) and \( n \) are the number of moles of \( A \) and \( B \) respectively, and \( k \) is the reaction rate constant which depends on the temperature and the nature of the reactants.

Since the partial pressures of the components in a gas mixture at a certain temperature are proportional to their concentrations, Eq. (5.3) can be written in the form

\[
r = kp_A^m p_B^n
\]  

(5.4)

In heterogeneous combustion, the concentration of the combustible substance (solid fuel) is constant and therefore, the rate of reaction depends only on the concentration of oxygen on the surface of solid fuel (\( c_s \)),

\[
r = k c_s
\]  

(5.5)

If the concentrations of the reactants do not vary with time, the reaction rate is determined by the reaction rate constant \( k \) which depends on the temperature and nature of the reactants, as given by Arrhenius equation,

\[
k = k_o e^{-E/RT}
\]  

(5.6)

where \( k_o \) is a constant, \( E \) is the activation energy, kJ/kg mol, \( \overline{R} \) is the universal gas constant, 8.3143 kJ/(kg mol K) and \( T \) is the absolute temperature.

Figure 5.3 shows the dependence of reaction rate on (a) temperature, (b) activation energy, and (c) concentration of combustible matter. The rate of reaction increases rapidly with temperature. A chemical reaction occurs due to the collisions of the molecules of reacting substances. If all collisions resulted in a reaction, combustion would occur at an enormous rate and \( k \) would be equal to \( k_o \). The energy that is sufficient to destroy the molecular bonds of the starting substances is called the activation energy, \( E \). If the activation energy is high, it is difficult to destroy the molecular bonds of the
original molecules, and so the reaction rate is low. Therefore, as $E$ increases, $r$ decreases. The activation energy of the reaction $C + O_2 = CO_2$ is $E_{CO_2} = 140$ kJ/mol and that of the reaction $C + \frac{1}{2}O_2 = CO$, $E_{CO} = 60$ kJ/mol. Thus, it follows that the rate of formation of CO in carbon oxidation is substantially higher than that of CO$_2$, and CO will be formed predominantly at the surface of burning carbon particles.

![Graphs](image)

**Fig. 5.3** Effect of (a) temperature, (b) activation energy and (c) concentration of combustible matter, on the reaction rate, $LL = lower$ limit, $UL = upper$ limit

Combustion cannot take place at any arbitrary concentration of fuel in the mixture, but only in a definite range of its concentrations in the air. There is a lower concentration limit below which combustion is impossible, and an upper concentration limit when any further increase of the concentration of the fuel prevents combustion. Thus, combustion is possible only in the concentration range between these two limits.

### 5.2 MECHANISM OF SOLID FUEL COMBUSTION

Upon heating, solid fuel particles first undergo a stage of *thermal preparation*, which consists in the evaporation of residual moisture and distillation of volatiles. Fuel particles are heated to a temperature at which volatiles are evolved rapidly (400–600 °C) in a few tenths of a second. The volatiles are then ignited, so that the temperature around a coke particle increases sharply and its heating is accelerated. The combustion of volatiles occurs in 0.2–0.5s. A high yield of volatiles produces enough heat to ignite coke particles. When the yield of volatiles is low, the coke particles must be heated additionally from an external source like a radiant furnace wall. The final stage is the combustion of coke particles at a temperature above 800–1000 °C. This is a heterogeneous reaction (gas–solid), the rate of which depends on the oxygen supply to the reacting surface. The burning of a coke particle takes up 1/2 to 2/3 of the total combustion time which is about 1-2.5 s (for pulverized coal).

In carbon–oxygen reaction, oxygen is first adsorbed from the gas volume on the surface of particles and reacts chemically with carbon to form complex
carbon–oxygen compounds of the type $C_xO_y$, which then dissociate to form $CO_2$ and CO. The resulting reaction at about 1200 °C can be written as

$$4C + 3O_2 = 2CO + 2CO_2$$  \hspace{1cm} (5.7)

The ratio of the primary products, CO/CO$_2$, increases sharply with the increasing temperature of burning particles. At 1700 °C, the resulting reaction becomes

$$3C + 2O_2 = 2CO + CO_2$$  \hspace{1cm} (5.8)

where the CO/CO$_2$ ratio is equal to 2.

The primary reaction products are continuously removed from the surface of particles to the environment. In this process, CO diffusing out encounters the oxygen diffusing into the reacting surface and reacts with it within the boundary layer of gas to form CO$_2$. Consequently, the concentration of oxygen decreases sharply as it approaches the reacting surface, while the concentration of CO$_2$ increases (Fig. 5.4a). At a high temperature, CO can consume all the oxygen supplied, which, consequently, will not reach the reacting solid surface (Fig. 5.4b) and the endothermic reduction of CO$_2$ to CO will occur, with the high combustion temperature maintained due to high heat release (Fig. 5.4).

**Fig. 5.4** Variation of concentration of gases near the burning carbon surface (a) at moderate temperature, (b) at high temperature; $\delta_f$ = thickness of boundary layer of gases

### 5.3 KINETIC AND DIFFUSION CONTROL

The rate of combustion of carbon particles is determined not only by the rate of chemical reaction on the carbon surface but also by the rate of oxygen supply to the reaction zone by molecular and turbulent diffusion. Ash is distributed uniformly throughout the coal substance. With the progress of combustion the reaction surface of the coke particle moves inward leaving behind a layer of porous inert ash, through which also oxygen diffuses for reaction (Fig. 5.5). The
ash layer will be assumed to shed off the particles due to turbulence. The burning particle is surrounded by a boundary layer or gas film across which there is a gradient of oxygen gas concentration (or partial pressure) and in which CO is oxidized to CO₂ while diffusing out from the reacting surface. Outside this gas film, in the main flow, there is no gradient in concentration of any gas or temperature due to turbulent mixing. It may be assumed that the oxygen concentration in the bulk flow, c_b, is equal to the oxygen concentration at the film surface, c_f.

![Diagram of coke particle combustion](image)

**Fig. 5.5  Burning of a coke particle**

The rate of diffusion of oxygen per unit surface area of particle is given by Fick's law:

\[ r_s^d = k_g (c_b - c_s) \]  \hspace{1cm} (5.9)

where \( k_g = D/\delta \) is the mass transfer coefficient, m/s, \( D \) is diffusion coefficient, m²/s, and \( \delta \) is the boundary layer thickness, m.

The diffusion rate of oxygen will be maximum when \( c_s = 0 \),

\[ (r_s^d)_{max} = k_g c_b \]  \hspace{1cm} (5.10)

Let us assume that all the oxygen supplied by diffusion reacts on the particle surface. Then the rate of reaction is given by the Eq. (5.5).

\[ r_s^r = k c_s \]  \hspace{1cm} (5.11)

The maximum reaction rate will occur at \( c_s = c_b \).

\[ (r_s^r)_{max} = k c_b \]  \hspace{1cm} (5.12)

Now, the oxygen consumption in the surface reaction must be equal to the oxygen supply through the boundary film, i.e. \( r_s^d = r_s^r = r_s \). From Eqs (5.9) and (5.11).

\[ r_s^d = r_s = k_g \left( c_b - \frac{r_s^r}{k} \right) \]
\[ r_s = \frac{k_g k}{k_g + k} c_b = \frac{1}{\frac{1}{k} + \frac{1}{k_g}} c_b \]  

(5.13)

Temperature has only a weak effect on mass transfer by diffusion \( (k_g \propto T^{0.5}) \), but it considerably influences the surface reaction rate which increases exponentially with temperature \( (k \propto e^{-E/RT}) \). The nature of variation of the maximum rates of oxygen diffusion to the surface and oxygen consumption by that surface (Fig. 5.6) suggests two principal regions of combustion of coke particles. At temperatures less than 1000 °C, the surface reaction proceeds at a slow rate and oxygen consumption is only a small fraction of the quantity of oxygen supplied to the surface, i.e. \( k \ll k_g \) and \( c_b = c_s \). Then Eq. (5.13) reduces to

![Diagram](image)

**Fig. 5.6** Variation of oxygen concentration near the burning coke particle surface. I = kinetic region, II = transition zone, III = diffusion region of burning

\[ r_s = k c_s \]  

(5.14)

Thus, the total reaction rate is controlled by the kinetics of the chemical reaction on the surface. This temperature region (I) of reactions is called the kinetic combustion zone, and the reaction is said to be kinetically controlled (which depends on the temperature of the reaction surface).

At high temperatures, above 1400 °C, the rate constant of the reaction on the surface increases rapidly and exceeds the maximum rate of oxygen supplied to the surface, which varies only slightly with temperature. Then \( k \gg k_g \) and from Eq. (5.13),

\[ r_s = k_g c_b \]  

(5.15)

In this zone, the reaction rate varies slowly inspite of increasing temperature. Oxygen supplied to the surface by diffusion reacts instantaneously (at high temperature) and its concentration at the surface becomes zero. This temperature region is called the diffusion combustion zone (III). With oxygen deficiency at the surface, the reduction of \( \text{CO}_2 \) to \( \text{CO} \) occurs at the incandescent
coke surface, while the diffusing oxygen is completely consumed in the gas film in oxidizing CO to CO$_2$. In this zone, the rate of combustion increases with the increasing rate of diffusion of gases, i.e. turbulence, and with the decreasing size of solid particles.

At intermediate temperatures (1000–1400 °C) the rate of reaction at the surface commensurates with the rate of oxygen supplied to the surface, and the total rate of the reaction is determined by both the processes. This is called the transition zone of combustion (II).

It can thus be inferred that when the reaction surface is incandescent and its temperature is very high, the chemical resistance at the surface is small and the reaction is almost instantaneous. Solid fuel combustion is most often diffusion controlled. It depends on the rate at which oxygen diffuses into the reaction surface. If the gas film resistance is reduced by inducing turbulence, the rate of diffusion of oxygen and hence the rate of reaction get enhanced. When the air-fuel mixing is very high, as in fluidized beds, the diffusional resistance becomes negligible and the combustion is then kinetically controlled with the combustion rate depending on the surface temperature.

Ignition of any fuel begins relatively at a low temperature with ample oxygen supply and it is essentially in the kinetic zone. As the temperature rises, oxygen consumption in the reaction zone rapidly increases and the process passes through the transition zone into the diffusion zone.

The end of combustion is usually well in the diffusion zone where the residual oxygen concentration becomes low, thus retarding the combustion of the remaining fuel.

### 5.4 COMBUSTION EQUIPMENT FOR BURNING COAL

Coal may be fed into the furnace for combustion in lump pieces or in powder form. Combustion of coal may occur in

- (a) Fuel bed furnaces (coarse particles)
- (b) Pulverized coal furnaces (fine particles)
- (c) Cyclone furnaces (crushed particles)
- (d) Fluidized bed furnaces (crushed small particles)

### 5.5 FUEL BED COMBUSTION

A grate is used at furnace bottom to hold a bed of fuel (Fig. 5.1). There are two ways of feeding coal on to the grate:

- (a) overfeeding
- (b) underfeeding

An overfed fuel bed section receives fresh coal on its top surface. It is characterized by the following distinct zones from top to the bottom (Fig. 5.7).
1. A layer of fresh or green coal
2. A layer of coal losing moisture—drying zone
3. A coking layer of coal losing its volatile content—distillation zone
4. A layer of incandescent coke, where the fixed carbon is consumed—combustion zone
5. A layer of ash progressively getting hotter.

From zone 4 where heat is released by combustion, heat transfer occurs upward and downward by conduction, while the air flow from below will tend to carry away the heat upward by convection.

Primary air ($N_2$, $O_2$, $H_2O$) gets warmed up as it flows through the ash layer. As it passes through the incandescent coke layer (at about 1200 °C), the reaction taking place initially is $C + O_2 = CO_2$. This is an exothermic reaction and provides the heat release for continuing the combustion process. It continues till all the oxygen is consumed. If the incandescent layer is thick, $CO_2$ may be partly or fully reduced to $CO$: $CO_2 + C = 2CO$. A slight water gas reaction may also take place with the moisture from air: $H_2O + C = H_2 + CO$. These are endothermic reactions and may bring down the temperature of the bed and gas stream considerably.

The stream then passes through the distillation zone where the volatile matter is added, and then through the drying zone where the moisture is picked up, and finally emerges above the fuel bed. Its contents are nitrogen, carbon dioxide, carbon monoxide, hydrogen, volatile matter and water vapour.

If the combustion of carbon monoxide, hydrogen and volatile matter is to be completed before the gases leave the furnace, the following have to be provided:

(a) A hot ignition point—in the range 1000–1390 °C. This takes the form of a fire brick-lined arch which stores up the heat and remains at a high temperature.
(b) Sufficient fresh air—secondary or overfire air.
(c) Turbulence—secondary air is fed at right angles to the upflowing gas stream from the fuel bed so as to penetrate the gas mass and secure thorough mixing of air and gases.
It does not help if secondary air is supplied along with primary air, since more primary air produces only more carbon monoxide.

The fixed carbon in coal leaves the fuel bed in the form of CO₂ or CO, both of which are colourless. But carbon combined with hydrogen in the volatile matter leaves the fuel bed as part of the hydrocarbon gases, the molecules of which break down or disintegrate at high temperature of the furnace releasing some free carbon in the form of finely divided particles suspended in the gas stream. If the coal feeding apparatus and the furnace are not properly designed, or if a well-designed coal burning unit is not properly operated, these fine carbon particles may not burn, in which case these will appear at the chimney top as black smoke, which is a certain indication of inefficient combustion and energy wastage.

In under-feeding, coal is fed from below the grate by a screw-conveyor or ram (Fig. 5.8). Primary air passing through holes in the tuyeres diffuses through spaces in the raw or green coal picking up moisture and then through the distillation zone where volatile matter is added. When the gas stream passes next through the incandescent coke region, the volatile matter breaks up and readily burns with secondary air fed at the top. In overfeeding, burning the volatile matter will be somewhat cooler and so need longer time to ignite and burn.

**Fig. 5.8 (a)** *Underfeed fuel bed section receives fresh coal from retort underneath between two tuyeres*

**Fig. 5.8 (b)** *Single-retort stoker showing contour of fuel bed and ash dumps*
5.6 MECHANICAL STOKERS

In small boilers, the grate is stationary and coal is fed manually by shovels. But for more uniform operating condition, higher burning rate and greater efficiency, moving grates or stokers are employed. Stokers may be of the following types:

(a) Travelling grate stoker  
(b) Chain grate stoker  
(c) Spreader stoker  
(d) Vibrating grate stoker  
(e) Underfeed stoker

5.6.1 Travelling Grate Stoker

The grate surface is made up of a series of cast-iron bars joined together by links to form an endless belt running over two sets of sprocket wheels with a surface as wide as needed (Fig. 5.9). A coal gate at the rear of the coal hopper regulates the depth of the fuel bed. The gate can be raised or lowered as needed. Simultaneous adjustment of grate speed, fuel bed thickness, and air flow controls the burning rate so that nothing but ash remains on the grate by the time it reaches the furnace rear. The ash falls into the ash pit as the grate turns on the rear sproket to make the return trip.

**Fig. 5.9 (a)** Travelling grate stoker moving like an endless belt carrying the fuel bed from right to the left
As the raw or green coal on the grate enters the furnace, the surface coal gets ignited from heat of the furnace flame and from radiant heat rays reflected by the ignition arch. The fuel bed becomes thinner toward the furnace rear as the combustible matter burns off. Undergrate air pressures are varied by dampers from front to rear of the stoker to admit gradually reduced quantity of primary air fed by the FD fan. The secondary air aids in mixing the gases and supplies oxygen to complete combustion.

**Fig. 5.9 (b) A chain grate stoker**

### 5.6.2 Chain Grate Stoker

It is similar to the travelling grate stoker except that it is made up of a series of CI links connected by bars or pins to form an endless chain, the principle of its operation being the same.

Overfeed stokers are suited for industrial power plants having steady demand. The grate heat release rate should be limited to a maximum of 1340 kW/m² of active grate area. A relatively wide range of coals can be burned on overfeed stokers, the size varying from 19 to 38 mm with the amount of fines limited to 50% passing through a 6 mm screen. Non-caking or free-burning coal is preferred. Coal having more than 20% ash is difficult to burn efficiently, incurring considerable unburnt carbon loss.
5.6.3 Spread Stoker

Coal from the hopper is fed by a rotating feeder, a drum fitted with short blades on its surface, to the spreader or distributor below, which projects the coal particles in a continuous stream on to the grate holding an ignited fuel bed (Fig. 5.10). The finer particles burn in suspension and the coarser particles are consumed on the grate. The speed of the feeder varies directly with the steam output of the boiler.

![Diagram of Spread Stoker]

**Fig. 5.10** Spread stoker hurling coal particles into furnace

Secondary or overfire air promotes turbulence and completes combustion. High capacity boiler may have a travelling grate in addition to the spreader.

The grate consists of CI bars. Links underneath the grate connect all the bars to a lever. Moving the lever back and forth makes the bars rock about their pivots, which makes the ash fall through to the ash pit below.

The solid fuels burned on spreader stokers include bituminous coal, subbituminous coal, lignite, wood waste from the forest products industry, bagasse from the sugar cane industry, peat from peat bogs, peanut shells, hydrocarbon from agricultural wastes, and refuse-derived fuels (RDF) of municipal solid waste.

It is necessary to size the fuel properly for spreader stokers. Coal should have 95% less than 32 mm. Waste fuels can have top sizes up to 102 mm. Spreader stokers can be applied to a wide range of boiler sizes which can go up to 155 MW for coal and 264 MW for certain waste fuels with steam ing rates of 50 kg/s and 75 kg/s respectively.
5.6.4 Vibrating Grate Stoker

The stoker shakes the fuel bed intermittently, the frequency and amplitude of vibration depending on boiler load. The fuel bed is inclined so that the fuel moves towards the rear of the boiler by gravity with the progress of combustion and then falls into the ash pit. The grate is water-cooled to prevent slagging.

5.6.5 Underfeed Stoker

Underfeed stokers, either single or multiple retort, consist essentially of a trough or troughs into which coal is pushed by rams or screws. Part of the combustion air is introduced into the fuel bed through tuyers. Movement of the fuel particles discourages the formation of large coke masses. Volatile matter is distilled off the coal and burns above the incandescent fuel bed. Coal forms a thick sloping pile (15 to 25°) over the whole surface of the grate and carbonization proceeds from above downward. The coal particles are pushed forward by the ram till the refuse is discharged to the ashpit (Fig. 5.11). Retorts may be set side by side according to width of furnace, up to 18 retorts being used in a furnace 9.6 m wide. Fuel bed is quite thick, which may be up to 0.6 m at the front of the grate.

![Diagram of retort-type stoker](image)

*Fig. 5.11 Principle of retort-type stoker*

In all mechanical stokers forced draught fans are used both for undergrate primary air and overfire secondary air. Zoned primary air admission is used to control excess air. Stoker firing is limited to relatively low capacities (up to 50 kg/s of steam) due to low efficiency and the physical size of the stoker. However, in their size range, stokers remain important in steam generator systems.

Beyond a certain size of boiler, stokers become impracticable and uneconomical due to increased floor area and boiler house cost. Modern high pressure boilers tend to grow vertically upward rather than horizontally, adopting pulverized coal firing through burners.
5.7 PULVERIZED COAL FIRING SYSTEM

The first commercial application of pulverized coal firing for steam generation was made in the early 1920s. Since then it has become almost universal in central utility stations using coal as fuel. Coal is first ground to dustlike size and powdered coal is then carried in a stream of air to be fed through burners into the furnace. As the entering coal particles get heated in high temperature flames in the furnace, the volatile matter is distilled off and this reduces the coal particles to minute sponge-like masses of fixed carbon and ash. The volatile gases mix with the oxygen of the air, get ignited and burn quickly. Oxygen of the hot air reacts with the carbon surface to release energy. The combustion products form a blanket on carbon particles, which is stripped off by turbulent mixing of these particles and air. Proper burning of fuel needs the supply of correct proportion of air, mixing of fuel and air, high temperature, and adequate time to complete combustion reactions. The ash resulting from combustion (i) partly falls to the furnace bottom and (ii) the rest is carried in gas stream as fly ash to flue-gas outlet, or (iii) is deposited on the boiler heating surfaces. Modern central station boiler furnaces have water-cooled walls that form part of the heat-absorbing surfaces in steam generation.

To burn pulverized coal successfully, the following two conditions must be satisfied:

1. Large quantities of very fine particles of coal, usually those that would pass a 200-mesh sieve must exist to ensure ready ignition because of their large surface-to-volume ratio.
2. Minimum quantity of coarser particles should be present since these coarser particles cause slagging and reduce combustion efficiency.

A typical screen analysis of a high volatile bituminous coal sample, pulverized to 80% 200 mesh (0.074 mm opening).

99.5%–50 mesh
96.5%–100 mesh
80.0%–200 mesh

This represents a surface area of roughly 150,000 mm²/g with 97% of the surface in the – 200 mesh portion. By overgrinding and poor classification, it would be possible to have a sample of the following analysis:

95%–50 mesh
90%–100 mesh
80%–200 mesh

This is not a satisfactory grind because of the high percentage retained on the 50 mesh, even though the surface area remains the same. Thus classification plays a major role in matching the particle size to the reactivity of the fuel.

Greater surface area per unit mass of coal allows faster combustion reactions because more carbon becomes exposed to heat and oxygen. This reduces the excess air needed to complete combustion. This also reduces the dry exhaust loss through chimney and raises the steam generator efficiency. However, the extra cost of the pulverizing equipment and grinding energy partly offset these advantages.
Advantages of pulverized coal firing

1. Low excess air requirement
2. Less fan power
3. Ability to use highly preheated air reducing exhaust losses
4. Higher boiler efficiency
5. Ability to burn a wide variety of coals
6. Fast response to load changes
7. Ease of burning alternately with, or in combination with gas and oil
8. Ability to release large amounts of heat enabling it to generate about 2000 t/h of steam or more in one boiler
9. Ability to use fly ash for making bricks etc.
10. Less pressure losses and draught need.

Disadvantages

1. Added investment in coal preparation unit
2. Added power needed for pulverizing coal
3. Investment needed to remove fly ash before ID fan
4. Large volume of furnaces needed to permit desired heat release and to withstand high gas temperature.

However, the advantages far outweigh the disadvantages in large utility central stations, and the net gain has led to the wide use of pulverized coal firing in such systems.

In modern plants the hot air for drying coal in the pulverizer is supplied from the forced draught fan and the air preheater, as shown schematically in Fig. 5.12. Most of the air (about 70%) leaving the air preheater goes directly to the burner windbox on the boiler and is known as secondary air. The remaining air, called the primary air, is used to dry the coal in the pulverizer and convey the powdered coal from the pulverizer to the burners (or storage bins). Burners make the mixing of coal, primary air, and secondary air.

In some instances, the temperature of the air leaving the air preheater is not high enough to dry the coal properly. Then, a primary air heater using steam coils may be used to raise the air temperature, or very hot flue gases from the steam generator may be mixed with the air to raise its temperature. The flue gases must be taken from a point of the steam generator where the concentration of CO₂ is low and must be well diluted with air in the mixing chamber. Otherwise, the presence of the CO₂ will be detrimental to the combustion process.

An advantage of coal pulverizers over stokers is the ability of pulverizers to use hot air at temperatures ranging from 260 °C to 420 °C, depending on the kind of coal. These high air temperatures promote good combustion and permit lower flue gas exit temperatures.
Fig. 5.12 Primary and secondary air in a pulverizer coal firing system

In order to maintain the desired air temperature leaving the pulverizer with variation in coal moisture and varying air temperature with boiler load (Table 5.1), tempering air from the boiler room or forced-draught fan exit is mixed with primary air at the pulverizer inlet to maintain the necessary outlet temperature. Again, if the temperature of the hot primary air is very high then in addition to moisture, some volatile matter may also be distilled off from the coal releasing heat, which may lead to a fire hazard in a cumulative effect. Tempering air may be used to offset this tendency.

Table 5.1 Allowable pulverizer outlet temperatures

<table>
<thead>
<tr>
<th>System</th>
<th>Storage (°C)</th>
<th>Direct (°C)</th>
<th>Semidirect (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. High-rank high-volatile bituminous</td>
<td>54</td>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>2. Low-rank high volatile bituminous</td>
<td>54</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>3. High-rank low volatile bituminous</td>
<td>57</td>
<td>82</td>
<td>82</td>
</tr>
<tr>
<td>4. Lignite</td>
<td>43</td>
<td>43–60</td>
<td>49–60</td>
</tr>
<tr>
<td>5. Anthracite</td>
<td>93</td>
<td>95</td>
<td>95</td>
</tr>
</tbody>
</table>
5.7.1 Crushers

Coal carried in conveyor belt passes through a magnetic separator to remove tramp iron pieces. Then coal is taken to crusher for being broken into the required feed size which in pulverizing mills is 30 mm or below. Two types of crushers are common: the ring crushers and the hammer mill (Figs 5.13 and 5.14). The coal is fed at the top and is crushed by the action of rings that pivot off centre on a rotor or by swinging hammers attached to it. Adjustable screen bars determine the maximum size of the discharged coal.

There is another type of crusher, called the Bradford breaker (Fig. 5.15) which is used for large capacity work. It consists of a large cylinder made up of perforated steel plates to which lifting shelves are attached on the inside. The cylinder rotating slowly at about 20 rpm receives coal at one end. The shelves lift the coal up and then the coal drops down by gravity.

![Diagram of a ring-type coal crusher](image1)

**Fig. 5.13** A ring-type coal crusher

![Diagram of a hammer-mill coal crusher](image2)

**Fig. 5.14** A hammer-mill coal crusher
Repeated lifting and dropping crushes the coal until its size becomes small enough to pass through the perforation (roughly 32 to 38 mm).

### 5.7.2 Pulverizers

There are three stages in the pulverizing process of coal: (a) feeding, (b) drying, and (c) grinding. The feeding system controls the fuel feed rate according to the boiler demand and the required air rate (primary air) for drying, and then transporting the pulverized fuel and primary air stream to the burner. Since coals have varying quantities of moisture and in order that lower-rank coals can be used, dryers are an integral part of the pulverizing equipment. Part of the hot air from the air preheater (primary air) is forced into the pulverizer at about 350 °C by the primary air fan. There it is mixed with the coal as it is being circulated and ground. The heart of the equipment is, however, the pulverizer or the tube mill. Grinding is performed by impact, attrition, crushing, or combinations of these. Pulverizers, commonly used, are classified by speed: (a) low speed (below 75 rpm), the ball tube mill; (b) medium speed (75 to 225 rpm), the ball-and-race mill and bowl mill; and (c) high speed (above 225 rpm), the impact or hammer mill.

The ball tube mill is a hollow horizontal cylinder with conical ends and wear-resistant liners revolving slowly at about 20 rpm with 20–35% of its volume being filled with forged steel balls of mixed size 30–60 mm (Fig. 5.16).
Coal is pulverized by attrition and impact as the balls and coal rise up and fall down with cylinder rotation. Primary air is blown over the charge to carry the pulverized coal to classifiers, which feed back the coarser particles for regrinding. The ball mill is reliable and requires low maintenance, but it is bulky and heavy in construction, consumes more power (about 22 kWh per tonne) and because of poor air circulation works less efficiently with wet coals.

The ball-and-race mill (Fig. 5.17) crushes coal between two moving surfaces, balls and races, by attrition. An upper stationary race and a lower rotating race, driven by a worm and gear, hold the balls between them. Coal enters the mill through the rotating table feeder to fall on the inner races. The moving balls and races catch coal between them to crush it to a powder. Springs hold the upper stationary race and adjust the force needed for crushing. Hot air from F.D. fan enters the mill and carries the coal dust to the classifier above. Mill, feeder and fan need about 14 kWh per tonne of coal pulverized.

**Fig. 5.17**  Ball-race mill (a) B and W type, (b) Principle of working

The bowl mill is widely used for grinding coal. Coal is fed to a rotating bowl and thrown outward by centrifugal force to the rotating ring where it is pulverized by rolling under spring-loaded rollers (Fig. 5.18). Air sweeps by the bowl and removes the product to the classifier, from which large particles are returned for further grinding. This is an intermediate speed pulverizer (vertical
Fig. 5.18 (a) Roller-type ring-roller mill, (b) Bowl mill, (c) Reymond bowl mill
shaft speed about 65–100 rpm) which, with its quiet, vibrationless operation, combines satisfactorily the desirable characteristics of low power consumption and low maintenance cost.

High speed pulverizers use hammer beaters that revolve in a chamber with wear-resistant liners. They are mostly used with low-rank coals with high moisture content and often use flue gas for drying.

### 5.7.3 Performance of Pulverizer

The power consumption of the pulverizer should be low to minimize operating costs. The main factor of performance depends on (a) the grindability of the coal, (b) the surface moisture on the coal, and (c) the fineness of the grind needed.

Hardgrove index is most often used to measure the grindability of a coal. Properly prepared and measured coal samples are ground in a small laboratory mill for 60 revolutions and then the amount of fines that passes through a 200-mesh sieve indicates the Hardgrove index, which normally varies from less than 26 to over 112. A low index number designates coals hard and difficult to grind, while a high index applies to coals soft and easy to grind.

The fineness of the powdered coal needed for successful burning depends largely on the ratio of volatile matter (VM) to fixed carbon. Since the carbon burns slower than the volatile matter, low-VM coals must be ground finer to expose more surface to permit burning more coal in a given residence time in the furnace. Thus, for anthracite (low VM) about 85% of the powdered coal should pass through a 200-mesh sieve, with less than 2% being retained on a 50-mesh screen, whereas only about 60% need pass through a 200-mesh sieve for a high-VM coal like lignite. The finer the grind, the greater the power input needed. Overgrinding is expensive in energy and maintenance incurred by extra wear. Moisture makes fine coal particles adhere to each other, defeating the purpose of grinding to expose more surface. Heated primary air evaporates this moisture in the mill and carries the vapour to the furnace with the coal.

### 5.7.4 Feeding Systems

A pulverized coal system comprises pulverizing, delivery, and burning equipment. It must be capable of continuous operation as well as rapid change as required by load demands. There are three main feeding systems:

1. Bin or storage system
2. Direct firing system
3. Semidirect firing system

**Indirect firing or storage system** In the early pulverized coal fired installations, pulverizers did not discharge directly to the furnace, and were often housed in a separate hall outside the boiler house. The coal was usually dried externally, then fed to the pulverizers, and the pulverized product conveyed to bunkers above the pulverized fuel feeders through which it was fed to the burners.
This direct or storage system has the advantage that the coal-feed and coal-air ratios to individual burners are under the direct control of the operator, and less mill capacity is needed for a given load since the mills always operate at full load. The mills may not be operated during peak load periods if auxiliary power saving is important, depending on the load factor and the pulverizer size selection.

Among its disadvantages are duplication of bunkers, conveying and feeding equipment and so on, which necessitate large capital expenditure, higher auxiliary power requirement, space for cyclone, disposal of dusty air from mill vents and safeguarding against fire hazard.

**Direct firing system**  Indirect firing or storage system has mostly been replaced by direct firing system in modern power plants. It continuously processes the coal from the storage receiver bunker through a feeder, pulverizer,
Fig. 5.20 Semidirect firing system of pulverized coal
and primary air fan to the furnace burners (Fig. 5.19). Fuel flow is suited to load demand by a combination of controls on the feeder and on the primary air fan in order to give air-fuel ratios suitable for various steam-generator loads. Large steam generators are provided with more than one pulverizer system, each feeding a number of burners, so that a wide control range is possible by varying the number of pulverizers and the load on each. This system has greater simplicity and hence greater safety, lower space requirements, lower capital and operating costs, and greater plant cleanliness.

**Semidirect firing system**  A cyclone separator between the pulverizer and furnace separates the conveying medium from the coal. The hot primary air separated in the cyclone is used by the exhauster or primary air fan to push the coal particles, falling by gravity from the cyclone, through the burners into the furnace (Fig.5.20).

### 5.7.5 Pulverized Coal Fired Furnaces

Pulverized coal burns in suspension in the furnace space. Heat released by combustion is transferred to the water wall tubes around the furnace by radiation. The combustion gases leave the furnace at a safe temperature which will not cause clinkering to the subsequent heating surfaces. Water walls consist of vertical tubes arranged mostly in tangent connected with the boiler drum at the top and headers at the bottom. These tubes receive water from the drum via downcomer tubes and discharge the water–steam mixture to the drum.

A furnace can be characterized geometrically by its linear dimensions: front width \( a \), depth \( b \), and height \( h_f \) (Fig. 5.21) which are estimated according to the rated fuel consumption and the thermal, physical and chemical properties of the fuel to be used. The hot furnace gases pass through the cross-sectional area of the furnace \((a_f = ab)\) at a rather high speed (7–12 m/s). The heating power of the furnace (kW) is given by

![Diagram of a furnace showing main dimensions](image)

**Fig. 5.21**  Main dimensions of a furnace
\[ Q_f = w_f \times \text{HHV} \]

where \( w_f \) is the fuel consumption rate, kg/s and HHV is the heating value, kJ/kg. The heat release rate per unit bed cross-section,

\[ q_f = \frac{w_f \times \text{HHV}}{a_f} \text{ kW/m}^2 \]  \( (5.16) \)

is an important characteristic of a furnace. The highest permissible value of \( q_f \) depends on the kind of fuel and the method of burning (arrangement and type of burners), and it may vary from 3500 kW/m\(^2\) for brown coals to 6400 kW/m\(^2\). An increase in \( q_f \) results in an increase in \( w_f \) and temperatures of gases as well as heat absorption by the furnace walls. The rated value of heat release \( q_f \) is taken to be somewhat lower than this highest permissible limit. If this is known, then for a certain heating power of a furnace \((Q_f)\) its cross-sectional area can be obtained from

\[ a_f = \frac{Q_f}{q_f} \]  \( (5.17) \)

The depth \( b \) of a furnace is chosen in the range of 6 m to 10.5 m so that the flame does not impinge the waterwall on the opposite side, which depends on the steaming or fuel rate of the boiler. The depth can be increased to 8–10.5 m when using burners of larger ports in several tiers on the furnace walls.

The front width of the furnace “\( a \)” varies from 9.5 m to 31 m depending on the steaming capacity of the boiler, which can be approximately estimated from

\[ a = 0.67 \sqrt{D} \]  \( (5.18) \)

where \( D \) is the steaming capacity in tonnes per hour.

The height \( h_f \) of the furnace is taken to be in the range 15–65 m to ensure complete combustion of fuel along the flame length and allow space for arranging the water tubes on the furnace walls to cool the combustion gases to the specified temperature and to ensure the desired natural circulation of water–steam mixture. If \( \bar{V}_g \) is the average velocity of gases in the furnace cross-section, m/s and \( \tau \) is the time a unit volume of gas is present in the furnace, s, then the furnace height required to ensure complete combustion is given by

\[ h_f = \bar{V}_g \cdot \tau \]  \( (5.19) \)

The furnace is also characterized by the allowable volumetric heat release rate, i.e. energy release per unit volume, kW/m\(^3\), given by

\[ q_V = \frac{Q_f}{V_f} = \frac{w_f \times \text{HHV}}{V_f} \]  \( (5.20) \)

where \( V_f \) is the volume of the furnace \((a \times b \times h_f)\). The value of \( q_V \) varies from 120 to 210 kW/m\(^3\), the magnitude of which determines the average time the gases are present in the furnace space, \( \tau \). As \( q_V \) increases, \( \tau \) decreases (Fig. 5.22).
Fig. 5.22 *Relation between heat release rate per unit volume* \( (q_v) \) *and the time of residence in furnace* \( (t) \)

The surface areas provided inside the furnace, i.e. the water walls and secondary superheater, absorb heat by radiation, whereas the surface areas of the primary superheater, reheater, economiser and air preheater absorb heat by convection. To obtain the best ratio between the radiant and convective heat transfer in boiler surfaces, it is advisable to maintain the temperature of gases at the furnace exit in the range 1100–1200 °C.

Depending upon the condition of ash coming out from the furnace bottom, pulverized coal furnaces may be of two types:

(a) Dry-bottom furnaces
(b) Wet-bottom or slag-tap furnaces

(a) **Dry-bottom furnace**  In a dry-bottom furnace ash or slag is removed in the solid or dry state. The exit temperature of gases leaving the furnaces must be below the ash fusion temperature. Due to suction of the ID fan, about 85% of the ash in coal, known as “fly ash” flows with the flue gases and is collected in the dust collectors and electrostatic precipitator (ESP) before the ID fan. The remaining ash (about 15%) falls through furnace bottom to the clinker-grinder and then to the hydraulic sluice to which also falls the ash collected in the dust collectors and ESP. The furnace has a hopper at the bottom which has its walls inclined at an angle of 50–60° (angle of repose) so that ash does not stay on the walls and falls by gravity. The minimum gap \( b \) is about 1.0 to 1.2 m (Fig. 5.23). The dry-bottom hopper intensively cools the furnace gases in the furnace bottom, so that molten slag particles which enter this zone are cooled quickly, solidify and fall along the hopper sides into a slag pit containing water. Granulated slag particles are continuously removed from the bath by a screw mechanism. The water bath also serves as the hydraulic seal preventing the suction of cold air from beneath into the furnace.
Fig. 5.23  
Dry bottom furnace: 1-dry bottom hopper; 2-slag bath with water; 3-hydraulic ash disposal channel; 4-burner; 5-water walls; 6-flame core; 7-slag removing screw mechanism; 8-electric motor

(b) Wet-bottom furnace  In a wet-bottom furnace, 40–60% of the ash in coal moves with the flue gases due to ID fan suction, and the rest falls through furnace bottom in a continuous molten stream (Fig. 5.24). The temperature of gases at the walls in the lower portion of the furnace should be higher than the melting temperature of ash to increase its fluidity by making the flame core closer to the furnace bottom. The molten ash is discharged into a slag tank containing water, quenched and shattered into a granular form. Water wall tubes at the furnace bottom are often studded and refractory coated to prevent molten ash from eating into the tubes. Coals having high ash content and low ash fusion temperature are only consumed in wet-bottom furnaces.

5.7.6 Pulverized Coal Burners

The efficient utilization of pulverized coal largely depends upon the ability of the burners to produce uniform mixing of coal and air. The air which carries the pulverized coal through the burner is primary air and the remaining secondary air required for complete combustion is admitted separately around the burner or elsewhere in the furnace. The pulverized coal burners should satisfy the following requirements:

1. It should prepare two individual flows, a coal dust air mixture and secondary air, for ignition and active burning in the furnace space, creating a turbulent environment for thorough mixing and sucking hot furnace gases for preheating.
2. It should not only maintain stable ignition of the fuel-air mix but also control the flame shape and travel in the furnace, since ignition depends on the rate of flame propagation.

3. To prevent flash back into the burner, the coal-air mixture must move away from the burner at a rate equal to flame-front travel.

4. Secondary air quantity is to be controlled since too much air supply can cool the mixture and prevent its heating to ignition temperature.

Pulverized coal burners can be divided into two main groups: circular and slot type burners, as shown in Fig. 5.25. The simplest circular burner consists of a cylindrical tube through which the pulverized coal and primary air mixture is blown into the furnace, the secondary air being delivered through a separate tube. The simplest slot-type burner differs from the circular burner only in the shape of its cross-section.
Fig. 5.25 Pulverized coal burners: (a) circular cross-section, (b) rectangular cross-section

Depending on the kind of flow of the two streams, air-fuel mixture and secondary air, within the burner and its method of mixing, burners can be of two types:

1. Straight-flow burner (external mixing of air-fuel mixture and secondary air in the furnace space).
2. Turbulent or vortex burner (internal mixing of air-fuel mixture and secondary air before entering the furnace).

In straight-flow burners, air–fuel mixture and secondary air are blown in as parallel jets. They may be either circular or rectangular in cross-section. Their intermixing takes place in the furnace space, not in the burners, by properly arranging the burners on the furnace walls. Efficient combustion is achieved by making the jets from various burners interact with one another in the furnace space. These burners can be either fixed or tilted, which assists combustion control. A typical straight flow burner with tilting nozzle is shown in Fig. 5.26. These burners are usually of a relatively low throughput capacity. In high-capacity boilers, a number of such burners are combined in a burner assembly (Fig. 5.27). They are suitable for high-reactive fuels like high-volatile coals. The velocity of pulverized coal-primary air mixture at the burner outlet is taken as 20–28 m/s to ensure keeping the powdered coal in suspension, and the optimal velocity of secondary air is 35–45 m/s.
Fig. 5.26  Straight-flow burners for pulverized coal: (a) with tilting nozzle at the exit of fuel-air mixture; (b) with central channel for hot air; 1-supply of fuel-air mixture; 2-supply of hot air; 3-fuel-air mixture outlet; 4-hot air outlet; 5-suction of furnace gases

Fig. 5.27  An assembly of three straight-flow pulverized coal burners; 1-supply of fuel-air mixture to burner; 2-secondary air supply; 3-pipe for mounting the fuel-oil burner with electric gas igniter; 4-tilting air pipes

In turbulent or vortex burners, a whirling rotary motion is imparted to the coal-air mixture by entering it tangentially to a central nozzle and also to the secondary air that flows through the annular space surrounding the nozzle (Fig. 5.28). This mixes the two streams as they enter the furnace to distribute the coal dust uniformly throughout the total air available for combustion. This is
what is called a *two-scroll burner*, where two scrolls are provided for whirling the dust air mixture and secondary air. In a straight scroll burner the dust air mixture is supplied through a central straight flow nozzle and spread by a deflector, and the secondary air is whirled in a scroll (Fig. 5.29). There are also vane-type burners in which whirling of the dust-air mixture and the secondary air is effected by axial and tangential vanes. A burner for horizontal firing of coal is also shown.

![Diagram showing a two-scroll burner](image)

**Fig. 5.28** Two scroll turbulent burner. 1-Scroll for fuel-air mixture; 2-secondary air scroll; 3-annular channel for discharging fuel-air mixture into furnace; 4-ditto, for secondary air; 5-main fuel oil burner; 6-furnace lining; AB-boundary of fuel-air mixture ignition; C-suction of gases to flame root

Turbulent burners have a throughput capacity of the range 1–3.8 kg/s. The completeness of fuel burning depends on the ratio of axial velocities of the primary and secondary air flows in a burner. The velocity of primary flow (dust-air mixture) is usually 16–25 m/s, while that of secondary air is 1.3 to 1.4 times the primary flow velocity.

![Diagram showing a straight-scroll burner](image)

**Fig. 5.29 (a)** Straight-scroll burner; 1-channel for primary air and coal; 2-scroll for secondary air; 3-mixing zone and ignition; C-suction of gases to the flames

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**Distributed mixing burner (DMB)**  In this type of burner, the total air supplied is distributed in such a way that complete combustion of fuel is achieved.
with minimum excess air (Fig 5.30a). This is a very efficient burner with the sequential stages of fuel-air mixing as shown in Fig 5.30b. Conceptually, there are three zones into which the total input is divided. In the first zone, the stoichiometry is maintained at a very rich condition when the devolatization starts. Supplementary secondary air is then added to the rich products so as to decay the volatilized fuel-nitrogen compounds. Tertiary air provides the oxygen necessary for complete combustion. This oxidizing zone around the rich primary zone prevents corrosion due to a reducing atmosphere in the furnace and limits the interactions between adjacent burners.

![Diagram of burner mixing concept](image)

**Fig. 5.30 (b) Distributed burner mixing concept**

**Low NO\textsubscript{x} burners** NO\textsubscript{x} is produced within the furnace combustion system by the oxidation of both atmospheric nitrogen (thermal NO\textsubscript{x}) and the nitrogen chemically combined with the fuel (fuel NO\textsubscript{x}). In order to limit the NO\textsubscript{x} output, it is necessary to reduce the oxygen available to the nitrogen in the primary combustion process (by fuel-rich operation) and subsequently to control the overall gas temperature to limit the further oxidation of nitrogen while completing combustion (fuel-lean operation). Thus, the main NO\textsubscript{x} reducing parameters are met by the following measures in a corner-fired unit: (1) Reduction of secondary air quantity injected through the fuel nozzles. (2) Diversion of the secondary air injected through the auxiliary secondary air nozzles towards the furnace walls. (3) Injection of the remaining secondary air through ports above the top fuel nozzles on the original firing circle (overfire air). Figure 5.31(a) shows the effect on NO\textsubscript{x} production of varying quantities of overfire air.

A CEGB 500 MW corner-fired boiler has been converted to this system and NO\textsubscript{x} reduction of up to 40% obtained without increase in carbon-in-ash losses and an improvement in the slagging characteristics of the boiler (Modern Power Station Practice, Vol. B, 1991).
Fig. 5.31  (a) Effect of overfire air on NO\textsubscript{x} production
(b) Babcock and Wilcox two-stage venturi furnace (for low NO\textsubscript{x} production)

Figure 5.31(b) shows a two-stage venturi furnace for reducing NO\textsubscript{x} emissions. Coal is burned in two separate chambers. In the initial chamber coal is burned at substoichiometric conditions, resulting in incomplete combustion. This partial burning also allows the desired N\textsubscript{2} producing reactions to occur. Secondary air is added at the entrance to the second chamber in order to achieve oxidizing conditions during transport through the second combustion chamber.

NO\textsubscript{x} emissions can also be reduced by flue gas recirculation where bulk flame temperature and flame oxygen concentration are reduced.

Fig. 5.32 (a) Multifuel burner for pulverized coal, oil and gas
**Combined burners**  Power stations may have to operate simultaneously or alternatively on a variety of fuels. Combined burners which can burn efficiently each kind of fuel are then employed. Figure 5.32 shows a combined burner in a high capacity boiler which can operate on three kinds of fuel: pulverized coal (basic), fuel oil and natural gas. The burner has an enlarged diameter in the central channel in which the main fuel oil burner and a whirling device for turbulizing the axial air flow are arranged. Natural gas is fed through feeder tubes in thin jets between the axial and secondary air flows ensuring thorough intermixing and stable burning of fuel.

![Combined Burner Diagram](image)

Fig. 5.32 (b)  Multifuel burner: Numbering as in Fig. 5.28 and 8-tertiary air inlet; 2¢-secondary air inlet; 7-swirl vanes for sec. air; 10-Swirler for axial air; 5¢-firing fuel oil burner; 13-natural gas inlet; 14-pipes for gas inlet; 15-gas to furnace; 16-electric gas igniter.

**Burner arrangement**  Burners are arranged on the furnace walls so as to ensure almost complete combustion of fuel in the flame core itself. Care must be taken for the removal of slag from the furnace (in either dry or molten state) and prevention of clinker formation on the furnace walls. Burners are optimally arranged depending on their type and characteristics.

1. **Turbulent burners**: These burners give a shorter and wider flame than do the straight-flow type. The primary and secondary air flows are intensively intermixed due to the energy of turbulent motion, which secures a thorough burning off of fuel in the flame cope.

Turbulent burners can be positioned for

(a) front firing  
(b) double-front or opposed firing  
(c) side firing  
(d) double-side firing

These arrangements are shown in Figure 5.33.
Fig. 5.33  Arrangement of turbulent burners on furnace walls: (a) front firing; (b) opposed firing; (c) opposed firing on side walls

The important parameter of turbulent burners is the diameter of their ports, \( D_p \). Burners are arranged \((2.2 \text{ to } 3) D_p\) from one another and at a distance of \((1.6 \text{ to } 2) D_p\) from the side walls of the furnace, which helps to prevent the flames from interacting prematurely and touching the walls. With a single wall firing (either front or side), the opposite water wall of the furnace has a high heat absorption (10–20% above the average value), so that the furnace depth in a dry-bottom furnace must not be less than \( b = (6 \text{ to } 7) D_p \) so as to prevent clinkering on this wall. The opposed firing, either front or side, is employed in high-capacity steam boilers where the required number of burners cannot be arranged on a single wall. With the opposite arrangement of burners, heat absorption by water walls in the furnace is much more uniform.

2. Straight-flow burners: Burners of this type secure complete combustion of fuel by the huge mixing of the flames from individual burners upon their collision in the furnace space. Typical arrangements of these burners are

(a) opposite displaced (offset) firing
(b) corner firing with encountering flame jets
(c) corner firing with tangential jets
(d) vertical firing

These are shown in Fig. 5.34.

With opposite displaced jets (a) peat and brown coal can be efficiently burnt as a result of the intensive turbulization of the flame in the main combustion zones, as a result of large velocity gradients between adjacent jets moving in opposite directions.

With corner burners and encountering jets (b), the flame core is highly turbulized. However, if the flame moves away from the furnace centre to both sides of the wall, clinkering of the walls may occur due to intense thermal loading near those walls.

The tangentially fired system (c) with corner burners and flame jets directed tangentially to an imaginary circle 1–2.5 m in diameter in the furnace centre has found wide application in pulverized coal fired boilers. Both fuel and air are
projected horizontally from the corners of the furnace along lines tangent to a vertical cylinder at the centre of the furnace. Intensive mixing occurs where the streams meet. A rotating motion, similar to that of a cyclone, is imparted to the flame body, which spreads out and fills the furnace area. Dampers control the air to each compartment according to the need. The vertical arrangement of fuel nozzles permits great fuel flexibility for multifuel burning. Fuel and air nozzles most commonly tilt in unison to raise and lower the flame in the furnace, to control furnace heat absorption and thus heat absorption in the superheater and reheater sections.

A tangential burner arrangement can, however, be employed in furnaces with a nearly square cross-section, i.e. with $a/b$ ratio equal to 1–1.2. This ensures good aerodynamics in the furnace space.

![Diagram](image)

**Fig. 5.34 (i)** Arrangement of straight flow burners on furnace walls: (a) opposed displaced firing, (b) corner firing with encountering jets; (c) corner firing with tangential jets; (d) vertical firing
Vertically fired systems (d) are used only to fire solid fuels which are difficult to ignite, like anthracite and low volatile coals. Pulverized coal and primary air mixture is discharged through vertical nozzles. Heated combustion air is introduced around the fuel nozzles and through auxiliary ports. High pressure jets help avoid short-circulating the fuel air streams to the furnace discharge. Tertiary air from the front and rear rows provides needed turbulence. Long travel (and so long residence time) in the combustion chamber helps in complete combustion.

Figure 5.34(ii) shows the flow pattern of flame in dry-bottom furnaces which can be straight, vertically turbulent or horizontally turbulent. The flow pattern of horizontal firing is shown in Fig. 5.34(e).

5.8 CYCLONE FURNACE

Cyclone furnace firing, developed in the 1940s, represents one of the most significant steps in coal firing since the introduction of pulverized coal firing in the 1920s. It is now widely used to burn poorer grades of coal having high ash and moisture content. Biofuels like rice husks can also be successfully burned in cyclone furnaces for steam generation.

The cyclone is essentially a water-cooled horizontal cylinder (Fig. 5.35) located outside the main boiler furnace, in which crushed coal (60 mm size or less) is fed and fired with very high rates of heat release. The cyclone is made
to a diameter of 1.8–4 m and its length is 1.2–1.3 times its diameter. The crushed coal is fed into the cyclone from the left along with primary air, which is about 20 per cent of combustion or secondary air. The coal air mixture is entered tangentially, thus imparting a centrifugal motion to the coal particles. The secondary air is also admitted tangentially at the top of the cyclone at high speed (80–120 m/s) imparting further centrifugal motion. A small quantity of air, called

Fig. 5.35  **Cyclone furnace**
tertiary air, is admitted at the centre. Combustion of the coal is completed before
the resulting hot gases enter the boiler furnace. The whirling motion of coal and
air results in large volumetric heat release rates ranging from 4.7 to 8.3 MW/m³
and high combustion temperatures, more than 1650 °C. These high temperatures
melt the ash into a liquid slag that covers the surface of the cyclone and eventually
drains through the slag-tap opening to a slag tank at the furnace bottom, where it
is solidified and fragmented for removal.

The main advantage of cyclone firing is the removal of ash, about 60 per
cent, as molten slag through the slag tank. Thus only 40 per cent ash leaves with
flue gases, compared with 85 per cent ash for dry-bottom pulverized coal
furnaces, this reduces erosion and fouling of boiler surfaces as well as the size of
dust-removal precipitators or bag houses at boiler exit. Another advantage is that
only crushed coal is used and no pulverization equipment is needed and that the
boiler size is reduced. The disadvantages are higher forced draught fan pressures
and therefore, higher power requirement, and formation of relatively more oxides
of nitrogen, NOₓ, which are air pollutants.

5.9 FLUIDIZED BED COMBUSTION

When air is passed through a fixed or packed bed of particles, air simply
percolates through the interstitial gaps between the particles. As the air flow rate
through the bed is steadily increased, a point is eventually reached at which the
pressure drop across the bed becomes equal to the weight of the particles per unit
cross-sectional area of the bed. This critical velocity is called the minimum
fluidization velocity, Umf, at which the bed is said to be incipiently fluidized. As
the air velocity is increased further, the particles are buoyed up and imparted a
violently turbulent fluidlike motion, with the drag forces exerted by the fluid on
the particles exceeding their weight. There is a high degree of particle mixing and
equilibrium between gas and particles is rapidly established. This is called a
fluidized bed.

In Fig 5.36 (a), air supplied by a centrifugal blower is passed through a
perforated or porous plate, called the distributor, and then a bed of particles of
wide size distribution. A few distributors are shown in Fig. 5.36(b). The air flow
rate is regulated by a bypass valve along with a control valve, and it is measured
by a rotameter. Dividing the mass flow rate, so measured, by the product of the
bed cross-sectional area and density of air, the superficial velocity of air, U, is
estimated. For each mass flow rate or superficial velocity, which is gradually
increased, the pressure drop across the bed is measured. Figure 5.37(a) demonstrates the variation of bed pressure drop with superficial velocity. The
pressure drop Δp varies with the superficial velocity linearly along AB till it
approaches W/At, where W is the weight of particles in the bed and At is the bed
cross-sectional area. This is the fixed bed regime. With further increase in air
flow, the particles are buoyed up by the drag force only to fall back into the bed
yielding high turbulence, with Δp = W/At remaining constant. This continues till
there is considerable pressure fluctuation and finally the superficial velocity
reaches the state $C$ when the particles start getting entrained in the air stream. The superficial velocity of air at which particle entrainment starts is called the \textit{terminal velocity}, $U_t$, of the particles. The state of bed in the range of superficial velocity from $B$ to $C$, i.e. between $U_{mf}$ and $U_t$ is the \textit{fluidized bed regime}. With further increase in velocity, the entrained particles are carried away, the weight of particles in the bed and so $\Delta p$ decrease till it reaches the state $D$ when all the particles are carried away by the air stream, the bed becomes empty and $\Delta p$ is zero.

![Diagram of fluidized bed](image)

\textbf{Fig. 5.36} (a) Measurement of $U_{mf}$ and $U_t$ of a fluidized bed 
(b) Few types of distributors; (i) sandwiching perforated plates, (ii) staggered perforated plates; (iii) grate bars, (iv) bubble cap type, (v) water-cooled type

At incipient fluidization, a small "hump" may be recorded. This arises because extra pressure is required to unlock the particles from their pattern of
packing. Once the particles have been released from this pattern, the voidage increases, and the pressure drop falls back to a value sufficient to support the weight of particles of the bed.

![Diagram showing different regimes of fluidization](image)

**Fig. 5.37 (a) Variation of bed pressure drop with superficial velocity**

### 5.9.1 Regimes of Fluidization

Fluidization, as explained above, is an operation by which fine solids are transformed into a fluid like state through contact with a gas (or a liquid). A fluidized bed displays characteristics similar to those of a liquid, e.g. (a) the static pressure at any height is approximately equal to the weight of bed solids per unit cross-section above that height, (b) the bed surface maintains a horizontal level, irrespective of how the bed is tilted, (c) the solids from the bed may be drained like a liquid through an orifice at the bottom or on the side, and (d) an object denser than the bulk of the bed (steel ball) will sink, while one lighter than the bed (wood piece) will float.

With an increase in gas velocity through a bed of granular solids, the bed moves from one regime to another. Table 5.2 gives a comparison of some characteristic features of different gas solid processes in various types of boilers.

Different commercial combustion systems operate under different gas–solid flow regimes, as shown in Fig. 5.37(b). A stoker-fired boiler burns coal in a fixed bed (E) having the highest density and lowest gas velocity. It is followed by a bubbling fluidized bed (A) boiler, an Ignifluid boiler, where the bed is in turbulent
fluidization \((B)\); a CFB boiler, where parts of the combustor are in fast fluidization \((C)\); and pulverized coal firing where coal burns under entrained or pneumatic transport condition \((D)\) (Basu and Fraser, 1991).

**Table 5.2 Comparison of characteristic features of gas–solid processes in various boilers**

<table>
<thead>
<tr>
<th>Property</th>
<th>Packed bed</th>
<th>Fluidized bed</th>
<th>Fast bed</th>
<th>Pneumatic transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Application in boilers</td>
<td>Stoker</td>
<td>Bubbling fluidized</td>
<td>Circulating fluidized</td>
<td>Pulverized coal fired</td>
</tr>
<tr>
<td>Mean particle diameter (mm)</td>
<td>&lt;300</td>
<td>0.03–3</td>
<td>0.05–0.5</td>
<td>0.02–0.08</td>
</tr>
<tr>
<td>Gas velocity through combustor zone (m/s)</td>
<td>1–3</td>
<td>0.5–3</td>
<td>3–12</td>
<td>15–30</td>
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<td>Typical U/Ut</td>
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<td>0.3</td>
<td>2</td>
<td>40</td>
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<td>Gas motion</td>
<td>Up</td>
<td>Up</td>
<td>Up</td>
<td>Up</td>
</tr>
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<td>Gas mixing</td>
<td>Near plug flow</td>
<td>Complex two phases</td>
<td>Dispersed plug flow</td>
<td>Near plug flow</td>
</tr>
<tr>
<td>Solids motion</td>
<td>Static</td>
<td>Up &amp; down</td>
<td>Mostly up, some down</td>
<td>Up</td>
</tr>
<tr>
<td>Solid-solids mixing</td>
<td>Negligible</td>
<td>Usually near perfect mixing</td>
<td>Near perfect mixing</td>
<td>Near plug flow</td>
</tr>
<tr>
<td>Overall voidage</td>
<td>0.4–0.5</td>
<td>0.5–0.85</td>
<td>0.85–0.99</td>
<td>0.98–0.998</td>
</tr>
<tr>
<td>Temperature gradient</td>
<td>Large</td>
<td>Very small</td>
<td>Small</td>
<td>May be significant</td>
</tr>
<tr>
<td>Typical bed-to-surface, heat transfer coefficient, (W/m²K)</td>
<td>50–150</td>
<td>200–550</td>
<td>100–250</td>
<td>50–100</td>
</tr>
<tr>
<td>Attrition</td>
<td>Little</td>
<td>Some</td>
<td>Some</td>
<td>Considerable</td>
</tr>
<tr>
<td>Agglomeration</td>
<td>Considerable</td>
<td>Some</td>
<td>No problem</td>
<td>No problem</td>
</tr>
</tbody>
</table>


1. **Packed bed** A packed or fixed bed consists of a bed of stationary particles on a perforated grid through which a gas is flowing (Fig. 5.38a). The pressure drop per unit height of a packed bed of uniformly sized particles, \(\Delta p/L\), is given by Ergun’s equation

\[
\frac{\Delta p}{L} = 150 \frac{(1 - \varepsilon)^2}{\varepsilon^3} \left(\frac{\mu U}{\phi d_p^2}\right) + 1.75 \frac{1 - \varepsilon}{\varepsilon^3} \frac{\rho_g U^2}{\phi d_p}
\]  

(5.21)

where \(U\) is the superficial velocity, \(\varepsilon\) is the void fraction in the bed (volume of gas or voids divided by total volume of bed), \(d_p\) and \(\phi\) are the diameter and sphericity of particles respectively. \(\mu\) is viscosity and \(\rho_g\) is the density of the gas. The voidage...
of a bed of particles, \( \varepsilon \), is the fraction of the bed volume which is occupied by the space between the solid particles. So,

\[
\text{Voidage, } \varepsilon = \frac{\text{volume of bed} - \text{volume of solids}}{\text{volume of bed}}
\]

**Fig. 5.37 (b)** Different commercial combustion systems operating under different gas-solid flow regimes

\[
= 1 - \frac{\text{volume of solids}}{\text{volume of bed}} = 1 - \frac{m_s}{\rho_s} \times \frac{\rho_b}{m_b}
\]

where \( m_s \) and \( m_b \) are the masses of solids and bed respectively and \( \rho_s \) and \( \rho_b \) are their corresponding densities. Since the voids may be regarded as empty spaces, \( m_s = m_b \). Therefore,

\[
\varepsilon = 1 - \frac{\rho_b}{\rho_s}
\]  

(5.22)

where \( \rho_b \) is the bulk density of the bed which is equal to the mass of solids in the bed divided by the bed volume.

The sphericity, \( \phi \), of a particle is defined as

\[
\phi = \frac{\text{surface area of a sphere of the same volume as the particle}}{\text{surface area of the particle}}
\]
In a moving packed bed, the solids move with respect to the wall, but not with respect to one another.

2. Bubbling fluidized bed When the superficial velocity of gas flowing through a fixed bed reaches the minimum fluidization velocity, \( U_{mf} \), the fixed bed transforms into an incipiently fluidized bed and the bed starts behaving as a liquid (Fig. 5.38b). Since the pressure drop across the bed is equal to the weight of the bed, the fluid drag \( F_D \) is given as

\[
F_D = \Delta p A = AL \left( 1 - \varepsilon \right) \left( \rho_s - \rho_g \right) g
\]

where \( A \) and \( L \) are the cross-section and height of the bed, respectively, and \( \rho_s \) is the density of solids.

\[
\Delta p/L = \left( 1 - \varepsilon_{mf} \right) \left( \rho_s - \rho_g \right) g
\]

(5.23)

The minimum superficial velocity, \( U_{mf} \), may be obtained by solving Eqs. (5.21) and (5.23) simultaneously, as given below; in terms of Reynolds number \( Re_{mf} \):

\[
Re_{mf} = \frac{\rho_g d_p U_{mf}}{\mu} = \left[ C_1^2 + C_2 Ar \right]^{0.5} - C_1
\]

(5.24)
\[ \text{Ar} = \text{Archimedes number} = \frac{\rho_g (\rho_s - \rho_g) g d_p^3}{\mu_e^2} \] (5.25)

\[ C_1 = 27.2 \text{ and } C_2 = 0.0408 \text{ (Grace, 1982)} \]

As increase in gas flow beyond minimum fluidization can cause the extra gas to flow in the form of bubbles. The section of the bed outside the bubbles is called the \textit{emulsion phase}. Bubbles are gas voids with very little or no solids. Due to the buoyancy force a bubble rises through the emulsion phase and its size increases with particle diameter \( d_p \), excess gas velocity \( U - U_{mf} \), and bed height. A bubble carrying some solid particles in its wake erupts at the bed surface throwing particles into the space above the bed, called the \textit{freeboard}. The particles entrained travel upwards due to their momentum and local drag of gas. Some of the particles may disengage from the gas and fall to the dense bed due to gravity. Beyond a certain height, called the \textit{transport disengaging height} (TDH), particle disengagement is negligible, and the flux rate of particles carried away is known as the \textit{elutriation rate}.

The velocity \( U_b \), at which a single isolated bubble rises in a large diameter vessel is given by

\[ U_b = 0.71 \left( g D_b \right)^{1/2} \] (5.26)

where \( D_b \) is the bubble diameter, taken as the diameter of a sphere having the same volume as the bubble.

In most practical situations, the fluidizing velocity is sufficiently greater than \( U_{mf} \) for bubbles to form continuously. However, the bubbles are not isolated, but they are in close proximity. They influence particle mixing, gas to particle contacting, bed expansion, gas bypass, elutriation and so on. In a bubbling bed, the bubble rise velocity may be obtained from

\[ U_b = k (U - U_{mf}) + 0.71 \left( g D_b \right)^{1/2} \] (5.27)

where the value of constant \( k \) is approximately 1.

3. \textbf{Slugging} If the bed is small in cross-section and deep, the bubble may increase to a size comparable to the diameter or width of the bed. The bubble then passes through the bed as a slug (Fig. 5.38c). This is known as slugging and there is a large fluctuation of pressure drop across the bed which is unstable.

4. \textbf{Turbulent bed} As the velocity of gas through a bubbling fluidized bed is increased, the bed expands, and a point is eventually reached when the emulsion walls become very thin, the bubbles constantly collapse and re-form, resulting in a violently active bed. The bed surface is highly diffused and particles are thrown into the freeboard above. Such a bed is called a \textit{turbulent bed} (Fig. 5.38d). The pressure drop across the bed fluctuates rapidly. The amplitude of pressure fluctuation reaches a peak at the velocity \( U_c \) and reduces to a steady value at the velocity \( U_k \) (Fig. 5.39). Grace (1982) gave the following correlations
Combustion Mechanism, Combustion Equipment and Firing Methods

![Amplitude of pressure fluctuation graph](image)

**Fig. 5.39** Transition from bubbling to turbulent bed

\[
U_c = 3(\rho_p d_p)^{1/2} - 0.17 \text{ m/s} \quad (5.28)
\]

\[
U_k = 7(\rho_p d_p)^{1/2} - 0.77 \text{ m/s} \quad (5.29)
\]

where \((\rho_p d_p)\) is in the range 0.05–0.7 kg/m². Finer particles become turbulently fluidized at \(U > U_p\), where coarser particles undergo this transition at \(U < U_t\). The gas-solid contact in this regime is very good.

In the regimes discussed so far, solids are generally retained within the bed and there is no large scale migration of particles from the bed, except for some entrainment. These regimes are said to be in the captive stage.

**5. Fast fluidized bed** It is a regime lying between the turbulent fluidized bed and pneumatic transport. Following Basu and Fraser (1991), a fast fluidized bed may be defined as a high-velocity gas-solid suspension where particles, elutriated by the fluidizing gas above the terminal velocity of single particles, are recovered and returned to the base of the furnace at a rate sufficiently high as to cause a degree of solid reflushing that will ensure a minimum level of temperature uniformity in the furnace. The term “fast bed” has become synonymous with the circulating fluidized bed (Fig. 5.40). High slip velocity \((U_g - U_s)\) between gas and solid, formation and disintegration of particle agglomerates, and a very good gas-solid mixing are the characteristic features of this regime. The cross-sectional average suspension density \(\rho_{sus}\) is given from

![Clusters](image)

**Fig. 5.40** A fast fluidized bed in which solid agglomerates move up and down in a very dilute dispersion of solids
\[ m = m_s + m_g \]
\[ \rho_{\text{sus}} V = \rho_s (V - V_g) + \rho_g V_g \]
\[ \rho_{\text{sus}} = (1 - \varnothing) \rho_s + \varnothing \rho_g \]  (5.30)

where \( V_g \) represents the volume of the voids and \( V \) the volume of the bed.

The main difference between bubbling beds and fast or circulating beds lies in the gas velocity used. While bubbling beds normally operate at gas velocities of around 1–3 m/s (depending on particle size), the circulating fluidized bed (CFB) is typically run at 5–10 m/s.

The circulating fluidized bed may be divided into a dense bottom region and a dilute upper region. The bottom region operates either in bubbling or turbulent mode of fluidization depending on the superficial velocity used. Above the bottom bed is the splash zone and then the upper dilute zone where the solids volumetric concentration is very low, typically about 0.3%.

The cross-section average suspension density above the entry level of the secondary air in a commercial CFB boiler generally decays exponentially along the height of the furnace, as it does in the freeboard region of a bubbling fluidized bed. The local gas-voidage at a particular height decreases towards the wall in the same way the local gas velocity through a tube decreases towards it.

The structure of the fast bed is better explained by the core-annulus model which divides it into two vertical zones, a relatively dilute up-flowing core in which solid particles are entrained upward by a high velocity gas stream, and a much denser annular layer near the column wall in which solid particles congregate and fall as dense structures similar to waves of strands or streamers (Fig. 5.41). In the core area the gas velocity is more and the particles are entrained in the up-flowing gas stream. Near the wall the gas velocity is less and the particles cohering to form clusters or streamers fall by gravity. After falling a certain distance along the wall the clusters disintegrate only to reform again. The solids concentration in the wall region is found to be 2–3 times the cross-sectional average solids concentration. The thickness of the annulus decrease towards the top of the bed, which may vary from a few mm in laboratory scale units to several decimeters in commercial units. The clusters are slender in shape and may have a voidage of the range 0.6–0.8. They move upwards in the core of the furnace, and when swept towards the low velocity region near the wall, start falling along the wall at the velocity of 1–2 m/s.

The axial voidage distribution in a CFB furnace (Fig. 5.41b) is found to be S-shaped. There is a point of inflection which separates the lean and dense regions of a fast bed and is a function of the solid circulation rate and the solids inventory in the system.

Figure 5.42 gives a pressure balance around a CFB loop. The lower section of the fast bed is denser and therefore it results in higher pressure drops per unit height of the bed than the upper leaner section. For stable operation the pressure balance around the loop may be written as

\[ \Delta p_{F-A} + \Delta p_{A-B} + \Delta p_{B-C} + \Delta p_{C-D} = \Delta p_{D-E} + \Delta p_{E-F} \]

This pressure balance depends on different operating parameters.
5.9.2 Terminal Velocity of Particles and Elutriation

If a single particle falls freely under gravity in the atmosphere, it will accelerate until its velocity is such that the drag exerted by the surrounding air is equal to the gravitational force. This velocity is called the "terminal velocity", \( U_t \), which for a sphere of diameter \( d_p \) is given by

![Diagram showing core-annulus structure and voidage variation](image)

**Fig. 5.41** (a) Core-annulus structure of a CFB: (a) upflowing core (dilute suspension of gas solid mixture) and downflowing annulus (dense phase of gas and solids), (b) voidage variation along height

![Diagram showing pressure balance](image)

**Fig. 5.42** Pressure balance around a CFB loop. The dotted lines indicate the pressure profile at a higher circulation rate
\[
(p_s - p_g)g \frac{\pi d_p^3}{6} = \frac{1}{2} \rho_g U_t^2 S C_D
\]  

where \( S \) is the frontal area of a sphere, \( \frac{\pi}{4} d_p^2 \), \( C_D \) is the experimentally determined drag coefficient, \( \rho_s \) and \( \rho_g \) are the densities of solids and gas, respectively.

\[
U_t = \left[ \frac{4(p_s - p_g)g d_p}{3 \rho_g C_D} \right]^{1/2}
\]

\( \therefore \)

The drag coefficient \( C_D \) depends upon the Reynolds number \( \text{Re}_p \) defined as

\[
\text{Re}_p = \frac{\rho_g U_t d_p}{\mu_g}
\]

No simple relationship expresses the variation of drag coefficient with Reynolds number. Two relationships often quoted are

\[
C_D = \frac{24}{\text{Re}_p} \quad \text{for} \quad \text{Re}_p < 0.4
\]

and

\[
C_D = \frac{10}{(\text{Re}_p)^{1/2}} \quad \text{for} \quad 0.4 < \text{Re}_p < 500
\]

Substituting these expressions into Eq. (5.30) yields

\[
U_t = \frac{(p_s - p_g)g d_p^2}{18 \mu_g} \quad \text{for} \quad \text{Re}_p < 0.4
\]  

and

\[
U_t = \left[ \frac{4(p_s - p_g)^2 g^2}{225 \mu_g \rho_g} \right]^{1/3} \cdot d_p \quad \text{for} \quad 0.4 < \text{Re}_p < 500
\]

The terminal velocity of the smallest particle in the bed is important conceptually in that it constitutes the maximum velocity of the gas at which a bubbling fluidized bed should be operated, if entrainment of particles is to be avoided. Fluidized beds, however, commonly operate at significantly higher gas velocities. Particulate solids are composed of a range of particle sizes and, perhaps densities, while many solid materials undergo attrition when fluidized, or fragmentation due to thermal stresses when put into a hot fluidized bed. Attrition and fragmentation lead to the production of fines. These fines become entrained in the gas leaving the bed.

5.9.3 Fluidized Bed Combustion of Solid Fuels

In its simplest arrangement, the fluidized bed consists of inert particles such as sand, through which fluidizing air can be passed via a distributor plate at the bottom (Fig. 5.43). The bed normally contains only a small percentage of burning
fuel, about 1 or 2%, so that the inert particles carry the heat away from the burning particles, thus keeping their temperature below that at which ash melts. The temperature of the bed when burning solid fuels is commonly controlled to about 850 °C. So, heat is required to be removed from the bed by installing cooling tubes. The bed temperature during start-up should be raised to the solid fuel ignition temperature, which may vary from 450 °C to 500 °C depending on the type of fuel by burning fuel gas or oil in the bed (Fig. 5.44). The initiated input to the bed by burning gas or oil is gradually reduced till the main fuel starts to deliver heat and self-sustaining combustion is established.

Fig. 5.43 Fluidized bed combustor with immersed cooling surfaces
Fig. 5.44 Start-up transient of a fluidized bed combustor

Flue gas desulphurization is achieved by adding a sorbent such as limestone or dolomite to the fluidized bed, where sulphur is absorbed in a solid form. This sorbent is used as the bed material into which fuel is fed. A combustion temperature of 850 °C is about the optimum for absorption of sulphur oxides by the lime, and the residence time of lime in the bed is long. The amount of sorbent required to absorb about 90% of the sulphur in the fuel needs a calcium-sulphur molar ratio of about 3 (Howard, 1989).

Fluidized bed systems are generally smaller for a given thermal output than conventional equipment. The firing rate (MW/m²) of a bubbling fluidized bed boiler having cooling tubes in the bed can be up to 50% larger. Fluidized beds can burn a wide variety of fuels quite efficiently. Indeed, with some low-grade fuels and wastes, e.g. colliery tailings, fluidized bed combustion is the only way they can be burned satisfactorily. The relatively good mixing of particles and gas promoted by bubbling action provides good conditions for both combustion and heat transfer.

5.9.4 Combustion of Fuel Particles in a Fluidized Bed

The sequence of events and temperature history of a single burning particle of solid fuel in a fluidized bed at 850 °C is shown in Fig. 5.48. Initially, the moisture is given off and the temperature of the solid fuel particle rises as it is heated by interparticle convection, gas convection and radiation. The temperature of the fuel particle soon reaches a value at which volatile matter is released. The residue left after devolatilization is char, which is essentially porous carbon with some ash bound to it, with the degree of porosity depending on the type of fuel. Once the ignition temperature is reached, the char begins to burn. As seen in Fig. 5.45, the time required to burn the particle out is much longer than the time required to devolatilize. The mass and size of the char particle diminishes with the progress of combustion, as well as by attrition and fragmentation till the particle becomes small enough to be elutriated from the bed.
5.9.5 Mechanism of Combustion of Char Particles

The general principle of coal combustion has been discussed earlier in Section 5.2. The rate of loss of mass of the burning particle per unit surface area is termed "specific burning rate", \( r_s \) (kg/m\(^2\)/s). The faster the burning rate, the shorter the time required for the char particle to burn out.

A char particle whose surface is at sufficiently high temperature will burn rapidly if there is a large mass flow rate of oxygen towards it. There are two resistances which impede the rate at which the char reacts, viz., the diffusional resistance to mass transfer of oxygen, \( R_g \), and the chemical resistance, \( R_c \), depending on the surface temperature or the chemical kinetics, as we have discussed in Section 5.3. To speed-up the burning rate of char, these resistances must be reduced.

**Mass transfer rate of oxygen**  
The mass transfer rate of oxygen towards the fuel surface may be characterized by Sherwood number, \( Sh \), given by

\[
Sh = \frac{k_g d_c}{D_g}
\]

where \( k_g \) is the mass transfer coefficient, \( d_c \) is the diameter of the char particle and \( D_g \) is the diffusivity of oxygen. Chakraborty and Howard (1981) suggested the following expansion

\[
Sh = 2e + 0.69 \text{Re}^{1/2} \text{Sc}^{1/3}
\]  

(5.35)

where \( \text{Re} \) is the carbon particle Reynolds number

\[
\text{Re} = \frac{\rho_g U d_c}{\mu_g}
\]
and Sc is the Schmidt number

\[ \text{Sc} = \frac{\mu_g}{\rho_g d_c} \]

The specific burning rate of carbon may be written as

\[ r_s = \lambda k_g (c_b - c_s) \quad (5.36) \]

where \( \lambda \) depends upon which particular reaction occurs at the surface, \( c_b \) is the oxygen concentration in the bulk particulate phase of the bed and \( c_s \) is the oxygen concentration at the particle surface. For the reaction,

\[ C + O_2 \rightleftharpoons CO_2 \]

\( \lambda = 3/8 \), and for the reaction,

\[ C + \frac{1}{2} O_2 \rightleftharpoons CO \]

\( \lambda = 3/4 \). The mass transfer coefficient, \( k_g \), is the inverse of the resistance, \( R_g \), to mass flow of oxygen:

\[ R_g = \frac{1}{k_g} \]

The diffusional resistance \( R_g \) can be reduced by increasing the Reynolds number or the velocity of the gas.

**Chemical kinetic rate** For a first order reaction, the specific burning rate \( r_s \) may be expressed as

\[ r_s = kc_s \quad (5.37) \]

where \( k \) is the reaction rate coefficient and \( c_s \) is the oxygen concentration at the carbon surface. The reaction rate coefficient \( k \) is given by Arrhenius equation,

\[ k = k_o e^{-E/RT} \]

The inverse relationship between the reaction rate coefficient \( k \) and the surface kinetic resistance \( R_c \) is

\[ R_c = 1/k_c \]

If the chemical kinetics controls the rate of reaction, the burning rate may be increased by increasing the surface temperature of carbon.

**Combustion control by combined influence of mass transfer and chemical kinetics** The oxygen concentration at the char surface, \( c_s \), is not known directly, so if it is eliminated between Eqs (5.36) and (5.37), the resulting expression for specific burning rate is

\[ r_s = Kc_s \quad (5.38) \]

where

\[ \frac{1}{K} = \frac{1}{\lambda k_g} + \frac{1}{k} \quad (5.39) \]

which is similar to Eq. (5.13).
During the residence time of burning carbon particle within a fluidized bed, both chemical kinetic and diffusional resistances influence the burning rate. However, it has been observed that for particles of size 1–3 mm, combustion is mainly controlled by diffusion of oxygen to the carbon surface, whereas for particles smaller than 1 mm, combustion is controlled primarily by chemical kinetics (Ross and Davidson, 1981). Larger char particles glow more brightly than the bulk of the bed with higher surface temperature and diffusion controlling the reaction. Finer particles are seen to glow with almost the same brightness as the bed with their temperature only slightly above that of the bed. Thus, the only way to speed up the burning rate of the fines is to raise the bed temperature.

5.9.6 Circulating Fluidized Bed Combustion System

The furnace is divided into three distinct zones: (1) Lower zone (below the secondary air entry), 2. Upper zone (above the secondary air entry), and (3) Hot gas/solid separator (Fig. 5.46).

The lower zone in the furnace is fluidized by primary combustion air, which constitutes about 40–80% of the stoichiometric air required for the coal feed: coal and sorbent (limestone or dolomite) as well as the unburnt char from the hot cyclone are received in this section. Devolatilization and partial combustion of char occur in this zone, which is oxygen deficient. So, to protect the boiler tubes from corrosion, this zone is refractory lined.

Fig. 5.46 Fast or circulating fluidized bed combustion system
As mentioned earlier, the lower zone of a CFB furnace is much denser than the upper zone. Thus, it serves also as an insulated storage of hot solids, providing the CFB boiler with a thermal "flywheel". When the boiler load increases, the proportion of primary to secondary air is increased. As a result, a greater amount of hot solids is transported to the upper zone of the furnace and the solid circulation rate increases. When the boiler load decreases, the reverse happens and the solid circulation rate decreases.

Char particles transported to the upper zone are exposed to an oxygen rich environment, where most of the combustion occurs. The upper zone is much taller than the lower zone. Unburned char particles that are entrained out of the furnace go around the refractory-lined cyclone. The residence time of char particles within the cyclone is short and the oxygen concentration is the lowest there. So the extent of combustion inside the cyclone is small compared to the rest of the combustion loop. However, carbon monoxide and the volatiles often burn in the cyclone.

Basu and Halder (1989) presented a model for estimation of the burning rate of a single char particle in a fast bed. The ratio of fixed carbon to volatile matter in fuel, the attrition rate, the intrinsic reactivity of char, the particle size and temperature, swelling and fragmentation, and finally, the temperatures of coal ignition and ash agglomeration affect the combustion characteristics in a CFB.

5.9.7 Pressurized Fluidized Bed Combustion (PFBC)

If a coal-fired fluidized bed combustor is operated at elevated pressure, the products of combustion can be expanded through a gas turbine to produce electricity. The products of combustion have to be sufficiently clean for a gas turbine to accept without excessive erosion, corrosion or fouling of the turbine. Pressurized fluidized bed combustion (PFBC) has been under extensive research and development, not only because of its potential as a coal fired gas turbine but also when used in conjunction with a steam plant (Fig. 5.47), a significantly higher efficiency of electricity generation is possible than that from either a gas turbine or steam turbine plant alone. The thermodynamic aspects of combined cycle power generation have been discussed in great details in Chapter 3.

The first PFBC plants with an electrical power of about 80 MW were almost simultaneously installed by ABB Carbon at Värtan (2 units) in Sweden, Escarton in Spain and TIDD in the USA. Their details are as follows.

<table>
<thead>
<tr>
<th>Location</th>
<th>Year</th>
<th>Electrical Power</th>
<th>Gas Turbine Inlet</th>
<th>Steam Condition</th>
<th>Coal Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Värtan</td>
<td>1991</td>
<td>2 x 220 MW (th)</td>
<td>830 °C, 12 bar</td>
<td>137 bar, 530 °C</td>
<td>Bituminous</td>
</tr>
<tr>
<td>Escarton</td>
<td>1991</td>
<td>79</td>
<td>840 °C, 12 bar</td>
<td>94 bar, 513 °C</td>
<td>Lignite</td>
</tr>
<tr>
<td>TIDD</td>
<td>1991</td>
<td>71</td>
<td>830 °C, 12 bar</td>
<td>90 bar, 496 °C</td>
<td>Bituminous</td>
</tr>
</tbody>
</table>

Operational success of these demonstration units gave an increased confidence in this technology. The Värtan PFBC plant in Stockholm is a
Fig. 5.47 A combined cycle plant with a pressurized fluidized bed combustor (PFBC)

cogeneration unit, which consists of two modules, each containing one boiler with gas turbine and associated fuel and ash systems (Fig. 5.48). The fuel is a mixture of coal, sorbent and water supplied as a paste by means of six piston pumps. Coal is crushed to 0–6 mm size and the sorbent (dolomite) to 0-3 mm size, before they are mixed in water to form the paste.

Figure 5.50 shows a simplified flow diagram of the TIDD plant at Ohio, USA, where the PFBC operates at 12 bar increasing the gas density and reducing the bed area for a given heat release. To further improve the overall cycle efficiency, there is a growing interest in combining PFBC with partial or mild gasification of coal to take advantage of a higher gas turbine inlet temperature. This is known as the second generation PFBC, or the Topping Cycle. Mild gasification is a devolatilization process designed to produce a series of alternate fuels by decomposing coal into simpler components at relatively mild temperatures (540°C–650°C) and pressure (1–2 bar). The coal heated by limited air gets pyrolyzed to yield a solid carbon char (20–40% of feed), a complex of hydrocarbon liquids and hydrocarbon gas. The char is combusted in a conventional fluidized bed boiler to raise steam and hence electricity. The gaseous and liquid hydrocarbon products are cleaned and then combusted in a gas turbine, also producing electricity. The plant (Fig. 5.50) is similar to the British Coal Topping Cycle (Fig. 3.24) and the CFBC topping cycle (Fig. 3.25). The significant advantage of partial gasification over total gasification is that devolatilization and gasification of coal are achieved relatively easily, whereas
Fig. 5.48  Simplified Flow Diagram for the PFBC Plant at Värtan (Sweden)
Fig. 5.49  TIDD PFBC simplified plant cycle
the total gasification of the char requires more energy and time. Consequently, by partially gasifying and then combusting the char in a fluid bed combustor to raise steam, an optimum can be achieved between the gasifier and the combustor sizing as well as the steam and gas turbines. Also, advantage can be made of higher GT inlet temperatures.

Table 5.3 shows the PFBC projects in Japan and Fig. 5.51 gives the flow diagram of the Wakamatsu plant, where the PFBC boiler is housed inside a thick pressure vessel and the combustion gases are de-dusted by cyclone and ceramic filter. The outline of the 85 MWe Tomatoh-Azuma (Atsuma-Cho) PFBC plant is given in Fig. 5.52. Coal fed together with limestone is burnt at an elevated pressure. Desulphurization of the flue gas takes place along with combustion. After being de-dusted in cyclones and ceramic filters, the flue gas, still at high pressure, expands in the gas turbine, generating electricity. The exhaust gas at about atmospheric pressure flows through SCR (Selective Catalytic Reduction), where NOx is removed and SGC (Stack gas cooler), where heat is removed, before leaving through the stack. The boiler is of bubbling fluidized bed drum-type with membrane water wall. The steam turbine is a single casing configuration which accommodates HP/IP and LP turbines within one cylinder. Gas and steam turbines generate electrical power in a ratio of 15 per cent and 85 per cent respectively.
Fig. 5.51  Simplified Cycle of Wakamatsu Plant, Japan

Table 5.3  PFBC Projects in Japan

<table>
<thead>
<tr>
<th>Project</th>
<th>MW</th>
<th>Commissioning</th>
<th>Manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wakamatsu</td>
<td>71</td>
<td>1994 Nov</td>
<td>IHI/ABB</td>
</tr>
<tr>
<td>Tomatoh-Azuma</td>
<td>85</td>
<td>1996 Oct</td>
<td>MHI</td>
</tr>
<tr>
<td>Karita</td>
<td>350</td>
<td>1997 July</td>
<td>IHI/ABB</td>
</tr>
<tr>
<td>Ohsaki</td>
<td>250×2</td>
<td>1999 Mar</td>
<td>Hitachi</td>
</tr>
</tbody>
</table>

The schematic diagram of the 250 MWe PFBC system at Ohsaki, Japan is shown in Fig. 5.52, while Table 5.4 gives its principal features.

Table 5.4  Principal Specifications of PFBC Plant at Ohsaki, Japan

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross electrical output</td>
<td>250 MW mty × 2 (ST : 210 MWe, GT : 40 MWe) × 2</td>
</tr>
<tr>
<td>Boiler</td>
<td>Pressurized bubbling fluidized bed combustion regenerative once-through type</td>
</tr>
<tr>
<td>Steam output at MCR</td>
<td>520 t/h × 2</td>
</tr>
<tr>
<td>Bed temperature</td>
<td>865°C</td>
</tr>
<tr>
<td>Combustion pressure</td>
<td>10 bar</td>
</tr>
<tr>
<td>Coal and limestone feed system</td>
<td>Wet paste feed</td>
</tr>
<tr>
<td>Gas turbine</td>
<td>Single shaft simple open cycle</td>
</tr>
<tr>
<td>Steam turbine</td>
<td>Tandem compound regenerative condensing type</td>
</tr>
<tr>
<td>Steam condition</td>
<td>166 bar, 566/593°C</td>
</tr>
<tr>
<td>Emission control</td>
<td>SO\textsubscript{x}</td>
</tr>
<tr>
<td>SO\textsubscript{x}</td>
<td>In-site limestone desulfurization</td>
</tr>
<tr>
<td>NO\textsubscript{x}</td>
<td>NH\textsubscript{3}-injection SNCR and SCR</td>
</tr>
<tr>
<td>Particulate</td>
<td>Cyclones and Ceramic filters</td>
</tr>
<tr>
<td>Stack</td>
<td>Height 200 m.</td>
</tr>
</tbody>
</table>
Fig. 5.52 Schematic diagram of the 250 MWe PFBC system at Ohsaki, Japan

Fuel is fed into the bed as paste. Raw coal is crushed and screened to less than 6 mm and fed into a mixer with limestone of 3 mm top size. With water added, the mixer makes a coal-water paste (CWP), which is then pumped into the pressurized fluidized bed through fuel nozzles using piston pumps. Hot flue gas leaving the bed is cleaned by cyclones and ceramic filters and is then delivered to the gas turbine. The heat recovery heat exchanger (HRHE) recovers heat from the GT exhaust of about 430 °C and heats up the boiler feedwater.

The 350 MW Karita Unit 1 (Fig. 5.53) at Fukuoka, Japan is the largest PFBC power plant in the world. The schematic PFBC hot loop is shown in Fig. 5.54 and an advanced PFBC process diagram is shown in Fig. 5.55.
Fig. 5.54  Schematic PFBC hot loop
5.10 COAL GASIFIERS

The gasification of coal is a very old technology. Before natural gas was introduced in the market, coal gasification was used to produce fuel gas (town gas) for distribution in urban areas. The basic principles of coal gasification have been discussed in section 4.8.1.

In auto thermal coal gasification, partial combustion of coal takes place to generate the heat required for the gasification process, the overall reaction of which is endothermic. The oxygen required for combustion is supplied to the gasifier by injecting air or oxygen into it.

Gasification process can, therefore, be classified as:

1. Air-blown gasifiers, which produce a gas with a low calorific value, typically 5000–6000 kJ/kg.
2. Oxygen-blown gasifiers, which produce a gas with a medium calorific value, typically 15,000 kJ/kg, approximately one-third of the calorific value of natural gas.

The typical gas composition, relative heating value and adiabatic flame temperature of natural and coal-derived gases are given in Table 5.5. Depending on the flow pattern in the gasifier itself, there are three main types of gasifiers:
Table 5.5  Typical gas composition, in % volume

<table>
<thead>
<tr>
<th>% volume</th>
<th>Natural gas</th>
<th>Oxygen-blown</th>
<th>Air-blown</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO</td>
<td>—</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>H₂O</td>
<td>—</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>CO₂</td>
<td>—</td>
<td>03</td>
<td>07</td>
</tr>
<tr>
<td>N₂</td>
<td>10</td>
<td>02</td>
<td>49</td>
</tr>
<tr>
<td>CH₄</td>
<td>90</td>
<td>05</td>
<td>01</td>
</tr>
<tr>
<td>Relative heating value</td>
<td>1 (reference)</td>
<td>1/3</td>
<td>1/9</td>
</tr>
<tr>
<td>Adiabatic flame temperature</td>
<td>2100 °C</td>
<td>2300 °C</td>
<td>1900 °C</td>
</tr>
</tbody>
</table>

1. Fixed bed gasifiers with counter-current flow
2. Fluidized bed gasifiers with bubbling or circulating bed
3. Entrained bed gasifiers with co-current flow

The gasifiers which are closer to large-scale commercial application for combined cycle power generation are either the fixed bed type (British Gas/Lurgi slagging gasifier) or the entrained type (Texaco, Shell, Dow, Prenflow), both oxygen-blown. Table 5.6 and Fig. 5.56 to 5.58 show the above types of gasifiers.

Table 5.6  Different types of gasifiers

<table>
<thead>
<tr>
<th>British gas/Lurgi</th>
<th>Texaco</th>
<th>Shell</th>
<th>Prenflow</th>
<th>Dow</th>
<th>KRW (Kellogg Rust Westinghouse)</th>
<th>High Temperature Winkler</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxidant</td>
<td>Oxygen</td>
<td>Oxygen</td>
<td>Oxygen</td>
<td>Oxygen</td>
<td>Oxygen/air</td>
<td>Oxygen/air</td>
</tr>
<tr>
<td>Coal gas value</td>
<td>Medium</td>
<td>Medium</td>
<td>Medium</td>
<td>Medium</td>
<td>Medium/low</td>
<td>Medium/low</td>
</tr>
<tr>
<td>Type of gasifier</td>
<td>Fixed bed</td>
<td>Entrained bed</td>
<td>Entrained bed</td>
<td>Entrained bed</td>
<td>Fluidized bed</td>
<td>Fluidized bed</td>
</tr>
<tr>
<td>Flow pattern</td>
<td>Counter current flow</td>
<td>Counter current flow</td>
<td>Counter current flow</td>
<td>Counter current flow</td>
<td>Bubbling or circulating bed</td>
<td>Bubbling or circulating bed</td>
</tr>
<tr>
<td>Raw gas temperature (°C)</td>
<td>500–600</td>
<td>1400–1700</td>
<td>1400–1700</td>
<td>1400–1700</td>
<td>850–900</td>
<td>850–900</td>
</tr>
<tr>
<td>Gasifier pressure</td>
<td>Compatible with gas turbine requirement</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Raw gas cooling</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Gasifiers can also be classified according to whether they operate at
1. atmospheric pressure, or
2. pressure above atmospheric.

Pressurized gasifiers are of interest for combined cycle plants since the gas pressure at inlet to the gas turbine must be at least 20 bar. So the gasifiers for combined cycle plants operate at a pressure between 20 to 30 bar.

By contacting coal, oxygen or air, and steam in a reactor at elevated temperatures and pressures, a product gas comprising CO, H₂, CH₄, CO₂, H₂O (g), N₂, H₂S and other trace gases (HCN, HCl) is liberated. The reactions within the gasifier are complex (see Section 4.8.1) and the basic ones are:
**Fig. 5.58 Fluidized bed gasifier**

1. \( C + \frac{1}{2} O_2 \rightarrow CO \) (partial combustion) \(- 124 \text{ kJ/mole (exothermic)}\)
2. \( C + O_2 \rightarrow CO_2 \) (complete combustion) \(- 406 \text{ kJ/mole (exothermic)}\)
3. \( C + H_2O \rightarrow CO + H_2 \) (gasification) \(+ 119 \text{ kJ/mole (endothermic)}\)
4. \( C + CO_2 \rightarrow 2CO + 162 \text{ kJ/mole (endothermic)}\)
5. \( CO + H_2O \rightarrow CO_2 + H_2 \) (water–gas shift) \(- 42 \text{ kJ/mole (exothermic)}\)
6. \( C + 2H_2 \rightarrow CH_4 \) (methanation) \(- 87 \text{ kJ/mole (exothermic)}\)

Carbon in the coal reacts with oxygen, steam and carbon dioxide as shown in the above equations. The products of these reactions then react with one another. Fixed bed (sometimes also called moving bed) gasifiers operate at 500–600 °C, while the entrained bed gasifiers operate at 1500 °C. Therefore, the latter are equipped with gas coolers to generate saturated or superheated steam, which is used in a combined cycle plant to generate more power.

The gas produced in a fixed bed gasifier contains tars and phenol which must be separated out and recycled to the gasifier. This is not true with an entrained bed gasifier because of the much higher process temperature which cracks the heavy hydrocarbon (tars and phenols).

Coal gasification is particularly interesting from the point of view of emission control. It is far easier to obtain a very clean fuel by cleaning fuel gas than by cleaning the combustion gas from a coal-fired boiler because:

1. The volume flow rate of coal gas is less than 1\% of that of the exhaust gas from the boiler.
2. It is much easier to remove H\(_2\)S than SO\(_2\).
3. Most other pollutants, i.e. heavy metals, chlorides and so on are also removed in the gasification process.
4. The major byproduct of the desulphurization process is elemental sulphur, which is easy to transport and sell.

5. Most oxygen-blown gasifiers produce unleachable slag. Waste disposal is thus easier.

Coal gasification is the most promising method to produce a truly clean coal burning plant. Table 5.6 gives the characteristics of different types of gasifiers.

5.10.1 IGCC

The integrated gasification combined cycle (IGCC) has been discussed in Chapter 3. The synthetic gas (syngas) exiting a typical gasifier is cooled by generating high pressure steam which is subsequently expanded in a steam turbine generator to produce electricity. The syngas is then cleaned of particulates, by cyclones, filters, and/or water scrubbing, followed by heat exchange to cool the gas to near ambient temperature (Fig. 5.59). Sulphur compounds in the gas, primarily, H₂S and some carbonyl sulphide (COS), are removed, which is ultimately converted to elemental sulphur for sale. The cleaned fuel is finally sent to the combined cycle where it is combusted and expanded in the gas turbine to produce electricity. Hot exhaust gases from the gas turbine are also used to generate steam for power generation.

Typical gasification plant (Fig. 5.59) includes the following blocks:

1. Coal preparation to prepare coal to meet gasifier specification.

![Basic IGCC plant arrangement](image_url)
2. Air separation—to generate 95% pure oxygen for feeding the gasifier, and to generate gaseous nitrogen for injection into the gas turbine and for plant inert gas uses.

3. Coal gasification—to convert the coal to gas by partial oxidation.

4. Ash/slag removal—includes the removal of ash/slag from the gasifier, dewatering and disposal.

5. Syngas cooling—to remove sensible heat from the gasifier effluent gas and to facilitate gas cleaning.

6. Particulate removal—to remove fine particulates in the raw gas stream.

7. Acid gas removal—to remove sulphur bearing gases in the syngas.

8. Sulphur recovery—to convert the sulphur bearing gases to elemental sulphur.

The system incorporates one advanced gas turbine–generator similar to a General Electric MS 7001F gas turbine (inlet temperature 1260 °C), which, apart from the engine, includes auxiliaries such as fuel system, lube oil system and control system. The unit also includes one heat recovery steam generator (HRSG) having superheaters, re heater and economiser, one steam turbine–generator, a surface condenser, an integral deaerator, boiler feedwater pumps, and a stack.

The steam turbine normally produces 30–40% of the total energy output of an IGCC plant. Typical steam conditions and approach temperature differences are given below (Blamire, 1994).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP steam pressure</td>
<td>101 bar</td>
</tr>
<tr>
<td>HP steam superheat temperature</td>
<td>540 °C</td>
</tr>
<tr>
<td>IP steam pressure</td>
<td>22 bar</td>
</tr>
<tr>
<td>Reheat temperature</td>
<td>540 °C</td>
</tr>
<tr>
<td>LP steam pressure</td>
<td>4 bar</td>
</tr>
<tr>
<td>Boiler pinch temperature difference</td>
<td>14 °C</td>
</tr>
<tr>
<td>Economiser approach</td>
<td>25 °C</td>
</tr>
<tr>
<td>Deaerator pressure</td>
<td>1.25 bar</td>
</tr>
<tr>
<td>Condenser pressure</td>
<td>0.047 bar</td>
</tr>
<tr>
<td>Stack temperature</td>
<td>120 °C</td>
</tr>
</tbody>
</table>

The gas turbine exhaust flows successively through the superheater, re heater, boiler and economiser sections.

A unique characteristic of an IGCC plant is its ability to take advantage of phased construction. Peaking turbines and/or a combined cycle plant of required capacities initially fuelled with fuel oil or natural gas may first be installed. It may then be followed by the addition of a coal gasification plant to refuel the combined cycle with coal-derived fuel gas. It has a higher overall efficiency and is more friendly with environment.

### 5.11 COMBUSTION OF FUEL OIL

Highly volatile petroleum fuels are too expensive for boiler firing. The heavy fuel oils, the residue left after refining of crude petroleum, are used. An oil burning installation is shown in Fig. 5.60.
Fig. 5.60 Schematic of an oil burning installation

Because of the high viscosity of fuel oil, steam heating coils are installed in the oil tank to facilitate pumping. Oil pumps, mostly of the reciprocating type, receive oil from the strainers and discharge it to the burners through heaters and a meter. Oil is required to be heated, either by steam or electricity, to further reduce viscosity and improve atomisation. Oil temperature should vary between 65 °C to 115 °C. It should not exceed 115 °C because then the oil may vaporize and even decompose or crack. Recirculation of oil is necessary initially to purge the line so that heavy oil does not collect and plug anywhere in the pipeline. Usually a light oil is used for half-an-hour or so when starting and stopping the boiler to wash the pipes clean of the heavy oil.

5.11.1 Mechanism of Fuel Oil Combustion

The sequential steps of spray combustion of fuel oil are:

1. Atomization
2. Evaporation
3. Mixing of evaporated spray with air
4. Combustion of spray

Fig. 5.61 Combustion of an oil droplet
Immediately on being atomized, the oil vaporizes and a film of vapour surrounds the oil droplet (Fig. 5.61). Around the vapour film there is a gas layer through which oxygen diffuses. Reaction occurs at the flame front where the concentration or partial pressure of oxygen is zero. The combustion products diffuse out from the flame front to the bulk fluid. The flame front is an area of very rapid chemical reaction and is a boundary between burned and unburned gas. The more rapid the combustion, the thinner the flame front.

### 5.11.2 Oil Burners

The major parts of an oil burner are the atomizer, air register, and necessary oil lines with strainers and valves. Atomization of the liquid exposes more surface area per unit mass, and thus promotes ignition and completes combustion. It is accomplished mechanically or with steam or air. In twin-fluid burners, high pressure steam or air is used to break up oil drops into fine droplets, enabling better mixing with the air for combustion (Fig. 5.62).

![Diagram of Oil Burners](image)

**Fig. 5.62 Steam- or air-atomised oil burners**

For large boilers, oil burners with pressure atomisation are suitable, in which oil is supplied under pressure (6–20 bar) and atomised by ejecting the oil through a small hole after passing through a swirl chamber (Fig. 5.63). A fine oil mist in the shape of a funnel forms in front of the nozzle outlet. The control of fuel oil consumption in pressure atomisers with load variation is done either by oil recirculation, part of the oil being returned to the oil tank from the swirl chamber through the return pipeline, or by means of a control piston which closes the tangential slots in the burner (Fig. 5.64).

Rotary cup atomisers are also important for the furnaces of large boilers (Morse, 1953). The oil flows through a hollow shaft into a rapidly rotating cup at the edge of which it is atomised under centrifugal force. The primary air is introduced by a fan around the outside of the cup and assists oil atomisation since it possesses swirl in the opposite direction of the rotation of the cup.
**Fig. 5.63** Pressure atomiser with tangential oil feed

**Fig. 5.64** Methods of output control in pressure atomisers

### 5.12 COMBUSTION OF GAS

Burning of gas is easy and clean. No atomization is required. Combustion of 1 m$^3$ of natural gas requires roughly 20 m$^3$ of hot air. Proper mixing of gas and air can be ensured by introducing the gas into the air flow in the form of thin jets of high penetrability (Fig. 5.65). Because of good mixing excess air required for combustion is less.

### 5.13 COMBINED GAS FUEL OIL BURNERS

One advantage of combined gas-fuel oil burners is that the change from one fuel to the other can be done quite easily. In addition, both fuels can be burned under almost optimal conditions. Natural gas flows from the central annular header through two rows of holes (Fig. 5.66). Air is supplied through a tangential vane register and controlled by disc valve. Fuel oil is atomized in a mechanical burner mounted in the central channel. The mixture is ignited by electric means.
Example 5.1  A bed of solid particles has a mass of 2500 kg. The density of solid is 2650 kg/m³. The mean particle size is 800 mm and the sphericity of particles is 0.84. Estimate the total surface area of the particles in the bed.

Solution  Surface-to-volume ratio of particles,

\[ S_v = \frac{A_s}{V_s} = \frac{N_p \pi d_p^2}{N_p \frac{\pi}{6} d_p^3} = \frac{6}{d_p} \]

where \( N_p \) is the number and \( d_p \) is the diameter of particles. For non-spherical particles, the surface-area of particles is

\[ A_s = \frac{6V_s}{\phi d_p} \]

where \( V_s \) is the volume and \( \phi \) is the sphericity of the particles.

\[ A_s = \frac{6 \times 2500}{2650 \times 0.84 \times 800 \times 10^{-6}} = 8423 \, \text{m}^2 \]

\( \text{Ans.} \)
Example 5.2 A bed of particles of mean size 427 μm is fluidized by air under the ambient conditions, where the air density is 1.21 kg/m³ and the viscosity is $1.82 \times 10^{-5}$ kg/m·s. The density of the loosely packed bed is 1620 kg/m³. If the density of solids is 2780 kg/m³, find (a) the voidage of the bed, and (b) the minimum fluidization velocity.

**Solution** (a) Bulk density of bed, $\rho_b = 1620$ kg/m³

Density of solids, $\rho_s = 2780$ kg/m³

\[
\varepsilon_{mf} = 1 - \frac{\rho_b}{\rho_s} = 1 - \frac{1620}{2780} = 0.417
\]

Ans.

(b) Archimedes number, $Ar = \frac{\rho_g (\rho_s - \rho_g) g d_p^3}{\mu_g^2}$

\[
Ar = \frac{1.21(2780 - 1.21) \times 9.81 \times (427 \times 10^{-3})^3}{(1.82 \times 10^{-5})^2} = 7753
\]

From Eq. (5.24),

\[
Re_{mf} = [C_1^2 + C_2 Ar]^{0.5} - C_1
\]

\[
= [27.2^2 + 0.0408 \times 7753]^{0.5} - 27.2 = 5.299
\]

Now,

\[
Re_{mf} = \frac{U_{mf} \times \rho_g \times d_p}{\mu_g}
\]

\[
U_{mf} = \frac{5.299 \times 1.82 \times 10^{-5}}{1.21 \times 427 \times 10^{-6}} = 0.185 \text{ m/s}
\]

Ans.

Example 5.3 For the above bed (Ex. 5.2), taking $U_{mf} = 0.14$ m/s, estimate the sphericity of particles.

**Solution** Using Ergun’s equation,

\[
\frac{\Delta p}{L} = 150 \left(1 - \varepsilon_{mf}\right)^2 \frac{\mu_g}{\varepsilon_{mf}^3} \frac{U_{mf}}{(\phi d_p)^2} + 1.75 \frac{(1 - \varepsilon_{mf}) \rho_g U_{mf}^2}{\varepsilon_{mf}^3} \frac{d_p}{(\phi d_p)}
\]

\[
= [1 - \varepsilon_{mf}] (\rho_s - \rho_g) g
\]

Multiplying each side by $\rho_g d_p^3 / m_g^2 (1 - \varepsilon_{mf})$ gives

\[
Ar = 150 \frac{1 - \varepsilon_{mf}}{\phi^2} Re_{mf} + \frac{1.75}{\phi^3} Re_{mf}^2
\]

Now,

\[
Re_{mf} = \frac{U_{mf} \rho_g d_p}{\mu_g} = \frac{0.14 \times 1.21 \times 427 \times 10^{-6}}{1.82 \times 10^{-5}} = 3.974
\]

\[
Ar = 7753
\]

\[
7753 = \frac{150(1 - 0.417)}{\phi^2 0.417^3} \times 3.974 + \frac{1.75}{\phi 0.417^3} \times (3.974)^2
\]
\[ \therefore \quad 7753 f^2 - 381.1 f - 4793 = 0 \]
\[ \therefore \quad f = 0.811 \quad \text{or} \quad -0.762 \]
\[ \therefore \quad \text{Sphericity} = 0.811 \quad \text{Ans.} \]

**Example 5.4** A fluidized bed combustion system having an output of 35 MW at 80% efficiency when using a coal of heating value 26 MJ/kg with a sulphur content of 5.6% requires a particular limestone to be fed to it at a calcium-sulphur ratio of 3.0, so as to limit emissions of SO\(_2\) adequately. The limestone used contains 85% CaCO\(_3\). Determine the required flow rate of limestone.

Solution  \( \frac{35}{0.80 \times 26} = 1.683 \) kg/s = 6057.7 kg/h

\[ \therefore \quad \text{Flow rate of sulphur} = \frac{6057.7 \times 0.036}{32} = 6.81 \text{ kmol/h} \]

For a Ca : S molar ratio of 3.0, the flow rate of calcium is \( = 6.81 \times 3.0 = 20.44 \) kmol/h

\[ \therefore \quad \text{Mass flow rate of CaCO}_3 \text{ required} = 20.44 \times 100 \]
\[ = 2044 \text{ kg/h (since molecular weight of CaCO}_3 \text{ is 100)} \]

\[ \therefore \quad \text{Mass flow of limestone} = \frac{2044}{0.85} = 2404.7 \text{ kg/h which is about 40% of the coal burning rate.} \]

**Example 5.5** A fluidized bed combustor burns a solid fuel of high volatile matter with a calorific value of 24 MJ/kg. The combustion conditions are such that 65% of the calorific value is released in the bed and the remainder in the above-bed zone from which the products leave at 850 °C. The air inlet temperature is 30 °C, the bed temperature is 850 °C and the air fuel ratio by mass is 13.5 : 1. The specific heat of the products leaving the bed surface is 1.035 kJ/kg K. If the burning rate of coal is 7000 kg/h, estimate the rate of heat removal (a) from the bed, and (b) from the above-bed zone.

Solution  (a) Heat removal from the bed per kg fuel
\[ = \text{heat released} - \text{heat carried away by the products} \]
\[ = 0.65 \times 24000 - 14.5 \times 1.035 \times (850 - 30) \]
\[ = 3294 \text{ kJ/kg fuel} \]

\[ \therefore \quad \text{Rate of heat removal from the bed} = 3294 \times \frac{7000}{3600} \]
\[ = 6405 \text{ kW} \quad \text{Ans.} \]

(b) Rate of heat removal from the above bed zone
\[ = \frac{7000}{3600} \times 0.35 \times 24000 = 16333 \text{ kW} \quad \text{Ans.} \]
Example 5.6  Fluidized bed is required to operate at atmospheric pressure and a
bed temperature of 850 °C. The fuel has a calorific value of 25 MJ/kg. The
stoichiometric air fuel ratio is 9.5 by mass and 20% excess air is used. The total
fueling rate is 4.8 MW. The density of air at 850 °C is 0.3145 kg/m³. Find the
planform area required if (a) the firing rate is 2 MW/m², and (b) the fluidizing
velocity is 2.7 m/s.

\[ \text{Solution} \]
(a) Planform area \( = \frac{4.8}{2} = 2.4 \text{ m}^2 \quad \text{Ans.} \)

(b) Fuel burning rate, \( \dot{m}_f = \frac{4.8 \times 1000}{25000} = 0.192 \text{ kg/s} \)

\[ \therefore \quad \text{Air flow rate, } \dot{m}_a = 9.5 \times 1.2 \times 0.192 = 2.1888 \text{ kg/s} \]

\[ \therefore \quad \text{Planform area } = \dot{m}_a/(\rho_a U) = \frac{2.1888}{0.3145 \times 2.8} = 2.48 \text{ m}^2 \quad \text{Ans.} \]

Example 5.7  Crushed coal particles, with diameters varying from 6 mm to 20
mm, and average density of 1250 kg/m³, are used in a fluidized bed combustor.
The coefficient of drag is 0.6 under fluidized conditions. Calculate (a) the
minimum gas velocity that fluidizes all the coal particles if the gas is at 850°C
and 9 bar pressure, and (b) the pressure drop in the bed assuming that the coal in
the collapsed state (\( \varepsilon_0 = 0.35 \)) has a height \( H_0 \) of 30 cm and voidage fraction
(\( \varepsilon = 0.50 \)) and the gas density is approximated by pure air.

\[ \text{Solution} \]
Minimum fluid velocity required for fluidization is calculated by
equating the drag force on a particle to the weight of the particle, or

\[ C_D \rho_f AV^{-2}/2 = \rho_s \times q_5 \times g \]

where \( A = \text{projected area} \), \( V = \text{velocity of flow} \), \( \rho_f = \text{fluid density} \), \( \rho_s = \text{solid}
density. If \( \gamma \) is the radius of the spherical particle, \( A = \pi \gamma^2 \) and \( V_s = \frac{4}{3} \pi r^3 \)

\[ \therefore \quad V = \left[ \frac{8}{3C_D} \times \frac{\rho_s}{\rho_f} \times gr \right]^{1/2} \]

For minimum \( r = 3 \text{ mm} = 3 \times 10^{-3} \text{ m} \),

\[ \rho_f = \rho_a = \frac{P}{RT} = \frac{9 \times 10^5}{287 \times (850 + 273)} = 2.792 \text{ kg/m}^3 \]

\[ \therefore \quad \bar{V} = \left[ \frac{8}{3 \times 0.6} \times \frac{1250}{2.792} \times 9.81 \times 3 \times 10^{-3} \right]^{1/2} \]

\[ = 7.65 \text{ m/s} \quad \text{Ans. (a)} \]

Height of fluidized bed, \( H \)

\[ H = H_0 \frac{1 - \varepsilon_0}{1 - \varepsilon} = 0.3 \times \frac{1 - 0.35}{1 - 0.5} \]

\[ = 0.39 \text{ m} \]
Pressure drop in the bed

\[
\Delta p = (1 - \varepsilon) H \rho_s g \\
= (1 - 0.5) \times 0.39 \times 1250 \times 9.81 \\
= 2.59 \text{ kPa}
\]

### SHORT-ANSWER QUESTIONS

5.1 On what factors does a chemical reaction depend?
5.2 What are the two broad types of furnaces?
5.3 Explain the physical and chemical factors determining the combustion rate of a fuel.
5.4 What are reversible and irreversible chemical reactions?
5.5 What is the law of mass action?
5.6 What is Arrhenius equation?
5.7 Explain what you understand by activation energy.
5.8 Explain why carbon monoxide is formed predominantly on the surface of burning carbon particles.
5.9 What is the average burning time of a pulverized coal particle?
5.10 Why does the ratio of CO/CO\textsubscript{2} increase with temperature?
5.11 Explain briefly the mechanism of burning of a carbon particle.
5.12 Explain what you understand by kinetically controlled reaction.
5.13 What is diffusion controlled reaction?
5.14 What do you mean by transition zone of combustion?
5.15 Explain diffusion zone and kinetic zone of combustion.
5.16 What are the different types of furnaces in which coal may be burnt?
5.17 What are the characteristics of an overfed fuel bed furnace?
5.18 Explain the characteristics of an underfed fuel bed furnace.
5.19 What is a stoker? What are the different types of stokers?
5.20 Explain the operation of a travelling grate stoker with the help of neat sketch.
5.21 Explain the operation of a spreader stoker with the help of a neat sketch.
5.22 What are the two conditions to be satisfied to burn pulverized coal successfully?
5.23 What are the advantages and disadvantages of pulverized coal firing?
5.24 What is the function of hot primary air?
5.25 What is the function of secondary air?
5.26 What is tempering air? When is it required?
5.27 What is the function of the coal crusher?
5.28 What are the three stages of pulverization process?
5.29 How are pulverizers classified?
5.30 Explain the characteristic features of a ball mill.
5.31 What is a bowl mill? What are its advantages?
5.32 On what factors does the performance of a pulverizer depend?
5.33 What is Hardgrove index?
5.34 What is the effect of ratio of VM/FC on the fineness of powdered coal?
5.35 What are the feeding systems of pulverized coal into the furnace?
5.36 Explain with a sketch the direct firing system? What are its merits and demerits?
5.37 What is the heat release rate of a p.f. - fired furnace?
5.38 Explain how the linear dimensions of a p.f. furnace are ascertained.
5.39 Explain the characteristics of a dry bottom furnace.
5.40 What is a wet bottom furnace? Where is it used? How is slag removed continuously?
5.41 What are the conditions to be satisfied by pulverized coal burners?
5.42 What are the different types of pulverized coal burners?
5.43 Distinguish straight flow and vortex burners.
5.44 Explain the distributed mixing burner.
5.45 Discuss the different arrangements of burners in a pulverized coal fired furnace.
5.46 What is tangential or corner firing? What are its merits and demerits?
5.47 What is a cyclone furnace? Where is it used? Mention its advantages and disadvantages.
5.48 Why is tertiary air required in a cyclone furnace? Where is it admitted?
5.49 What is a fluidized bed? What is minimum fluidization velocity?
5.50 What is a distributor? Why is it needed in a fluidized bed?
5.51 How is superficial velocity estimated?
5.52 What are the various regimes of fluidization?
5.53 Define (a) Voidage, (b) Sphericity of a particle.
5.54 How would you estimate $U_{mf}$?
5.55 What are particulate and emulsion phases?
5.56 What is transport disengaging height (TDH)?
5.57 What do you mean by elutriation rate?
5.58 What is slugging? How is it to be avoided?
5.59 Explain a turbulent fluidized bed.
5.60 What is the captive stage of a fluidized bed?
5.61 What is fast fluidization? On what factors does it depend?
5.62 Explain with a sketch a circulating fluidized bed (CFB).
5.63 Explain the variation of voidage, both radially and axially, in the riser column of a CFB.
5.64 Explain the core–annulus model of the structure of a CFB riser.
5.65 What is a cluster? What is its average voidage? With what velocity does it fall along the wall?
5.66 Explain the pressure balance along a CFB loop.
5.67 What do you understand by terminal velocity of particles? How does it depend on particle Reynolds number?
5.68 What is drag coefficient? On what does it depend?
5.69 What do you understand by attrition and fragmentation?
5.70 How is the fluidized bed combustion in a furnace started?
5.71 What is the advantage of using limestone or dolomite as the bed material in a fluidized bed? Why are they called "sorbents"?
5.72 What are the optimum combustion temperature and the approximate calcium-sulphur molar ratio for maximum absorption of sulphur oxides?
5.73 What is a PFBC? Discuss its scope, advantages and disadvantages.
5.74 Briefly explain the sequence of events of burning of a fuel particle in a fluidized bed.
5.75 What are the two resistances which impede the reaction of a char particle?
5.76 What is Sherwood number? What is Schmidt number? How are they related in diffusional mass transfer of oxygen?
5.77 On what factors does the specific burning rate depend?
5.78 Explain the influence of mass transfer by diffusion and chemical kinetics in controlling combustion rate in a fluidized bed.
5.79 What are the three distinct zones of combustion in a CFB furnace?
5.80 How does the lower zone of a CFB furnace act as a thermal flywheel with respect to boiler load variation?
5.81 What are the different factors which influence the combustion characteristics of coal in a CFB?
5.82 What is the distinction between air-blown and oxygen-blown gasifiers?
5.83 What are the different types of gasifiers? Briefly explain their characteristics.
5.84 What are the basic reactions taking place in coal gasifiers?
5.85 Why does an entrained bed gasifier require gas coolers?
5.86 How is coal gasification important from emission viewpoint?
5.87 Explain the IGCC plant with the help of certain relevant blocks.
5.88 What is the unique characteristic of an IGCC plant?
5.89 Explain with a schematic diagram the oil-burning installation of a steam generator.
5.90 Why is fuel oil required to be heated? Is there any restriction to its heating?
5.91 Explain the mechanism of combustion of a drop of fuel oil.
5.92 Discuss the different types of oil burners.
5.93 How is the fuel consumption in a pressure atomiser-cum-burner controlled?
5.94 What are the advantages of using combined gas-oil burners?

PROBLEMS

5.1 The pressure drop (N/m²) across the distributor when fluidized at ambient pressure and temperature is related to the superficial velocity \( U \) (m/s) by the equation

\[ \Delta p = 0.13 \, U + 0.03 \, U^2 \]

Silica sand particles are loosely packed in a bed to a depth of 0.3 m. The bulk density of the bed is 1600 kg/m³ and the bed diameter is 1.7 m. Determine the pressure drop across the bed and distributor, when the bed is incipiently fluidized. Take \( U_{mf} = 0.19 \) m/s. Also find the pumping power required by the system when fluidized with ambient air at 0.5 m/s.
5.2 A fluidized bed is operating at a velocity 2.8 times the minimum fluidizing velocity. Visual observation suggests that the largest bubbles formed are of 10 mm diameter. Estimate the residence time of a single, isolated bubble if the bed depth when fluidized is 650 mm. Take $U_{mf} = 0.15 \text{ m/s}$.

5.3 A bed of particles of mean size 483 mm is fluidized by air under the ambient conditions, where the air density is 1.22 kg/m$^3$ and the viscosity is $1.87 \times 10^{-5}$ kg/ms. The bulk density of the loosely packed bed is 1680 kg/m$^3$. If the density of solids is 2650 kg/m$^3$, estimate (a) the voidage of the bed, (b) the minimum fluidization velocity, and (c) the sphericity of particles.

5.4 A fluidized bed combustion system having an output of 40 MW operating at 78% efficiency with a coal of heating value of 25 MJ/kg with a sulphur content of 3.8% requires a particular limestone to be fed to it at a calcium-sulphur ratio of 3.1, so as to limit emissions of $\text{SO}_2$ adequately. The limestone used contains 85% $\text{CaCO}_3$. Determine the consumption rate of limestone.

5.5 A fluidized bed furnace operating at ambient conditions produces a stream of hot combustion products at a bed temperature of 850 °C. The mass flow rate of combustion products is to be 7.5 kg/s and the heat losses from the system are such that the air fuel ratio by mass is 25 : 1. Estimate the planform area required for the bed if the fluidizing velocity is 1.2 m/s. At 850 °C, take the density of air as 0.3145 kg/m$^3$. What is the firing rate in $\mu\text{W/m}^2$?

**BIBLIOGRAPHY**

A steam generator generates steam at the desired rate at the desired pressure and temperature by burning fuel in its furnace. Steam generators are used in both fossil-fuel and nuclear-fuel electric generating power stations. This chapter will, however, cover only fossil-fuel steam generators. Nuclear fuel steam generators are of very different design and will be covered in Chapter 9.

A steam generator is a complex integration of furnace, superheater, reheater, boiler or evaporator, economiser, and air preheater along with various auxiliaries such as pulverizers, burners, fans, stokers, dust collectors and precipitators, ash-handling equipment, and chimney or stack. The boiler (or evaporator) is that part of the steam generator where phase change (or boiling) occurs from liquid (water) to vapour (steam), essentially at constant pressure and temperature. However, the term “boiler” is traditionally used to mean the whole steam generator.

6.1 BASIC TYPES OF STEAM GENERATORS

Classification of steam generators or boilers can be made in different ways. From the point of view of applications, they can be

(a) utility steam generators
(b) industrial steam generators
(c) marine steam generators

Utility steam generators are those used by utilities for electric-power generating plants. Depending on whether the pressure of steam is below or above the critical pressure (221.2 bar), they can be either subcritical or supercritical units. The subcritical steam generators are water tube–drum type and they usually operate at between 130 and 180 bar steam pressure. The supercritical steam generators are drumless once-through type and operate at 240 bar pressure or higher. Majority of the utility steam generators are of the 170–180 bar water tube–drum variety, which produce superheated steam at about 540–560 °C with one or two stages of reheating. The steam capacities of modern
utility steam generators range from 120 to 1300 kg/s with power plant unit output ranging from 125 to 1300 MW. Pulverized coal is mainly the fuel used. Some circulating fluidized bed steam generators have also been installed recently in some utilities.

*Industrial steam generators* are those used in process industries like sugar, paper, jute and so on, and institutions like hospitals, commercial and residential building complexes. They are smaller in size. They can be pulverized coal fired, fluidized bed or stoker fired units, with coal mostly as the fuel. They can also be heat recovery types which use waste heat from various industrial processes, and are termed waste heat steam generators. They operate at pressures ranging from 5 to 105 bar with steam capacities upto 125 kg/s. Normally, they do not produce superheated steam. They supply wet or saturated steam, and sometimes even only hot water.

*Marine steam generators* are used in many marine ships and ocean liners driven by steam turbines. They are usually oil-fired. They produce superheated steam at about 60–65 bar and 540 °C.

Steam generators or boilers can also be classified according to the relative flows of products of combustion or flue gases and water. In one type of boiler, called the *fire-tube boiler*, the hot flue gases flow through tubes surrounded by water in a shell. In another type of boiler, called the *water-tube boiler*, water from a drum flows through the tubes and hot flue gases flow over them.

### 6.2. FIRE-TUBE BOILERS

Since the late eighteenth century, fire-tube boilers have been used in various forms to produce steam for industrial purposes and also for hauling railway locomotives and river launches. They are no longer used in utility power plants and steam locomotives have also mostly disappeared. However, they are still often used in industrial plants to produce saturated steam at the upper limits of about 18 bar pressure and 6.2 kg/s steaming capacity.

For small steam requirements, fire-tube boilers are suitable. They have certain inherent advantages like (1) low first cost, (2) reliability in operation, (3) need of only unskilled labour, (4) less draught required, and (5) quick response to load changes.

A fire-tube boiler is so named because the products of combustion pass through its tubes or flues, which are surrounded by water. They may be either:

(a) Externally fired (e.g. locomotive type boilers, Lancashire boilers, horizontal return tubular (HRT) boiler etc.), or
(b) Internally fired (e.g. Scotch-marine boilers, package boilers etc.).

Figure 6.1 shows a typical externally fired fire-tube boiler in which the furnace is outside the boiler shell. Coal is entered manually by shovels on to the grate by opening the fire-door. The products of combustion flow through the tubes which are immersed in the shell containing water. A fusible plug made up of a low melting point alloy (lead-based) is installed on the roof of the crown in the furnace. If the water-level in the shell falls below a certain level, the fusible
plug melts due to overheating and water pours down through the hole formed and puts out the fire. There is a spring-loaded safety-valve provided to keep the boiler pressure within the safety limit. The spring is set in such a way that the upward thrust of steam against the lid is balanced by the downward thrust of the spring. If the operating steam pressure exceeds this value, the upward thrust of steam will then be greater than the downward spring thrust and the difference will force open the lid upward, as a result of which steam will be released with a hissing sound, the steam pressure inside the shell will go down till the lid is forced down to be back on its seat.

![Diagram of a boiler](image)

**Fig. 6.1 A typical fire-tube boiler**

As the hot flue gases flow through the tubes, heat is transferred from gas to water all along the length. The gas is cooled and the water is heated till there is nucleate boiling around the tubes and steam is formed. Steam is taken out at the required rate by opening the main stop valve. Auxiliary steam may also be taken to operate a steam jet water injector to feed water into the shell. The shell is insulated all around by asbestos and 85% magnesia to reduce heat loss to the surroundings. Air flow from below the grate is regulated by operating dampers according to the requirement of combustion. An elliptic manhole is provided for a man to go in to do cleaning or repair as the need arises.
An oil-fired pressurized package type boiler is shown in Fig. 6.2. It is an internally fired fire-tube boiler, since the furnace is within the cylindrical shell. Oil is filtered, adequately heated either by electricity or steam, and is then fed through the burner. Air is supplied by an FD fan. Excess oil is returned to the main feedline. Combustion occurs in a horizontal corrugated flue, and the combustion gases flow through the tubes in two passes till they are let out through a chimney to atmosphere. The corrugations of the flue increase strength and help in allowing thermal expansion and contraction, and also in cracking and removing the soots deposited. The gases get cooled as they flow through the tubes and water is heated enough to form steam. The wet steam taken from the top flows through a bank of coils for superheating and the superheated steam is taken out as desired. Since the pressure inside the furnace is above atmospheric, caution is required to make the casing sealed against leakage. If it is not tightly sealed, hot gases will exfiltrate causing energy losses and polluting the atmosphere.

Fig. 6.2 An oil-fired package boiler

The package-type boiler has certain inherent advantages: (1) It is highly compact. A large surface area of heat transfer is provided in a small volume. The furnace volume is less since it is pressurized. (2) No ID fan is required. A short chimney is provided just for disposal of flue gases. (3) The entire unit is mostly fabricated in the factory itself. It is transported to the site and installed there with relative ease. (4) It occupies less space. (5) It is easy to operate.
Because of these advantages, package boilers were favoured in process industries. But the paucity of oil supply has reduced its demand giving way to the use of fluidized bed boilers mostly based on coal.

As stated earlier, fire-tube boilers are relatively inexpensive. They have large water storage capacity and can thus meet relatively large and sudden load demands with only small pressure changes. Reduction in pressure leaves the stored water superheated and causes part of it to flash into steam. However, the large water storage increases the explosion hazard of the unit, and also because of it, a longer period of time is required to bring the unit to steaming from a cold condition.

The major shortcoming of a fire-tube boiler is that definite size and pressure limitations are inherent in its basic design, i.e. the maximum size of the unit and the maximum operating pressure are limited. The tensile stress on the drum wall is a function of the drum diameter and the internal pressure given by

\[
\sigma = \frac{pd}{2t}
\]  

(6.1)

where \( \sigma \) = tensile stress, N/m\(^2\)

\( p \) = gauge pressure, N/m\(^2\)

\( d \) = internal diameter of shell, m

\( t \) = thickness of wall, m

The growing needs for increased quantities of steam at higher and higher pressures could not be met by fire-tube boilers, for as Eq. (6.1) shows, both higher pressures and larger sizes mean larger wall thicknesses. Thus, high pressures and large diameters lead to prohibitively thick shells, and the thicker the shell, the higher the cost.

### 6.3 WATER-TUBE BOILERS

Water-tube boilers were developed to permit increases in boiler pressure and capacity with reasonable metal stresses. As mentioned above, with higher steam pressures and capacities, fire-tube boilers would need large-diameter shells, and with such large diameters, the shells would have to operate under such extreme pressure and temperature stresses that the thicknesses would become very large. They are also subjected to large scale deposit and susceptible to boiler explosions, and become very costly.

The water-tube boiler, where water flows through the tubes and flue gases flow outside them, puts the pressure in the tubes and the relatively small-diameter drums, which are capable of withstanding extreme pressures of the modern steam generator.

#### 6.3.1 Straight-Tube Boiler

The water-tube boiler went through several stages of development. The earliest design employed straight tubes rolled into headers at each end, since straight
tubes could be made, installed and replaced easily. The tubes were of 75 to 100 mm O.D., inclined upward at about 15° to the horizontal and staggered. Nearly saturated water leaving the drum flowed through one header, called the downcomer, into the tubes. While flowing upward in the tubes some of this water on being heated by flue gases flowing outside get transformed to vapour and the two-phase water-steam mixture went back to the drum through the other header, called the riser. The density of nearly saturated water in the downcomer was larger than the density of the two-phase mixture in the riser, and this density difference caused natural circulation. The drum was arranged either parallel to the tubes (the longitudinal drum, Fig. 6.3a) or perpendicular to them (the cross drum, Fig. 6.3b). The feedwater from the feedwater heater flowed through an economiser to the drum, which supplied saturated steam to the superheater. The lower end of the downcomer was connected to a mud drum, which collected sediments from the circulating water and discharged through blowdown.

![Diagram of a water-tube boiler with a longitudinal drum](image)

**Fig. 6.3** Early water-tube boiler: (a) longitudinal drum

A single longitudinal drum, about 1.2 m in diameter, could accommodate only a limited number of tubes and hence a limiting heating surface. Depending on capacity, more than one drum could be installed in parallel. Longitudinal drum boilers were limited to low pressures (12–23 bar) and steam capacities (0.63–10 kg/s), having heating surfaces of 93–930 m².
Cross-drum boilers could accommodate many more tubes than longitudinal drum boilers because of their geometry and were built with heating surfaces of 93–2200 m², pressures 12–100 bar and steam capacities of 0.63–63 kg/s.

To ensure maximum exposure of tubes to the hot flue gases, baffles were installed across the tubes in both kinds of boilers to permit up to three gas passes.

### 6.3.2 Bent-tube Boilers

Straight-tube boilers had many disadvantages, like:

1. They had less accessibility and poorer inspection capability. Considerable time, labour and expense were required to open up or close the bolts in the headers, and to remove or replace the gaskets and handhole caps for tube inspection and replacement.
2. Inadequate design and imperfect fabrication of handhole caps resulted in much leakage, which was a continuous source of annoyance to the operators.
3. Circulation was sluggish due to low head, and limited steam disengaging surface made inadequate separation of steam and water reducing steaming rate.
Bent-tube boilers introduced in the 1880's offered many advantages over the straight-tube ones, the notable among them being greater accessibility for inspection, cleaning, and maintenance, and ability to operate at higher steaming rates and to deliver drier steam.

In a bent-tube boiler, the tubes were so bent that they entered and left the drums radially. As an example, a four-drum Stirling boiler is shown in Fig. 6.4. There was easy access to every tube. The combustion gases flowed upward from the furnace through the first bank of tubes connecting the front steam drum and the mud drum, through the superheater, and by proper baffling through the second and third tube banks connecting to the second and rear steam drums. The gases then flowed past an economiser in counterflow arrangement. Feedwater from the economiser entered the rear steam drum. Water circulated from the rear to the lower drum through the rear bank of tubes (downcomers) and then up through both the centre and front banks of tubes (risers) to the centre and front drums. All three drums had their steam and water regions interconnected at top and bottom. The tubes were typically 75–77 mm in OD and spaced 125–175 mm on centres, for easy replacement of defective tubes without removing neighbouring tubes.

Fig. 6.4 A four-drum bent tube boiler
More recent designs of Stirling boilers used water cooled furnace walls by lining the interior of the walls with tubes carrying the same boiler water, where partial boiling of water occurs. Fig. 6.5 shows a schematic of a natural circulation four-drum Stirling boiler. Water flows downwards from the mud drum to headers feeding the tubes lining the walls of the radiant furnace. The low-density steam-water mixture rises up to the steam drum at the upper right. The steam is separated and flows to the central drum, where it is removed. Feedwater enters the drum at the left and mixes with the sturred liquid in the left and central drum. The cooled liquid flows down to the mud drum.

Fig. 6.5  *Natural circulation four-drum Stirling boiler*
The four-drum Stirling boiler was superseded by a simpler two-drum design in which the steam drum was directly above the water drum with one bank of bent tubes to the front and another to the rear. Later designs of the two-drum boiler had a single gas-path. The Stirling boiler was generally capable of meeting conditions of rapidly varying loads and was adaptable to various fuels. It was employed in both stationary and marine applications.

Modern pulverized coal-fired boilers stand up vertically with a furnace, a steam drum, downcomers, risers, superheaters, reheaters, economiser, air preheater, tube mills, FD and ID fans, coal feeders and burners, windbox, various control systems, ductwork and all accessories and mountings. The penthouse encloses the drum, high-pressure headers and connecting links on top of the steam generator. The floor of the penthouse is formed by the roof tubes of the boiler. The principal structural members consist of vertical structural steel columns, a top support grid and beams (buckstays) which provide stiffness to the furnace, combustion air and flue gas flow enclosures. Igniters and warmup burners ignite and stabilize the coal flame. A ductwork system directs the flow of combustion air and flue gas. Soot blowers are devices that direct a high pressure fluid (steam or water) onto heat transfer surfaces to remove ash or slag deposits. Selected components of modern steam generators are shown in Figs 6.6 (a) and (b).

![Boiler Nomenclature](image_url)
6.3.3 Heat Absorption in Water-Tube Boilers

In a water-tube boiler, feedwater is heated in three kinds of heat exchangers, viz., economizer, evaporator (downcomer-riser circuit) and superheaters (Fig. 6.6). Feedwater from the h.p. heater enters the economiser where it is heated by the outgoing flue gases till it is saturated liquid at that pressure and then it is fed to the drum. Saturated water falls through the downcomer into the bottom header and moves up through the riser where water is partially boiled back into the drum (Fig. 6.7(a)). Saturated steam from the drum goes to the superheaters for being heated to the desired temperature. If the pressure drop during the entire heating process and any external heat loss are neglected, the line $1 - 4$ represents the heating process in $T-s$ diagram (Fig. 6.7(b)). For each kg of steam formed, the heats absorbed in the economiser (in the liquid phase), in the evaporator (liquid to vapour transition or the latent heat of vaporization), and in the superheater (in the vapour or gas phase) are given by
**Fig. 6.7(a)**  Heat absorption in a water-tube boiler

**Fig. 6.7(b)**  Heat absorption in different heat exchangers of a water-tube boiler

\[
\begin{align*}
Q_{\text{eco}} &= h_2 - h_1 \\
Q_{\text{eva}} &= h_3 - h_2 \\
Q_{\text{SH}} &= h_4 - h_3
\end{align*}
\]  \hspace{1cm} (6.2)
The percentage of total heat absorbed in the economiser, evaporator and superheater are:

\[
\text{% economiser} = \frac{Q_{\text{eco}}}{Q_{\text{eco}} + Q_{\text{eva}} + Q_{\text{SH}}} \times 100 = \frac{h_2 - h_1}{h_4 - h_1} \times 100
\]

\[
\text{% evaporator} = \frac{Q_{\text{eva}}}{Q_{\text{eco}} + Q_{\text{eva}} + Q_{\text{SH}}} \times 100 = \frac{h_3 - h_2}{h_4 - h_1} \times 100
\]

\[
\text{% superheater} = \frac{Q_{\text{SH}}}{Q_{\text{eco}} + Q_{\text{eva}} + Q_{\text{SH}}} \times 100 = \frac{h_4 - h_3}{h_4 - h_1} \times 100
\]  \hspace{1cm} (6.3)

If a reheater is used, the fraction of the total heat absorbed in the reheater can similarly be estimated. The use of a large number of feedwater heaters (up to seven or eight) means a smaller economiser, and a high pressure means a smaller boiler surface (risers) because the latent heat of vaporization decreases rapidly with pressure. Thus, a modern high pressure steam generator requires more superheating and reheating surfaces and less boiler surface than older units. The fraction of total heat absorbed in superheaters alone may be as high as 60% in a modern boiler.

### 6.3.4 Circulation

The flow of water and steam within the boiler circuit is called circulation. Adequate circulation must be provided to carry away the heat from the furnace. If circulation is caused by density difference, the boiler is said to have natural circulation. If it is caused by a pump, it has forced or controlled circulation.

A simple downcomer–riser circuit connecting a drum and a header is shown in Fig. 6.8. The downcomer, which is insulated, is outside the furnace, and the riser is inside it.

![Natural circulation in a downcomer–riser circuit](image)

**Fig. 6.8** Natural circulation in a downcomer–riser circuit
Nearly saturated water falls by gravity from the drum through the downcomer into the bottom header. From the header water flows up along the riser where it partially boils with the formation of bubbles and then back into the steam drum. The density of steam–water mixture in the riser is less than that of saturated water in the downcomer, and as a result of this density difference a circulation current is set up within the downcomer–riser circuit. The feedwater from the economiser enters the drum and saturated steam is taken out of the drum to the superheater.

The pressure head available for natural circulation is given by

$$\Delta p = gH(\rho_D - \rho_m)$$  \hspace{1cm} (6.4)

where $H$ is the height of the riser (~ furnace), $\rho_D$ is the density of saturated water in the downcomer, and $\rho_m$ is the mean density of steam–water mixture in the riser. For simplicity, the mean density, $\rho_m$, may be taken as the arithmetic mean of the densities at the bottom and top of the riser, or

$$\rho_m = \frac{\rho_{\text{bottom}} + \rho_{\text{top}}}{2}$$  \hspace{1cm} (6.5)

Now, $\rho_{\text{bottom}} = \rho_D$ and $\rho_{\text{top}} = 1/v_{\text{top}}$, $v_{\text{top}} = v_f + x_{\text{top}}v_{fg}$, where $x_{\text{top}}$ is the quality of the mixture at the top of the riser and is often referred to as the top dryness fraction (TDF).

The maximum height of the furnace ($H$) is fixed by the rate of burning of fuel and the heat release rate. For a certain $H$,

$$\Delta p \propto (\rho_D - \rho_m)$$  \hspace{1cm} (6.6)

Higher is the density difference, more will be the pressure head available for natural circulation. However, the density differential decreases as pressure increases (Fig. 6.9). It is observed in the figure that the difference in specific volume ($v_g - v_f$) or the difference in density ($\rho_f - \rho_g$) of saturated liquid and saturated vapour decreases with increase in pressure. At the critical pressure, $v_g = v_f$ and $\rho_g = \rho_f$ and there can be no natural circulation.

![Fig. 6.9 Differentials in specific volume and density varying with pressure](image)

(a)
Risers installed all around the four walls of the furnace act as cooling tubes or a water wall and carry away the heat from the furnace at the same rate at which heat is released in it by the burning of fuel. For this reason, adequate circulation must be provided in the circuit. If the circulation is not adequate, the rate at which heat is carried away will be less than the rate of heat release, and the difference will be stored in the metal of the riser tubes leading to their overheating and ultimately rupturing when the tube temperature exceeds the melting point of the metal.

If the boiler pressure is greater than 30 bar, the downcomers are placed outside the furnace so as to secure more density difference for natural circulation. If both downcomers and risers are placed inside the furnace and heated, the density difference will be less. However, for pressures less than 30 bar, both are placed inside, with the risers installed in the hotter zone. The tubes which are heated more will act as risers and the tubes which are heated less act as downcomers.

If the pressure of steam exceeds 180 bar, the density difference \( (\rho_f - \rho_g) \) becomes so small (Fig. 6.9) that natural circulation cannot be relied upon and then forced circulation is used where a pump circulates saturated water through all the risers around the furnace walls (Fig. 6.10).

![Diagram of forced circulation system](image)

**Fig. 6.10** Forced circulation system

There is a term called “circulation ratio” (CR) used in this connection. It is defined as

\[
\text{Circulation ratio} = \frac{\text{Flow rate of saturated water in downcomers}}{\text{Flow rate of steam released from the drum}}
\]

It refers to the amount of saturated water to be circulated through the downcomer–riser circuit per kg of steam released from the drum. Therefore,
\[ CR = \frac{m}{m_g} = \frac{m_g + m_l}{m_g} = \frac{1}{m_g/(m_g + m_l)} = \frac{1}{TDF} \]  

(6.7)

where \( m \) = mass of saturated water flowing through the downcomer–riser circuit during a certain time,

\( m_g \) = mass of steam released from the drum during the same time,

\( m_l = m - m_g \) = mass of liquid water (saturated) at the riser exit

\( TDF \) = top dryness fraction, i.e. the quality of liquid–vapour mixture discharged from the riser into the drum.

The circulation ratio in a natural circulation boiler varies from one riser tube to another. The riser tubes located opposite to the burners have more thermal loading and generate more steam. So, these will have less circulation ratio. While the risers located in the corners of the furnace are relatively cooler, and so these will have more circulation ratio. However, the circulation ratio in any tube should not be less than 6, i.e. at least six kg of saturated water must circulate in the downcomer–riser circuit for each kg of steam released. Otherwise, the tube will get overheated and fail prematurely. The circulation ratio should again not be higher than 25 for effective utilization of the tube in steam generation.

Too much steaming (or steam formation) is not desired in a riser tube and circulation ratio needs to be maintained above 6. From heat transfer consideration a wetted surface is always desired and nucleate boiling should always persist. Bubbles originate on the heated surface. If there is a high rate of heat transfer to the riser, there will be too much bubble formation with the result that the bubbles may coalesce and first form an unstable vapour film which continually collapses and reforms. With a still higher heat transfer, the vapour film may be stable. Since a vapour film has a much lower thermal conductivity than a liquid film, it will offer a large thermal resistance, almost blanketing the surface where it forms, as a result of which heat absorbed and carried away \((Q_2)\) will be much less than the heat transferred to the wall \((Q_1)\). The difference \((Q_1 - Q_2)\) will be stored in the metal of the tube with the increase in its internal energy (Fig. 6.11). Consequently, the temperature of the metal may exceed the melting point and the tube may rupture allowing tube leakage.

![Fig. 6.11 Nucleate and film boiling in a riser tube](image)

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Downcomers are fewer in number and bigger in diameter, whereas risers are more in number and smaller in diameter. Downcomers are meant to make the water fall by gravity. Bigger the diameter, less the pressure drop due to friction, since pressure drop is inversely proportional to the tube diameter.

\[ \Delta p = \frac{fL}{D} \rho \frac{V^2}{2} \]  

(6.8)

So, the downcomers are made bigger in diameter, which may vary from 150 to 200 mm or even higher. Now,

\[ m_f = \left( \frac{n \pi}{4} D^2 \right) \rho_f V \]  

(6.9)

where  
- \( m_f \) = mass flow rate of saturated water in the downcomers, kg/s
- \( n \) = number of tubes
- \( D \) = diameter of downcomer, m
- \( \rho_f \) = density of saturated water at the boiler drum pressure, kg/m³
- \( V \) = average velocity of water in downcomers, m/s

The circulation velocity \( V \) depends on the circulation ratio in the tube, and may vary from 0.4 to 1.4 m/s depending on capacity. Thus, the number of downcomers required can be estimated from Eq. (6.9). It can be 6, 8 or as high as 20, again depending on the capacity of the steam generator.

Risers absorb heat from the furnace. For the same total cross-sectional area, the smaller the diameter, the larger the surface area exposed to hot gas for heat transfer. Therefore, the risers are of smaller diameter, 62.5 to 76.5 mm, and larger in number compared to downcomers.

It is required that nucleate boiling should occur in each riser tube and film boiling is avoided. Too much steaming and less circulation ratio in a riser may cause a departure from nucleate boiling (DNB) and an onset of film boiling. Recently, the water tubes in high heat absorbing areas of the furnace are provided with internal twisters and springs which break the vapour film and retard the onset of DNB. Tubes ribbed helically on their inside surface are often used now. The ribbing creates a centrifugal action that directs the water droplets to the vapour film clinging to the surface and to wash it away.

The void fraction (\( \alpha \)) of a two-phase mixture, often called volumetric quality, is defined as

\[ \alpha = \frac{\text{volume of vapour}}{\text{volume of liquid + volume of vapour}} \]

Thus, \( \alpha \) is the volume fraction of vapour in a liquid vapour mixture. The quality (\( x \)) is the mass fraction of vapour in the two-phase mixture. While the specific volume of a mixture of quality \( x \) is given by

\[ v = (1 - x)v_f + x v_g = v_f + x v_{fg} \]

the density of the mixture of voidage \( \alpha \) is given by

\[ \rho = (1 - \alpha)\rho_f + \alpha \rho_g = \rho_f + \alpha \rho_{fg} \]
Let us consider the two-phase flow in a riser (Fig. 6.12). Let $A_f$ = cross-section occupied by saturated liquid

![Diagram of two-phase flow in a riser]

**Fig. 6.12** Two phase flow in a riser

$A_g$ = cross-section occupied by vapour bubbles

Then, $\alpha = \frac{A_g}{A_f + A_g}$ and $1 - \alpha = \frac{A_f}{A_f + A_g}$

$$\frac{A_f}{A_g} = \frac{1 - \alpha}{\alpha} \quad (6.10)$$

When a two-phase mixture is moving, the vapour, because of its buoyancy, has a tendency to slip past the liquid, i.e. move at a higher velocity than a liquid. The slip ratio $S$ is defined as

$$S = \frac{\overline{V}_g}{\overline{V}_f} = \frac{\text{velocity of vapour}}{\text{velocity of liquid}} \quad (6.11)$$

at any cross-section of the riser, and it is greater than unity. Now, the quality $x$ is given by

$$x = \frac{m_g}{m} = \frac{\rho_g A_g \overline{V}_g}{m} = \frac{A_g \overline{V}_g}{m v_g}$$

$$\therefore \quad \overline{V}_g = \frac{m \times v_g \times x}{A_g} \quad (6.12)$$

Similarly,

$$1 - x = \frac{A_f \overline{V}_f}{m v_f}$$

$$\overline{V}_f = \frac{(1 - x) m v_f}{A_f} \quad (6.13)$$

From Eqs (6.12) and (6.13)

$$\frac{\overline{V}_g}{\overline{V}_f} = \frac{x v_g A_f}{A_g (1 - x) v_f} \quad (6.14)$$
Therefore,
\[ S = \frac{x}{1-x} \times \frac{A_f}{A_g} \times \frac{v_g}{v_f} \]

Substituting \( A_f/A_g \) from Eq. (6.10),
\[ S = \frac{x}{1-x} \times \frac{1-\alpha}{\alpha} \times \frac{v_g}{v_f} \] \hspace{1cm} (6.15)

The slip ratio has been measured experimentally and found to vary between 1 and 10, approaching 1 at high pressures (where the liquid and vapour densities approach each other). As pressure increases, the slip ratio decreases.

From Eq. (6.15), by re-arranging
\[ \alpha = \frac{x v_g}{x v_g + (1-x) v_f S} = \frac{1}{1 + \frac{1-x}{x} \times \frac{v_f}{v_g} \times S} \]
or,
\[ \alpha = \frac{1}{1 + \frac{1-x}{x} \psi} \] \hspace{1cm} (6.16)

where \( \psi = \frac{v_f}{v_g} S \) \hspace{1cm} (6.17)

Similarly,
\[ x = \frac{1}{1 + \frac{1-\alpha}{\alpha} \times \frac{1}{\psi}} \] \hspace{1cm} (6.18)

The axial heat flux distribution to the riser can be used to determine the quality distribution. Using a reasonable value of \( S \) (between 1 and 2), a void fraction \( (\alpha) \) distribution along the height can be obtained. The mixture density distribution \( \rho \) is now found from
\[ \rho = (1-\alpha)\rho_f + \alpha \rho_g \] \hspace{1cm} (6.19)

The average mixture density in the riser is now obtained from
\[ \rho_m = \frac{1}{H} \int_0^H \rho(z) \, dz \] \hspace{1cm} (6.20)

where \( z \) is the axial distance from the bottom of the riser.

In the case of uniform axial heating, the above equation has been solved (ElWakil, 1985) to yield
\[ \rho_m = \rho_f - \rho_g \left\{ 1 - \left[ \frac{1}{\alpha_e (1-\psi)} - 1 \right] \ln \frac{1}{1-\alpha_e (1-\psi)} \right\} \] \hspace{1cm} (6.21)

where \( \alpha_e \) is the void fraction at the riser exit.

The total pressure head produced due to density difference, Eq. (6.4), is balanced by the pressure losses in the loop.
\[ \Delta p = gH(\rho_D - \rho_m) = \Delta p_D + \Delta p_R + \Delta p_{bends} + \Delta p_{header} \] \hspace{1cm} (6.22)
where  
\[ \Delta p_D = \text{pressure loss in the downcomer, Pa} \]
\[ \Delta p_R = \text{pressure loss in the riser, Pa} \]
\[ \Delta p_{\text{bends}} = \text{pressure loss in the bends etc., Pa} \]
\[ \Delta p_{\text{header}} = \text{pressure loss in the header, Pa} \]

provided there is no conversion of pressure to kinetic energy, as the fluid is discharged from the riser into the drum.

Figure 6.13 shows three downcomers, a lower (collecting) drum or header from which water enters the riser tubes, nine heated risers, an upper collecting drum or header, and four pipes for delivery of the steam. Each circulation circuit, operating at the designed boiler pressure, has a definite circulation rate associated with it, and this depends on the thermal loading of that circuit. Variation of this loading causes automatic variation in the circulation and it is impossible to alter it by any external means. The resistance to fluid motion along a pipe is proportional to the square of the velocity, to the configuration (vertical or horizontal) and dimensions of the pipe, and on the density of the fluid. With a given system, the configuration and dimensions are given and so is the density of water and steam, and for a given thermal load, the steam formation rate is known. The circulation rate and the velocity of steam–water mixture are unknown for each circuit. However, there exist definite circulation rates for which the expression, Eq. 6.22, applies.

![Diagram of a natural circulation boiler with downcomers, risers, heating, and headers.](image-url)

**Fig. 6.13** Circulation circuit in a natural circulation boiler
6.3.5 Steam Drum

Risers discharge into the drum a mixture of water, steam, foam and sludge. Steam must be separated from the mixture before it leaves the drum. Any moisture carried with steam to the superheater tubes contains dissolved salts. In the superheater, water evaporates and the salts remain deposited on the inside surface of the tubes to form a scale, which is difficult to remove. This scale reduces the rate of heat absorption, ultimately leading to the failure of the superheater tubes by overheating and rupture. The superheater tubes are exposed to the highest steam pressure and temperature on the inside and the maximum gas temperature on the outside. They are made of the costliest material. So, utmost care is taken so that no damage is done to it by the excessive carryover of moisture with steam. Some of the impurities in the steam may be vaporized silica, which may cause turbine blade deposits.

No vapour bubble should flow along with saturated water from the drum to the downcomers. This will reduce the density difference and the pressure head for natural circulation. The bubbles tending to flow upward may also impede the flow in the downcomer and thus affect circulation. The drum has to secure moisture-free steam going to the superheater and bubble-free water going to the downcomer (Fig. 6.14).

![Separation of two-phase mixture in the drum](image)

Water is adequately treated prior to feeding it to the boiler as make-up. Still, it has some impurities in the form of total dissolved solids (TDS). These solids are expressed in ppm (parts per million). One ppm has one part impurity in million parts of water, by mass. Saturated water from the economiser is continuously entering the drum. Steam is separated in the drum and is taken to the superheater. So the solid content of water (TDS) in the drum goes on increasing. To maintain a certain ppm in the drum, blowdown is necessary. Blowdown may be intermittent or continuous. In utility boilers, continuous
blowdown is preferred for proper control of solid concentration and for energy
recovery from it, if possible. Solids precipitate at the drum bottom and are
removed along with some hot pressurized water by opening the blowdown
valve at the drum bottom. Trisodium phosphate or tannin is injected into the
drum periodically in suitable doses to help precipitate salts at the drum bottom.
If the steam generator is viewed as a steady flow device, continuity of mass
requires that the total solids entering be equal to the total solids leaving. Therefore, at steady state for continuous blowdown,

\[ \text{Impurities in} = \text{Impurities out} \]
\[ \omega_{fw} \times \text{ppm}_{fw} = \text{BD(ppm)}_{\text{drum}} + (1-x)\omega_s \text{ppm}_{\text{carryover}} \]  

(6.23)

where

\[ \omega_{fw} = \text{rate of feedwater entering the drum, kg/s} \]
\[ \text{BD} = \text{rate of blowdown necessary, kg/s} \]
\[ \omega_s = \text{rate of steam flow from the drum, kg/s} \]
\[ x = \text{quality of steam leaving the drum} \]

The ppm of drum water depends on the pressure and varies from 1000 to
2000. The ppm of moisture carryover is maintained at 1–3 ppm.

The functions of the drum in a water-tube boiler can then be summarized as
follows:

1. To store water and steam sufficiently to meet varying load requirement.
2. To aid in circulation.
3. To separate vapour or steam from water–steam mixture, discharged by
   the risers.
4. To provide enough surface area for liquid–vapour disengagement.
5. To maintain a certain desired ppm in the drum water by phosphate
   injection and blowdown.

Separation of steam from steam–water mixture discharged by the risers is
one of the important functions of the drum. At low pressures (up to 20 bar)
gravity separation is simply used, if sufficient disengaging surface is provided
(Fig. 6.15a). Water being heavier separates out by gravity and steam moves up
\( \rho_f \gg \rho_g \). But at high pressures, separating force due to gravity is low. As
pressure increases, the density difference \( \rho_f - \rho_g \) decreases and so the
separating force also decreases. At 100 bar, e.g. the separating force is only
35% of that at 20 bar, and it is difficult to achieve adequate separation and there
is considerable moisture carried with steam. An improvement of the situation by
increasing the drum size to provide greater disengaging area definitely is not
economical in view of the large cost of high pressure drums. Therefore, a
positive mechanical means of steam–water separation is required.

Mechanical separators like baffles, screens and cyclones which are housed
inside the drum for separation of steam–water mixture are known as drum
internals. Baffle plates (Fig. 6.15b) act as primary separators. They change or
reverse the steam flow direction, thus assisting gravity separation, and act as
Fig. 6.15 Steam drum separation (a) gravity, (b) baffle and screen, (c) cyclone and scrubber

impact plates that cause water to drain off. Screens made of wire mesh act as secondary separators where the individual wires attract and intercept the fine droplets, just as fabric filters attract dust from gases. The accumulating water drops then fall by gravity back to the main body of water.

At high pressures, where the density differential between water and steam diminishes, centrifugal forces, much greater than the gravity forces, are used. Cyclone separators (Fig. 6.15c) utilize the centrifugal forces for separation of two-phase mixture, which is entered tangentially to direct the water downward and to make the steam flow upward. The steam then goes through the zig-zag path in corrugated plates, called the scrubber or dryer, on the way out to help
remove the last traces of moisture. Finally, perforated plates or screens under the drum exit provide the final drying action. Drum internals of a controlled circulation boiler are shown in Fig. 6.15(d).

Figure 6.15(e) shows the drum of a CEGB boiler (UK) where water from the economiser is supplied to the drum via a feedpipe along its length. A small pipe with a number of small holes along its length is used to distribute chemicals into the drum at intervals. The bottom of the cyclone discharges below the water level to prevent bubbles from going along with saturated water into the downcomers. Steam, while flowing through the scrubbers gets completely dried by a number of sharp changes in direction along its flow path.

\[ \omega_i \text{ (ppm)} + \omega_{CO} \text{ (ppm)}_{\text{drum}} = (\omega_i + \omega_{CO}) \text{ (ppm)}_{\text{CO}} \]  

(6.24)

where

- \( \omega_i \) = rate of low-ppm water injection, kg/s
- \( \omega_{CO} \) = rate of moisture carry-over, kg/s

**Fig. 6.15 (d) Drum internals of a controlled circulation boiler**
with the suffices “i” and “CO” denoting injection and carry-over respectively. The moisture carry-over will have the same ppm as the drum water. Steam washing also helps in reducing silica concentration in steam. The silica vapour is absorbed in the feedwater and so silica content in steam is reduced to acceptable levels.

A boiler drum is said to be “priming” if there is too much moisture carry-over because of a high water level or a high steaming rate. Erratic feedwater control and rapid changes in steaming rate can induce priming. A certain water level in the drum should be maintained to give adequate steam space and reduce priming.

Another phenomenon encountered in the delivery of impure steam is foaming. Foaming is a condition resulting from the formation of bubbles on the water surface. It is caused by the presence of saponification agents in the boiler water, like oil, certain dissolved salts and high alkalinity. Frequently, a surface blowdown connection is located on the drum at about the normal water level to allow some of the foam to be drawn off the water surface and wasted.

Steam drums in utility boilers have diameters as high as 5 m, lengths more than 30 m and weigh a few hundred tonnes. They are made in cylindrical sections, called courses, which are welded together, and two nearly hemispherical heads which are welded to the ends. Drum size largely depends on the space requirements of all the internals to perform the steps of purification.

**6.3.6 Modern Water Tube Boilers**

It is now usual in public utilities to have only one boiler per turbine. This has made it possible to build even the largest power plant in unit design thus simplifying the piping systems and facilitating boiler and turbine control, especially in plants using steam reheating.

The appearance of water-cooled furnace walls, called water walls, eventually led to the integration of furnace, economiser, boiler, superheater, reheater, and air preheater into the modern steam generator. Water cooling is also used for superheater and economiser compartment walls and various other components, such as screens, dividing walls, etc.

Three design concepts of water tube boilers are illustrated in Fig. 6.16. Type A is a boiler with natural circulation as is type (a). Heat transfer to the water tubes around the walls is mostly by radiation from the fuel flame and less by convection from flue gases. Natural circulation is used up to steam pressures of approximately 180 bar, with separation of the steam from the water taking place in the boiler drum. Boilers with forced circulation by a special pump, originally known as La Mont boilers, are shown schematically as type B and also (b) in Fig. 6.16. They offer a certain amount of freedom in the arrangement of evaporator tubes and the boiler drum. Such boilers can be adapted to limitations in height and space. They are suitable for steam pressures up to 200 bar. Boilers operating at subcritical pressures (<221.2 bar) which rely on a drum and recirculation, either natural or forced, are commonly known as drum boilers. Circulation in the boiler is often controlled by orifices, usually located at the
Fig. 6.16 Types of boiler circulation: A, (a) Natural circulation; B, (b) Forced circulation; C, (c) Once-through; D, (d) Once-through with recirculation (forced)
bottom of the tubes. They ensure even distribution of flow through the tubes in the waterwalls.

Boilers can be designed to operate at pressures above the critical pressure (221.2 bar). No drum is necessary in such boilers since no separation of the water and steam occurs, the two being at the same density. Thus, there is no recirculation. The water enters the bottom of the tubes and is completely transformed to steam by the time it reaches the top, passing through the tubes only once. For this reason it is known as a once-through boiler, often referred to as drumless boiler. It is illustrated in type C as well as (c) and (d) of Fig. 6.16.

Figure 6.17 shows a schematic flow diagram of a common natural circulation steam generator. Water at 170–260°C (depending on boiler pressure) from the high pressure feedwater heater enters the economiser and leaves saturated or as a two-phase mixture of low quality. It then enters the steam drum at midpoint (lengthwise). Water from the steam drum flows through insulated downcomers, which are situated outside the furnace, to headers. The headers are connected to the water tubes that line the furnace walls and act as risers. The water in the riser tubes receives heat mainly by radiation from the combustion gases and boils. The density differential between the water in the downcomers and that in the risers helps circulation. Steam is separated from the bubbling water in the drum and goes to the convective or primary superheater (CSH) where heat is absorbed by convective mode. From the convective superheater steam goes to the radiant superheater (RSH) installed at the top of the furnace, where heat is absorbed by radiation. Steam leaving the radiant superheater goes to the desuperheater where water of high purity is sprayed on to the steam to bring down the steam temperature to its desired value, if it exceeds the latter. From the desuperheater, steam finally goes to the pendent superheater (PSH) for further superheating before it leaves through the main stop valve to the high pressure (HP) turbine. Both RSH and PSH are often termed as secondary superheaters. Steam exiting the HP turbine goes back to the furnace for being re-superheated to the desired temperature in the reheater. Steam after being reheated flows to the intermediate pressure (IP) turbine for further expansion.

Atmospheric air fed by a forced draught (FD) fan is preheated by the outgoing flue gases in a heat exchanger called air preheater (APH). Air heated to the desired temperature, which varies from 250 °C to 400 °C depending on the type of coal burnt, is split into two streams, primary and secondary. Hot primary air (15–30% of total air) flows to the pulverizer to dry the coal for better grinding and also to help push the coal particles through the burners into the furnace. The hot secondary air flows into the windbox and gets distributed to the burners to help combustion. Fuel burns in the furnace with great intensity to some 1350 °C or even higher, depending on the quality of coal. The combustion gases impart portions of their energy to the water tubes (risers) and then to the superheaters, reheater, and economiser, and leave the latter at about 450 °C. From there they preheat the atmospheric air in the APH, leaving it at about 160°C. Dust collectors and electrostatic precipitators (ESP) collect the entrained solid particles (mostly ash) from the flue gas stream. An induced
Fig. 6.17  A typical natural circulation water-tube boiler
draught (ID) fan draws the flue gases from the furnace and sends them through the stack to the atmosphere. The exiting gas temperature (160 °C) represents an availability loss to the plant but it is accepted because of the following reasons.

1. The gas temperature should be kept well above the dew point temperature of the water vapour in the gases (equal to saturation temperature at the partial pressure of water vapour) to prevent condensation which would form acids (with SO₂ or SO₃) that would corrode metal components in its path.

2. The flue gases must have enough buoyancy to rise in a high plume above the stack for proper atmospheric dispersion.

From the dry bottom furnace the clinkers fall to the clinker grinder to be broken to pieces and then carried by water in a hydraulic sluice where the ash also falls. The dust collectors and the precipitator.

Another modern natural circulation two-drum boiler is shown in Fig. 6.18, where bulk of the steam is generated in the radiant boiler tubes along the furnace walls. The steam water mixture flows upward to the main steam drum (upper drum), where a centrifugal separator separates the steam and water. After passing through a dryer, the steam proceeds to the two convective superheaters.

**Fig. 6.18** A two drum natural circulation steam generator

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In the superheater, the hot gases leaving the furnace superheat the steam to the desired temperature. Feedwater is fed to the drum from which water flows downward through the tubes at the far left to the lower drum. This drum feeds liquid to the radiant tubes all around the furnace, where it is partially vaporized. The left hand tube bank acts as an economiser. The exiting gases then proceed to an air heater, where they preheat the combustion air.

Figure 6.19 shows another typical single drum water tube boiler with natural circulation. For this type of boiler the capacity varies from 136 to 3200 t/h. The steam conditions are usually subcritical with throttle pressure 126–175 bar and the temperature around 540 °C. The boiler can use coal, lignites, oil and natural gas as the fuel.

![Radiant boiler for pulverized coal](image)

**Fig. 6.19** Radiant boiler for pulverized coal

The general arrangement of a modern water-tube steam generator showing all the essential components is shown in Fig. 6.20 (a). Figure 6.20(b) shows the cross-section of the 210 MW Babcock downfired boiler installed in Bokaro “B” thermal power station (DVC), which burns low volatile high ash coal. The back pass is divided into two parallel sections: one consisting of horizontal re heater tubes and the other horizontal primary superheaters. The economiser is located below both the banks across the flow path.
The 500 MW NTPC units have single drum natural circulation reheat units with steam condition at 170 bar, 540 °C/540 °C, while the 210 MW units have steam condition at 135 bar, 540 °C/540 °C. The CEGB (UK) has 660 MWe boilers operating at 165 bar outlet pressure with superheater and reheater temperatures of 568 °C, and designs are now being offered of 900 MWe capacity with steam conditions of 182 bar, 568 °C/568 °C. The superheater outlet pressure of these latter boilers represents the upper limit of drum-type natural circulation boilers. Figure 6.20(c) shows the schematic of a 660 MWe CEGB boiler, while Fig. 6.20(d) shows the gas and steam/water flow paths between the boiler components. The back pass is divided into two parallel gas paths, one containing primary superheater and the other convective reheater with an economiser across both. Figure 6.20(e) gives a comparison of fossil-fired boilers over about 25 years, from 1952 to 1974, with evaporation capacity increasing from 55 to 560 kg/s and the furnace height from 32 to 61.4 m.

**Fig. 6.20** (a) General arrangement of a modern water-tube steam generator
AT₁ First stage attemperator
AT₂ - 2nd stage attemperator (L.H.S.)
AT₃ - 2nd stage attemperator (R.H.S.)

Fig. 6.20 (b) Cross-section of a 210 MW Babcock down-fired steam generator (Bokaro 'B' thermal power plant, DVC)
1. Feed water inlet
2. Economisers
3. Bottom bank
4. Top bank
5. Reheater bottom bank
6. Reheater top bank
7. Primary superheater outlet legs
8. Reheat outlet legs
9. Risers
10. Upflow cage rear outlet
11. Reheat outlet manifold
12. Platen superheater
13. Final superheater
14. Reheat final loop
15. Inlet boxes
16. Rear screen
17. Final superheater outlet manifold
18. Front wall
19. Rear wall
20. Side wall bottom boxes

Fig. 6.20 (c) Section through a 660 MW CEGB steam generator
**Boiler water wall** The water wall consists of a number of water or evaporation tubes (risers) spaced all around the walls of the furnace and carries away the heat released in the furnace by the burning of fuel. The water tubes could be installed in various ways as shown in Fig. 6.21 such as (a) tangent tubes touching the refractory wall, (b) tubes embedded in the refractory, (c) studded tubes, and (d) membrane tubes. The membrane water wall is now more common. It consists of tubes spaced on centres having centre-to-centre distance varying from 1.2 to 1.3 times the tube diameter. The tubes are connected by bars or membranes welded to them at their centre lines. The membranes act as fins to increase the heat transfer as well as to strengthen the
furnace construction. Insulation is provided on the outer side of the wall, and then a metal lagging is given to protect the insulation. The tube diameter varies from 68 to 76.2 mm.

![Diagram of a steam generator](image)

**Fig. 6.20** (c) Comparison of fossil fuel-fired steam generators over 25 years

The water wall absorbs heat mainly by radiation. Heavier triatomic gases like CO₂, H₂O vapour, SO₂ etc. are selective radiators emitting and absorbing radiation in certain ranges of wavelengths, mostly outside the visible range. The radiant energy absorbed by the water wall depends on the \((T_g^4 - T_w^4)\), \(T_g\) and \(T_w\) being the gas and wall temperature (K) respectively, the partial pressures of the gases \((P_{H_2O}, P_{CO_2}\) etc.) and the shape and size of the gases (view factor). This heat is conducted through the membranes and tube walls and is then convected to the two-phase mixture inside the tubes by nucleate boiling heat transfer. The heat transfer resistance of the latter is much smaller than the others and is often neglected in design calculations.

**Forced circulation and once-through boilers** A number of modern boilers use forced circulation rather than natural circulation. One scheme (La Mont type) is illustrated in Fig. 6.22. The operation is basically similar to a natural circulation unit, with a pump being used below the down-
**Fig. 6.21** Different types of water-wall: (a) tangent tubes touching the refractory wall, (b) tangent tubes embedded in the refractory, (c) studded tubes, (d) membrane tubes

**Fig. 6.22** Modern forced circulation steam generator of La Mont type
comer for circulating water through the risers. These are used at very high pressures (often above 160 bar), where the reduced density difference between liquid and vapour limits the natural circulation. They are often called pump-assisted circulation units since the natural circulation head is still significant. Assisted circulation boilers can supply between 130 to 3000 t/h of steam, depending on the design, at pressures ranging from 115 to 165 bar.

Both natural circulation and assisted circulation steam generators are designated as recirculating units, since only a fraction of water is evaporated in the riser tubes, and the steam separated in the drum proceeds to the superheater (Fig. 6.23a). In an once-through (OT) boiler, the feedwater goes through the economiser, furnace walls, and superheater sections, changing sequentially to saturated water, saturated steam, and superheated steam in one continuous pass (Fig. 6.23b). No steam drum is required to separate saturated steam from boiling water and no water recirculation takes place. Reheating of steam after it is expanded in the h.p. turbine is performed in a re heater in the usual manner.

Fig. 6.23 Schematic flow diagram of (a) drum type and (b) once-through steam generators. SU = superheater, EC = economiser

The once-through boiler is the only type suited to supercritical pressure operation (above 221.2 bar) because there is no latent heat of vaporization involved, and water on being heated at constant (assumed) supercritical pressure suddenly flashes into vapour, there being no saturation temperature as such, and the vapour is further heated to the desired temperature in the superheaters. Although there is no drum, but there should be a transition section where the water is likely to flash in order to accommodate the large increase in volume. Water enters the boilers at one end of a number of parallel paths and emerges at the outlet as superheated steam. Movement of the water is forced by the boiler feed pump.
A large number of pioneering super-critical pressure once-through boilers were built for the utility industry, many with double reheat, with steam pressures of the range 310–345 bar and steam temperatures 620–650 °C and cross compound (two-shaft) turbines. To reduce operational complexity and improve equipment reliability, moderate steam conditions of 240 bar and 540–565 °C were used. However, the use of higher steam conditions lowers the heat rate. Figure 6.24 shows a simplified view of a C.E. Sulzer Monotube Steam Generator with a triflux re heater where steam is reheated partly by steam and partly by the gas. The name derives from the feature of using continuous tubes from the inlet to the outlet of the evaporating section. This avoids the distribution problem inherent in a design employing intermediate headers.

Fig. 6.24 Simplified diagram of a sulzer monotube steam generator with triflux re heater

Subcritical boilers may also use the once-through design, incorporating steam and water separation and removal of a small amount (about 5%) of the water to permit blowdown of concentrated solids. The economical range of pressure varies from 138 to 276 bar and the steam output varies from 3.8 to 1260 kg/s.

A principal advantage of the once-through boiler is that it does not require circulating pumps or drums. Energy required for circulation is provided by the feed pump. The design is well suited to quick starts and rapid load changes, and
since it utilizes small diameter tubes, it has no drum, and provides positive circulation under all operating conditions. The prime disadvantage is the requirement of extremely pure boiler water, about 0.1 ppm TDS in the supercritical design, since all solids present are deposited in the tubes or carried along with steam to turbine. In the subcritical pressure range, the limitation is slightly less severe since there is some blowdown of boiler water. The capital costs for a supercritical steam generator are a few per cent higher than those of a drum type subcritical one of the same capacity, but because of the increased efficiency of the power plant the capital costs of the turbogenerator, as well as other components (condenser, cooling towers, feedwater heaters, etc.) are lower. The net effect is that the total production costs of electricity are lower for the supercritical cycle with double reheat.

Forced circulation is often adopted in low pressure waste heat boiler (HRSG) to enhance heat transfer (absorption) from waste gases since the heat transfer coefficient in forced circulation riser is more than that in natural circulation.

6.4 ECONOMISERS

An economiser is a heat exchanger which raises the temperature of the feedwater leaving the highest pressure feedwater heater to about the saturation temperature corresponding to the boiler pressure. This is done by the hot flue gases exiting the last superheater or reheater at a temperature varying from 370 °C to 540 °C. The term “economiser” was used historically because the throwing away of such high temperature gases involved a great deal of energy loss. By utilizing these gases in heating feedwater, higher efficiency and better economy were achieved, and hence the heat exchanger was called “economiser”.

Modern economisers are often designed to allow some boiling of the feedwater in the outlet sections, up to 20 per cent quality at full power, less at part loads. They are often termed as “steaming economisers”.

Economiser tubes are commonly 45–70 mm in outside diameter and are made in vertical coils of continuous tubes connected between inlet and outlet headers with each section formed into several horizontal paths connected by 180° vertical bends. The coils are installed at a pitch of 45 to 50 mm spacings, which depends on the type of fuel and ash characteristics.

Figure 6.25 shows an economiser coil and the temperature profiles of flue gas and water heated to saturation temperature.

The rate of heat transfer from the flue gases to feedwater is given by

\[ Q_{ECO} = \omega_g c_{pg} (t_{g1} - t_{g2}) \]
\[ = \omega_{fw} c_{pfw} (t_{sat} - t_{fw}) \]
\[ = U_o A_o \Delta t_{1,m} \]

(6.25)

where \( \Delta t_{1,m} = \text{log–mean temperature difference} \)
Fig. 6.25  Economiser coil and the temperature profiles of flue gas and water

\[
\Delta t_{l,m} = \frac{\Delta t_i - \Delta t_c}{\ln \frac{\Delta t_i}{\Delta t_c}}
\]

\[
\Delta t_i = t_{g1} - t_{sat}, \Delta t_c = t_{g2} - t_{fw} \quad \text{and}
\]

\[
\frac{1}{U_o A_o} = \frac{1}{h_{sc} A_i} + \frac{1}{h_i A_i} + \frac{x_w}{k_w A_{1,m}} + \frac{1}{h_{fo} A_o} + \frac{1}{h_o A_o}
\]

(6.26)

where \(U_o\) = overall heat transfer coefficient, \(h_{sc}\) = heat transfer coefficient due to scale formed on the inside of the tubes, \(h_i\) = inside heat transfer coefficient (water side), \(k_w\) = thermal conductivity of wall material, \(A_i = n \pi d_i l, A_o = n \pi d_o l, l = \text{length of one coil}, n = \text{number of coils}, A_{1,m} = \text{the log-mean area} = (A_o - A_i)/(\ln A_o/A_i), x_w = \text{thickness of tube wall}, h_{fo} = \text{heat transfer coefficient due to solids (ash and soot) on the outside surface of tubes, and} h_o = \text{heat transfer coefficient of the outside film (gas-side). If the resistances due to scale (1/h_{sc} A_i) and fouling (1/h_{fo} A_o) are neglected (considering them to be clean), and if the wall thickness is small, then the overall heat transfer coefficient is}

\[
\frac{1}{U_o} = \frac{1}{h_i} + \frac{x_w}{k_w} + \frac{1}{h_o}
\]

(6.27)

The gas-side heat transfer coefficient is much less than the water-side heat transfer coefficient, i.e. \(h_o << h_i\). To compensate this, the outer surface of the tubes may be finned to increase the surface area of heat transfer. Both gas and liquid flow under forced convection, for which

\[\text{Nu} = f(\text{Re, Pr})\]

where, \(\text{Nu} = \text{Nusselt number} = h d/k_f\), \(\text{Re} = \text{Reynolds number} = \bar{V} d/\nu_f\), \(\nu_f = \text{kinematic viscosity of fluid, Pr} = \text{Prandtl number} = (c_p \mu_f)/k_f, \mu_f = \text{dynamic viscosity}\)
viscosity of fluid, and $k_f$ = thermal conductivity of fluid film. If the flows are turbulent, Dittus-Boelter equation can be used to evaluate heat transfer coefficient as given below

$$\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^n$$

(6.28)

where $n = 0.4$ if the fluid is heated and $n = 0.3$, if the fluid is cooled.

With the help of Eqs (6.27) and (6.28), $U_0$ can be estimated, and from Eq. (6.25) the surface area $A_o$ needed can be ascertained, where $A_o = n \pi d_o l$.

Since feedwater is flowing through the tubes, at exit from the economiser, with water being saturated,

$$\omega_{fw} = \left( n \frac{\pi}{4} d_i^2 \right) \frac{V_{fw}}{v_f}$$

(6.29)

where $V_{fw} =$ velocity of fluid at exit and $v_f =$ specific volume of saturated water.

Higher is the fluid velocity (or Reynolds number), less will be the film thickness and more will be the heat transfer coefficient. But higher velocity means more pumping power required, i.e. more power is required to be imparted to the fluid ($V \Delta p$), where $V$ is the volume flow of fluid ($= \omega_{fw} v_f = A_i V_{fw}$) the power required is thus proportional to the cube of fluid velocity (since $\Delta p \propto V_{fw}^2$). These two conflicting considerations of higher heat transfer and more pumping power lead to the optimization of fluid velocity. On the gas-side, the velocity of flue gas is about 10-12 m/s and on the water-side, the water velocity is limited to 1-1.2 m/s. If the water velocity $V_{fw}$ is prescribed, from Eq. (6.29), the number of parallel coil paths required to accommodate the given flow can be estimated, and since $A_o$ has already been computed, the length of one coil ($l$) can be determined.

If the clearance $C$ is given on two sides of the gas duct of width $B$ (Fig. 6.25), then the number of turns, $n_t$, of one coil can be estimated from

$$n_t = \frac{l}{B - 2C}$$

(6.30)

Again, if the vertical pitch or the centre-to-centre distance of the consecutive horizontal tubes forming the coil, $p$, (Fig. 6.26) is assumed, then the height of duct occupied by the economiser is equal to

**Fig. 6.26** Section x-y in Fig. 6.25(a): height of the economiser duct
\[ h_{\text{ECO}} = n_x p \]  \hspace{1cm} (6.31)

The economiser is generally placed between the convective superheater and the air preheater.

### 6.5 SUPERHEATERS

The superheater is a heat exchanger in which heat is transferred to the saturated steam to increase its temperature. It raises the overall cycle efficiency. In addition, it reduces the moisture content in the last stages of the turbine and thus increases the turbine internal efficiency.

In modern utility high pressure boilers, more than 40% of the total heat absorbed in the generation of steam takes place in the superheaters. So, large surface area is required to be provided for superheating of steam.

Superheaters are commonly classified as either convective superheaters, radiant superheaters or combined superheaters, depending on how heat is transferred from the gases to steam. Convective superheaters are located in the convective zone of the furnace, usually ahead of the economiser. Earlier designs screened the convective superheater from the furnace radiation by a bank of water filled tubes which not only protected them from combustion flames and high temperature, but also intercepted the slag particles and reduced slagging problems. The convective superheaters are often termed as primary superheaters where saturated steam from the drum is admitted. Although the disposition of heating surface varies widely with the design of the steam generator, a typical arrangement is shown in Fig. 6.17. After convective superheater (CSH), the steam proceeds to the radiant superheater (RSH) which is placed in the radiant zone of the furnace near the water-wall to absorb heat by radiation. Steam leaving the radiant superheater goes to the desuperheater where highly pure water is directly sprayed on to steam in such a quantity that the temperature of steam after the last stage of superheating in the pendant superheater (PSH) does not exceed the rated value. The pendant superheater is a combined superheater in the sense that it receives heat partly by convection and partly by radiation. The radiant and combined superheaters together are often termed secondary superheaters.

Figure 6.27 shows the heat absorption in the three kinds of superheaters.

The energy balance of the convective superheater (Fig. 6.28) gives

\[ Q_{\text{CSH}} = \omega_g c_p g (t_{g1} - t_{g2}) = \omega_s (h_2 - h_1) \]

\[ = U_o A_o (\Delta t)_{\log-mean} \]  \hspace{1cm} (6.32)

where

\[ \frac{1}{U_o} = \frac{1}{h_i} + \frac{x_w}{k_w} + \frac{1}{h_o} \]  \hspace{1cm} and \hspace{1cm} \( A_o = n \pi d_o l \) \hspace{1cm} \( l = \text{length of one coil} \)

and \( n = \text{number of coils} \). Since it is a gas-to-gas heat exchanger with low \( h_i \) (steam) and \( h_o \) (gas), the overall heat transfer coefficient \( U_o \) is also low. Now,

\[ \omega_s = \left( n \frac{\pi}{4} d_i^2 \right) \frac{V_s}{v_g} \]  \hspace{1cm} (6.33)
Fig. 6.27  Heat absorption in the three kinds of superheater

Fig. 6.28  Convective superheater coil and the temperature profiles

where \( V_s \) is the average velocity of steam in the superheater. For intermediate pressure boilers, the steam velocity may be about 22 m/s, for high pressure boilers it is about 12 m/s and for very high pressure boilers it is 10 m/s (approximate).

Superheaters and reheaters are made of tubes 50 to 75 mm OD in utility boilers. The smaller diameters have lower pressure stresses and withstand them better. The larger diameters have lower steam–flow pressure drops and are easier to align. The number of coils (\( n \)) needed for the flow is estimated from Eq. (6.33) and the length of one coil (\( l \)) is determined after \( A_o \) is computed from Eq. (6.32).

Higher is the gas velocity (\( V_g \)), higher will be the heat transfer coefficient on the gas side (\( h_o \)) and higher also will be the pressure drop of gas (\( \alpha V_g^2 \)) and hence, the pumping power (\( \alpha V_g^3 \)). Calculations involving the cost of heating surfaces, electrical power and so on in driving fans have given a figure of 20 m/s as an economic gas velocity (Kholodovskii, 1965). However, this
economic gas velocity is not achieved in boiler plant because damage to heating surfaces by erosion from the fly ash in the flue gas increases as the cube of gas velocity, i.e.,

\[ \text{Erosion rate } \alpha V_g^3 \]  

(6.34)

Thus, the greater the ash content of the fuel, the less is the allowable gas velocity in the system. For low-ash coals, the gas velocity in the superheaters and economisers should not exceed 12 m/s, while in tubular air preheaters it should be below 16 m/s. For high-ash coals, the gas velocity should be limited to 8 m/s in superheaters and economisers, and in air preheaters it should be limited to 13 m/s.

As demand for steam increases with increase in load, fuel and air flow and hence, combustion gas flow are increased. The convective heat transfer coefficients \((h_i \text{ and } h_o)\) increase both inside and outside the tubes, increasing the overall heat transfer coefficient \((U_o)\) between gas and steam faster than the increase in mass flow rate of steam alone. The combustion temperatures do not materially change with load. Thus, the steam receives greater heat transfer per unit mass flow rate, and its temperature increases with load (Fig. 6.29).

![Graph of Steam-outlet temperature response of convective, radiant, and combined (in series) superheater](image)

**Fig. 6.29** Steam-outlet temperature response of convective, radiant, and combined (in series) superheater

The radiant superheater located in the radiant zone of the furnace permits greater heat absorption (Fig. 6.27) by the radiative mode of heat transfer.

\[ Q_{RSH} = \sigma A_T F_{fw} (T_f^4 - T_w^4) = w_s(h_3 - h_2) \]  

(6.35)

where, \(\sigma = \text{Stefan-Boltzmann constant} = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4\), \(A_T = \text{total surface area of superheater exposed to the flame, m}^2\), \(F_{fw} = \text{view factor with respect to flame and wall, } T_f = \text{flame absolute temperature, } K, \text{ and } T_w = \text{wall absolute temperature, } K\). Now, \(t_w = t_{sat} + 50\) to 75°C.
Since $T_r >> T_w$, $Q_{RSH} \propto T_r^4$

Now, $T_r$ is not strongly dependent on load. As the steam flow increases with load, the heat transfer per unit mass flow decreases. Thus, an increase in steam flow due to an increased load demand would result in a reduction in exit steam temperature, the opposite effect of a convection superheater (Fig. 6.29). The combination of convective and radiative superheaters in series gives a relatively flat curve for steam outlet temperature and is thus insensitive to load change.

The steam velocity in radiant superheater is approximately 7.5 m/s, coefficient of heat transfer is about 3140 W/m$^2$ °K and heat flux 116.28 kW/m$^2$.

Pendant type superheaters (PSH) are hung or suspended from above (Fig. 6.30). They have the advantage of firm structural support but the disadvantage of flow blockage by condensed steam after a cold shut down. It needs a slow restart to purge the water that accumulates in the bottom of each coil (by condensation, from previous run).

![Diagram of a superheater](image)

Fig. 6.30 Pendant (mixed flow) superheater

In a PSH, about 3/4th of the heating surface is made counter flow, while the remaining 1/4th acts in parallel flow (Fig. 6.30). If the entire coil were in counter flow, the exit end of the superheater coil will be subjected on the inside to the maximum steam temperature (say, 560 °C) and on the outside to the maximum gas temperature which may be about 1100 °C. This departure from
the most economical arrangement (counter flow) is done by making a few turns act in parallel flow so as to avoid the exit end of the superheater being damaged by metal failure. The velocity of steam in the tubes must range from 15 to 25 m/s depending on the permissible pressure drop.

Radiant and pendant superheaters (RSH and PSH) can be in the form of coils or platens. Figure 6.31 shows different types of superheater coils. The bank of tubes becomes most compact when double-coiling is used (b and c). Platens are also hung from above and can be in various forms (Fig. 6.32). They are also compact heat exchangers, providing large surface area per unit volume.

![Types of superheater coils](image)

**Fig. 6.31** Types of superheater coils: (a) single coil, (b) double coil one tube, (c) double coil two-tube

![Types of platens](image)

**Fig. 6.32** Types of platens (SH): (a) triangular, (b) rectangular

Essentially, the modern boiler is designed with two parallel gas flow paths at the rear of the boiler prior to the air preheater (Fig. 6.20b). One gas path contains convective superheater surface, while the other contains convective
reheater surface. A gas-tight baffle separates the two flow-paths. A common economiser is usually situated across both the two flow paths. The quantity of gas flowing down each of the two parallel flow paths is regulated by the gas dampers usually situated below the economiser surfaces. The reheat steam temperature control system is linked to control the damper position regulating the gas flow down the reheat gas pass.

6.6 REHEATERS

The design considerations for reheaters are similar to those for superheaters. Although the steam outlet temperatures are about the same, the overall temperatures are lower and the steam pressures are about 20–25% of those in the superheaters (Fig. 6.33). The pressure stresses are, therefore, lower and a lower grade steel alloy is tolerated. In addition, larger tube diameter with higher stresses may be used to reduce the pressure losses in the re heater.

![Diagram of reheat system](image)

Fig. 6.33 Convective reheater

The rate of heat absorption in the reheater (Fig. 6.33) is

$$Q_{RH} = \omega_s(h_4 - h_3) = \omega_s c_{pg} (t_{g1} - t_{g2})$$

$$= U_o A_o \Delta t_{1,m}$$  \hspace{1cm} (6.36)

where \(A_o = n \pi d_o l\), \(n = \) number of coils, \(l = \) length of one coil, and

$$\frac{1}{U_o} = \frac{1}{h_i} + \frac{x_w}{k_w} + \frac{1}{h_o}$$

Again,

$$\omega_s = \left( n \frac{\pi d_i^2}{4} \right) \frac{V_s}{v_s}$$  \hspace{1cm} (6.37)

where \(V_s\) is the steam velocity, say, at inlet, and \(v_s\) is the specific volume at that section (inlet).
The reheater is usually located above the primary or convective superheater in the convective zone (Fig. 6.17) of utility boilers. It consists of a number of vertical coils of horizontal tubes connected between two headers, similar to the economiser and CSH. Steam comes from the h.p. turbine and after being reheated it goes to the i.p. turbine for further expansion.

In modern high pressure boilers, reheaters are normally in two sections. The primary section is placed in the convective zone of the backpass and the secondary section is placed just at the furnace exit hanging from the top, known as pendant reheater.

### 6.7 STEAM GENERATOR CONTROL

The object of steam generator control is to provide the steam flow required by the turbine at design pressure and temperature. The variables that are controlled are fuel firing rate, air flow, gas flow distribution, feedwater flow and turbine valve-setting. The key measurements that describe the plant performance are steam flow rate, steam pressure, steam temperature, primary and secondary air flow rates, fuel firing rate, feedwater flow rate and steam drum level, and electrical power output. The control system must act on the measurement of these plant parameters so as to maintain plant operation at the desired conditions.

Most modern control instrumentation employs closed loop control. In this mode, the actual output of the system is measured and compared to some demand signal (set point). The difference between the measurement and demand, called the *error signal*, is then used to reduce the difference between measurement and set point to zero. Proportional control is the simplest type of closed loop control. In this mode, the controller output is proportional to the error signal. The control signal will be either directly or inversely proportional to the error signal, depending on the control action required.

Only a few basic control systems related to feedwater and drum level, steam pressure and steam temperature are discussed below.

#### 6.7.1 Feedwater and Drum Level Control

Feedwater and, therefore, steam flow is controlled to meet load demand by the turbine and at the same time maintain the level of water in the steam drum within relatively narrow limits. Normally, the water level in the drum is maintained half-full up to the diametral plane. A high steam consumption by the turbine, combined with low feedwater supply would lower the water level in the drum. A three-element automatic control system, of which the drum level is one element, is shown in Fig. 6.34. The drum level sensor responds to the error between actual drum level and its set point, such as in the case of high steam consumption and low feedwater supply, and acts on the controller to increase the feedwater valve opening to meet the steam flow demand. This action may be too slow and is supplemented by sensors for feedwater and steam flow. The
difference between the signals from these two sensors anticipates changes in drum level and sends a signal to the controller to actuate the valve in the desired direction.

![Diagram of a three-element feedwater control system](image)

**Fig. 6.34** *Schematic of a three-element feedwater control system*

### 6.7.2 Steam Pressure Control

The steam pressure control system, sometimes called the “boiler master”, maintains steam pressure by adjusting fuel and combustion air flows to meet the desired pressure (Fig. 6.35). When pressure drops, the flows are increased. A steam pressure sensor acts directly on the fuel flow and air flow controls, such as the pulverized coal power drives and forced draught fan, to affect the desired changes. A trimming signal from fuel flow and air flow sensors maintains the proper fuel–air ratio. Since it is often difficult to obtain accurate fuel flows, a steam flow sensor is sometimes substituted for the fuel flow sensor. Usually, about a 5 – s delay is allowed when changing coal flow and air flow to ensure the prevention of a momentary rich mixture with high fuel–air ratio and thus assure smoke–free combustion.

### 6.7.3 Steam Temperature Control

An accurate control of superheat temperature is important for efficient power plant operation. The principal variables affecting superheat temperature are: (1) furnace temperature, (2) cleanliness of radiant and pendant superheaters, (3) temperature of gases entering the convective superheater, (4) cleanliness of convective superheater, (5) mass flow rate of gases through the convective superheater, (6) feedwater temperaure, and (7) variation of load on the unit. These variables are not unrelated but the inter-relationships among them are not
simple. The last one, i.e. the effect of load variation on superheat temperature is, however, the most important.

![Diagram of steam pressure control system]

**Fig. 6.35  Schematic of a steam pressure control system**

A reduction in steam temperature results in loss in plant efficiency, e.g., a drop of about 20 °C in steam temperature results in about 1% increase in heat rate. On the other hand, a rise in steam temperature above design value may result in overheating and failure of superheater and reheater tubes and turbine blades.

The temperature of the saturated steam leaving the drum corresponds to the boiler pressure and remains constant if the steam-pressure controls are in working order. It is the superheater–reheater responses to load changes which need to be corrected. There are several ways of doing this as given below.

1. **Combined radiant-convective superheaters** Radiant and convectives superheaters are arranged in series to yield a relatively flat final steam temperature over a wide load range (Fig. 6.29), by taking advantage of their opposite steam temperature responses to load variation, as explained in Section 6.5.

2. **Desuperheating and attemperation** Control by attemperation means that the steam temperature is reduced by removing energy from the steam. In a tubular type, a portion of steam \( w_1 \) taken out through tubes from a point between the primary and secondary superheaters by an automatic valve diverted to a shell-and-tube heat exchanger where boiled water from the
drum may be circulated. The steam gives up some of its energy to that water and re-mixes with the primary steam ($\omega_2$) before entering the secondary superheater (Fig. 6.36).

**Fig. 6.36**  Control of steam temperature by attemperation

A spray type attemperator, also called desuperheater (Fig. 6.37) reduces the steam temperature by spraying low temperature water from the boiler drum or

**Fig. 6.37**  A spray attemperator for steam temperature control
economizer exit into the line between the primary and secondary superheaters (Fig. 6.38) or the radiant and pendant superheaters (Fig. 6.17). The spray nozzle injects water into the throat of a mixing venturi, where the water mixes with high velocity steam in the throat, vaporizes and cools the steam. The venturi and a thermal sleeve (a high chrome steel) protect the main steam pipe from thermal shock caused by any unvaporized water droplets that otherwise might impact on the pipe. The water used for spray must be of high purity so that no deposits (scale) are added on the superheater tubes, pipes, and turbine blades. The spray-type desuperheater or attemperator has been quite satisfactory in its service. It provides a rapid and sensitive means for temperature control. By regulating the amount of spray water, steam temperature is controlled to produce a flat temperature curve beyond point $a$ (Fig. 6.39). Steam is heated from $m$ to $n$ in the primary superheater, cooled from $n$ to $o$ in the desuperheater by spray water, and again heated from $o$ to $p$ in the secondary superheater.

![Diagram of primary and secondary superheaters](image)

**Fig. 6.38** *Spray-type desuperheater*

![Diagram of steam temperature curve](image)

**Fig. 6.39** *Regulation of spray water results in a flat temperature curve*
By energy balance at steady state (Fig. 6.38),

\[ \omega_s h_{s1} + \omega_w h_w = (\omega_s + \omega_w) h_{s2} \]  

(6.38)

where,

\[ \omega_s \text{ and } \omega_w = \text{mass flow rates of steam and water, respectively, kg/s} \]

\[ h_s \text{ and } h_w = \text{specific enthalpies of steam and water, kJ/kg.} \]

If the desuperheater is located after the last stage superheater, the steam temperature exceeds the maximum desired temperature before attemperation which is harmful. For example, if the maximum permissible steam temperature is 560 °C and the desuperheater is after the last stage superheater, then at a reduced load the steam temperature may shoot up to 570 °C, which is brought down by water spray to 560 °C. However, the temperature has to be limited to 560 °C and cannot be allowed to exceed it. On the other hand, the desuperheater is located before the last stage superheater, then at rated load with no attemperation the steam temperature at the location should be, say, 510 °C, which increases to 560 °C by absorption of heat in the last stage superheater. If at a reduced load the steam temperature at this location shoots up to 520 °C, then water will be sprayed to bring the temperature down to 510 °C, by which heat absorption in the last superheater rises to 560 °C before exiting to the steam turbine. The control is done one stage ahead and it is the preferred practice.

3. **Gas by-pass or damper control** If at any time the flow of gases through a convective superheater is reduced without changing the steam flow, then the final steam temperature will be reduced. Gas by-passing of the superheater or damper control utilizes this principle. If it is desired to have constant steam temperature for 3/4th steam flow and higher, then the superheater is designed to give the proper steam temperature at 3/4th steam flow. At greater steam flows, some of the flue gases are by passed around the superheater to maintain the desired steam temperature by partial closing of the dampers under the superheater (Fig. 6.40).

![Steam superheat by gas by-pass or damper control](image)
4. Gas recirculation  In this system, gas from some point downstream of the superheater–reheaters, mostly from the economiser outlet, but sometimes from the air preheater outlet, is recirculated back to the furnace by means of a gas recirculation fan (Fig. 6.41). The function of the recirculated gas is to reduce furnace heat absorption by diluting the furnace gas and lowering the

![Diagram of gas recirculation system]

**Fig. 6.41**  Gas recirculation for steam temperature control

furnace zone temperature (radiant zone). The total gas enthalpy leaving the furnace tends to be increased, however, because of the greater gas mass. Thus the gas velocity and the heat transfer rate in the convective superheater are increased by recirculation. As load falls, a greater gas mass is recirculated to maintain full superheat. Figure 6.42 shows how the gas recirculation amount influences the heat absorption in the economiser, primary and secondary super-

![Graph showing effect of gas recirculation on heat absorption]

**Fig. 6.42**  Effect of gas recirculation on the heat absorption pattern at a constant firing rate
heaters, reheater and the furnace. The range of control possible with recirculation is, however, limited largely by the power consumption of the recirculation fans. In some gas recirculation systems, recirculated gas is admitted near the furnace exit. This is called gas tempering. The furnace exit temperature is here reduced without affecting the furnace heat absorption.

Gas recirculation is sometimes used in series with desuperheating or attemperation for effective steam temperature control (Fig. 6.43).

![Diagram of steam temperature and load](image)

**Fig. 6.43** Gas recirculation and attemperation in series

5. **Excess air** Increased excess combustion air has the effect of lowering furnace temperature. Since radiant heat transfer is a function of the difference in the fourth power of the temperatures, furnace heat absorption will be materially reduced when furnace temperatures are lowered by the use of excess air. Thus, the gas temperature entering the superheater is actually increased. This increase in gas temperature along with the increase in mass flow raises the steam temperature.

   Although increased excess air is a means for steam temperature control, the increased mass of stack gas materially lowers the boiler efficiency.

6. **Tilting burners** Tilting or vertically adjustable burners (Fig. 6.44) change the gas temperature entering the superheater section by changing the elevation of the fireball within the furnace. At low loads (steam flow), the burners are tilted upward (the usual range $\pm 20^\circ$) so that the lower portion of the furnace becomes less effective in absorbing energy. Thus, the gases enter the superheater at a higher temperature than if the burners were fixed, and the steam temperature leaving the convection superheater can be maintained constant from less than one-half (50%) load to full load (100%). This is a satisfactory and economical method of temperature control.

7. **Burner selection** Large steam generators have several burners that are arranged in horizontal rows. Since only some of the burners are needed at partial load, use of the upper row of burners will increase steam temperature in the same way as when the burners are tilted upward.
Fig. 6.44  Use of tilting burners to vary effective furnace volume for steam temperature control
The effective furnace volume is reduced by not firing the upper burners at low loads (or by tilting the burners downward). The heat absorption at low loads is reduced and the gas temperature entering the convective superheater is kept near the desired level.

8. Separately fired superheater Separately fired furnaces for superheat regulation give a good degree of control over a wide range. The rate of firing is adjusted to yield a flat steam temperature load curve. This system is not generally economical for large utility boilers and is usually used in the chemical process industry.

Reheater outlet steam temperature is required to be controlled for the same reasons as those for regulating superheater outlet temperature, and the methods used are generally the same.

6.8 AIR PREHEATER

The temperature of feedwater in present day’s power plants with five or more feedwater heaters goes up to 170–280 °C. It means that the temperature of flue gases leaving the economiser cannot be below 270–480 °C (because for gas-to-gas heat exchange, the temperature difference between gas and air is, say, of the order of 100 °C). At such a high temperature of exit gases, the loss of energy with stack exhaust is high, and so the efficiency of the boiler will be low.

It is possible to cool the exit gases in the heating of air supplied for the combustion of fuel. The use of hot air makes the combustion process more efficient by making it more stable and lessening the energy losses due to incomplete combustion and unburnt carbon.

The application of hot air also leads to the increase of theoretical flame temperature and to the intensification of radiant heat transfer in the furnace. As a result of the increased furnace exit temperature, gas temperatures throughout the steam generator are raised above the levels which would prevail in the absence of an air preheater. This also enhances convective heat transfer in downstream heat exchangers. The gas temperatures exiting the air preheater vary from 135 to 180 °C, not only for the prevention of acid corrosion problems (to remain above dew point temperature) but also for proper dispersion of exhaust gases from the stack to the atmosphere.

While flue gases enter the air preheater temperatures varying between 315 to 450 °C and are cooled to 135–180 °C, air is heated from FD fan outlet temperature (40–60°C) to 280–400 °C and sometimes still higher. For the same energy input to the furnace, the preheating of air saves fuel. This saving in fuel and the consequent increase in boiler efficiency are approximately proportional to the air temperature rise in the air preheater. Typical fuel savings are 4% for a 110 °C rise in air temperature and above 11% for a 280 °C rise in air temperature in the preheater.

Preheated air is also required for the operation of pulverized coal furnaces. Air (primary) of the temperature range 150 – 420 °C is needed for drying coal in the pulverizer. Air is also used for transporting pulverized coal to the furnace.
and burning it there. Small stoker fired units do not need preheated air. However, it is beneficial to use preheated air in large stoker fired boilers, but the air temperature should not exceed 180 °C to prevent damage to the stoker moving parts.

Air preheaters are in general divided into two types: the recuperative and regenerative. In recuperative air preheaters, heat is directly transferred from the hot gases to the air across the heat exchanging surface. They are commonly tubular, although some plate types are still in use. Tubular units are essentially counter-flow shell-and-tube heat exchangers in which the hot gases flow inside the vertical straight tubes and the air flows outside (Fig. 6.45 (a) and (b)). Baffles are provided to maximize air contact with the hot tubes. The tubes are mechanically expanded into top and bottom tube sheets. The rate of heat transfer from gas to air is given by

\[ Q_{\text{APH}} = \omega_g c_p \left( t_{g_1} - t_{g_2} \right) = \omega_a c_p \left( t_{a_2} - t_{a_1} \right) = U_o A_o (\Delta t)_{\text{log-mean}} \]  

(6.39)

where

\[ \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{1}{h_d A_i} + \frac{x_w}{k_w A_{lm}} + \frac{1}{h_o A_o} \]  

(6.40)

and \( h_d \) = heat transfer coefficient due to deposits on the inside heating surface, W/m²K. The temperature profiles of gas and air are shown in Fig. 6.46.

Since the gas is flowing through the tubes,

\[ \omega_g = \frac{n \pi d_i^2}{4} \rho_i (V_g)_i \]  

(6.41)
Fig. 6.45  (b) Air heater arrangement

where

\[ n = \text{number of tubes}, \]
\[ d_i = \text{inside diameter of tubes; about 50 mm} \]
\[ \rho_i = \text{density of gas at inlet to the air preheater (may be approximately obtained from } \rho_i = \frac{p}{R_g (T_g)_i} \]

\[ R_g = \text{characteristic gas constant } = 0.287 \text{ kJ/kg K} \]

\[ p = 1.03 \text{ bar} \]

\[ (T_g)_i = \text{inlet gas temperature (K)} \]

\[ (V_g)_i = \text{inlet gas velocity, which is about 12–14 m/s.} \]

From Eq. (6.41), \( n \) can be determined.

If the fouling of the heating surface (inside) is not considered and the thickness of the tubes is small,

\[ \frac{1}{U_o} = \frac{1}{h_i} + \frac{x_w}{k_w} + \frac{1}{h_o} \]

Fig. 6.46  Temperature profiles of gas and air in APH

Both \( h_i \) and \( h_o \) being small, the overall heat transfer coefficient \( U_o \) is low, ranging between 30 and 60 W/m² K. From Eq. 6.39, \( A_o \) can be estimated and since \( A_o = n \pi d_o l \), \( d_o \) being the outside tube diameter, the length of the tubes, \( l \), can be computed.

Cost, cleaning requirements and fuel type determine the diameter of the tubes used. Modern steam generators use 50–65 mm OD heater tubes.

Regenerative air preheaters, known as storage type heat exchangers, have an energy storage medium, called the matrix, which is alternately exposed to the hot and cold fluids. When the hot flue gases flow through the matrix in the first half of the cycle, the matrix gets heated and the gas is cooled. In the next half of the cycle when air flows through the matrix, air gets heated and the matrix is cooled. The cycle repeats itself. The heat transfer from gas to air is thus periodic, the amount depending on the number of heating–cooling cycles executed per second. The most common type is the rotary air preheater, known as Ljungstrom air phreheater, which consists of a rotor driven by an electric motor through reduction gearing at a low speed with 2–4 rpm. The rotor is
divided into a number of radial sectors, 12 to 24 in number, each sector being filled with steel sheets, either flat or corrugated (Fig. 6.47). These sheets form the heat transfer surfaces. Two blank opposite sectors act as seals to prevent leakage of air to the gas path and vice-versa. As the rotating sectors enter the hot gas zone, they are progressively heated by the gas storing energy as sensible heat. When they enter the air zone, they progressively give up this heat to the air.

**Fig. 6.47** (a) Details of a regenerative air preheater (Ljungstrom APH) (b) Another view of a regenerative air preheater
Rotary air preheaters are compact heat exchangers with a large heat transfer surface being accommodated in a small volume. They are relatively cheap and have good heat transfer characteristics. They are almost as popular as tubular air preheaters in power station practice. However, there are certain drawbacks, *viz.*, (a) flow openings being plugged by fly ash, (b) sealing of gas to air path and vice-versa, and (c) large pressure drops for both gas and air.

If air is required to be heated to 400 °C or above, two stages of air preheating are used with the economiser installed in between the two air preheaters. The first air preheater (from the gas side) may be tubular, while the second one may be rotary.

### 6.9 Fluidized Bed Boilers

Fluidized bed boilers produce steam from fossil and waste fuels by using a technique called fluidized bed combustion which has been discussed in the last chapter. These can be of two types:

1. Bubbling fluidized bed (BFB) boilers
2. Circulating fluidized bed (CFB) boilers

In BFB boilers, crushed coal (6–20 mm) is injected into the fluidized bed of limestone just above an air-distribution grid at the bottom of the bed (Fig. 6.48).

![Schematic of bubbling fluidized bed boiler](image)

**Fig. 6.48 Schematic of bubbling fluidized bed boiler**

The air flows upwards through the grid from the air plenum into the bed, where combustion of coal occurs. The products of combustion leaving the bed contain a large proportion of unburnt carbon particles which are collected in cyclone separator and fed back to the bed. The boiler water tubes are located in the furnace.
Since most of the sulphur in coal is retained in the bed by the bed material used (limestone), the gases can be cooled to a lower temperature before leaving the stack with less formation of acid (H₂SO₄). As a result of low combustion temperatures (800–900 °C), inferior grades of coal can be used without slugging problems and there is less formation of NOₓ. Cheaper alloy materials can also be used, resulting in economy of construction. Further economies are achieved since no pulverizer is required. The volumetric heat release rates are 10 to 15 times higher and the surface heat transfer rates are 2 to 3 times higher than a conventional boiler. This makes the boiler more compact.

Figure 6.49 shows a bubbling fluid bed boiler system operating at atmospheric pressure, similar to the one of 160 MWe Tennessee Valley Authority (TVA) project at Shawnee, USA, recently installed (1993).

![Diagram of CFB boiler system]

Fig. 6.49 An atmospheric bubbling fluid bed boiler system

The CFB boiler is said to be the second generation fluidized bed boiler (Fig. 6.50 (a)). It is divided into two sections. The first section consists of (a) furnace or fast fluidized bed, (b) gas–solid separator (cyclone), (c) solid recycle device (loop seal or L–valve), and (d) external heat exchanger (optional). These components form a solid circulation loop in which fuel is burned. The furnace enclosure of a CFB boiler is generally made of water tubes as in pulverized coal-fired (PC) boilers. A fraction of the generated heat is absorbed by these heat transferring tubes. The second section is the back-pass, where the remaining heat from the flue gas is absorbed by the reheater, superheater, economiser, and air preheater surfaces (as in a conventional PC boiler).

The lower part of the first section (furnace) is often tapered. Its walls are lined with refractory up to the level of secondary air entry. Beyond this the furnace walls are generally cooled by evaporative, superheater, or reheater
Fig. 6.50 (a) Schematic of a circulating fluidized bed boiler

surfaces. The gas–solid separator and the non-mechanical valve are also lined with refractory. In some designs, a part of the hot solids recycling between the cyclone and the furnace is diverted through an external heat exchanger, which is a bubbling fluidized bed with heat transfer surfaces immersed in it to remove heat from the hot solids.

Coal is generally injected into the lower section of the furnace. It is sometimes fed into the loop-seal, from which it enters the furnace along with returned hot solids. Limestone is fed into the bed in a similar manner. Coal burns when mixed with hot bed solids.

The primary combustion air enters the furnace through an air distributor or grate at the furnace floor. The secondary air is injected at some height above the grate to complete the combustion. Bed solids are well mixed throughout the height of the furnace. Thus, the bed temperature is nearly uniform in the range 800–900 °C, though heat is extracted along its height. Relatively coarse particles of sorbent (limestone) and unburned char, larger than the cyclone cut-off size, are captured in the cyclone and are recycled back near the base of the furnace. Finer solid residues (ash and spent sorbents) generated during combustion and desulphurization leave the furnace, escaping through the cyclones, but they are collected by a bag-house or electrostatic precipitator located further downstream.

Figure 6.50 (b) shows the CFB boiler of the 165 MWe Pt. Aconi Power Plant at Nova Scotia, Canada.
Fig. 6.50  (b) CFB boiler of 165 MWe Point Aconi Power Plant at Nova Scotia, Canada
A schematic diagram of a CFB along with the flow regimes is shown in Fig. 6.51.


1. Furnace (below secondary air level)—turbulent/bubbling fluidized bed
2. Furnace (above secondary air level)—fast fluidized bed
3. Cyclone—Swirl flow
4. Return leg (stand pipe)—moving packed bed
5. Loop seal/external heat exchanger—bubbling fluidized bed
6. Back pass—pneumatic transport

6.9.1 Non-mechanical Valves

Non-mechanical valves are devices that allow the flow of solids between the return leg (stand pipe) and the furnace without any external mechanical force. Some of such valves (Fig. 6.52) are: (1) L-valve, (2) J-valve, (3) reverse seal valve, (4) loop seal, and (5) seal pot.

Solids in non-mechanical valves are moved by air. The air is added at a short height above the exit of the valve. The L-valve consists of a right-angled, bent (L-shaped) pipe connecting the two vessels between which the solids are to be transferred. It has a moving packed bed in the return leg and a less dense bed in the furnace into which the solid is fed. A small amount of air injected into the dense leg moves solids to the dilute bed. The L-valve can be designed from a pressure balance across the CFB loop.
Fig. 6.52  Non-mechanical valves: (a) controllable, (b) automatic

The operation of J-valve and reverse seal is similar to the L-valve. The solid flow rate is controlled by changing the aeration rate.

A seal pot consists of a bubbling fluidized bed with an overflow pipe and the standpipe of the cyclone. Higher pressure in the seal pot prevents furnace air from entering the standpipe.

The loop seal is very similar to the seal pot, except that it is smaller and here solids move horizontally around a vertical partition separating the standpipe and the overflow part of the seal pot. It also requires less air than that in the seal pot.

6.9.2 Cyclone Separators

Cyclone separators are gas cleaning devices that utilize the centrifugal force created by a spinning gas stream to separate particles from a gas. A standard tangential inlet vertical cyclone separator is shown in Fig. 6.53. The gas flow is forced to follow the curved geometry of the cyclone, while the inertia of particles in the flow causes them to move toward the outer wall, where they collide and are collected. A particle of mass $m_p$ moving in a circular path of radius $r$ with a tangential velocity $v_\theta$ is acted upon by a centrifugal force, $F_c = \frac{m_p v_\theta^2}{r}$. At a typical velocity of $v_\theta = 10$ m/s, $r = 0.5$ m, $F_c = 200$ $m_p$, whereas the gravity force $F_g = m_p g = 9.81$ $m_p$. Thus, the centrifugal force is 20.4 times the gravity force on the same particle.
Fig. 6.53 A tangential inlet vertical cyclone separator

In a cyclone, the particles in the spinning gas stream move progressively closer to the outer wall as they flow through the device. The gas stream may execute several complete turns as it flows from one end of the device to the other.

The particle-laden gas is entered tangentially to the cyclone. The particles are forced to the wall by centrifugal force and then fall down the wall by gravity. At the bottom of the cyclone the gas flow reverses to form the inner core that leaves at the top of the unit.

Out of $m$ kg of particles entering the cyclone with the gas stream, if $m_1$ kg (fines) leave with the gas at the top and is collected, say, in a bag filter, then $(m - m_1)$ kg of particles are separated by the cyclone from the gas–solid mixture, and the expression \( \frac{m - m_1}{m} \times 100 \) is called the collection efficiency of the cyclone. The cyclone collection efficiency increases with increasing (a) particle size, (b) particle density, (c) inlet gas velocity, (d) cyclone body length, (e) number of gas revolutions, and (f) smoothness of cyclone walls. The collection efficiency decreases with increasing (a) cyclone diameter, (b) gas outlet duct diameter, and (c) gas inlet area.

For any specific cyclone whose ratio of dimensions is fixed, the collection efficiency increases as the cyclone diameter is decreased. The design of a cyclone separator represents a compromise among collection efficiency, pressure drop, and size. Higher efficiency requires higher pressure drops (i.e. inlet gas velocity) and larger sizes (i.e. body length).
Shepherd and Lapple determined optimum dimensions for cyclones (Fig. 6.54) (Perry and Chilton, 1973). All dimensions were related to the body diameter $D_c$.

\[
\begin{align*}
B_c &= D_c/4 \\
D_c &= D_c/2 \\
H_c &= D_c/2 \\
L_c &= 2D_c \\
S_c &= D_c/8 \\
Z_c &= 2D_c \\
J_c &= D_c/4 \\
\end{align*}
\]

(Perry and Chilton, 1973)

**Fig. 6.54** Optimum dimensions for cyclones

The cyclone in CFB boilers handles a large volume of gas at high temperatures. Small diameter cyclones have higher collection efficiencies. Multiple numbers of cyclones of smaller size and higher efficiency can be used in parallel instead of a large cyclone of lower efficiency.

Cyclones used in CFB boilers often contain a large mass of refractory, which takes a long time to heat up and cool down. As a result, CFB boilers take a much longer time to start up, which is a drawback.

The outer skin temperature of the cyclone is relatively high, entailing high energy losses by natural convection and radiation. To overcome this problem, some cyclones are being made water or steam cooled. Typical performance parameters of a cyclone in a CFB boiler are given below:

- Inlet gas velocity: 25–43 m/s
- Solid concentration at inlet: 0.5–5 kg/m$^3$ gas
- Particles size range: $< 1$–1000 μm
- Gas throughput: $< 250,000$ Nm$^3$/h
- Cut-off particle diameter for 50% efficiency: $\sim 10$ μm
- Pressure drop: 100–200 mm water gauge
6.9.3 Advantages of CFB Boilers

Circulating fluidized bed boilers have a number of unique features that make them more attractive than other solid fuel fired boilers as given below.

1. Fuel Flexibility This is one of the most attractive features of CFB boilers. Fuel particles constitute less than 1–3% by weight of all bed solids in the furnace of a typical CFB boiler. The rest of the solids are non-combustible: sorbents (limestone or dolomite) and fuel-ash. The special hydrodynamic condition in the CFB furnace allows an excellent gas–solid and solid–solid mixing. Thus fuel particles fed to the furnace are quickly dispersed into the large mass of bed solids, which rapidly heat the fuel particles above their ignition temperature without any significant drop in the temperature of bed solids. This feature of a CFB furnace allows it to burn any fuel without the support of an auxiliary fuel. It can thus burn a wide variety of fuels very efficiently in the same boiler, either simultaneously or separately. High ash bituminous coals, anthracite, coal washery rejects, peat, wood chips, petroleum coke, oil shale and so on have been fired in CFB units.

To maintain the combustion temperature within an optimum range (800–900 °C), a certain portion of the generated heat is required to be absorbed from the combustion zone. This fraction varies from one fuel to another. It is controlled by adjusting the heat extraction from the recirculating solids outside the furnace by means of an external heat exchanger. If the external heat exchanger is absent, the hydrodynamic condition in the furnace is adjusted (say, by regulating the solid recirculation rate and superficial air velocity) so as to change the solid concentration (or suspension density) in the furnace, which in turn, alters the heat absorption in the furnace. The solid circulation rate can be adjusted by regulating the ratio of primary and secondary air. By increasing the primary air the solid circulation rate can be increased.

2. High combustion efficiency Superior mixing in the CFB, large reaction space (combustion zone extending up to the top of the furnace, about 40 m and above in large utility boilers, and further beyond into the hot cyclone) and consequently long residence time of combustion afford high combustion efficiencies, which can be as high as 99%, for a wide variety of fuels under different operating conditions.

3. Efficient sulphur removal When coal burns, sulphur is oxidized primarily to sulphur dioxide

\[ S + O_2 = SO_2 + 9260 \text{ kJ/kg} \]

Some part of SO\(_2\) may be converted to SO\(_3\)

\[ SO_2 + \frac{1}{2}O_2 = SO_3 \]

This SO\(_2\) and SO\(_3\) if thrown out to atmosphere undergo chemical reactions with moisture, catalyzed by sunlight, to form acids. These acids are precipitated
on the earth through rain as acid rain. This is lethal to aquatic life, trees and plants and destroys the nutrients in soil.

Limestone (CaCO₃) and dolomite (CaCO₃, MgCO₃) are two principal sorbents used for the absorption of SO₂ in fluidized bed combustors. For limestone directly fed into the CFB furnace as bed material, there is calcination when the limestone is decomposed into CaO and CO₂ through an endothermic reaction

\[
\text{CaCO}_3 = \text{CaO} + \text{CO}_2 - 180 \text{kJ/kg CaCO}_3
\]

In a sufficiently oxidizing environment, the porous calcined limestone reacts as

\[
\text{CaO} + \text{SO}_2 + \frac{1}{2} \text{O}_2 = \text{CaSO}_4 + 15,141 \text{kJ}
\]

CaSO₄ retains the sulphur in the bed in the solid form, rather than allowing it to escape as gaseous SO₂.

Thermodynamically, the formation of CaSO₄ is favoured at lower temperatures. The rate of formation is maximized at about 850 °C. At temperatures exceeding 900 °C, the sulphur capture is greatly reduced. This is one of the reasons why fluidized bed combustion processes are constrained to the 800–900 °C temperature range.

Good contact between the gas and solid and the long residence time in the CFB combustor afford better sulphur capture at a given Ca/S ratio than in a bubbling fluidized bed. Industrial CFB boilers show 90% sulphur capture at Ca/S ratio of 1.5 to 2.0.

Limestone particles below 3 mm size are typically used. The SO₂ capture efficiency is a function of temperature, Ca/S ratio, stone structure and composition, FBC design and pressure. Representative results of the effects of Ca/S ratio and temperature for atmospheric bubbling beds are shown in Fig. 6.55 (a).

The effect to pressure on sulphur retention for limestone and dolomite is shown in Fig. 6.55 (b). Unlike atmospheric FBCS, the sulphur capture efficiency does not pass through a maximum, but continues to increase with combustor temperature.

4. **Low NOₓ emission** Owing to low combustion temperature and the staged combustion process, NOₓ emission in a CFB boiler can be kept low, in the range 50–150 ppm. Figure 6.55(c) shows the combustion zones together with the entry points of primary and secondary air. Primary air fed from below through the distributor contains 50–60% of the stoichiometric requirements. Thus, the fuel nitrogen, when released, does not find oxygen in the immediate vicinity to produce oxides. This strategy of low NOₓ formation is more effective when the fuel contains a high level of volatile organic compounds. Secondary air is added above the reducing zone. Since the fuel nitrogen is already transformed into molecular nitrogen, there is limited opportunity for the formation of NO₂ above this zone. The nitrogen in the combustion air is not normally oxidized to NO₂ at the relatively low temperatures (800–900 °C) of CFB combustors.
**Fig. 6.55 (a)** Sulphur capture in atmospheric BFBC at various Ca/S molar ratios as a function of bed temperature

**Fig. 6.55 (b)** Effect of pressure on sulphur retention for reactive limestone and dolomites sized 0-1600 μm, normalized to Ca/S = 2 and reaction time = 0.5 s
5. Simpler fuel handling and feed system  In contrast to the conventional pulverized coal fired boiler, CFB boilers receive solid fuels in fairly coarse sizes. This simplifies the upstream feed preparation. But the most important advantage is the need for only one (industrial boiler) or few (utility boiler) feedpoints. The high degree of lateral solid mixing in turbulent zone at the bottom ensures uniform feed distribution within the bed. Besides, the large height corrects any maldistribution.

6. Small furnace cross-section  A high heat release rate per unit furnace cross-section (about 5 MW/m²) is a major advantage of the CFB boiler. It results in less floor area. A high superficial velocity (4–7 m/s) along with an intense gas–solid mixing promotes a high heat release and heat dispersion in the bed.

7. Good turndown and load following capability  The relatively high fluidizing gas velocity and the use of staged combustion permit a fairly good turndown ratio by simply reducing the proportionate amounts of fuel and air. The turndown ratio of 5 : 1 has been achieved (20% load).

CFB boilers respond quickly to load changes. The given load can be readily adjusted by changing the ratio of primary and secondary air, and by controlling the solid circulation rate.

8. High availability  The availability records of CFB boilers are highly impressive. It is more than 90%.

The major disadvantages of CFB boilers are, however, (i) erosion of reactor walls, (ii) attrition of particles, (iii) difficulty of immersing internals due to possible erosion, and (iv) complexities of the hydrodynamics.
6.9.4 Status of CFB Boilers

The world’s first commercial CFB boiler went into service at Pihlava, Finland in 1979. The pioneering company was Ahlstrom and the aim was to convert an oil-fired unit into a CFB boiler producing 20 t/h of steam from wood waste. Successful operation of this unit led to the start-up in 1981 of a 90t/h (steam) CFB boiler at Kauttua, Finland, that was designed to burn coal, wood waste and peat.

Lurgi built up in 1982 a 84 MW (thermal) CFB boiler at Leunen, Germany, burning coal washery rejects with ash content of up to 55%. In the relatively short span of its commercial life, the CFB boiler along with its “bubbling” counterpart, has established itself as a reliable and attractive option for industrial boilers to supply process steam. A few CFB utility boilers in the 100–165 MWe range are already in operation and some are in various stages of construction. The 165 MWe Point Aconi Power Station at Cape Breton in Nova Scotia, Canada, has now the largest CFB boiler in operation. The biggest CFB boiler in the erection stage is for a 250 MWe power unit at Lyon in France.

Table 6.1 gives a comparison of circulating fluidized bed boilers with other types of boilers (Basu and Fraser, 1991).

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Stoker</th>
<th>Bubbling</th>
<th>Circulating</th>
<th>Pulverized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of bed or fuel burning zone (m)</td>
<td>0.2</td>
<td>1–2</td>
<td>15–40</td>
<td>27–45</td>
</tr>
<tr>
<td>Superficial velocity (m/s)</td>
<td>1.2</td>
<td>1.5–2.5</td>
<td>4–8</td>
<td>4–6</td>
</tr>
<tr>
<td>Excess air (%)</td>
<td>20–30</td>
<td>20–25</td>
<td>10–20</td>
<td>15–30</td>
</tr>
<tr>
<td>Grate heat release rate (MW/m²)</td>
<td>0.5–1.5</td>
<td>0.5–1.5</td>
<td>3–5</td>
<td>4–6</td>
</tr>
<tr>
<td>Coal size (mm)</td>
<td>32–6</td>
<td>6–0</td>
<td>6–0</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Turndown ratio</td>
<td>4:1</td>
<td>3:1</td>
<td>5:1</td>
<td></td>
</tr>
<tr>
<td>Combustion efficiency (%)</td>
<td>80–85</td>
<td>90–96</td>
<td>95–99</td>
<td>99</td>
</tr>
<tr>
<td>NOₓ emission (ppm)</td>
<td>400–600</td>
<td>300–400</td>
<td>50–200</td>
<td>400–600</td>
</tr>
<tr>
<td>SO₂ capture in furnace (%)</td>
<td>none</td>
<td>80–90</td>
<td>80–90</td>
<td>small</td>
</tr>
</tbody>
</table>

6.9.5 Pressurized Fluidized Bed Boiler

The combustion process takes place in a pressurized environment resulting in a compact furnace and improved combustion efficiency. There is in situ sulphur capture and low NOₓ generation with the bed operating at about 850 °C like an atmospheric fluidized bed combustion (AFBC) furnace. The process allows the use of a gas turbine, driven by pressurized hot combustion gases. Thus, in
addition to having the ability to burn high sulphur and low grade coal in an environmentally acceptable manner and the compactness of the steam generator, PFBC also results in the ability to power a gas turbine by combusting inexpensive coal.

Table 6.2 lists the representative conditions in four major classes of fluidized bed combustors.

**Table 6.2 Representative conditions in major classes of FBCS**

<table>
<thead>
<tr>
<th></th>
<th>Atmospheric</th>
<th></th>
<th>Pressurized (~ 17 atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BFBC</td>
<td>CFBC</td>
<td>BFBC</td>
</tr>
<tr>
<td>1. Gas velocity (m/s)</td>
<td>1.3–3.5</td>
<td>4–9</td>
<td>~ 1</td>
</tr>
<tr>
<td>2. Bed height (m)</td>
<td>1.0–1.4</td>
<td>~ 35</td>
<td>3.5–4.0</td>
</tr>
<tr>
<td>3. Coal feed points/m²</td>
<td>0.5–1.0</td>
<td>0.06–0.08</td>
<td>0.25–6</td>
</tr>
<tr>
<td>4. Bed residence time</td>
<td>~ 0.5</td>
<td>~ 5</td>
<td>~ 4</td>
</tr>
</tbody>
</table>

Figure 6.56 shows three schemes of turbo-charged, combined and hybrid advanced cycles, each equipped with a PFBC unit.

Like AFBC units, there are two types of pressurized combustors:

1. Pressurized bubbling fluidized bed combustor (PBFBC).
2. Pressurized circulating fluidized bed combustor (PCFBC).

In a pressurized bubbling fluidized bed combustor (PFBC), the granular solids are fluidized in bubbling fluidized mode. Here, the superficial gas velocity generally lies between the minimum bubbling velocity of coarser particles and terminal velocity of finer particles. A part of the heat generated in the fluidized bed is extracted by water carrying tubes in the bed. The hot gas is cleaned and then expanded through the gas turbine (Fig. 6.57). The steam produced in the
combustor or the waste heat recovery heat exchanger at the gas turbine exit produces further power by expanding through a steam turbine.

**Fig. 6.56** PFBC units in (a) combined cycle, (b) turbocharged cycles, and (c) hybrid advanced cycle
Fig. 6.57 Schematic of a pressurized bubbling fluidized bed combustor

The advantages of PBFBC are as follows:

1. There is an increase in its specific power output and hence, a potential reduction in capital cost. At a typical fluidizing velocity of 2 m/s the bed area of AFBC at 1 bar is 2 m²/MW, while it is 0.2 m²/MW at 10 bar.

2. By combining a gas turbine with a steam turbine, the overall efficiency of power generation system can be increased from 33% for conventional power plants to more than 40% in PBFBC plants.

3. The emission of nitrogen oxides can be substantially reduced.

4. The PBFBC has a higher combustion efficiency than an AFBC.

5. The gas residence time in an AFBC is about 0.5 s, whereas it is about 5 s in a PBFBC (due to low fluidizing velocity of about 1 m/s for a higher gas density). Therefore, the sulphur capture is more.

The solids in a pressurized circulating fluidized bed (PCFB) are kept in “fast fluidized” condition (Fig. 6.58). A major feature of PCFB is that it approaches a condition where solids are fully mixed throughout the combustor, but there is very little mixing of gas in the axial direction. The gas flows upward with negligible back mixing. Primary air, less than stoichiometric amount, enters the combustor through the grid at the bottom of the combustor. The secondary air enters at some distance above the grid. Due to the absence of back mixing of the air, a substoichiometric condition prevails in the lower section of the bed. The boiler tubes are generally located above this zone. This staging of air reduces NO<sub>x</sub> emission and also the chance of corrosion of the tubes.

As compared to PBFB, PCFB offers the following advantages:

1. Reducing space requirement: PCFB has a higher velocity, higher heat release per unit area and fewer components inside the pressure vessel. So, the pressure vessel diameter is smaller.
Fig. 6.58  A pressurized circulating fluidized bed combustor

2. Less erosion and more accessibility: PCFB has a low particle density fluidized bed with fine particles distributed throughout the furnace. In PBFB, submerged heat transfer surfaces are used. So in PCFB (a) the potential for erosion is greatly reduced with no immersed surfaces, (b) platens for superheating are more accessible for inspection and maintenance.

3. PCFB has more vigorous mixing in the furnace which results in fewer feedpoints and results in a simpler, more reliable system.

4. PCFB does not require a bed material handling system to control the bed temperature.

5. The exit gas temperature of PCFB is higher since there is no freeboard where the flue gas may be cooled down. This is favourable to the gas turbine.

6. PCFB has lower NOx emission due to enhanced staged combustion.

7. Load following is more effectively accomplished by varying the fuel and air flow and the proportion of primary and secondary air.
8. PCFB has **less bed inventory and less total metal and refractory.** This results in a shorter start-up and cool-down period and a quicker response to load changes.

9. At higher pressures, the partial pressure of carbon dioxide, \( P_{\text{CO}_2} \), in a bubbling bed is higher, which requires higher temperature for calcination of CaCO\(_3\). One has to use dolomite instead of limestone in PBFBC.

In PCFB air is added in stages. In substoichiometric zone \( P_{\text{CO}_2} \) is lower. So, CaCO\(_3\) in limestone is easily calcined increasing pore volumes for sulfation reaction. So, PCFB units can use more readily available limestone than PBFBC.

The boiler, which is integral with combustor could use either of the three types of circulation (Fig. 6.59).

(a) *Once-through circulation:* The feed pump forces water through the entire steam-water circuit. It is suitable at high load. When water-flow rate decreases, the evaporator surface can perform superheater duty. This increases the metal temperature, which, in turn, reduces the heat absorption—a requirement at low load. A vertical steam-water separator feeds saturated water to a circulation pump below, which maintains sufficient flow through the boiler.

![Fig. 6.59 Different steam-water circulation systems in PFBC boilers](image)

(b) *Natural circulation:* This is caused by density difference, as explained earlier. It is convenient at low load. The steam drum is installed outside to provide adequate hydrostatic head for natural circulation.

(c) *Forced circulation:* Water from the economiser is fed by the boiler feed pump into the boiler drum. A forced circulation pump circulates the water from the drum through the downcomer–riser circuits back to the drum. Superheater and reheater steams flow under their own pressure.

Two–stage cyclones are used to clean the hot flue gas adequately for expansion through commercial gas turbines. The entry velocities of gas at entry to the first and second stage cyclones are less than 30 m/s and 42 m/s respectively. The dust loading in the cleaned gas is about 250 ppm. The gas requires to be further cleaned by passing it through ceramic filters.
**6.10 ELECTROSTATIC PRECIPITATOR**

In 1905, Dr. F.G. Cottrell, Professor of Physical Chemistry at the University of California, conducted a series of laboratory experiments that resulted in the development of the first commercial electrostatic precipitator. It was an immediate success and the precipitator soon came to be widely used in power plants, smelters, steel plants, paper mills and many other industries.

The principal components of an electrostatic precipitator (ESP) are two sets of electrodes insulated from each other. The first set is composed of rows of electrically grounded vertical parallel plates, called the *collection electrodes*, between which the dust-laden gas flows. The second set of electrodes consists of wires, called the *discharge* or *emitting electrodes* that are centrally located between each pair of parallel plates (Fig. 6.60). The wires carry a unidirectional negatively charged high-voltage (between 20 and 100 kV) current from an external dc source. The applied high voltage generates a unidirectional, non-uniform electrical field whose magnitude is greatest near the discharge electrodes. When that voltage is high enough, a blue luminous glow, called a *corona*, is produced around them. Electrical forces in the corona accelerate the free electrons present in the gas so that they ionize the gas molecules, thus forming more electrons and positive gas ions. The new electrons create again more free electrons and ions, which result in a chain reaction.

![Image of electrostatic precipitator](image)

**Fig. 6.60 Basic elements of an electrostatic precipitator**

The positive ions travel to the negatively charged wire electrodes. The electrons follow the electrical field toward the grounded electrodes, but their velocity decreases as they move away from the corona region around the wire electrodes toward the grounded plates. Gas molecules capture the low velocity electrons and become negative ions. As these ions move to the collecting electrode, they collide with the fly ash particles in the gas stream and give them
negative charge. The negatively charged fly ash particles are driven to the collecting plate by the force which is proportional to the product of this charge and the strength of the electric field (Fig. 6.61).

![Diagram of ESP process]

**Fig. 6.61** Vertical electrodes and grounded plates in an ESP showing four basic operations

When the particles collect on the grounded plates, they lose their charge on the ground. The electrical resistivity of the particles, however, cause only partial discharging, and the retained charge tends to hold the particles to the plates. High resistivity causes retention of most of the charge, which increases the forces holding the particles to the plates and makes removal more difficult. This can be rectified either by operating at high gas temperatures (before APH) or by superimposing a high voltage pulse on the base voltage to enhance ESP performance during operation under high-resistivity conditions.

Collected particulate matter must be removed from the collecting plates on a regular schedule to ensure efficient collector operation. Removal is usually accomplished by a mechanical hammer scrapping system. The vibration knocks the particulate matter off the collecting plates and into a hopper at the bottom of the precipitator (Fig. 6.62).
An electrostatic precipitator, like a cyclone separator, has an overall collection efficiency, $\eta_0$, defined by

$$\eta_0 = \frac{\text{mass of all particles retained by collector (ESP)}}{\text{mass of all particles entering collector}}$$

It is easier to collect large particles than smaller particles. It can be approximately estimated by the Deutsch equation as given below

$$\eta_0 = 1 - \exp\left(-\frac{AV_{\text{mo}}}{Q}\right)$$  \hspace{1cm} (7.39)

where $A =$ area of the collector plate, m$^2$,

$V_{\text{mo}} =$ effective migration velocity of particles, m/s,

$Q =$ flue gas volume flow rate for each plate, m$^3$/s.
6.11 FABRIC FILTERS AND BAGHOUSES

Fabric filters are used in power plants to remove dust particles from a gas stream. They are made of porous material which retains particulate matter while the carrier gas flows out through the voids.

A fabric filter element is usually made in the form of a long, hollow cylindrical tube that provides a large surface area per unit of gas volumetric flow rate (m²/m³ gas flow). The reciprocal of this parameter, called the air-to-cloth ratio (A/C value). It is a fundamental fabric filter descriptor denoting the ratio of the flue gas flow to the amount of fabric, or filtering surface area. It ranges typically between 5 to 40 mm/s.

A fabric filter system usually contains a large number of fabric filter elements arranged in parallel rows. It is called a baghouse (Fig. 6.63). A power plant baghouse might contain several thousand vertical hollow cylinders, each ranging in diameter from 125 to 350 mm and up to 12.5 m in height. The elements have, in general, an open bottom and closed top. They rest on a tube sheet above a dirty air plenum.

![Diagram of a fabric filter]

**Fig. 6.63** Typical baghouse with mechanical shakers
Fabric filters come in three basic types: reverse-air, shake-deflate, and pulse-jet units (Fig. 6.64). Reverse air fabric filters are characterized by low A/C values and by the flow in which dirty gas flows from inside the bags to outside. Clean air is then forced through the filter elements in a reverse direction, from outside into the bag, which collapses the bag and fractures the dust cake. When the bag is brought back on line with dirty gas flowing in to out, it re-inflates and removes the broken dust cake into the hopper below (Fig. 6.64(a)). This type of baghouse is found to be economical for large power plants.

![Diagram](image)

**Fig. 6.64 (a) Operating Schematic Reverse Air Fabric Filter**

A shake-deflate unit is also of low A/C value and collects dust on the inside of the bags as in the reverse-air design. To clean the bags, the top end is shaken by a drive linkage.

In pulse-jet units, gas flow is from outside the bags inward. Usually A/C ratios are much higher than the previous two types. Cleaning is accomplished with a high-pressure burst of air into the open end of the bag at the top. Because of compactness, these are preferred for industrial boilers (Fig. 6.64(b)).

Pressure drop and bag life are the criteria for selecting the bag filter material, which can fail as a result of high temperature burning, caking, erosion, chemical attack and aging. Fabrics considered include wool, cotton, nylon, glass fibre, polyester and aromatic polyamides. Wool and cotton can be used only up to about 80 or 95 °C. Over 90% of the bags in the USA utility service are the woven fibreglass variety. Bag performance is rated in terms of permeability, cleanability and durability. A baghouse is normally placed after the air preheater.
ASH HANDLING SYSTEM

Boilers burning pulverized coal (PC) have dry bottom furnaces. The large ash particles are collected under the furnace in a water-filled ash hopper. Fly ash is collected in dust collectors with either an electrostatic precipitator or a baghouse. A PC boiler generates approximately 80% fly ash and 20% bottom ash. Ash must be collected and transported from various points of the plants as shown in Fig. 6.65. Pyrites, which are the rejects from the pulverizers, are disposed with the bottom ash system. Three major factors should be considered for ash disposal systems.

1. Plant site
2. Fuel source
3. Environmental regulation

Needs for water and land are important considerations for many ash handling systems. Ash quantities to be disposed depend on the kind of fuel source. Ash storage and disposal sites are guided by environmental regulations.

Fig. 6.65 Ash collection and transportation
The sluice conveyor system (Fig. 6.66a) is the most widely used for bottom ash handling, while the hydraulic vacuum conveyor (Fig. 6.66b) is the most frequently used for fly ash systems.

![Sluice conveyor diagram](image)

**Fig. 6.66** (a) Bottom ash sluice conveyor, (b) Fly ash hydraulic vacuum conveyor

Bottom ash and slag may be used as filling material for road construction. Fly ash can partly replace cement for making concrete. Bricks can be made with fly ash. These are durable and strong.

### 6.13 FEEDWATER TREATMENT

Boiler make-up water to the extent of 1.5–2 per cent of the total flow rate is required to replenish the losses of water through leakage from fittings and bearings, boiler blowdown, escape with non-condensable gases in the deaerator, turbine glands, and other causes. This make-up water needs to be treated prior to feeding it to the boiler for

1. Prevention of hard scale formation on the heating surfaces
2. Elimination of corrosion,
3. Control of carry-over to eliminate deposition on superheater tubes, and
4. Prevention of silica deposition and corrosion damage to turbine blades.
Raw water is, therefore, first pre-treated and then demineralized. For once-through boilers and boiling water nuclear reactors, which require high water-purity, a condensate polishing system is used to further polish the water. Raw water contains a variety of impurities, such as (a) suspended solids and turbidity, (b) organics, (c) hardness (salts of calcium and magnesium), (d) alkalinity (bicarbonates, carbonates, hydrates), (e) other dissolved ions (sodium, sulphate, chloride, etc.), (f) silica, and (g) dissolved gas (O₂, CO₂). The extent of pre-treatment depends on the source of raw water.

6.13.1 External Treatment

The first step of pre-treatment of boiler feedwater is clarification, in which the water is chlorinated to prevent biofouling of the equipment. The suspended solids and turbidity are coagulated by adding special chemicals (like aluminium sulphate, Al₂(SO₄)₃) and agitated. The coagulated matter settles at the bottom of the clarifier and is removed.

If the turbidity of clarified effluent is high, positive filtration is needed. Both gravity filters and pressure-type filters are used, but the latter is preferred. A granular medium like sand is commonly used for filtration. The pressure difference across the filtering medium is an indication of solid accumulation. When it reaches a given limit, the solids are removed from the bed by backwashing. Further filtration by activated carbon can absorb organics and remove residual chlorine from the chlorination process.

The dissolved salts of calcium and magnesium give to water a quality called hardness. Hardness is characterized by the formation of insoluble precipitates or curds with soaps, and is usually measured with a standard soap test. All natural waters are hard and contain scale-forming impurities which are mainly the salts of calcium and magnesium in the form of carbonates, bicarbonates, chlorides and sulphates. The hardness is expressed in ppm of dissolved salts. Softening of water, i.e. removal of hardness from water, can be done by lime-soda process, phosphate process, zeolite process and demineralization.

6.13.2 Lime–Soda Process

In lime–soda softening, calcium and magnesium salts are removed using lime (calcium hydroxide) and soda ash (sodium carbonate). When this process is carried out at normal raw-water temperature, it is called a “cold process” softening; and when carried out at or near the boiling point, it is referred to as a “hot process” softening. Since heating greatly accelerates the necessary reactions, the hot process is preferred for boiler water treatment, where most of the energy used in heating the water may be retained in the cycle. The representative reactions are given below:

\[
Ca(HCO₃)₂ + Ca(OH)₂ = 2CaCO₃ + 2H₂O
\]
\[
\begin{align*}
\text{MgCl}_2 + \text{Ca(OH)}_2 &= \text{Mg(OH)}_2 + \text{CaCl}_2 \\
\text{MgSO}_4 + \text{Ca(OH)}_2 &= \text{Mg(OH)}_2 + \text{CaSO}_4 \\
\text{CaSO}_4 + \text{Na}_2\text{CO}_3 &= \text{CaCO}_3 + \text{Na}_2\text{SO}_4 \\
\text{CaCl}_2 + \text{Na}_2\text{CO}_3 &= \text{CaCO}_3 + 2\text{NaCl}
\end{align*}
\]

The products, calcium carbonate and magnesium hydroxide are insoluble in water and settle to the bottom of the vessel. The softened effluent is then passed through sand or charcoal filters before usage.

6.13.3 Hot Phosphate Softening

In the hot phosphate softening process, calcium and magnesium hardness is removed using phosphate and caustic soda, Tricalcium phosphate \((\text{Ca}_3\text{(PO}_4\text{)}_2)\) and magnesium hydroxide are precipitated. The process is carried out at a temperature of 100 °C or above. Since the hot phosphate process requires more expensive chemicals than the lime–soda process, it is used where the initial water hardness is 60 ppm or less. Where hardness is greater than this, a lime–soda process may be used first, followed by a phosphate clean-up. The representative reactions are given below:

\[
\begin{align*}
3\text{Ca(HCO}_3\text{)}_2 + 6\text{NaOH} &= 2\text{CaCO}_3 + 2\text{Na}_2\text{CO}_3 + 6\text{H}_2\text{O} \\
\text{CaCO}_3 + 2\text{Na}_3\text{PO}_4 &= \text{Ca}_3\text{(PO}_4\text{)}_2 + 2\text{Na}_2\text{CO}_3 \\
\text{Mg(HCO}_3\text{)}_2 + 4\text{NaOH} &= \text{Mg(OH)}_2 + 2\text{Na}_2\text{CO}_3 + 2\text{H}_2\text{O} \\
3\text{CaSO}_4 + 2\text{Na}_3\text{PO}_4 &= \text{Ca}_3\text{(PO}_4\text{)}_2 + 3\text{Na}_2\text{SO}_4
\end{align*}
\]

6.13.4 Sodium Zeolite Softening

Water can be softened by passing it through a bed of sodium zeolite, which may be natural compounds of sodium aluminium silicate, with the cations of calcium and magnesium removed in the process.

\[
\begin{align*}
\text{CaCO}_3 + \text{Na}_2\text{Z} &= \text{CaZ} + \text{Na}_2\text{CO}_3 \\
\text{MgSO}_4 + \text{Na}_2\text{Z} &= \text{MgZ} + \text{Na}_2\text{SO}_4 \\
\text{Ca(HCO}_3\text{)}_2 + \text{Na}_2\text{Z} &= \text{CaZ} + \text{Na}_2\text{(HCO}_3\text{)}_2
\end{align*}
\]

The softening capacity of the bed gets exhausted in course of time, and the bed can be regenerated by flushing it with brine (NaCl),

\[
\begin{align*}
\text{CaZ} + 2\text{NaCl} &= \text{Na}_2\text{Z} + \text{CaCl}_2 \\
\text{MgZ} + 2\text{NaCl} &= \text{Na}_2\text{Z} + \text{MgCl}_2
\end{align*}
\]

Zeolite softening is not ideal for the following reasons: (i) Water of high or low pH have a deleterious effect on zeolites, (ii) high temperatures also have a bad effect, (iii) turbid waters coat the zeolite material, reducing its efficiency, (iv) there is no reduction in alkalinity or total solids, (v) there can be silica gain
in water from the zeolite, (vi) with low content of calcium, the water can be corrosive. Sodium zeolite softening in conjunction with the use of evaporators may be more effective.

6.13.5 Hydrogen Zeolite Softening

When water containing calcium, magnesium, and sodium ions is passed through a hydrogen zeolite, these ions are exchanged for hydrogen and the bicarbonate, sulphate, chloride and nitrate radicals are converted to their respective acids. Typical reactions are:

(Ca) + (Mg) (HCO₃)₂ + H₂Z = (Mg) Z + 2H₂CO₃
(Ca) + (Na₂) + (Ca) + (Na₂)
(Mg) SO₄ + H₂Z = (Mg) Z + H₂SO₄
(Na₂)
(Na₂)

When the hydrogen zeolite becomes exhausted, it is backwashed and regenerated with acid. After being rinsed, it is ready for use again. Sulphuric acid is generally used for regeneration because of its relatively low cost.

MgZ + H₂SO₄ = H₂Z + MgSO₄
CaZ + H₂SO₄ = H₂Z + CaSO₄

6.13.6 Anion Exchangers

Anion exchangers can remove the anions like chlorides, sulphates and nitrates (acid forms) present in hydrogen zeolite effluent by resinous materials which adsorb them. Typical reactions are:

2HCl + RCO₃ = RCl₂ + H₂CO₃
H₂SO₄ + RCO₃ = RSO₄ + H₂CO₃
2HNO₃ + RCO₃ = R(NO₃)₂ + H₂CO₃

Carbonic acid is removed by aeration. When the acidic water is sprayed in a shower to expose large surface area, the carbon dioxide gas is released.

H₂CO₃ → CO₂ + H₂O

When the anion exchanger is exhausted, it is regenerated by backwashing with soda ash

RCl₂ + Na₂CO₃ = RCO₃ + 2NaCl
RSO₄ + Na₂CO₃ = RCO₃ + Na₂SO₄

6.13.7 Demineralizing Plant

The process of removing dissolved solids in water by ion exchange is called demineralization. Two types of resins, cation and anion, are used. The cation
resin is the hydrogen zeolite where the hydrogen ion is exchanged for the cations calcium, magnesium and sodium, and the anion resin adsorbs the anions chlorides, nitrates and sulphates, as discussed above. Both ion-exchange processes are reversible, and the resins are restored to their original form by regeneration.

A typical dimineralizing plant consisting of a cation exchanger, an anion exchanger, a degasifier and a silica adsorber in series is shown in Fig. 6.67. In the degasifier, carbon dioxide gas is removed by aeration. Silica in water is very detrimental at high pressure. It vaporizes at high pressure and flows with steam, condenses on turbine blades in the form of hard glassy scales which are difficult to remove. Magnesium hydroxide is often used to adsorb silica from water.

The membrane treatment for removing the total dissolved solids from make-up water is also an energy efficient process and is gradually gaining more acceptance. It uses the principle of either reverse osmosis or electrodialysis. The driving force for reverse osmosis is the application of counter pressure to normal osmotic pressure, driving water molecules through the membranes in preference to dissolved salts. The basis of electrodialysis is the reverse of that of reverse osmosis in that it moves dissolved salts away from the water accruing a practical advantage because the quantity of salts is far less than the water volume. The membranes used in either process are expensive, and coagulation, settling and filtration are used first to protect them.

6.13.8 Condensate Polishing

A high quality make-up water can be produced for the plant by using demineralizing systems as discussed above. However, this treated water while flowing through the cycle can pick up impurities due to condenser leakage from the circulating water through the tubes as well as metallic ions, such as iron and copper, from pipelines. Condensate polishing is accomplished by passing the condensate through large demineralizing vessels, called mixed bed units, which contain both cation and anion resins. The resins not only remove dissolved salts in the above manner, but also act as filters for impurities or suspended solids. Power plants using once-through boilers and nuclear reactors generally require high quality water and use condensate polishing systems.

6.14 DEAERATION

Deaeration (degasification) is one of the most important steps in boiler water treatment. It depends on the decrease in solubility of dissolved gases, notably O₂ and CO₂ as the water temperature is increased. The deaerator is a direct contact feedwater heater which has been described in Section 2.14.
Fig. 6.67  A typical demineralizing plant

- Raw water
- Cation exchanger
- Anion exchanger
- Degasifier
- Demineralized water to boiler as make-up
- Silica adsorber [Mg(OH)₂]
- CO₂
6.15 EVAPORATION

Evaporation of pre-treated feedwater \( (w) \) by motive steam \( (m) \) bled from the turbine is often used where the boiler make-up water requirement is not large. Both evaporated water \( (v) \) and condensed steam \( (c) \) exiting the evaporator are fed back as make-up into the plant, normally the deaerator (Fig. 6.68). Prior to the evaporator, the boiler feedwater is softened by passing it through a sodium zeolite bed or by lime-soda process.

![Diagram of a single-effect submerged evaporator]

**Fig. 6.68** Schematic of a single-effect submerged evaporator

6.16 INTERNAL TREATMENT

The dissociation and recombination of water go on till equilibrium is reached,

\[ \text{HOH} = \text{H}^+ + (\text{OH})^- \]

The presence of excess hydrogen ions makes the water acidic and hence corrosive. The pH is the logarithm of the reciprocal of the hydrogen ion concentration. If the hydrogen ion concentration in water is \( 10^{-7} \), the pH value of water would be \( \log \left( \frac{1}{10^{-7}} \right) \) or 7. The pH of the water varies with temperature (Fig. 6.69). At 15 °C, the pH of neutral water is 7. If pH > 7, water is alkaline and if pH < 7, it is acidic. Trisodium phosphate, \( \text{Na}_3\text{PO}_4 \), is injected to water in suitable doses to increase alkalinity. Monosodium phosphate, \( \text{NaH}_2\text{PO}_4 \), is used to decrease alkalinity. A pH meter can be used to measure the pH of water.
The essential factors required for corrosion are hydrogen ions (low pH) and dissolved oxygen. Scale formation and corrosion are reciprocal phenomena: conditions for one preclude the other, and the pH value of water determines it. A pH 10.5 is usually maintained for boiler water in order to minimize corrosion. It will, however, cause scale formation. Two principal steps in scale prevention are (i) periodic or continuous blowdown, and (ii) external or internal treatment to eliminate the scale producers by chemical means. The external treatments have already been discussed.

*Internal treatment* to prevent scale deposition is usually carried out with some form of sodium phosphate (trisodium, disodium or monosodium). Once in the boiler, all these phosphates react with the calcium salts to form tricalcium phosphate, which is rather a flocculent precipitate. The magnesium salts are also readily precipitated due to alkalinity of boiler water. Tannins or starches are sometimes used to prevent the precipitates from adhering to the boiler surfaces.

**Fig. 6.69 pH of water varying with temperature**

Concentrated solid precipitates at drum bottom (by internal treatment) are removed through blowdown, which may be either intermittent or continuous (see section 6.3.5), the amount of blowdown being governed by the allowable solids concentration and the purity of feedwater. It is usually expressed as a percentage. Thus,

\[
\% \text{ blowdown} = \frac{\text{quantity of water blown down}}{\text{quantity of feed water}} \times 100
\]
For instance, 3% blowdown means that 3 per cent of the total amount of feedwater supplied is removed as liquid by blowdown of the drum. The continuous blowdown has the advantage of closer control of concentration and the recovery of part of the energy of the blowdown.

Since water purity is controlled by blowdown in drum type boilers, condensate polishing is not required in such boilers.

6.18 STEAM PURITY

The purity of steam is determined by the quality of the steam delivered by the boiler and the boiler salt concentration. Solids in the steam tend to cause scale deposits on superheater tubes or on turbine blades. Scrubbers and steam washers in the drum (see Section 6.3.5) help achieving the desired steam purity. By measuring the electrical conductivity of condensed steam samples, the steam purity can be determined. Pure water has a high electrical resistance. Presence of solids in water decreases the resistance. So the electrical conductivity is proportional to the solids in the water.

Foaming and priming often cause excessive carry-over of water with steam leaving the drum. Foaming is the condition that results from the formation of bubbles on the drum water surface by the saponification agents like oil, high alkalinity and so on. Priming is a violent, spasmodic action which results in the throwing of slugs of water with steam. It is caused by too high a water level, uneven water circulation and rapid changes in steaming rate. For the desired steam purity, both foaming and priming have to be adequately controlled.

Example 6.1 A furnace riser, 18 m long, 76.2 mm OD and 6.1 mm thick receives saturated water at 80 bar and 1.5 m/s velocity. Assuming a circulation ratio of 12.5 and a slip ratio of 1.2, determine (a) the pressure head developed, (b) the void fraction at riser exit, and (c) the heat transfer rate per unit projected area of the riser tube.

Solution The pressure head developed due to natural circulation (Fig. E6.1),

$$\Delta p = Hg(\rho_f - \rho_m)$$

where $\rho_m$ = average density of liquid–vapour mixture in the riser tube

$$x_{top} = \frac{1}{TDF} = \frac{1}{12.5} = 0.08$$

The quality of steam at the top of the riser is 8%.

At 80 bar,

$$v_f = 0.001384 \text{ m}^3/\text{kg}$$

$$v_{fg} = 0.02214 \text{ m}^3/\text{kg}$$

$$\rho_f = \frac{1}{v_f} = 722.543 \text{ kg/m}^3$$
\[ v_{\text{top}} = v_f + x_{\text{top}} v_{fg} = 0.00138 + 0.08 \times 0.02214 = 0.0031552 \text{ m}^3/\text{kg} \]

\[ \rho_{\text{top}} = \frac{1}{v_{\text{top}}} = 316.937 \text{ kg/m}^3 \]

\[ \rho_m = \frac{\rho_{\text{top}} + \rho_f}{2} = \frac{316.937 + 722.543}{2} = 519.74 \text{ kg/m}^3 \]

(a) \[ \Delta p = H g (\rho_f - \rho_m) \]

\[ = 18 \text{ m} \times 9.806 \text{ m/s}^2 \times \frac{m}{s^2} (722.54 - 519.74) \text{ kg/m}^3 = 35.8 \text{ kPa} \quad \text{Ans.} \]

(b) \[ \psi = \frac{v_f}{v_g} S = \frac{0.001384}{0.02352} \times 1.2 = 0.0706 \]

\[ \alpha = \frac{1}{1 + \frac{1 - x}{\psi}} = \frac{1}{1 + \frac{0.92}{0.08} \times 0.0706} = 0.5519 \quad \text{Ans.} \]

(c) \[ h_{fg} = 1441.3 \text{ kJ/kg} \quad \text{and} \quad d_i = 76.2 - 12.2 = 64.0 \text{ mm} \]

Mass flow rate of saturated water entering the riser,

\[ \omega = \rho_f A_i V = 722.543 \times \frac{\pi}{4} \times (0.064)^2 \times 1.5 \]

\[ = 3.4866 \text{ kg/s} \]

Rate of steam formation in the riser tube,

\[ \omega_s = 0.08 \times 3.4866 = 0.2789 \text{ kg/s} \]
Heat transfer rate per unit projected area

\[ \frac{\omega_s h_{ig}}{D_o H} = \frac{0.2789 \times 1441.3}{0.0762 \times 18} = 293.1 \text{ kW/m}^2 \]  

**Ans.**

**Example 6.2**  A spray type desuperheater is supplied with water at 60 °C. It is connected in a steam line carrying 200 t/h of steam at 35 bar. Calculate the amount of water that must be sprayed per hour to maintain steam at 400 °C when the boiler load causes steam to leave at 450 °C.

**Solution**  Let \( \omega \) be the rate of water spray required to bring down the steam temperature from 450 °C to 400 °C at 35 bar. We would neglect any pressure drop due to mixing and heat loss to the surroundings. By energy balance of the control volume containing the desuperheater (Fig. E6.2),

\[ \omega_s h_1 + \omega h_2 = (\omega_s + \omega)h_3 \]
\[ \omega_s (h_1 - h_3) = \omega (h_3 - h_2) \]

or

\[ \omega = \frac{\omega_s (h_1 - h_3)}{h_3 - h_2} \]

Now,

\[ h_1 = 3337.2 \text{ kJ/kg}, \ h_2 = 252 \text{ kJ/kg and } h_3 = 3222.3 \text{ kJ/kg} \]

**Fig. E6.2**

\[ \omega = \frac{200(3337.2 - 3222.3)}{3222.3 - 252} = \frac{200 \times 114.9}{2970.3} = 7.733 \text{ t/h} \]

\[ = 2.148 \text{ kg/s} \]  

**Ans.**

**Example 6.3**  A 15 m high downcomer–riser circuit operates at 160 bar. The riser receives uniform heat flux and saturated water. The exit quality is 50 per cent. Calculate the pressure head developed due to natural circulation. Take the slip factor as 1.2.

**Solution**  Using Eq. (6.17),

\[ \psi = \frac{v_f}{v_g} S = \frac{0.001711}{0.009306} \times 1.2 = 0.2206 \]
Using Eq. (6.16),
\[
\alpha_e = \frac{1}{1 + \frac{1 - x_e}{x_e} \nu} = \frac{1}{1 + \frac{0.5}{0.5} \times 0.2206} = 0.8193
\]

Using Eq. (6.21) for uniform axial heating,
\[
\rho_m = \rho_i - \rho_e \left[ 1 - \left( \frac{1}{\alpha_e (1 - \nu)} - 1 \right) \ln \frac{1}{1 - \alpha_e (1 - \nu)} \right]
\]
\[
= 584.45 - \frac{584.45 - 107.45}{1 - 0.2206} \left[ 1 - \left( \frac{1}{0.8193(1 - 0.2206)} - 1 \right) \ln \frac{1}{1 - 0.8193(1 - 0.2206)} \right]
\]
\[
= 584.45 - 612 \{ 1 - 0.566 \times 1.0178 \}
\]
\[
= 325 \text{ kg/m}^3
\]

Pressure head developed due to natural circulation,
\[
\Delta p = g(\rho_i - \rho_m)h = 9.806(584.45 - 325) \times 15
\]
\[
= 38162 \text{ N/m}^2 = 38.162 \text{ kPa} \quad \text{Ans.}
\]

**Example 6.4** A power plant producing 120 MW of electricity has steam condition at boiler outlet as 100 bar, 500 °C and the condenser pressure is 0.1 bar. The boiler efficiency is 90%. The boiler consumes coal of calorific value 25.7 MJ/kg. The feedwater temperature at boiler inlet is 160 °C. The steam generator has risers in the furnace wall 40 m high and unheated downcomers. The quality at the top of the riser is 8% and a minimum exit velocity of mixture leaving the riser and entering the drum is required to be 2 m/s. The risers have 60 mm o.d. and 3 mm wall thickness. Neglecting any pressure drop and heat loss, as well as the pump work, estimate (a) the steam generation rate, (b) the fuel burning rate, (c) the evaporation factor, (d) the pressure head available for natural circulation, (e) the circulation ratio, (f) the number of risers required, and (g) the heat absorption rate per unit projected area of the riser.

**Solution** With reference to Fig. E6.4,
\[ h_1 = 3374.8 \text{ kJ/kg} \]
\[ s_1 = 6.6011 \text{ kJ/kg K} = 0.6479 + x_2 (7.5055) \]
\[ x_2 = 0.7932 \]
\[ h_2 = 191.46 + 0.7932 \times 2393.29 = 2089.82 \text{ kJ/kg} \]
\[ h_3 = 191.46 \text{ kJ/kg} = h_4, \quad h_5 = 675.5 \text{ kJ/kg} \]
\[ \omega_s (h_1 - h_2) = 120 \times 10^3 \text{ kW} \]
\[ \omega_s = \frac{120 \times 10^3}{1285} = 93.385 \text{ kg/s} \quad \text{Ans.}(a) \]
\[ \eta_{st, gen} = 0.90 = \frac{93.385 \times (3374.8 - 675.5)}{\omega_t \times 25700} \]
\[ \omega_t = 10.898 \text{ kg/s} \quad \text{Ans.}(b) \]
Evaporation factor \[ \omega_s / \omega_t = 93.385 / 10.898 = 8.57 \quad \text{Ans.}(c) \]
Pressure head available for natural circulation,
\[ \Delta p = g H (\rho_t - \rho_m) \]
\[ v_{top} = 0.0014523 + 0.08 \times 0.0165884 = 0.002779 \text{ m}^3/\text{kg} \]
\[ \rho_{top} = 359.8 \text{ kg/m}^3, \quad \rho_t = 1 / v_t = 688 \text{ kg/m}^3 \]
\[ \rho_m = \frac{\rho_t + \rho_{top}}{2} = 523.9 \text{ kg/m}^3 \]
\[ \Delta p = 40 \times 9.81 (688 - 523.9) = 64.327 \text{ kPa} \]
\[ = 0.6433 \text{ bar} \quad \text{Ans.}(d) \]
Circulation ratio (CR) = \[ 1 / x_{top} = 1 / 0.08 = 12.5 \quad \text{Ans.}(e) \]
\[ d_i = 60 - 6 = 54 \text{ mm} \]
Rate of steam formation in a riser,
\[ \omega = \left[ \frac{\pi}{4} d_i^2 \rho_{top} V_i \right] x_{top} = \frac{\pi}{4} (54 \times 10^{-3})^2 \times 2 \times 359.8 \times 0.08 \]
\[ = 0.131 \text{ kg/s} \]
Number of risers = \[ 93.385 / 0.131 = 712 \quad \text{Ans.}(f) \]
Heat absorption rate per unit projected area of the riser
\[ = \frac{\omega h_{fg}}{D_o H} = \frac{0.131 \times 1319.8}{0.06 \times 40} = 72.47 \text{ kW/m}^2 \quad \text{Ans.}(g) \]

**Example 6.5** Steam leaves the drum of a boiler at the rate of 64 kg/s at 60 bar, with 2% moisture. The feedwater from the economiser enters the drum at the rate of 62 kg/s and has 3 ppm. The makeup water at the rate of 2 kg/s is fed into the drum and it has 50 ppm. Effective steam washing reduces the solids in the moisture, leaving the drum with steam, to 5 ppm. If the solid concentration in the drum water is to be maintained at 1000 ppm, find (a) the blowdown required in kg/s, (b) heat loss in blowdown as a percentage of total heat released in the furnace, if blowdown heat recovery is not used, when the
fuel burning rate is 7 kg/s and its heating value 23 MJ/kg, and (c) the deposition of scale in superheater tube in kg/day. Take the room temperature as 30 °C.

**Solution** Making an impurity balance of the drum at steady state (Fig. E6.5).

\[
\text{FW from economiser} \quad \text{To superheater} \\
\text{Blowdown}
\]

**Fig. E6.5**

(Impurities entering with feedwater + impurities entering with make-up = impurities leaving with blowdown + impurities leaving with moisture carry-over with steam)

\[
62 \times 3 \times 10^{-6} + 2 \times 50 \times 10^{-6} = \text{BD} \times 1000 \times 10^{-6} + 0.02 \times 64 \times 5 \times 10^{-6}
\]

The rate of blowdown is

\[
\text{BD} = \frac{62 \times 3 + 2 \times 50 - 0.02 \times 64 \times 5}{1000} = 0.2796 \text{ kg/s} \quad \text{Ans.(a)}
\]

The energy loss in blowdown as a percentage of total heat released in furnace

\[
\frac{\text{BD}(h_{f,60 \text{ bar}} - h_{\text{ambient}})}{\omega_f \times \text{HHV}} \times 100
\]

\[
= \frac{0.2796(1213.35 - 125.79)}{7 \times 23000} \times 100 = 0.19 \% \quad \text{Ans.(b)}
\]

Scale deposition in superheater tubes

\[
= 0.02 \times 64 \times 5 \times 10^{-6} \times 3600 \times 24
\]

\[
= 0.553 \text{ kg/day} \quad \text{Ans.(c)}
\]

**Example 6.6** Feedwater from the high pressure heater enters the inlet header of the economiser at the rate of 600 kg/s and at 140 bar, 170 °C. It is heated by flue gases till it becomes saturated liquid at that pressure and leaves the outlet header to flow into the drum. Flue gases flow over the economiser coils at the rate of 1250 kg/s and leave at 450 °C. To restrict the erosion rate by fly ash, the flue gas velocity should not exceed 12 m/s, while the optimum water velocity leaving the coils is 1.2 m/s. The tubes are of 70 mm od and 60 mm id. The overall heat transfer coefficient may be taken as 70 W/m²K. Determine the number of coils needed in the economiser and the length of one coil. Take \( c_p \) of flue gases as 1.12 kJ/kg K. If the vertical pitch of the coil is 80 mm, and the clearance on the two sides of the duct of width 4.8 m is 5 mm, find the vertical height of the economiser coils.

**Solution** At 140 bar, \( h_f = 1571.1 \text{ kJ/kg}, \)

\[
t_{\text{sat}} = 336.75 \text{ °C}, \quad \nu_f = 0.001611 \text{ m}^3/\text{kg}
\]
At 170 °C,
\[ h_f = 719.21 \text{ kJ/kg} \]
\[ v_f = 0.001114 \text{ m}^3/\text{kg} \]

The rate of heat transfer in the economiser (Fig. E6.6)
\[ Q = \omega_g (h_f - h_{170°C}) = \omega_g C_{pg} (t_{g1} - t_{g2}) = U_o A_o \Delta t_{1,m} \]

or
\[ 600(1571.1 - 719.21) = 1250 \times 1.12 (t_{g1} - 450) \]

or
\[ 511134 = 1400(t_{g1} - 450) \]

\[ t_{g1} = 815°C \]

\[ \Delta t_{1,m} = \frac{478.25 - 280}{478.25} \ln \frac{280}{280} = 370.32°C \]

\[ A_o = \frac{Q}{U_o \Delta t_{1,m}} = \frac{511134 \text{ kW}}{70 \times 10^{-3} \times 370.32 \text{ kW/m}^2} = 19.72 \times 10^3 \text{ m}^2 \]

\[ \omega_g = \left( n \frac{\pi}{4} d_i^2 \right) \frac{V_w}{v_f} = 600 \text{ kg/s} \]

\[ n \times \frac{\pi}{4} \times (60 \times 10^{-3})^2 \times \frac{1.2}{0.001611} = 600 \]

\[ n \times 4 \times 0.001611 = \frac{600 \times 4 \times 0.001611}{\pi \times 3600 \times 10^{-6} \times 1.2} = \frac{4 \times 1611}{\pi \times 6 \times 1.2} = 285 \quad \text{Ans.} \]

\[ A_o = n \pi d_o l = 19.72 \times 10^3 \text{ m}^2 \]

Fig. E6.6
\[ l = \frac{19.72 \times 10^3}{285 \times \pi \times 70 \times 10^{-3}} = 314.6 \text{ m} \quad \text{Ans.} \]

Number of turns of one coil,
\[ n_t = \frac{1}{B - 2C} = \frac{314.6}{4.8 - 2 \times 0.005} = 65.6 \approx 66 \]

The vertical height of the duct occupied by the economiser coils
\[ = 66 \times 0.08 = 5.28 \text{ m} \quad \text{Ans.} \]

**Example 6.7** For the above steam generator there is a tubular type air preheater following the economiser, where the flue gases flow through the tubes and are cooled to 160 °C and air entering at 35°C flows outside the tubes at the rate of 1167 kg/s. The inlet velocity of flue gases is 13 m/s and the tubes are of 60/65 mm. If the overall heat transfer coefficient is 30 W/m²K, find the length and the number of tubes. Assume that the flue gases behave as an ideal gas having \( c_p = 1.10 \) and \( R = 0.287 \text{ kJ/kg K} \).

**Solution** For the air preheater (Fig. E6.7), the given particulars are:

\[ \omega_g = 1250 \text{ kg/s}, \quad t_{g_1} = 450 \degree C, \quad t_{g_2} = 160 \degree C, \quad t_{a_1} = 35 \degree C, \]
\[ t_{a_2} = 375 \degree C, \quad U_0 = 30 \text{ W/m²K}, \quad V_g = 13 \text{ m/s} \]
\[ \nu_{g_1} = \frac{RT_{g_1}}{p} = \frac{0.287 \times (450 + 273)}{101.325} = 2.048 \text{ m³/kg} \]
\[ Q = \omega_g c_{pg}(t_{g_1} - t_{g_2}) = \omega_a c_{pa}(t_{a_2} - t_{a_1}) = U_0 A_0 \Delta t_{1,m} \]

**Fig. E6.7**

\[ 1250 \times 1.1 (450 - 160) = 1167 \times 1.005 (t_{a_2} - 35) \]
\[ t_{a_2} = 375 \degree C. \]
\[ \Delta t_{1,m} = \frac{75 - 125}{\ln \frac{75}{125}} = 97.5 \degree C \]
\[ 1250 \times 1.1 \times 290 = 0.03 \times A_o \times 97.5 \quad \text{or} \quad A_o = 136325 \text{m}^2 \]

\[ \omega_g = \left( n \frac{\pi d_i^2}{4} \right) \frac{V_{gi}}{v_{gi}} = 1250 \text{ kg/s} \]

\[ n = \frac{1250 \times 4 \times 2.048}{\pi \times (0.06)^2 \times 13} = 69,647 \quad \text{Ans.} \]

\[ A_o = n \pi d_o l = 69647 \times \pi \times 0.065 \times 1 = 136325 \]

\[ l = \frac{136325}{69,647 \times \pi \times 0.065} = 9.59 \text{ m} \quad \text{Ans.} \]

**Example 6.8** Find the number and length of superheater coils of 50 mm id and 5 mm thickness to be provided if steam at exit is at 60 bar, 500 °C and flows with a velocity of 10 m/s and mass flow of 80 kg/s. Due to restriction by materials, the heat flux in the superheater coils is to be limited to 140 kW/m².

**Solution** From steam tables, at state points shown in Fig. E6.8,

\[ h_1 = h_g = 2784.3 \text{ kJ/kg} \]

\[ h_2 = 3422.2 \text{ kJ/kg}, \quad v_2 = 0.05665 \text{ m}^3/\text{kg} \]

Heat absorption rate in superheater coils

\[ = \omega_s (h_2 - h_1) = 80 (3422.2 - 2784.3) = 51032 \text{ kW} \]

Surface area required

\[ = \frac{51032}{140} = 364.5 \text{ m}^2 \]

Now,

\[ \omega_s = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \left( n \frac{\pi d_i^2}{4} \right) \frac{V_2}{v_2} = 80 \text{ kg/s} \]

---

**Fig. E6.8**
Number of coils, \[ n = \frac{80 \times 4 \times 0.05665}{\pi \times (0.05)^2 \times 10} = 231 \quad \text{Ans.} \]

Now, \[ A_o = 364.5 \text{ m}^2 = n \pi d_o l \]

Length of one coil, \[ l = \frac{364.5}{231 \times \pi \times 0.06} = 8.37 \text{ m} \quad \text{Ans.} \]

**Example 6.9** A boiler plant incorporates an economiser and air preheater and generates steam at 40 bar and 300°C with fuel of heating value 33,000 kl/kg burned at a rate of 500 kg/h. The temperature of feed water is raised from 40°C to 125°C in the economiser and the flue gases are cooled at the same time from 395°C to 225°C. The flue gases then enter the air preheater in which the temperature of combustion air is raised by 75°C. A forced draught fan delivers the air to the air preheater at a pressure of 1.02 bar and a temperature of 16°C with a pressure rise across the fan of 180 mm of water. The power input to the fan is 5 kW and it has a mechanical efficiency of 78%. Neglecting heat losses and taking \( C_p \) as 1.01 kl/kg K for flue gases, calculate (a) the mass flow rate of air, (b) the temperature of flue gases leaving the plant, (c) the mass flow rate of steam and (d) the efficiency of the boiler.

**Solution**

Power input to the FD fan, \[ P = \frac{\dot{V} \times \Delta p}{\eta_{\text{mech}}} \]

\[ \dot{V} = \frac{P \times \eta_{\text{mech}}}{\rho_w g h} = \frac{5 \times 10^3 \times 0.78}{1000 \times 9.81 \times 0.180} = 2.21 \text{ m}^3/\text{s} \]

Density of air, \[ \rho_a = \frac{P}{RT} = \frac{1.02 \times 10^5}{287 \times (273 + 16)} = 1.23 \text{ kg/m}^3 \]

Mass flow rate of air, \[ \dot{m}_a = \rho_a \times \dot{V} = 1.23 \times 2.21 = 2.72 \text{ kg/s} \quad \text{Ans. (a)} \]

Assuming complete combustion of fuel, the mass flow rate of flue gases,

\[ \dot{m}_g = \dot{m}_a + \dot{m}_f = 2.72 + \frac{500}{3600} = 2.855 \text{ kg/s} \]

Energy lost by flue gas = energy gained by air in air preheater

\[ 2.855 \times 1.01 \times \Delta T = 2.72 \times 1.005 \times 75 \]

\[ \Delta T = \text{temperature drop of flue gases} \]

\[ = 71°C \]

\[ \therefore \] Temperature of flue gases leaving the plant

\[ = 225 - 71 = 154°C \quad \text{Ans. (b)} \]

Making an energy balance for the economiser

\[ \dot{m}_w \times c_w (125 - 40) = \dot{m}_g \times c_{pg} \times (395 - 225) \]

\[ \therefore \quad \dot{m}_w = \frac{2.855 \times 1.01 \times 170}{4.182 \times 85} = 1.379 \text{ kg/s} \quad \text{Ans. (c)} \]
Water enters the boiler as feedwater at 40°C
\[ h_f = 167.4 \text{ kJ/kg} \]
Steam leaves the boiler at 40 bar, 300°C.
\[ h = 2962.3 \text{ kJ/kg} \]
:. 
Boiler efficiency = \( \frac{(2962.3 - 167.4) \times 1.379}{33000 \times 500/3600} = 0.84 \text{ or } 84\% \)

**SHORT-ANSWER QUESTIONS**

6.1 What is the function of a steam generator? Enlist its basic components.
6.2 How does an industrial steam generator differ from a utility boiler?
6.3 Explain the characteristic features of a fire tube boiler. How is it different from a water-tube boiler.
6.4 What is a fusible plug? Why is it used?
6.5 How does a spring-loaded safety valve operate?
6.6 What are the characteristic features of a package boiler?
6.7 Mention the merits and demerits of fire-tube boilers.
6.8 How does a cross-drum straight tube boiler differ from a longitudinal drum boiler?
6.9 What are the shortcomings of straight tube boilers?
6.10 What is a Stirling boiler? What are its main features?
6.11 What is circulation? What is the difference between natural circulation and forced circulation?
6.12 What is the pressure head developed for natural circulation?
6.13 Why and when are downcomers placed outside the furnace?
6.14 Under what situation is natural circulation not relied upon and forced circulation adopted?
6.15 What is circulation ratio? Mention the range in which the circulation ratio should vary.
6.16 What is TDF?
6.17 Why is too much steaming not desired in a riser tube?
6.18 When does a riser tube become vulnerable to rupture?
6.19 Why are downcomers fewer in number and bigger in diameter, while risers are more in number and smaller in diameter?
6.20 What is DNB? Why is it avoided?
6.21 What is void fraction? What is slip ratio? How are these two related to quality of a liquid–vapour mixture?
6.22 Why should no moisture flow along with steam from the drum to the superheater?
6.23 Why is no vapour bubble desired to flow along with saturated water from the drum to the downcomers?
6.24 What is blowdown? Why is it needed?
6.25 Why is trisodium phosphate injected into the drum?
6.26 What are the functions of the steam drum in a water tube boiler?
6.27 What are drum internals? Why are they required?
6.28 What are primary and secondary separators inside the drum?
6.29 What is steam-washing?
6.30 What do you mean by priming and foaming?
6.31 What are “courses”?
6.32 What is a water wall? What do you mean by a membrane water wall?
6.33 What are once-through boilers? How do they differ from drum boilers?
6.34 Explain the functions of primary air and secondary air.
6.35 What is an assisted circulation steam generator?
6.36 What is a La Mont boiler?
6.37 What is a supercritical boiler? What are its merits and demerits?
6.38 What is the function of an economiser? What are steaming and non-steaming economisers?
6.39 Why are the economiser tubes often finned or gilled on the gas-side?
6.40 How is the water velocity inside the economiser tube optimised?
6.41 What are primary and secondary superheaters?
6.42 What is a pendant superheater?
6.43 How are the gas velocity on the outside and steam velocity on the inside of a superheater coil decided?
6.44 How is the erosion rate of fly ash related to gas velocity?
6.45 Why does the exit steam temperature increase with the increase in load in a convective superheater?
6.46 Why does the exit steam temperature decrease with the increase in load in a radiant superheater?
6.47 What are platens? Where are they used?
6.48 Where is reheater located?
6.49 What is a closed loop control? What is proportional control?
6.50 How is the water level in steam drum controlled?
6.51 What is a “boiler master”? How is steam pressure controlled?
6.52 What are the different methods of controlling the superheat (and reheat) temperature of steam?
6.53 Explain the operation of a spray-type desuperheater.
6.54 Why is the desuperheater located before the last stage of superheating?
6.55 Explain the steam temperature control by gas recirculation. What is gas tempering?
6.56 What are tilting burners?
6.57 What is the function of air preheater? How does air preheating save fuel?
6.58 What are recuperative air preheaters? Why are baffles provided? Why is the overall heat transfer coefficient in APH low?
6.59 Explain the operation of Ljungstrom (regenerative) air preheaters. What are their advantages and disadvantages?
6.60 When are two stages of air-preheating required?
6.61 What is a bubbling fluidized bed combustor? What are its merits?
6.62 Explain the characteristics of a circulating fluidized bed boiler. What is the backpass?
6.63 What is the function of the external fluid bed heat exchanger?
6.64 What is a non-mechanical valve? Name some such valves.
6.65 How does the L-valve operate?
6.66 What is a loop seal? How is it different from a seal pot?
6.67 What are cyclone separators? Explain the principle of gas solid separation in a cyclone.
6.68 Define the collection efficiency of a cyclone. What is the cut-off size?
6.69 On what factors does the collection efficiency of a cyclone depend?
6.70 Enlist the advantages and disadvantages of CFB boilers.
6.71 Explain the aspect of fuel flexibility in a CFB boiler. How is the combustion temperature (800–900 °C) maintained in the furnace at varying loads?
6.72 What is acid rain? How is sulphur removed in a fluidized bed combustor? At what temperature is sulphur capture maximized? What should be the Ca/S ratio for high sulphur capture?
6.73 How is low NOx emission possible in a CFB combustor? Explain the staged combustion process.
6.74 What do you mean by turndown ratio? What is the turndown ratio achieved in a CFB boiler?
6.75 Which is the world's first CFB boiler? Which is now the biggest CFB boiler in operation? Which is the biggest boiler in the erection stage?
6.76 What is a pressurized fluidized bed boiler? What are its merits and demerits?
6.77 How does a PFBC differ from a PCFB?
6.78 Explain briefly the characteristic features of a PCFB boiler. How does it fit into the combined cycle power generation?
6.79 What are the circulation systems in a CFB boiler?
6.80 Explain the operation of an electrostatic precipitator.
6.81 What is a fabric filter? What do you mean by the term ‘air-to-cloth ratio’?
6.82 What are the different types of fabric filters?
6.83 What is a baghouse?
6.84 What are the factors to be considered for an ash disposal system.
6.85 What are the fractions of fly ash and bottom ash in a pc boiler?
6.86 What are the uses of fly ash?
6.87 What are the basic objectives of feedwater treatment?
6.88 What are the impurities in raw water?
6.89 What do you mean by clarification?
6.90 How are gravity filters different from pressure filters?
6.91 What are the basic processes of water softening?
6.92 Explain the lime–soda process for softening of feedwater.
6.93 What do you understand by hot phosphate softening?
6.94 Explain how a bed of sodium zeolite softens feedwater. How is the bed regenerated?
6.95 What are the basic components of a demineralization plant?
6.96 What do you mean by ion exchange?
6.97 What is the harmful effect of silica in water? How is it removed?
6.98 What is reverse osmosis? What is electrodialysis?
6.99 What do you understand by condensate polishing? Where is it required?
6.100 What is pH value? How is it controlled in water?
6.101 What is internal treatment of feedwater? How is it done?
6.102 What do you mean by steam purity?
6.1 A boiler plant comprises a boiler, superheater, economiser and air preheater. The feedwater enters the economiser at 45 °C. Air is preheated from a temperature of 15 °C to 150 °C. The steam is generated in the boiler at a pressure of 35 bar, 0.98 dry and leaves the superheater at 350 °C. When using oil of calorific value 42 MJ/kg, the evaporation rate is 10 kg steam per kg fuel and the air to fuel ratio is 20 : 1 by mass. Neglecting heat losses and pressure drops, estimate the heat transfer per kg fuel in each component and the plant efficiency. Assume \( c_p \) for air and flue gas as 1.005 and 1.045 kJ/kg K, respectively.

\[ \text{Ans. } \eta_{\text{plant}} = 0.696 \]

6.2 A boiler drum has a concentration of 1250 ppm. What percentage of water in the boiler must be wasted through the blowdown to reduce the concentration to 1000 ppm when the feedwater concentration is 11 ppm? If it is desired to have not more than 1 ppm in the steam from the boiler having a concentration of 1000 ppm, what will be the maximum moisture in the steam?

\[ \text{Ans. } 20.2\%, 0.1\% \]

6.3 The following data apply to a steam generator with economiser and air preheater. Steam output: 100 t/h; blowdown 3%; gas flow: 130 t/h; gas temperature to economiser: 410°C; gas temperature leaving air preheater: 150 °C; air flow 100 t/h; water temperature to economiser: 210 °C; water temperature from economiser: 225 °C; air temperature to air preheater: 44 °C; heat transfer surface area of economiser: 380 m²; heat transfer surface area of air preheater: 5000 m². Calculate, making needed assumptions, (a) the gas temperature leaving economiser, (b) the air temperature leaving air preheater, (c) the overall heat transfer coefficients for both economiser and air preheater. Take \( c_p \) for air, gas and feedwater as 1.005, 1.05 and 4.2 kJ/kg K, respectively.

\[ \text{Ans. (a) } 362.5 \degree C, (b) 332.62 \degree C, (c) 0.1015, 0.0965 \text{ kW/m}^2 \text{K} \]

6.4 A drum type steam generator operates at 160 bar. Water from the drum, subcooled at 7.3 °C flows down the downcomer at the rate of 1250 kg/s. The downcomers and risers are all 15 m high. The average density of water—steam mixture in the risers is 350 kg/m³. The pressure losses in the downcomer and risers is 0.52 bar. Calculate the power needed to drive a forced circulation pump of 70% efficiency.

6.5 A 15 m long, 75 mm diameter riser tube receives saturated water at 160 bar and at a velocity of 0.7 m/s. Heat is added to it uniformly. The slip ratio is 1.7. Estimate the maximum heat added to the tube in kJ/m if the exit void fraction is not to exceed 0.80.

6.6 Saturated steam at 160 bar enters a convective superheater. At a given load it leaves at 480 °C. The convective heat transfer coefficients \( h_i \) and \( h_o \) are proportional to \( \omega_{s}^{0.8} \) and \( \omega_{g}^{0.6} \) respectively, where \( \omega_{s} \) and \( \omega_{g} \) are the mass flow rates of steam and gas. Find the exit temperature of steam if both steam and gas flows are doubled. The gas temperature \( T_g \) remains constant at 1500 °C. Assume for simplicity that the heat transferred \( Q \) is proportional to \( U (T_g - T_s) \), where \( T_s \) is the average of the exit and inlet steam temperature.
6.7 Steam enters a radiant superheater in a dry saturated state at 160 bar. The convective heat transfer coefficient inside the tubes is proportional to $\omega_s^{0.8}$, where $\omega_s$ is the steam flow rate. The radiative heat transfer between the combustion gases and the tube surface is proportional to $(T_f^4 - T_w^4)$, where $T_f$ and $T_w$ are the flame and tube wall temperatures, respectively. At a given load, the superheater outlet temperature is 560 °C, and $h_o$ is 5-times $h_i$. Neglecting the conductive resistance of the wall, calculate the exit temperature of steam, if the steam mass flow is doubled.

6.8 A 24-section Ljungstrom air preheater contains 1.6 mm thick corrugated steel sheet and rotates at 2 rpm. The hot gases enter at 315 °C and leave at 175 °C. The air enters at 25 °C. For simplicity, assume that both gases and air have equal mass flow rates of 12.8 kg/s and the same specific heat of 1.008 kJ/kg K. The preheater has an effectiveness of 0.8 (ratio of actual to maximum temperature rise of air). Steel has a density of 7850 kg/m³ and a specific heat of 0.418 kJ/kg K. Calculate (a) the air exit temperature, and (b) the total area provided by the steel sheets.

6.9 A certain steam generator, operated at 40 bar, has a horizontal 1.5 m id steam drum which is 6.2 m long. The normal water level is at the mid-point. If the water level falls more than 0.30 m below this point, serious “starvation” of some of the downcomers will result. When the steaming rate is 12 kg/s, how long an interruption in feedwater supply could be tolerated?

6.10 A forced circulation boiler delivering 36 kg/s at 130 bar is operated with a circulation ratio of 5 : 1. The circulation pumps impart a head rise of 2.8 bar, with suction conditions of 350 °C and 140 bar. What would the ideal pump work amount to per kg of steam delivered?

6.11 A certain steam generator is to deliver 84 kg/s at 500 °C and 100 bar. The condition of steam after the primary superheater is expected to be 495 °C and 101 bar. It will be necessary to reduce the temperature to 450 °C prior to entering the secondary superheater if the desired final temperature is to result. What quantity of desuperheater water at 115 bar, 175 °C must be supplied per second to the desuperheater spray nozzles?

6.12 Calculate the efficiency of a marine boiler operating under the following conditions: feedwater at 50 bar and 120 °C is supplied to the boiler at the rate of 30 kg/s. Of this, 25 kg/s leaves the superheater outlet at 40 bar and 450 °C and the remainder leaves the steam drum as dry saturated steam at 42 bar for auxiliary purposes. The fuel used is oil having a calorific value of 40,414 MJ/m³ and it is supplied at the rate of 7.2 m³/h.

6.13 The boiler in Prob. 6.12 has an integral non-steaming economiser from which water leaves at a temperature of 175 °C. Of all the heat absorbed by the water and steam, determined (a) the percentage absorbed in the economiser, (b) the percentage absorbed in the evaporator, and (c) the percentage absorbed in the superheater.

6.14 It is proposed to install an economiser on a steam generator which at present generates saturated steam at 20 bar while consuming 3 tonnes of coal per hour. The coal has a HHV of 31.4 MJ/kg and contains a total of 5% H₂ as fired. Addition of the proposed economiser surface would result in a final gas
temperature of 195 °C entering the stack compared to the present temperature of 260 °C. There is no reason to believe that addition of the economiser would change appreciably the losses due to unburnt carbon, CO or radiation. At present, these losses total 6.5% of the HHV. Combustion air is supplied at 30 °C and may be considered to be dry. There are 14 kg of dry flue gas per kg of fuel. (a) Calculate the new steam generator efficiency (after addition of the economiser). (b) Calculate the reduction in fuel consumption due to this addition.

6.15 A boiler plant incorporates an economiser and an air preheater, and generates steam at 40 bar, 300 °C with fuel of calorific value 33 MJ/kg burned at a rate of 0.111 kg/s. In the economiser, the feedwater is heated from 40 °C to 125 °C by cooling flue gases from 395 °C to 225 °C. The flue gases then enter the air preheater in which air is heated and its temperature rises by 75 °C. An FD fan delivers air at 1.02 bar and 16 °C producing a pressure head of 180 mm of water. The power input to the fan is 5 kW and it has a mechanical efficiency of 76%. Estimate (a) the mass flow rate of air, (b) the temperature of flue gases leaving the plant, (c) the mass flow rate of steam, and (d) the efficiency of the boiler plant. Neglect heat losses and take $c_p = 1.01 \text{ kJ/kg K}$ for the flue gases.

[Ans. (a) 2.72 kg/s, (b) 150.4 °C, (c) 1.25 kg/s, (d) 83%]

6.16 A 210 MW power plant has steam condition at boiler outlet as 150 bar, 550 °C and the condenser pressure is 0.1 bar. The boiler efficiency is 90 per cent. The boiler consumes coal of calorific value 25 MJ/kg. The feedwater temperature at boiler inlet is 170 °C. The steam generator has risers in the furnace wall 43 m high and unheated downcomers. The boiler operates on natural circulation and to ensure wetted surface on the riser tubes it has a circulation ratio of 16. A minimum exit velocity of water-steam mixture leaving a riser and entering the drum is required to be 1.8 m/s. The risers have 60 mm outer diameter and 3 mm thickness. Taking $\eta_{Gen} = 0.94$ and $\eta_T = 0.92$ and neglecting any heat loss and pressure drop, as well as the pump work, estimate (a) the steam generation rate, (b) the fuel burning rate, (c) the evaporation factor, (d) the pressure head developed due to natural circulation, (e) the quality of steam at the top of the riser, (f) the void fraction of steam at the riser exit, if slip ratio is 1.2, (g) the number of risers required, (h) the heat absorption rate per unit projected area of a riser, (i) the overall plant efficiency, if 5 per cent of the total power is used to drive the auxiliaries.

6.17 In continuation of the above problem, feedwater enters the economiser inlet header at 150 bar, 170 °C and leaves in the saturated vapour state. The economiser consists of tubes 70 mm outer diameter and 60 mm inner diameter made in the form of vertical coils. To restrict the erosion relate by flyash, the flue gas velocity should not exceed 12 m/s, while the velocity of water leaving the outlet header is 1.2 m/s. The ash content of coal is 30 per cent by mass and 9.8 kg air is supplied per kg of coal. Flue gases leave the economiser coils at 450 °C. Taking the overall heat transfer coefficient as 80 W/m²K, estimate the number of coils needed in the economiser and the length of a coil. If the vertical pitch of the coils is 80 mm and the clearance on the two sides of the duct having width 4.8 m, is 5 mm, find the vertical height of the economiser.
6.18 For the above steam generator there is a tubular type air preheater following the economiser, where the flue gases flow through the tubes and are cooled to 150°C. Air entering at 35°C flows outside the tubes. The outlet velocity of flue gases is 14 m/s and the tubes are of 60/65 mm. If \( U_0 = 30 \text{ W/m}^2\text{K} \), find the number of tubes of the air preheater and the length of tubes. Assume that the flue gases behave as an ideal gas having \( c_p = 1.10 \) and \( R = 0.286 \text{ kJ/kgK} \).

**BIBLIOGRAPHY**

7
Steam Turbines

7.1 INTRODUCTION

A steam turbine is a prime mover which continuously converts the energy of high-pressure, high temperature steam supplied by a steam generator into shaft work with the low temperature steam exhausted to a condenser. This energy conversion essentially occurs in two steps:

1. The high-pressure, high-temperature steam first expands in nozzles and comes out at a high velocity.
2. The high velocity jets of steam coming out of the nozzles, impinge on the blades mounted on a wheel, get deflected by an angle and suffer a loss of momentum which is absorbed by the rotating wheel in producing torque.

A steam turbine is basically an assemblage of nozzles and blades.

The Greek inventor Hero of Alexandria built the first prototype of a steam turbine in 120 BC which operated on the reaction principles. A simple closed spherical vessel, mounted on bearings, carrying steam from a cauldron or boiler with four tangential pipes discharging steam is driven around by the reaction of the steam jets (Fig. 7.1 a). Many centuries later, Giovanni Branca made the prototype of the impulse steam turbine in 1629, which is often referred to as the Branca’s wheel.

In 1878, a Swedish engineer, Carl Gustav Patrik de Laval (1845-1913), developed a simple impulse turbine to separate cream from milk, using a convergent-divergent (supersonic) nozzle which ran the turbine to a maximum speed of 100,000 rpm. He constructed in 1897 a velocity-compounded impulse turbine (a two-row axial turbine with a row of guide vane stators between them, all fed by a single set of high velocity nozzles).

In France, Auguste Rateau (1863-1930) experimented with a de Laval turbine in 1894, and developed the pressure compounded impulse turbine by 1900.
Fig. 7.1 (a)  *Hero's turbine*

In the USA, Charles G. Curtis (1860-1953) patented in 1896 the velocity-compounded turbine similar to a two-stage de Laval turbine, but after further development he sold all his rights to General Electric in 1901.


Steam turbines are not only employed to operate electric generators in thermal and nuclear power plants to produce electricity, but they are also used (a) to propel large ships, ocean liners, submarines and so on, and (b) to drive power absorbing machines like large compressors, blowers, fans and pumps.

Turbines can be condensing or non-condensing, depending of whether the back pressure is below or equal to the atmospheric pressure. Figure 7.1(b) shows different types of turbines, in regard to steam flow through the units.

For small units without reheat, the steam turbine may consist of a single turbine when the steam expanding through the turbine exhausts to a condenser or to a process line. This is a straight-flow turbine as shown in Fig. 7.1 (b) (A) and (B). For a large unit without reheat, the steam may expand through an initial section and then exhaust to another turbine. This later turbine may then exhaust to a condenser or to a process. The initial turbine is designated as the high-pressure (HP) turbine and the second turbine the low-pressure (LP) turbine, as shown in Fig. 7.1(b) (C).

For a single reheat cycle, the steam from the boiler flows to the HP turbine where it expands and is exhausted back to the boiler for reheating. The reheat steam coming from the boiler flows to the intermediate-pressure (IP) or reheat turbine where it expands and exhausts into a crossover line that supplies steam
to the double-flow LP turbine. The steam expands through the LP turbine and exhausts to a condenser as shown in Fig. 7.1(b) (D).

Figure 7.1(b) (E) shows double-reheat cycle where steam is reheated twice, having four turbine sections, viz. HP, IP, reheat and double-flow LP. Flow designations of triple-flow, four-flow, six-flow and eight-flow are also used. A triple-flow designation indicates the use of one double-flow LP turbine and a single-flow LP turbine, as shown in Fig. 7.1(b) (F). Four-flow designation indicates the use of two double-flow LP turbines (Fig. 7.1(b)(G)).

Turbines are also designated by the type of extraction involved, if any. Steam extracted for feedwater heating varies in pressure and flow as a function of load. This variation is acceptable and so no effort is made to regulate the pressure. This is called uncontrolled, simple or nonautomatic extraction, as shown in Fig. 7.1(b)(H). However, turbines supplying steam for process applications must supply steam at a constant pressure. Since the pressure varies with load, regulating valve is used to control the pressure. This is called a controlled or automatic extraction turbine, as shown in Fig. 7.1(b)(I).

The overall steam turbine generator arrangement of a power plant is designated as tandem-compound or cross-compound on the basis of shaft orientation, as shown in Fig. 7.1 (c). The tandem-compound unit has all turbines and the generator in-line, connected to the same shaft and thus operate at the same speed. The cross-compound arrangement typically consists of HP and IP turbines operating at 3600 rpm (for frequency of 60 Hz and a bipolar generator, \[ \text{rpm} = (120 \times 60)/2 = 3600 \text{ rpm} \] as in the USA, while in India with 50 Hz frequency, it is 3000 rpm) driving a generator. The exhaust steam from the IP turbine crosses over to a LP turbine that operates at 1800 rpm (1500 rpm for 50 Hz and 4 poles), driving a separate generator. The LP turbine operating at slower speed allows the use of longer last-stage turbine blades with expansion to higher moisture percentages and less exhaust losses, resulting in higher turbine efficiencies.

Large cross-compound units in the 1300 MW range with both shaft orientations operating at 3600 rpm have been built. The dual-shaft arrangement was used to minimize shaft length and any single generator size.

Power plant steam turbines are thus typically designated by shaft orientation, number of LP turbine steam flow paths and the last stage blade length of the LP turbine. A turbine designated as TC4F30, for example, indicates a unit that is tandem-compound (TC) having two double-flow (4F) LP turbines with 30 in. (76.2 cm) last stage blade length. A CC2F23 indicates a unit that is cross-compound (CC) having one double-flow (2F) LP turbine with 23 in. (58.4 cm) last stage blade length.

### 7.2 FLOW THROUGH NOZZLES

A nozzle is a duct by flowing through which the velocity of a fluid increases at the expense of pressure drop. A duct which decreases the velocity of a fluid and causes a corresponding increase in pressure is called a diffuser. The same duct
(A) Straight flow condensing

(B) Straight flow noncondensing

(C) Two section condensing

(D) Single reheat cycle

(E) Double reheat cycle

(F) Triple flow low pressure turbine cycle
Fig. 7.1 (b)  **Turbine Types**

(A) Tandem compound unit

(B) Cross compound unit

**Fig. 7.1 (c)  Turbine shaft orientations**
may be either a nozzle or a diffuser depending upon the end conditions across it. If the cross-section of a duct decreases continuously from inlet to exit, the duct is said to be convergent, and if it increases from inlet to exit, the duct is said to be divergent. If the cross-section initially decreases and then increases, the duct is called convergent-divergent. The minimum cross-section of this duct is referred to as throat. A fluid is said to be compressible if its density changes with a change in pressure (or temperature). If the density does not changes or changes very little, the fluid is said to be incompressible. Gases and vapours are compressible, whereas liquids are incompressible.

### 7.2.1 Velocity of Pressure Pulse in a Fluid

Let us consider an infinitesimal pressure wave pulse initiated by a slight movement of a piston to the right (Fig. 7.2) in a pipe of uniform cross-section. The pressure wave front propagates steadily with a velocity \( c \), which is known as the velocity of sound, sonic velocity or acoustic velocity. The fluid near the piston will have a slightly increased pressure and will be slightly more dense than the fluid, away from the piston.

**Fig. 7.2** Diagram illustrating sonic velocity: (a) stationary observer, (b) observer travelling with wave-front

To simplify the analysis, let an observer be assumed to travel with the wave front to the right with the velocity \( c \). Fluid flows steadily from right to left as it passes through the wave front, the velocity is reduced from \( c \) to \( c - dV \). At the same time, the pressure rises from \( p \) to \( p + dp \) and the density from \( \rho \) to \( \rho + d\rho \). The continuity equation for the control volume gives

\[
\rho A c = (\rho + d\rho) A (c - dV)
\]

or

\[
\rho c = \rho c - \rho dV + c d\rho - d\rho dV
\]
Neglecting the product $dp \cdot d\bar{V}$, both being very small,

$$\rho d\bar{V} = c \, dp$$  \hspace{1cm} (7.1)

The momentum equation for the control volume gives

$$[p - (p + dp)]A = \rho Ac [(c - d\bar{V}) - c]$$  \hspace{1cm} (7.2)

From Eqs (7.1) and (7.2).

$$\frac{dp}{c} = cdp \quad \therefore \quad c = \left[ \frac{dp}{d\rho} \right]^{1/2}$$

Since the variations in pressure and temperature are negligibly small and the change of state is so fast as to be essentially adiabatic, and in the absence of any internal friction or viscosity, the process is reversible and isentropic. Hence, the sonic velocity is given, by

$$c = \sqrt{\frac{\gamma p}{\rho}}$$  \hspace{1cm} (7.3)

No fluid is truly incompressible, although liquids show little change in density. The velocity of sound in common liquids is of the order of 1650 m/s.

7.2.2 Velocity of Sound in Ideal Gas

For an ideal gas, in an isentropic process

$$p\gamma = \text{constant} \quad \text{or} \quad \frac{p}{\rho^\gamma} = \text{constant}$$

By logarithmic differentiation (i.e., first taking logarithm and then differentia-ting),

$$\frac{dp}{p} - \gamma \frac{dp}{\rho} = 0$$

$$\frac{dp}{d\rho} = \gamma \frac{p}{\rho}$$

Since

$$c^2 = \frac{dp}{d\rho} \quad \text{and} \quad p = \rho RT \quad \text{for an ideal gas},$$

$$c = \sqrt{(\gamma RT)}$$  \hspace{1cm} (7.4)

where $R$ is the characteristic gas constant, which is equal to the universal gas constant divided by the molecular weight of the gas.

The Mach number, $M$ is defined as the ratio of the actual velocity $V$ to the sonic velocity $c$

$$M = \frac{V}{c}$$
When $M > 1$, the flow is supersonic, when $M < 1$, the flow is subsonic, and when $M = 1$, the flow is sonic.

### 7.2.3 Stagnation Properties

The isentropic *stagnation state* is defined as the state a fluid in motion would reach if it were brought to rest isentropically in steady flow, adiabatic, zero work output device. This is a reference state in compressible fluid flow and is commonly designated with the subscript zero. The stagnation enthalpy $h_o$ (Fig. 7.3) is related to the enthalpy and velocity of the moving fluid by

\[ h_o = h + \frac{V^2}{2} \]

For an ideal gas, $c_p (T_o - T) = \frac{V^2}{2}$

The properties without subscript denote static properties.

Since

\[ c_p = \frac{\gamma R}{\gamma - 1}, \quad T_o/T = 1 + \frac{V^2 (\gamma - 1)}{2\gamma RT} \]

Using Eq. (7.4) and the Mach number,

\[ \frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \]  \hspace{1cm} (7.5)

The stagnation pressure $p_o$ is related to the Mach number and static pressure in the case of an ideal gas by the following equation:

\[ \frac{p_o}{p} = \left( \frac{T_o}{T} \right)^{\frac{\gamma}{\gamma - 1}} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \]  \hspace{1cm} (7.6)
7.2.4 Subsonic and Supersonic Nozzles

For isentropic flow through a nozzle,

\[ h_o = h + \frac{\bar{V}^2}{2} \]

Since stagnation enthalpy does not change,

\[ dh = -\bar{V} \ d\bar{V} \]  \hfill (7.7)

From the property relation

\[ T \ ds = dh - \nu \ dp \]

For isentropic flow,

\[ dh = \frac{dp}{\rho} \]  \hfill (7.8)

From Eqs (7.7) and (7.8)

\[ dp = -\rho \bar{V} \ d\bar{V} \]  \hfill (7.9)

or

\[ \frac{dp}{d\bar{V}} < 0 \]  \hfill (7.10)

Therefore, as pressure decreases for flow through a nozzle, the velocity increases, and vice versa. The continuity equation gives

\[ \omega = \rho AV \]

By logarithmic differentiation,

\[ \frac{dp}{\rho} + \frac{dA}{A} + \frac{d\bar{V}}{\bar{V}} = 0 \]

or

\[ \frac{dA}{A} = -\frac{d\bar{V}}{\bar{V}} - \frac{dp}{\rho} \]

Substituting from Eq. (7.9),

\[ \frac{dA}{A} = \frac{dp}{\rho \bar{V}^2} - \frac{dp}{\rho} = \frac{dp}{\rho \bar{V}^2} \left[ 1 - \bar{V}^2 \frac{dp}{dp} \right] \]

or

\[ \frac{dA}{A} = \frac{dp}{\rho \bar{V}^2} (1 - M^2) \]  \hfill (7.11)

Also,

\[ \frac{dA}{A} = (M^2 - 1) \ \frac{d\bar{V}}{\bar{V}} \]  \hfill (7.12)

When \( M < 1 \), i.e. the inlet velocity is subsonic, as flow area \( A \) decreases, the pressure decreases and the velocity increases. So, for subsonic flow, a convergent passage becomes a nozzle (Fig. 7.4a) and a divergent passage becomes a diffusor (Fig. 7.4b). When \( M > 1 \), i.e. when the inlet velocity is supersonic, as flow area \( A \) decreases, pressure increases and velocity decreases,
and as flow area \( A \) increases; pressure decreases and velocity increases. So, for supersonic flow, a convergent passage is a diffusor (Fig. 7.4c) and a divergent passage is a nozzle (Fig. 7.4d).

**Fig. 7.4** Effect of area change on subsonic and supersonic flow: (a) subsonic nozzle, (b) subsonic diffuser, (c) supersonic diffuser, (d) supersonic nozzle

### 7.2.5 Critical Pressure ratio and Choked Flow

Let us consider the isentropic flow of an ideal gas through a convergent-divergent nozzle (Fig. 7.5).

The mass flow rate per unit area is given by

\[
\frac{\dot{m}}{A} = \rho \bar{V} = \rho C_M = \frac{P}{RT} \sqrt{(\gamma RT) \cdot M} = \sqrt{\frac{\gamma}{R}} \frac{p_0 M}{\sqrt{T_0}} \left( \frac{T_0}{T} \right)^{1/2} \frac{p}{p_0}
\]
\[
\omega/A = \sqrt{\frac{\gamma}{R}} \frac{p_0 M}{\sqrt{T_0}} \left( \frac{T_0}{T} \right)^{1/2} \frac{1}{\left( \frac{T_0}{T} \right)^{(\gamma/\gamma-1)}}
\]

\[
(7.13)
\]

Since \(p_0\), \(T_0\), \(\gamma\) and \(R\) are constant, the discharge per unit area, \(\omega/A\), is a function of Mach number only. There is a particular value of \(M\) when \(\omega/A\) is maximum. Differentiating Eq. (7.13) with respect to \(M\) and equating it to zero.

\[
\frac{d(\omega/A)}{dM} = \sqrt{\frac{\gamma}{R}} \frac{p_0 M}{\sqrt{T_0}} \frac{1}{\left( 1 + \frac{\gamma-1}{2} M^2 \right)^{(\gamma+1)/(2(\gamma-1))}}
\]

\[
+ \sqrt{\frac{\gamma}{R}} \frac{p_0 M}{\sqrt{T_0}} \left[ -\frac{\gamma+1}{2(\gamma-1)} \right] \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma+1}{2(\gamma-1)}} \left( \frac{\gamma-1}{2} 2M \right) = 0
\]

or

\[
1 - \frac{M^2 (\gamma + 1)}{2 \left( 1 + \frac{\gamma - 1}{2} M^2 \right)} = 0
\]

or

\[
M^2 (\gamma + 1) = 2 + (\gamma - 1)M^2
\]

\[
M^2 = 1 \text{  or  } M = 1
\]

So, the discharge \(\omega/A\) is maximum when \(M = 1\).

Since \(\bar{V} = cM = (\gamma RT)^{1/2}M\), by logarithmic differentiation,

\[
\frac{d\bar{V}}{\bar{V}} = \frac{dM}{M} + \frac{1}{2} \frac{dT}{T}
\]

(7.14).

Again,

\[
\frac{T}{T_0} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{-1}
\]

By logarithmic differentiation,

\[
\frac{dT}{T} = \frac{(\gamma - 1)M^2}{1 + \frac{\gamma - 1}{2} M^2} \times \frac{dM}{M}
\]

(7.15)

From Eqs (7.14) and (7.15),

\[
\frac{d\bar{V}}{\bar{V}} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2} \times \frac{dM}{M}
\]

(7.16)

From Eqs (7.12) and (7.16),
\[
\frac{dA}{A} = \frac{(M^2 - 1) dM}{M \left(1 + \frac{\gamma - 1}{2} M^2\right)}
\]

(7.17)

By substituting \(M = 1\) in any of the Eqs (7.11), (7.12) and (7.17), \(dA = 0\), or \(A = \text{constant}\), So \(M = 1\) occurs only at the throat and nowhere else, and this happens only when the discharge is the maximum.

If the convergent-divergent duct (Fig. 7.5) acts as a nozzle, in the divergent part also, the pressure will fall continuously to yield a continuous rise in velocity. In other words, since \(\bar{v} = \omega / \rho A\), for a certain mass flow rate \(\omega\) with the decrease of pressure, density decreases at a rate faster than the rate at which area increases, as a result of which velocity continues to increase. This is true only for a compressible fluid. The velocity of the gas is subsonic before the throat, becomes sonic at the throat, and then supersonic till its exit, provided the exhaust pressure is low enough. The reverse situation prevails when the inlet velocity is supersonic. The whole duct then becomes a diffuser For an incompressible fluid, the convergent-divergent duct acts as a venturi, with the convergent part acting as the nozzle and the divergent part acting as the diffuser. The maximum limiting speed of such a fluid is the sonic velocity.

When \(M = 1\) occurs at the throat, the discharge is maximum and the nozzle is said to be choked. It is incapable of allowing more discharge even with further decrease in exhaust pressure (Fig. 7.6). This discharge is known as critical discharge and the properties at the throat under this condition are called critical properties designated by superscript asterisks (*).

![Fig. 7.6 Choking of nozzle](image)

In a converging nozzle, the velocity at exit from the nozzle is sonic when the discharge through it is maximum, and the exit properties are critical properties. Substituting \(M = 1\) in Eq. (7.5),
\[
\frac{T_0}{T^*} = 1 + \frac{\gamma - 1}{2} \left( \frac{T^*}{T_0} \right)^2 = \frac{\gamma + 1}{2}
\]

or
\[
\frac{T^*}{T_0} = \frac{2}{\gamma + 1}
\]  

(7.18)

The ratio of the temperature at the throat where the velocity is sonic (T) to the inlet temperature (T_0) is called the critical temperature ratio.

The critical pressure ratio, \( p^*/p_0 \), defined as the ratio of the pressure at the throat to the inlet pressure, for choked flow when \( M = 1 \) occurs at the throat, is given by,

\[
\frac{p^*}{p_0} = \left( \frac{2}{\gamma + 1} \right)^{\gamma/(\gamma - 1)}
\]  

(7.19)

Eqs (7.18) and (7.19) hold good only for an ideal gas. It is thus seen that the critical pressure ratio for an ideal gas depends only on the specific heat ratio \( \gamma \). For air, \( \gamma = 1.4 \), and

\[
\frac{p^*}{p_0} = \left( \frac{2}{1.4 + 1} \right)^{1.4/0.4} = 0.528
\]

Hence, for air at 10 bar, a convergent nozzle requires a back pressure of 5.28 bar so that the flow is sonic at exit. For a correctly designed convergent-divergent nozzle with inlet pressure of 10 bar, the pressure at the throat is 5.28 bar for the maximum discharge. For helium, \( \gamma = 1.67 \) and

\[
\frac{p^*}{p_0} = \left( \frac{2}{1.67 + 1} \right)^{1.67/0.67} = 0.487
\]

Hence, for helium at 10 bar, a convergent nozzle requires a back pressure of 4.87 bar for sonic flow at exit, and the pressure at the throat of a convergent-divergent nozzle with inlet pressure of 10 bar is 4.87 bar.

The critical velocity at the throat of a convergent-divergent nozzle or at exit of convergent nozzle is given from Eq. (7.4),

\[
\bar{V}^* = (\gamma RT^*)^{1/2}
\]  

(7.20)

For steam, Eqs (7.18), (7.19) and (7.20) are not strictly valid. However, a close approximation is obtained if it is assumed that steam expands isentropically in the nozzle following the law \( p v^k = \) constant, where \( k \) is the isentropic index and it is not a ratio of specific heats as is \( \gamma \). For steam initially dry, saturated, \( k = 1.135 \); for steam initially superheated, \( k = 1.3 \).

Eq. (7.19) can then be written as

\[
\frac{p^*}{p_0} = \left( \frac{2}{k + 1} \right)^{k/(k - 1)}
\]  

(7.21)

The proof of this equation in a steam nozzle is shown in the next Section. Therefore, when the steam entering a nozzle is dry, saturated,
\[ \frac{p^*}{p_0} = \left( \frac{2}{1.135 + 1} \right)^{1.135/0.135} = 0.577 \]

and when the steam entering a nozzle is superheated,

\[ \frac{p^*}{p_0} = \left( \frac{2}{1.3 + 1} \right)^{1.3/0.3} = 0.546 \]

For wet steam, Zeuner’s relation

\[ k = 1.035 + 0.1 x, \]

where \( x \) is the quality of steam, may be used. When \( x = 1 \), \( k \) becomes 1.135, as given earlier for dry, saturated steam.

### 7.2.6 Flow in Steam Nozzle

Let us consider the flow of steam through a convergent-divergent nozzle (Fig. 7.7). If the flow is adiabatic, \( Q = 0 \), and since there is no work interaction involved, \( W = 0 \), and neglecting the potential energy change, the steady flow energy equation for the nozzle (Fig. 7.7 a) reduces to

\[ h_0 + \frac{\bar{V}_0^2}{2} = h_1 + \frac{\bar{V}_1^2}{2} \]  

(7.22)

If the velocity of approach, i.e. the inlet velocity \( \bar{V}_0 \) is small, the velocity at exit from the nozzle is given by,

\[ \bar{V}_1 = [2(h_0 - h_1) \times 10^3]^{1/2} = 44.72(h_0 - h_1)^{1/2} \text{ m/s} \]  

(7.23)

where \( (h_0 - h_1) \) is the enthalpy drop across the nozzle in kJ/kg.

For isentropic flow in a nozzle,

\[ T ds = dh - v dp = 0 \]
or,
\[
\int_0^1 dh = \int_0^1 v dp
\]

\[
h_0 - h_1 = -\int_0^1 v dp
\]

(7.24)

From Eq. (7.22),
\[
h_0 - h_1 = \frac{\bar{V}_1^2 - \bar{V}_0^2}{2} = -\int_0^1 v dp
\]

Assuming that the pressure and volume of steam during expansion obey the law \(p_0 v^k = \) constant, where \(k\) is the isentropic index, and neglecting the inlet velocity \(V_0\),
\[
\frac{\bar{V}_1^2}{2} = -\int_0^1 \left( p_0 v_0^k \right)^{1/k} p^{-1/k} dp = -\int_0^1 \left( p_1 v_1^k \right)^{1/k} p^{-1/k} dp
\]

\[
= \frac{k}{k-1} (p_0 v_0 - p_1 v_1)
\]

\[
= \frac{k}{k-1} p_0 v_0 \left[ 1 - \left( \frac{p_1}{p_0} \right)^{(k-1)/k} \right]
\]

or
\[
\bar{V}_1 = \left[ \frac{2k}{k-1} p_0 v_0 \left[ 1 - \left( \frac{p_1}{p_0} \right)^{(k-1)/k} \right] \right]^{1/2}
\]

(7.25)

The rate of flow of steam is given by
\[
\omega = \frac{A_1 \bar{V}_1}{v_1} = A_1 \left[ \frac{2k}{k-1} \frac{p_0 v_0}{v_1^2} \left[ 1 - \left( \frac{p_1}{p_0} \right)^{(k-1)/k} \right] \right]^{1/2}
\]

(7.26)

By analogy, changing \(A_1\) to \(A\) and \(p_1\) to \(p\),
\[
\omega = \left[ \frac{2k}{k-1} \frac{p_0}{v_0} \left( \frac{p}{p_0} \right)^{2/k} - \left( \frac{p}{p_0} \right)^{(k+1)/k} \right]^{1/2}
\]

(7.27)

Equation (7.27) shows that for given values of \(p_0\) and \(v_0\), the mass flow rate of steam depends upon the pressure ratio \(p/p_0\). There is a certain pressure ratio for which discharge \((\omega/A)\) is maximum. Therefore, by differentiating the term in square brackets and equating it to zero,
\[
\frac{d}{d(p/p_0)} \left[ \left( \frac{p}{p_0} \right)^{2/k} - \left( \frac{p}{p_0} \right)^{(k+1)/k} \right] = 0
\]

or
\[
\frac{2}{k} \left( \frac{p}{p_0} \right)^{(2-k)/k} - \frac{k+1}{k} \left( \frac{p}{p_0} \right)^{1/k} = 0
\]

\[
\therefore \frac{p^*}{p_0} = \left( \frac{2}{k+1} \right)^{(k/(k-1))}
\]

This is the same equation as given in Eq. (7.21). This is the critical pressure ratio.

If \( p_{\text{exit}} < p^* \) a convergent-divergent nozzle is used, and if \( p_{\text{exit}} > p^* \), a convergent nozzle is used.

The velocity of steam at the throat or the critical velocity is given by
\[
\nu^* = 44.72 \ (h_0 - h^*)^{1/2}
\]  
(7.28)

where \((h_0 - h^*)\) is the enthalpy drop of steam in the converging portion of the nozzle. The enthalpy drops \((h_0 - h_1)\) and \((h_0 - h^*)\) are read from the Mollier (or \(h-s\)) chart.

If the nozzle is only convergent, the minimum pressure to which steam can expand is the critical pressure when the discharge is maximum and the exit velocity of steam is the sonic velocity, which is given by Eq. (7.28).

The critical velocity can also be obtained from Eq. (7.25) on substituting the critical pressure ratio,
\[
\bar{\nu}^* = \left[ \frac{2k}{k-1} p_0 \right] \left[ 1 - \frac{2}{k+1} \right]^{1/2}
\]
\[
= \left[ \frac{2k}{k+1} p_0 \right]^{1/2}
\]  
(7.29)

For superheated steam \( k = 1.3 \),
\[
\bar{\nu}^* = 1.06 (p_0 \nu_0)^{1/2}
\]  
(7.30)

For dry, saturated steam, \( k = 1.135 \)
\[
\bar{\nu}^* = 1.03 (p_0 \nu_0)^{1/2}
\]  
(7.31)

where \( p_0 \) is in the N/m\(^2\) and \( \nu \) is in m\(^2\)/kg.

### 7.2.7 Nozzle Efficiency

Due to friction between the fluid and walls of the nozzle, the expansion process is irreversible, although still approximately adiabatic. In nozzle design, the usual practice is to base all calculations on isentropic flow and then to make an allowance for friction using a coefficient or efficiency.
The nozzle efficiency, \( \eta_n \), is defined as the ratio of the actual enthalpy drop to the isentropic enthalpy drop (Fig. 7.8),

\[
\eta_n = \frac{h_0 - h_1}{h_0 - h_{1s}}
\]  
(7.32)

For an ideal gas,

\[
\eta_n = \frac{c_p (T_0 - T_1)}{c_p (T_0 - T_{1s})} = \frac{T_0 - T_1}{T_0 - T_{1s}}
\]  
(7.33)

![Diagram](image)

**Fig. 7.8** Nozzle efficiency

If the actual velocity at exit from the nozzle is \( \bar{V}_1 \) and the velocity at exit when the flow is isentropic is \( \bar{V}_{1s} \), then

\[
\eta_n = \frac{\bar{V}_1^2 - \bar{V}_0^2}{\bar{V}_{1s}^2 - \bar{V}_0^2}
\]  
(7.34)

When the inlet velocity is very small, then

\[
\eta_n = \frac{\bar{V}_1^2}{\bar{V}_{1s}^2}
\]  
(7.35)

Sometimes, a velocity coefficient \( \phi \) is defined as the ratio of actual exit velocity to the exit velocity when the flow is isentropic between the same pressures,

\[
\phi = \frac{\bar{V}_1}{\bar{V}_{1s}}
\]  
(7.36)

Therefore, \( \phi = (\eta_n)^{1/2} \)

Another coefficient which is frequently used is the ratio of the actual mass flow through the nozzle, \( \omega \), to the mass flow which would be passed if the flow were isentropic, \( \omega_s \). This is called the coefficient of discharge, \( C_d \)

\[
C_d = \frac{\omega}{\omega_s}
\]  
(7.37)
The convergent part of a nozzle is usually sharp, while the divergent part is gradual. Most of the friction loss occurs in the divergent portion of the nozzle. If the semi-divergence angle $\alpha$ is large (Fig. 7.9), there will be flow separation from the wall with the formation of eddies, which entails energy loss. If the angle $\alpha$ is small, the length of nozzle becomes large to provide the desired exit flow area, causing more energy loss due to friction. Usually, $\alpha$ varies from 5° to 8°.

![Fig. 7.9 Semi-divergence angle $\alpha$](image)

### 7.2.8 Nozzle Types

Probably the most common physical conception of the nozzle is the converging-diverging type, although its application in the turbine design is somewhat limited. More generally, the converging or subsonic nozzles are used. There are two types of nozzles:

1. Reamed or round nozzles
2. Foil nozzles

Reamed nozzles are used principally in the high pressure impulse stages of steam turbines. The principal advantages of the reamed nozzle are low cost, ease of manufacture, and adaptability to standardization. The principal disadvantages are lower efficiency, somewhat greater length, and an inability to utilize effectively for flow purposes the area of a given flow annulus. The angle of divergence in reamed nozzles, as mentioned earlier, is usually about 12° and rarely over 15°, in order to avoid flow separation with its attendant loss. The converging part is normally sharp and a good fillet or rounded entrance is provided.

The foil nozzle is formed by curved airfoil sections. The nozzle is short with well-rounded entrance edges and sharp exit edges affording a good issuing jet. These nozzles have high efficiency and are costlier. A foil nozzle block is made by welding the individual foils between sections of concentric rings. The block is then welded in place between the outer and inner shrouds. The foil nozzle finds wide application in large steam turbines.

### 7.2.9 Flow Area of Nozzle

If the condition of steam at inlet to the nozzle is known, the critical pressure $p^*$ can be estimated by multiplying the inlet pressure $p_0$ by 0.546 or 0.577
depending on whether the inlet steam is superheated or dry saturated, respectively. If the exit pressure is given and the nozzle efficiency is known, the exit velocity of steam, as well as the critical velocity of steam at the throat can be estimated from

\[ \bar{V}_1 = 44.72 \left[ (h_0 - h_{1s}) \eta_n \right]^{1/2} \]

and

\[ \bar{V}^* = 44.72 \left[ (h_0 - h_{s^*}) \eta_n \right]^{1/2} \]

where the enthalpy drops are read from the Mollier chart (kJ/kg). With the actual states of steam at exit and the throat being fixed on the Mollier diagram, the specific volumes can be obtained from it. Knowing the mass flow rate of steam (\(\omega\)), the flow areas required at the nozzle exit, as well as at the throat can be estimated from

\[ \omega = \frac{A^* \bar{V}^*}{\bar{V}^*} = \frac{A_1 \bar{V}_1}{\bar{V}_1} \]

(7.38)

\[ A^* = \frac{\omega \bar{V}^*}{\bar{V}^*} \quad \text{and} \quad A_1 = \frac{\omega \bar{V}_1}{\bar{V}_1} \]

If the nozzles are circular in cross-section, \(A_1 = n \frac{\pi}{4} \times d_{\text{exit}}^2\), where \(n\) is the number of nozzles and \(d_{\text{exit}}\) is the exit diameter of the nozzles. The cross-section may be rectangular or square also. Foil type nozzles are mounted in a nozzle diaphragm which is horizontally split into two halves and joined by vertical flange bolts (Fig. 7.10a). The top halves of the diaphragms, which form the stator, can be lifted by cranes thus exposing the rotor for inspection and repair, if needed.

Fig. 7.10  *Horizontally split diaphragm with nozzles*
If \( D_m \) = mean diameter of the nozzle diaphragm,
\( h_n \) = height of nozzles,
\( p \) = pitch of nozzles on the mean diameter,
\( o \) = minimum opening for flow,
\( t \) = thickness of nozzles,
\( n \) = number of nozzles,

and \( \alpha \) = nozzle angle, the angle subtended by the nozzle axis with the plane of rotation of the wheel (Fig. 7.10b), then the minimum area of nozzle open for steam flow,

\[
A_n = o \cdot h_n \cdot n \tag{7.39}
\]

Now,
\[
o = p \sin \alpha - t \quad \text{and} \quad n = \frac{\pi D_m}{p},
\]

\[
\therefore \quad A_n = (p \sin \alpha - t) h_n \cdot \frac{\pi D_m}{p} = \pi D_m h_n \sin \alpha \left(1 - \frac{t}{p \sin \alpha}\right)
\]

\[
= \pi D_m h_n \sin \alpha \times k_m \tag{7.40}
\]

where \( k_m \) is the nozzle thickness factor.

### 7.2.10 Nozzles Operating in the Off-Design Condition

Let us consider a convergent nozzle as shown in Fig. 7.11, which also shows the pressure ratio \( p/p_0 \) varying along the length of the nozzle. The inlet condition of the fluid is the stagnation state at \( p_0, T_0 \), which is assumed to be constant. The pressure at the exit plane of the nozzle is denoted by \( p_E \) and the back pressure is \( p_B \), which can be varied by a valve. As the back pressure \( p_B \) is varied, the mass flow rate \( \omega \) and the exit plane pressure \( p_E/p_0 \) also vary, as shown in Fig. 7.12.
Fig. 7.12  Mass flow rate and exit pressure as a function of back pressure in a convergent nozzle

When \( p_B/p_0 = 1 \), there is no flow and \( p_E/p_0 = 1 \), as designated by point 1. If the back pressure is now decreased to 2, such that \( p_B/p_0 > p^*/p_0 \), \( p_E = p_B \) and \( M_E < 1 \). Now, the back pressure is lowered to the critical pressure, point 3, when \( M_E = 1 \) and \( p_E = p_B \). When \( p_B \) is decreased below the critical pressure, point 4, there is no increase in the mass flow rate, and \( p_E = p^* \), \( M_E = 1 \). The drop in pressure from \( p_E \) to \( p_B \) occurs outside the nozzle exit. This is the choking limit which means that for given stagnation conditions the nozzle is passing the maximum possible mass flow.

Fig. 7.13  Pressure distribution in a convergent-divergent nozzle

Let us next consider a convergent–divergent nozzle, as shown in Fig. 7.13. Point 1 designates the condition when \( p_B = p_0 \) and there is no flow. When \( p_B \) is lowered to the pressure denoted by 2, so that \( p_B/p_0 < 1 \), but \( p_B/p_0 > p^*/p_0 \), the velocity increases in the convergent section, but \( M < 1 \) occurs at the throat. The divergent section acts as the subsonic diffuser in which pressure increases and velocity decreases. Point 3 indicates the back pressure at which \( M = 1 \) occurs at the throat but the diverging section acts as a subsonic diffuser. Point 4
indicates one other back pressure for which the flow is isentropic throughout and the diverging section acts as a supersonic nozzle with a continuous decrease in pressure and a continuous increase in velocity, and $p_{E4} = p_{B4}$. This condition of supersonic flow indicates the design pressure ratio of the nozzle. If the pressure ratio is lowered to 5, no further decrease in exit pressure occurs and the drop of pressure from $p_E$ to $p_B$ occurs outside the nozzle.

Between the back pressure between 3 and 4, flow is not isentropic in the diverging part, and it is accompanied by an irreversible phenomenon known as shocks. Shocks occur only when the flow is supersonic, and after the shock the flow becomes subsonic and the rest of the duct acts as a diffusor. Properties vary discontinuously across the shock. When the back pressure is as indicated by point $b$, the flow throughout the nozzle is isentropic, with pressure continuously decreasing the velocity increasing, but a shock appears just at the exist of the nozzle. When the back pressure is increased from $b$ to $a$, the shock moves upstream as indicted. When the back pressure is further increased, the shock moves upstream and disappears at the nozzle throat where the back pressure corresponds to 3. Since the flow throughout is subsonic, no shock is possible.

### 7.2.11 Supersaturated Flow

When a superheated vapour expands slowly and isentropically, condensation within the vapour begins to form when the saturated vapour line is reached. As the expansion continues below this line into the wet region, condensation proceeds gradually with the progressive decrease of quality ($x$) and increase in the degree of wetness. Point $B$ represents the point at which condensation within the vapour just begins (Fig. 7.14).

![Fig. 7.14 Slow and isentropic expansion of steam from the superheated into the two phase region](image)

However, when steam expands from superheated state to the two-phase wet region in a nozzle, the expansion occurs so rapidly that the vapour does not condense immediately as it crosses the dry saturated line, but somewhat later (at $x = 0.96$ to 0.97), when all the vapour suddenly condenses into liquid. Beyond the dry saturation line till the state when the vapour condenses, the flow is said to be supersaturated and the system is in metastable equilibrium, which means that it is stable to small disturbances but unstable to large disturbances. Wilson line ($x = 0.96$ to 0.97) is the locus of states below the dry saturation line where condensation within the vapour occurs at different pressures (Fig. 7.15).
**Fig. 7.15 Wilson line**

In Fig. 7.16 (a and b), line 0–1 represents isentropic expansion (slow) with equilibrium at every stage of the expansion path. Line 0–R represents supersaturated expansion in which the vapour expands as if the saturated vapour line does not exist. It can be seen that the temperature of the supersaturated vapour at $p_1$ is $t_R$, which is less than the saturation temperature $t_1$ corresponding the pressure $p_1$. The vapour is said to be supercooled, where the *degree of supercooling* = $t_1 - t_R$. Sometimes a *degree of supersaturation* is defined as the ratio of the actual pressure $p_1$ to the saturation pressure at $t_R$ (i.e. $p_1 / p_R$).

**Fig. 7.16** Supersaturated flow on (a) $p$–$v$, (b) $T$–$s$ and (c) $h$–$s$ diagrams
The exit velocity for isentropic flow is \( \bar{V}_1 = 44.72 \ (h_0 - h_1)^{1/2} \), whereas the exit velocity for supersaturated flow is \( \bar{V}_{1\text{sup}} = 44.72 \ (h_0 - h_R)^{1/2} \), which is less but not to a great extent since the difference in enthalpy drop is small.

If the expansion from 0 to B obeys the law \( p\nu^{1.3} \) = constant, and that from B to 1 obeys the law \( p\nu^{1.135} \) = constant, the isentropic and supersaturated expansion lines are shown on a \( p\nu \) diagram in Fig. 7.16 (a). It is seen that the specific volume at exit with supersaturated flow, \( \nu_R \), is considerably less than the specific volume at exit with isentropic flow, \( \nu_1 \). Now, the mass flow of steam through a given exit area \( A_1 \) is given by:

(a) For isentropic flow,
\[
\omega = \frac{A \bar{V}_1}{\nu_1}
\]

(b) For supersaturated flow,
\[
\omega_{\text{sup}} = \frac{A \bar{V}_{1\text{sup}}}{\nu_R}
\]

Since
\[
\bar{V}_1 = \bar{V}_{1\text{sup}} \quad \text{and} \quad \nu_1 > \nu_R,
\]
\[
\therefore \omega < \omega_{\text{sup}}
\]

The mass flow with supersaturated flow is greater than the mass flow with isentropic flow. This fact, proved experimentally, led to the discovery of the phenomenon of supersaturation.

### 7.3 TURBINE BLADING

Depending upon the types of blades used and the method of energy transfer from the fluid to the rotor wheel, the turbines may be of two types:

(a) Impulse turbines

(b) Reaction turbines

### 7.3.1 Impulse Turbines

In impulse turbines, all pressure drops of steam occur in the nozzles and there is no pressure drop as steam flows through the passage between two blades. In Fig. 7.17(a), steam enters the nozzles at pressure \( p_0 \) with velocity \( \bar{V}_0 \), undergoes expansion to pressure \( p_1 \) with velocity increased to \( \bar{V}_1 \). High velocity jets of steam impinge upon the blades with velocity \( \bar{V}_1 \), get deflected by an angle, and come out at a lower velocity \( \bar{V}_2 \) impressing a torque on the blades. The pressure of steam \( p_1 \) remains essentially constant as steam flows through the blades.

From the principle of conservation of momentum, Momentum of steam jets at inlet to the blades – momentum of jets at exit from the blades (both resolved in the direction of motion of the wheel) = momentum (angular) absorbed by the wheel in producing shaft work.

In Figs 7.17 (b) to 7.17 (d), it is seen that the wheel rotates only due to the impulsive effect of the jets (i.e., the difference of momenta of the jets deflected by the blades). The blades of such a wheel are called impulse blades.
Fig. 7.17  Flow through impulse blades
The mean peripheral velocity of the blades, also called the mean blade velocity, \( \bar{V}_b \), is given by

\[
\bar{V}_b = \frac{\pi D_m N}{60}
\]

where \( D_m \) is the mean diameter of the wheel and \( N \) is its rpm. The area of flow or blade annulus (Fig. 7.17(e)), \( A_b \), is given to be

\[
A_b = \frac{\pi}{4} \left( D_2^2 - D_1^2 \right) = \pi \frac{D_2 + D_1}{2} \frac{D_2 - D_1}{2} = \pi D_m h_b
\]

where \( D_1 \) is the root diameter, \( D_2 \) is the tip diameter, and \( h_b \) is the height of the blades.

1. **Velocity diagrams, diagram work and diagram efficiency** Steam coming out from the nozzles at absolute velocity \( V_1 \) strikes the blades with relative velocity \( V_{r_1} \) while the blades rotate with mean peripheral velocity \( \bar{V}_b \). Steam leaves the blades with relative velocity \( V_{r_2} \) while its absolute velocity is \( V_2 \). As shown in Fig. 7.18(a), \( \alpha \) is the nozzle angle subtended by the nozzle axis with the direction of rotation of the wheel, \( \beta_1 \) is the inlet blade angle and \( \beta_2 \) is the exit blade angle. The inlet and exit velocity triangles of Fig. 7.18(a) have been superposed on a common \( \bar{V}_b \) in Fig. 7.18(b). If all the angles are measured clockwise, then \( \gamma \) is the exit blade angle (\( \gamma = 180 - \beta_2 \)) and \( \delta \) is the angle made by absolute exit velocity of steam leaving the blades with the plane of rotation of the wheel.

![Fig. 7.18 Velocity diagrams for impulse blading](image)

It is the difference of the cosine components of the velocities of steam which drives the wheel and produces the torque. It is called the change in the velocity of whirl, \( \Delta \bar{V}_\omega \), which is given by
\( \Delta \bar{V}_\omega = \text{velocity of whirl at inlet-velocity of whirl at outlet} \)
\[
= \bar{V}_{\omega_1} - \bar{V}_{\omega_2} = \bar{V}_1 \cos \alpha - \bar{V}_2 \cos \delta
\]  
(7.42)

If \( \delta > 90^\circ \), \( \bar{V}_{\omega_1} \) and \( \bar{V}_{\omega_2} \) become additive in estimating \( \Delta \bar{V}_\omega \) (Fig. 7.18(b)). If \( \delta < 90^\circ \), \( \bar{V}_{\omega_2} \) is to be subtracted from \( \bar{V}_{\omega_1} \) to determine \( \Delta \bar{V}_\omega \) (Fig. 7.18(c)).

From the inlet velocity triangle \( ABC \) (Fig. 7.18 (b)).
\[
\bar{V}_1 \cos \alpha - \bar{V}_b = \bar{V}_1 \cos \beta_1
\]
\[
\bar{V}_1 \sin \alpha = \bar{V}_1 \sin \beta_1
\]

Therefore, on division,
\[
\tan \beta_1 = \frac{\bar{V}_1 \sin \alpha}{\bar{V}_1 \cos \alpha - \bar{V}_b}
\]  
(7.43)

From the exit velocity triangle \( DBC \) (Fig. 7.18 (b))
\[
\bar{V}_2 \cos (180 - \delta) + \bar{V}_b = \bar{V}_{f_2} \cos (180 - \gamma)
\]
\[
\bar{V}_b - \bar{V}_2 \cos \delta = - \bar{V}_{f_2} \cos \gamma
\]
\[
\therefore \bar{V}_2 \cos \delta = \bar{V}_b + \bar{V}_{f_2} \cos \gamma
\]  
(7.44)

The same result can also be obtained from Fig. (7.18 (c)).
\[
\bar{V}_b - \bar{V}_2 \cos \delta = \bar{V}_{f_2} \cos (180 - \gamma) = - \bar{V}_{f_2} \cos \gamma
\]
\[
\bar{V}_2 \cos \delta = \bar{V}_b + \bar{V}_{f_2} \cos \gamma
\]  
(7.45)

The ratio of relative velocity at exit to that at inlet is called “the blade friction factor”, \( k_b \),
\[
k_b = \frac{\bar{V}_{f_2}}{\bar{V}_1}
\]  
(7.46)

The energy loss due to friction in the blades is equal to
\[
\frac{\bar{V}_{f_2}^2 - \bar{V}_{f_1}^2}{2}
\]

Now, from Eqs (7.42) and (7.45)
\[
\Delta \bar{V}_\omega = \bar{V}_1 \cos \alpha - \bar{V}_2 \cos \delta = \bar{V}_1 \cos \alpha - \bar{V}_b - k_b \bar{V}_1 \cos \gamma
\]
\[
= \bar{V}_1 \cos \alpha - \bar{V}_b - k_b \frac{\bar{V}_1 \cos \alpha - \bar{V}_b}{\cos \beta_1} \cos \gamma
\]
\[
= \left( \bar{V}_1 \cos \alpha - \bar{V}_b \right) \left[ 1 - k_b \frac{\cos \gamma}{\cos \beta_1} \right]
\]  
(7.47)
Since 
\[ \gamma = 180 - \beta_1; \cos \gamma = -\cos \beta_1, \]
\[ \Delta V_\omega = (\bar{V}_1 \cos \alpha - \bar{V}_b) (1 + k_b) \]

Also,
\[ \Delta V_\omega = V_{r_1} \cos \beta_1 + V_{r_2} \cos \beta_2 = V_{r_1} \cos \beta_1 + k_b V_{r_1} \cos \beta_1 \]
\[ = V_{r_1} \cos \beta_1 (1 + k_b) = (V_1 \cos \alpha - V_b) (1 + k_b) \] (7.48)

Blades are said to be symmetrical if blade angles are equal, i.e. if \( \beta_1 = \beta_2 \). Impulse turbines mostly have symmetrical blades.

The tangential thrust impressed by the jets on the blades, \( P_t \), is
\[ P_t = \omega_s \cdot \Delta V_\omega \] (7.49)

where \( \omega_s \) is the steam flow rate.

The axial thrust, i.e. the thrust produced by the difference in the axial components of the velocities (along the axis of the shaft),
\[ P_a = \omega_s \cdot \Delta \bar{V}_a \] (7.50)

where 
\[ \Delta \bar{V}_a = V_{r_1} \sin \beta_1 - V_{r_2} \sin \beta_2 \] (7.51)

The rate at which work is done by the jets on the blades is called the blading work or diagram work, \( W_D \), which is given by
\[ W_D = P_t \times \bar{V}_b = \omega_s \cdot \Delta V_\omega \bar{V}_b \] (7.52)

The energy input to the blades is the kinetic energy of jets issuing out from the nozzles = \( \omega_s \cdot \left( \bar{V}_1^2 / 2 \right) \).

Therefore, the blading or diagram efficiency 
\[ \eta_b = \frac{\text{rate of work done on the blades}}{\text{rate of energy input to the blades}} = \frac{\omega_s \Delta V_\omega \bar{V}_b}{1/2 \omega_s \bar{V}_1^2} \]

or
\[ \eta_b = \frac{2 \Delta V_\omega \bar{V}_b}{\bar{V}_1^2} \] (7.53)

It is the fraction of kinetic energy of jets of steam issuing from nozzles which is converted to shaft work.

2. Optimum velocity ratio Substituting \( \Delta V_\omega \) from Eq. (7.48) in Eq. (7.53), the diagram efficiency is given by
\[ \eta_b = \frac{2 \left( \bar{V}_1 \cos \alpha - \bar{V}_b \right) (1 + k_b) \bar{V}_b}{\bar{V}_1^2} = \frac{2 \bar{V}_b^2 \left( \frac{\bar{V}_1 \cos \alpha}{\bar{V}_b} - 1 \right) (1 + k_b)}{\bar{V}_1^2} \]

Defining the velocity ratio \( \rho \) as \( \bar{V}_b / \bar{V}_1 \), the ratio of mean blade velocity \( V_b \) to the jet velocity \( \bar{V}_1 \),
\[ \eta_D = 2 \rho^2 \left( \frac{\cos \alpha}{\rho} - 1 \right) (1 + k_b) = 2(\rho \cos \alpha - \rho^2) (1 + k_b) \quad (7.54) \]

There is a certain value of \( \rho \) which makes \( \eta_D \) the maximum. Differentiating \( \eta_D \) with respect to \( \rho \) and making it equal to zero.

\[ \frac{d\eta_D}{d\rho} = 2(\cos \alpha - 2\rho) (1 + k_b) = 0 \]

Therefore, the optimum velocity ratio for impulse blading is

\[ \rho_{\text{opt}} = \frac{\cos \alpha}{2} \quad (7.55) \]

Substituting this optimum value of \( \rho \) in Eq. (7.54), the maximum blading efficiency is

\[ (\eta_D)_{\text{max}} = 2 \left[ \frac{\cos^2 \alpha}{2} - \frac{\cos^2 \alpha}{4} \right] (1 + k_b) = \frac{1 + k_b}{2} \cos^2 \alpha \quad (7.56) \]

If the energy loss due to friction in blades is small, \( k_b \approx 1 \), then

\[ (\eta_D)_{\text{max}} = \cos^2 \alpha \quad (7.57) \]

where \( \alpha \) is the nozzle angle. The lower is the nozzle angle, higher is the blading efficiency. However, too low a nozzle angle may cause energy loss at blade inlet. Therefore, the nozzle angle has to be maintained within a certain range, which varies from 16° to 22°.

### 3. Graphical method

The diagram work and the diagram efficiency under a certain operating condition can also be estimated graphically by drawing to scale the inlet and exit velocity triangles. If the state of steam at inlet to the nozzles, the exit pressure and nozzle efficiency are known, then

\[ \bar{V}_1 = 44.72 \left[ (h_0 - h_{1s}) \eta_n \right]^{1/2} \]

can be estimated. Knowing \( D_m \) and \( N \), \( \bar{V}_b \) is found out from

\[ \bar{V}_b = \frac{\pi D_m N}{60} \]

With \( \alpha \) and \( k_b \) being given and with the blades being symmetrical (\( \beta_1 = \beta_2 \)), a scale for the diagram is chosen, say, 1 cm = 50 m/s. A horizontal line \( AB \) is drawn, the length of which is proportional to \( \bar{V}_b \) (Fig. 7.19). A straight line is drawn from \( B \) making an angle \( \alpha \) and a length \( BC \) proportional to \( \bar{V}_1 \) is cut off. \( CA \) is joined. The angle \( \angle CAD \) is measured. This is the inlet blade angle \( \beta_1 \). \( CA \) is measured, which represents \( \bar{V}_1 \). Again, a straight line \( AE \) is drawn from \( A \) making an angle \( \beta_1 (= \beta_2) \) and a length \( AE \) is cut off proportional to \( \bar{V}_2 (= k_b \bar{V}_b) \). \( EB \) is joined. The velocity diagram is thus completed. From this diagram, \( \Delta \bar{V}_a \), \( \bar{V}_{a_1} \) and \( \bar{V}_{a_2} \) are measured. Then the desired particulars,
Fig. 7.19 Construction of velocity diagram

\[ P_t = \omega_1 \Delta V_\omega ; \quad P_a = \omega_2 (\vec{V}_{a_1} - \vec{V}_{a_2}) ; \quad W_D = P_t \vec{V}_b ; \quad \eta_D = (2 \Delta V_\omega \vec{V}_b) / \vec{V}_{c_1}^2 \] are estimated.

4. **Alternative way of drawing velocity diagrams** The velocity diagrams with symmetrical impulse blading can also be drawn in a different way as shown in Fig. 7.20. The inlet velocity triangle ABC is drawn first as before. A length CD equal to \((k_b \vec{V}_{r_2})\) is cut off from C to represent \(\vec{V}_{r_2}\) since \(\beta_1 = \beta_2\). ED is drawn from D, the length being proportional to \(\vec{V}_b\). CE is joined. The absolute exit velocity \(\vec{V}_2\) is the vector sum of \(\vec{V}_b\) and \(\vec{V}_{r_2}\), as shown.

Then, \(\vec{V}_{\omega_h}, \vec{V}_{\omega_b}, \vec{V}_{a_1}\) and \(\vec{V}_{a_2}\) are measured from the diagram to estimate the different output parameters.

If the blade friction factor \(k_b\) is unity, i.e., there is no energy loss due to friction as steam flows through the blades, and the discharge is axial, i.e., parallel to the axis of the shaft, for which \(\delta = 90^\circ\), the resultant velocity diagram becomes as shown in Fig. 7.21. The horizontal line AB (= 2 \(\vec{V}_b\)) is drawn such that \(AC = CB = \vec{V}_b\). From A, a perpendicular is drawn to AB and from B, a straight line is drawn making an angle \(\alpha\), which cuts the perpendicular at D. Then \(\Delta DCA\) is the exit velocity triangle.

5. **Compounding of steam turbines** One row of nozzles followed by one row of blades is called a stage of a turbine. If steam is allowed to expand from boiler condition down to condenser vacuum in a single row of nozzles, due to a large enthalpy drop, the velocity at exit from the nozzles \((\vec{V}_1)\) is very large (Fig. 7.22). Since \(\vec{V}_b / \vec{V}_1 = \cos \alpha / 2\), the blade velocity \((\vec{V}_b)\) also becomes very large. Again \(\vec{V}_b\) is equal to \(\pi D_m N / 60\). Therefore, if \(D_m\) is fixed at some economic value, \(N\) becomes very large. Thus single-stage impulse turbines, also called de Laval turbines, have inherently very high rotative speeds \((N)\).
Fig. 7.20  Velocity diagram with symmetrical impulse blading

Fig. 7.21  Velocity diagram for frictionless symmetrical impulse blading with axial discharge.

Fig. 7.22  Single stage expansion of steam

Such high rotational speeds cannot be properly utilized. It entails large friction losses. The centrifugal stresses also become very large. Alternatively, if
N is fixed, the diameter of the wheel becomes too large. Moreover, with a single stage, the velocity of steam at exit is sufficiently high and there is a considerable loss of kinetic energy with exiting steam. To obviate these difficulties the turbines are compounded or staged, where steam instead of expanding in a single stage is made to expand in a number of stages, whereby the turbine speed is reduced while securing the same enthalpy drop of steam.

Basically, there are two ways of compounding steam turbines:

1. Pressure compounding or Rateau staging
2. Velocity compounding or Curtis staging

6. **Pressure compounding or Rateau staging** The pressure compounding or Rateau staging corresponds to putting a number of simple impulse stages in series (Fig. 7.23). The total enthalpy drop is divided equally among the stages (Fig. 7.24). The pressure drops only in the nozzles. There is no pressure drop (theoretically) while steam flows through the blades. The kinetic energy of steam increases in the nozzles at the expense of the pressure drop and it is absorbed (partially) by the blades in each stage, in producing torque. In Fig. 7.24, the total isentropic enthalpy drop of steam \((h_0 - h_4)\) is divided equally among the four stages of the turbine. In Mollier diagram, the enthalpy drop \((h_0 - h_4)\) is measured, then the enthalpy drop, \(h_0 - h_1 = h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = (h_0 - h_4)/4\) are computed and inserted on the isentropic line. The interstage pressures noted from the diagram are \(p_1\), \(p_2\) and \(p_3\), i.e. the pressure after first stage is \(p_1\), the pressure after second stage is \(p_2\) and so on.

![Diagram](image)

**Fig. 7.23 (a)** Pressure-compounded impulse turbine
Fig. 7.23 (b) Three pressure (or Rateau) stages in series

Fig. 7.24 Enthalpy drop per stage in a 4-stage turbine

The velocity of steam at exit from the first row of nozzles is

\[ \bar{V}_1 = 44.72 \left( h_0 - h_1 \right)^{1/2} = 44.72 \left[ \frac{(h_0 - h_4)}{4} \right]^{1/2} \]

or,

\[ \bar{V}_1 = \frac{1}{2} \left( 44.72 \right) \left[ \frac{(h_0 - h_4)}{4} \right]^{1/2} \]  \hspace{1cm} (7.58)

If the KE of steam at inlet to each subsequent row of nozzles is small and neglected, then the velocity of steam leaving the nozzles in each row will be the same as given in Eq. (7.58). For a single-stage turbine the jet velocity would have been
\[ \bar{V}_1 = 44.72 \left[ h_0 - h_4 \right]^{1/2} \]  

(7.59)

Therefore, for a 4-stage turbine, the velocity of steam leaving the nozzles in each stage is half of that for a single-stage turbine. For a 9-stage turbine, it will be one-third.

For each impulse stage operating at its maximum blading efficiency, the blade velocity is given by

\[ \bar{V}_b = \frac{\cos \alpha}{2} \bar{V}_1 \]  

(7.60)

Therefore, along with \( \bar{V}_1, \bar{V}_b \) also gets halved for a 4-stage turbine, and for a given \( D_m \), \( N \) gets halved or for a given \( N, D_m \) gets halved. With more stages, \( N \) or \( D_m \) would further decrease.

If there are \( n \) stages in series in the turbine, the isentropic enthalpy drop per stage would be

\[ (\Delta h)_{\text{stage}} = \frac{\Delta h_{\text{total}}}{n} \]  

(7.61)

From Eq. (7.60), and for an impulse stage,

\[ \bar{V}_1 = \frac{2}{\cos \alpha} \bar{V}_b = 44.72 \left[ \Delta h_{\text{stage}} \right]^{1/2} \]

\[ \therefore (\Delta h)_{\text{stage}} = \left[ \frac{2 \bar{V}_b}{44.72 \cos \alpha} \right]^2 = 4 \left[ \frac{\bar{V}_b}{44.72 \cos \alpha} \right]^2 \]  

(7.62)

From Eqs (7.61) and (7.62), the number of impulse stages required for a certain enthalpy drop \((\Delta h)_{\text{total}}\) can thus be estimated under ideal condition, i.e.,

\[ n = \frac{(\Delta h_s)_{\text{total}}}{(\Delta h_s)_{\text{stage}}} \]

### 7. Velocity compounding or Curtis staging

In velocity compounding or Curtis staging, all the pressure drop and hence, enthalpy drop of steam take place in a single row of nozzles and the resultant kinetic energy of steam is absorbed by the wheel in a number of rows of moving blades with guide blades in between two such rows. As shown in Fig. 7.25, steam expands in the single row of nozzles from \( p_0 \) to \( p_1 \) with velocity increasing from \( \bar{V}_0 \) to \( \bar{V}_1 \). The pressure \( p_1 \) remains essentially constant thereafter. The KE of steam jets \( \left( \frac{1}{2} \omega_s \bar{p}_1^2 \right) \) at nozzle exit is partially converted to shaft work in the first row of moving blades with velocity decreasing from \( \bar{V}_1 \) to \( \bar{V}_2 \). The exiting steam jets are then deflected by the stationary guide blades to the next row of moving blades where part of the remaining kinetic energy \( \left( \frac{1}{2} \omega_s \bar{p}_2^2 \right) \) is converted to shaft work. It is a two-row Curtis or velocity stage having two rows of moving blades with one row of guide blades in between. In a three row Curtis stage, the two-row stage is followed by a second row of guide blades and then a third row
or moving blades, so that the energy conversion from fluid to rotor takes place in three stages. Steam with high KE exiting the nozzles works on the first row of moving blades, gets deflected by the first row of guide blades, works on the second row of moving blades, again gets deflected by the second row of guide blades and finally does work on the third row of moving blades.

The velocity diagrams for the first row of moving blades and the second row of moving blades are shown respectively in (a) and (b) of Fig. 7.26. The blade friction factor $k_b$ may be assumed the same for both moving and guide blades.

Here,

$$\frac{\bar{V}_1}{\bar{V}_2} = \frac{\bar{V}_3}{\bar{V}_4} = \frac{\bar{V}_5}{\bar{V}_6} = k_b$$

$\alpha_1$ = exit angle of the guide blades
$\beta_1, \beta_2$ = inlet and exit angles of the first row of moving blades
$\beta_3, \beta_4$ = inlet and exit angles of the second row of moving blades

$\Delta \bar{V}_{o_1}, \Delta \bar{V}_{o_2}$ = changes in the velocity of whirl in the first and second rows of moving blades

$\Delta V_{a_1}, \Delta V_{a_2}$ = changes in the axial components of velocity in the first and second rows of moving blades.

*Fig. 7.25 (a) A two-row Curtis stage, (b) Pressure and velocity variation in a two-row Curtis stage*
**Fig. 7.25** (c) Nozzles and blades in a two-row Curtis stage

**Fig. 7.26** Velocity diagrams for a 2-row Curtis stage: (a) first row of moving blades, (b) second row of moving blades

The tangential thrust,

\[ P_t = \omega_s \sum \Delta V_{\omega_1} = \omega_s \left( \Delta V_{\omega_1} + \Delta V_{\omega_2} \right) \]

The axial thrust,

\[ P_a = \omega_s \sum \Delta V_a = \omega_s \left( \Delta V_{a_1} + \Delta V_{a_2} \right) \]

Blading or diagram work, \( W_D = P_t \times \bar{V}_b \)

Blading or diagram efficiency,

\[ \eta_D = \frac{2 \sum \Delta V_{\omega} \bar{V}_b}{\bar{V}_1^2} \]

These parameters can be estimated either graphically by drawing to scale the velocity diagrams or trigonometrically.
For symmetrical blades \((\beta_1 = \beta_2 \text{ and } \beta_3 = \beta_4)\), the velocity triangles can also be drawn as shown in Fig. 7.27.

![Fig. 7.27 Two-row Curtis stage with symmetrical blades: (a) first row of moving blades, (b) second row of moving blades](image)

8. **Effectiveness of moving rows in a Curtis stage** Let us consider a two-row Curtis stage. Let us assume for simplicity symmetrical blading \((\beta_1 = \beta_2, \beta_3 = \beta_4)\), the blade friction factor as unity and axial discharge \((\delta = 90^\circ)\). The velocity diagrams of the first and second rows of moving blades can be combined into one diagram (Fig. 7.28). The horizontal line \(AB(= 4\bar{V}_b)\) is drawn such that \(AC = CD = DE = EB (= \bar{V}_b)\). At \(A\), a perpendicular is drawn, and from \(B\) a line is drawn making an angle \(\alpha\) with \(AB\), which cuts the perpendicular at point \(F\). \(FC, FD\) and \(FE\) are joined. The directions of velocities are shown in the diagram.

![Fig. 7.28 Velocity diagram of a two-row Curtis stage having frictionless symmetrical blading with axial discharge of steam](image)

Since, \(\delta_1 > 90^\circ\), \(\Delta \bar{V}_{\omega b} = \bar{V}_1 \cos \alpha + \bar{V}_2 \cos \alpha_1\)

\[= 4\bar{V}_b + 2\bar{V}_b = 6\bar{V}_b\]

Now, \(\Delta \bar{V}_{\omega_2} = \bar{V}_3 \cos \alpha_1 - \bar{V}_4 \cos \delta = 2\bar{V}_b\)

Work done by the steam jets on the first row of moving blades,

\[W_{D_1} = \omega_s \Delta \bar{V}_{\omega_1} \bar{V}_b = \omega_s 6 \bar{V}_b \bar{V}_b = 6 \omega_s \bar{V}_b^2\]

Work done by the steam jets on the second row of moving blades,

\[W_{D_2} = \omega_s \Delta \bar{V}_{\omega_2} \bar{V}_b = \omega_s 2 \bar{V}_b \bar{V}_b = 2 \omega_s \bar{V}_b^2\]
Thus
\[ W_{D_1} : W_{D_2} = 6 : 2 = 3 : 1 \quad (7.63) \]
It means that three-fourth of the total work is done by the steam jets on the first row of moving blades and one-fourth of the total work is done on the second row of moving blades.

In a 3-row Curtis stage, it can similarly be shown that
\[ W_{D_1} : W_{D_2} : W_{D_3} = 5 : 3 : 1 \quad (7.64) \]
Only one-ninth of the total work in the stage is done in the 3rd row of moving blades. Therefore, as the number of rows of moving blades in a Curtis stage increases, the effectiveness of the later rows decreases. In conventional designs only two rows of moving blades are used in a Curtis stage. It is not justifiable to use more than two rows.

9. **Optimum velocity ratio for a Curtis stage** Let us again consider a two-row Curtis stage having symmetrical blading but with discharge not being axial. The velocity diagram is shown in Fig. 7.29.

![Velocity diagram of a two-row Curtis stage with frictionless symmetrical blading and non-axial discharge](image)

Fig. 7.29 Velocity diagram of a two-row Curtis stage with frictionless symmetrical blading and non-axial discharge

\[
\Delta \bar{V}_{\omega_i} = \bar{V}_1 \cos \alpha + \bar{V}_2 \cos \alpha_1 = \bar{V}_1 \cos \alpha + \bar{V}_1 \cos \alpha - 2 \bar{V}_b \\
= 2(\bar{V}_1 \cos \alpha - \bar{V}_b)
\]
\[
\Delta \bar{V}_{\omega_2} = \bar{V}_3 \cos \alpha_1 + \bar{V}_4 \cos \alpha_2 \\
= \bar{V}_1 \cos \alpha - 2 \bar{V}_b + \bar{V}_1 \cos \alpha - 4 \bar{V}_b \\
= 2(\bar{V}_1 \cos \alpha - 3 \bar{V}_b)
\]

\[
\therefore \quad \Sigma \Delta \bar{V}_{\omega} = \Delta \bar{V}_{\omega_1} + \Delta \bar{V}_{\omega_2} = 4(\bar{V}_1 \cos \alpha - 2 \bar{V}_b)
\]

The rate of energy transfer from fluid to rotor,
\[ W_D = \omega_s \Delta \bar{V}_{\omega} \bar{V}_b \]
and the diagram efficiency (fraction of fluid energy converted to shaft work),
\[ \eta_D = \frac{2 \Sigma \Delta \bar{V}_{\omega} \bar{V}_b}{\bar{V}_1^2} = \frac{2 \times 4(\bar{V}_1 \cos \alpha - 2 \bar{V}_b) \bar{V}_b}{\bar{V}_1^2} = \frac{8 \bar{V}_b^2 \left( \frac{\bar{V}_1 \cos \alpha}{\bar{V}_b} - 2 \right)}{\bar{V}_1^2} \]
\[ 8 \rho^2 \left( \frac{\cos \alpha}{\rho} - 2 \right) = 8(\rho \cos \alpha - 2 \rho^2) \]

\[ \frac{d\eta_p}{d\rho} = 8(\cos \alpha - 4 \rho) = 0 \]

\[ \therefore \rho_{opt} = \frac{\cos \alpha}{4} \quad (7.65) \]

For a three-row Curtis stage, it can similarly be shown that

\[ \rho_{opt} = \frac{\cos \alpha}{6} \]

Therefore, for a Curtis stage having \( Z \)-rows of moving blades

\[ \rho_{opt} = \frac{\cos \alpha}{2Z} \quad (7.66) \]

If \( Z = 1 \), i.e. for a simple impulse stage, \( \rho_{opt} = \frac{\cos \alpha}{2} \) as obtained earlier.

For a two-row Curtis stage, \( Z = 2 \),

\[ \rho_{opt} = \frac{\bar{V}_b}{\bar{V}_1} = \frac{\cos \alpha}{4} \quad \therefore \bar{V}_b = \frac{1}{2} \left( \frac{\cos \alpha}{2} \bar{V}_1 \right) \]

Therefore, in a 2-row Curtis stage, the blade velocity is half of the value for a single-stage turbine. For a certain \( D_m \), \( N \) gets halved. Substituting in Eq. (7.64).

\[ (\eta_D)_{max} = 8 \left( \frac{\cos^2 \alpha}{4} - \frac{\cos^2 \alpha}{8} \right) = \cos^2 \alpha \quad (7.67) \]

This holds good for the conditions as specified before, i.e. there is no friction and blades are symmetrical, and the discharge is axial. The same efficiency (i.e. \( \cos^2 \alpha \)) was also obtained for symmetrical and frictionless simple impulse blading (Eq. (7.57)). However, had friction been taken into account the blading efficiency of the Curtis stage would have been considerably lower than the efficiency of the simple impulse blading.

\[ \bar{V}_1 = \frac{4 \bar{V}_b}{\cos \alpha} = 44.72 \left[ (\Delta h_s)_{Curtis} \right]^{1/2} \]

\[ \therefore (\Delta h_s)_{Curtis} = 16 \left[ \frac{\bar{V}_b}{44.72 \cos \alpha} \right]^2 \quad (7.68) \]

This is the isentropic enthalpy drop in the 2-row Curtis stage.

### 7.3.2 Reaction Turbines

In these turbines, pressure drop occurs both in the nozzles or the fixed row of blades, as well as in the moving row of blades (Fig. 7.30), since the moving
Fig. 7.30 Variation of pressure and velocity in (a) two reaction stages, (b) in a two-row Curtis stage followed by two reaction stages
blade channels are also of the nozzle shape. Due to the expansion of steam while flowing through the blades, there is an increase in kinetic energy, which gives rise to reaction in the opposite direction (by Newton’s third law of motion). Blades rotate due to both the impulse effect of the jets (due to change in their momentum) and the reaction force of the exiting jets impressed on the blades in the opposite direction. Such turbines are called impulse-reaction turbines, or to distinguish them from impulse turbines, simply reaction turbines. The degree of reaction \( R \) of these turbines is defined as

\[
R = \frac{\Delta h_{mb}}{\Delta h_{fb} + \Delta h_{mb}} \quad (7.69)
\]

where the subscripts “mb” and “fb” represent moving blades and fixed blades, respectively.

If \( (\Delta h)_{mb} = 0 \), \( R = 0 \), which is the case of pure impulse turbines where there is no enthalpy drop of steam in the moving blades, and all the enthalpy drop of the stage take place only in nozzles.

If \( \Delta h_{fb} = 0 \), \( R = 1 \), which is the case of a pure reaction \( (R = 100\%) \) turbine, e.g., Hero’s turbine.

If equal enthalpy drops occur in the fixed and moving blades, i.e., \( \Delta h_{fb} = \Delta h_{mb} = (\Delta h_{stage})/2 \), \( R = 1/2 \) or 50%. Sometimes, 50% reaction turbines are also called Parsons turbines.

The velocity diagrams for the moving blades of a 50% reaction turbine are shown in Fig. 7.31. With a simple impulse type the value of \( V_{r2} (= k_b V_{r1}) \) would be given by \( BE \), but in the reaction turbine this velocity is increased to \( BC \) by further expansion of the steam in the nozzle-shape blade channels.

\[ \text{Fig. 7.31 Velocity diagram of a 50\% reaction turbine} \]

For manufacturing advantage, both fixed blades and moving blades are made similar in shape so that they can be extruded from the same set of dies.

Since \( \Delta h_{fb} = \Delta h_{mb} \), \( V_1 = V_{r2} \)

Again, for similar geometry, \( \alpha = \beta_2 = 180^\circ - \gamma \)

The triangle \( ABD \) and \( DBC \) are similar.
\[ V_{r_1} = V_2 \quad \text{and} \quad \beta_1 = 180^\circ - \delta \]

Since \( \beta_1 \neq \beta_2 \), the blades are unsymmetrical. Again, since \( \Delta V_a = 0 \), there is no axial thrust imposed on the blades due to change in axial velocity in a 50% reaction turbine. However, there will be considerable axial thrust produced due to the pressure difference across the blades in each rotor disc since there is pressure drop of steam across the moving blades.

\[ \Delta V_\omega = V_1 \cos \alpha - V_2 \cos \delta = V_1 \cos \alpha + V_{r_2} \cos \beta_2 - V_b \quad (\because \delta > 90^\circ) \]

\[ = 2V_1 \cos \alpha - V_b \]

Also,

\[ \Delta V_\omega = V_{r_1} \cos \beta_1 + V_{r_2} \cos \beta_2 = V_1 \cos \alpha - V_b + V_1 \cos \alpha \]

\[ = 2V_1 \cos \alpha - V_b \quad (7.70) \]

The diagram work per kg of steam,

\[ W_D = \Delta V_\omega V_b = (2V_1 \cos \alpha - V_b)V_b \quad (7.71) \]

Energy input to blades per kg of steam

\[ = \frac{V_1^2 + V_{r_1}^2 - V_2^2}{2} + \frac{V_{r_1}^2}{2} = \frac{V_1^2}{2} - \frac{V_b^2}{2} \]

Now,

\[ V_{r_1}^2 = V_1^2 + V_b^2 - 2V_1 V_b \cos \alpha \]

\[ \therefore \quad \text{Energy input} = V_1^2 - \frac{V_1^2 + V_b^2 - 2V_1 V_b \cos \alpha}{2} \]

\[ = \frac{V_1^2 - V_b^2 + 2V_1 V_b \cos \alpha}{2} \quad (7.72) \]

Diagram efficiency of the blades,

\[ \eta_D = \eta_b_1 = \frac{2(2V_1 \cos \alpha - V_b)V_b}{V_1^2 - V_b^2 + 2V_1 V_b \cos \alpha} = \frac{2V_b^2 \left[ \frac{2V_1 \cos \alpha}{V_b} - 1 \right]}{V_1^2 \left[ 1 - \left( \frac{V_b^2}{V_1^2} \right) + 2 \left( \frac{V_b}{V_1} \right) \cos \alpha \right]} \]

Putting \( V_b/V_1 = \rho \), the velocity ratio,

\[ \eta_D = \frac{2 \rho^2 \left[ \frac{2 \cos \alpha}{\rho} - 1 \right]}{1 - \rho^2 + 2 \rho \cos \alpha} = \frac{2(2 \rho \cos \alpha - \rho^2)}{1 - \rho^2 + 2 \rho \cos \alpha} \quad (7.73) \]

There is a particular value of \( \rho \) for which \( \eta_D \) is a maximum. Differentiating \( \eta_D \) with respect to \( \rho \) and equating it zero,

\[ \frac{d\eta_D}{d\rho} = \frac{(1 - \rho^2 + 2 \rho \cos \alpha)(2 \cos \alpha - 2 \rho) - 2 \rho (2 \cos \alpha - \rho) (-2 \rho + 2 \cos \alpha)}{(1 - \rho^2 + 2 \rho \cos \alpha)^2} = 0 \]

\[ 4(1 - \rho^2 + 2 \rho \cos \alpha)(\cos \alpha - \rho) - 4 \rho (2 \cos \alpha - \rho)(\cos \rho - \rho) = 0 \]

\[ 4(\cos \alpha - \rho)(1 - \rho^2 + 2 \rho \cos \alpha - 2 \rho \cos \alpha + \rho^2) = 0 \]
\[ \rho_{opt} = \cos \alpha \]  
\[ V_b = V_1 \cos \alpha \]  

(7.74)

or

Substituting the value of \( \rho_{opt} \) in Eq. (7.73),

\[ (\eta_D)_{max} = \frac{2 \cos^2 \alpha}{1 + \cos^2 \alpha} \]  

(7.75)

From Eqs (7.71) and (7.74), the specific blading work corresponding to maximum blading efficiency,

\[ W_D = (2V_b - V_b)V_b = V_b^2 \]  

(7.76)

The velocity diagrams for a 50% reaction turbine operating with maximum blading efficiency are shown in Fig. 7.32.

**Fig. 7.32** Velocity diagram of a 50% reaction turbine operating with maximum blading efficiency

\[ \Delta V_\omega = V_1 \cos \alpha = V_b \]

\[ W_D = \Delta V_\omega V_b = V_b^2 \]

It is no longer convenient to talk of nozzles and blades, since in the reaction turbine both fixed and moving blades act as nozzles. It is usual to refer to the two sets of blades as the stator or fixed blades and the rotor or moving blades.

Since the isentropic enthalpy drop of the stage is equally divided between the fixed blades and moving blades (Fig. 7.33),

**Fig. 7.33** Enthalpy drop in a 50% reaction stage
\[(\Delta h_s)_{fb} = h_1 - h_{2s} = \frac{V_1^2 - V_2^2}{2}\]

\[(\Delta h_s)_{mb} = h_{2s} - h_{3s} = \frac{V_{r_2}^2 - V_1^2}{2} = \frac{V_1^2 - V_2^2}{2}\]

\[\therefore (\Delta h_s)_{fb} = (\Delta h_s)_{mb}\]

\[(\Delta h_s)_{stage} = h_1 - h_{3s} = 2(h_{2s} - h_{3s}) = 2(\Delta h_s)_{mb}\]

\[\therefore R = \frac{h_{2s} - h_{3s}}{h_1 - h_{3s}} = \frac{\Delta h_{mb}}{2(\Delta h_{mb})} = \frac{1}{2}\]

1. **Carry-over efficiency** For symmetrical staging \((R = 50\%\)) , the isentropic enthalpy drop across the stage is evenly distributed between the stationary and moving rows of blades (Fig. 7.33) so that

\[(\Delta h_s)_{fb} = (\Delta h_s)_{mb} = \frac{(\Delta h_s)_{stage}}{2} = (\Delta h_s)_{row}\]

Assuming the same nozzle efficiency for both fixed and moving rows, the kinetic energy of fluid gained per row, fixed as well as moving, is

\[\eta_n \cdot \frac{(\Delta h_s)_{stage}}{2}\]

Now, the kinetic energy associated with the leaving velocity of the preceding stage is available to do work in the following stage. Of course, some loss is involved in the journey from one stage to the next and is taken into consideration by what is known as *carry-over efficiency*. Hence, the kinetic energy leaving one stage and available to the next is given by \(\eta_{CO} V_2^2/2\), where \(\eta_{CO}\) is the carry-over efficiency.

The energy balance for the fixed row of blades gives

\[(V_1^2/2) + h_1 = \eta_{CO} \frac{V_2^2}{2} + h_2\]

\[\frac{V_1^2 - \eta_{CO} V_2^2}{2} = h_2 - h_1 = \eta_n \frac{(\Delta h_s)_{stage}}{2}\]

\[\therefore (\Delta h_s)_{stage} = \frac{V_1^2 - \eta_{CO} V_2^2}{\eta_n}\]  \(\text{(7.77)}\)

This is the energy available for conversion in one stage. The energy output or diagram work per kg of steam is given by Eq. (7.71). Therefore, the combined nozzle and blade efficiency \(\eta_{nb}\) is given by

\[\eta_{nb} = \frac{(2V_1 \cos \alpha - V_1) V_b}{(V_1^2 - \eta_{CO} V_2^2)/\eta_n}\]
or
\[ \eta_{nb} = \eta_n \frac{V_b^2}{V_1^2} \left( \frac{2V_1 \cos \alpha}{V_b} - 1 \right) \frac{1}{1 - \eta_{CO} \left( \frac{V_2^2}{V_1^2} \right)} \]  
\[ (7.78) \]

Again,
\[ V_2^2 = V_{r2}^2 + V_b^2 - 2V_{r2} V_b \cos \alpha \]
\[ \therefore \]
\[ \frac{V_2^2}{V_1^2} = 1 + \rho^2 - 2\rho \cos \alpha, \text{ where } \rho = V_b/V_1 \]

Substituting in Eq. (7.78),
\[ \eta_{nb} = \eta_n \frac{\rho^2 \left( \frac{2 \cos \alpha}{\rho} - 1 \right)}{1 - \eta_{CO} (1 + \rho^2 - 2\rho \cos \alpha)} \]

or
\[ \eta_{nb} = \eta_n \frac{2 \rho \cos \alpha - \rho^2}{1 - \eta_{CO} (1 + \rho^2 - 2\rho \cos \alpha)} \]  
\[ (7.79) \]

When \( \eta_{CO} = 1, \eta_{nb} = \eta_n, \) i.e. the losses are confined to the nozzle friction only.

When \( \eta_{CO} = 0, \) there is no carry-over of kinetic energy to the next stage (velocity of approach is zero), and
\[ \eta_{nb} = \eta_n (2\rho \cos \alpha - \rho^2) \]  
\[ (7.80) \]

which is the efficiency of a single row of blades.

If \( \eta_n = 1, \)
\[ \eta_{nb} = \eta_{b1} = \eta_0 = 2\rho \cos \alpha - \rho^2 \]  
\[ (7.81) \]

For maximum blading efficiency,
\[ \frac{d\eta_0}{d\rho} = 2 \cos \alpha - 2\rho = 0 \]
\[ \rho_{opt} = \cos \alpha \]

which is the same as Eq. (7.74).
Substituting in Eq. (7.81)
\[ (\eta_{b1})_{max} = \cos^2 \alpha \]  
\[ (7.82) \]

which is somewhat less than that obtained from Eq. (7.75).

A comparison of Eqs (7.57), (7.67) and (7.82) reveals that the optimum efficiencies for simple impulse, Curtis and reaction blading are all equal. However, when friction is taken into account, the reaction stage is found to be the most efficient, followed by Rateau and Curtis staging in that order. The friction losses are less significant in the reaction stage since the flow velocities are lower.
2. Comparison of enthalpy drops in various stages

Assuming the blades operating at maximum efficiency, for a symmetric (50% R) reaction stage,

\[ \rho_{opt} = \cos \alpha = \frac{V_b}{V_1} \]

\[ \therefore V_1 = \text{velocity of steam exiting the nozzles or fixed blades} \]

\[ = V_b / \cos \alpha = 44.72 \left( \frac{(\Delta h_s)_{stage}}{2} \right)^2 \]

\[ \therefore (\Delta h_s)_{stage} = 2 \left( \frac{V_b}{44.72 \cos \alpha} \right)^2 \]  \hspace{1cm} \text{(7.83)}

This is the isentropic enthalpy drop in a 50% reaction stage. If we compare this with the isentropic enthalpy drop for a simple impulse stage, Eq. (7.59) and that for a 2-row Curtis stage, Eq. (7.68),

\[ (\Delta h_s)_{impulse\ stage} = 4 \left( \frac{V_b}{44.72 \cos \alpha} \right)^2 \]

and \[ (\Delta h_s)_{2\text{-row Curtis\ stage}} = 16 \left( \frac{V_b}{44.72 \cos \alpha} \right)^2 \]

we find for the same values of \( \bar{V}_b \) and \( \alpha \),

\[ (\Delta h_s)_{50\%\ reaction\ stage} : (\Delta h_s)_{simple\ impulse\ stage} : (\Delta h_s)_{2\text{-row\ Curtis\ stage}} = 1 : 2 : 8 \]  \hspace{1cm} \text{(7.84)}

Therefore, a 2-row Curtis stage is equivalent (ideally) to four simple impulse stages and eight 50% reaction stages.

Since the number of stages required for a certain total isentropic enthalpy drop is

\[ n = \frac{(\Delta h_s)_{total}}{(\Delta h_s)_{stage}} \]

the lower is the enthalpy drop per stage, the higher is the number of stages required for the same output.

For \( V_b = 320 \text{ m/s} \) and \( \alpha = 16^o \),

\[ (\Delta h_s)_{impulse\ stage} = 221.64 \text{ kJ/kg} \]
\[ (\Delta h_s)_{50\%\ reaction\ stage} = 110.82 \text{ kJ/kg} \]
\[ (\Delta h_s)_{2\text{-row Curtis\ stage}} = 886.56 \text{ kJ/kg} \]
Thus, for the same output of a turbine the number of reaction (50%) stages required will be about twice the number of impulse or pressure (Rateau) stages and about 8 times the number of 2-row Curtis stages. So, reaction turbines are costlier than impulse turbines for the same output.

Again, for the same mean blade speed \( V_b \),

\[
V_1 = V_b / \cos \alpha \quad \text{for a 50% reaction stage}
\]

\[
V_1 = 2V_b / \cos \alpha \quad \text{for a simple impulse stage, and}
\]

\[
V_1 = 4V_b / \cos \alpha \quad \text{for a 2-row Curtis stage.}
\]

The energy loss due to friction is proportional to the square of the velocity of a fluid (steam). Since the fluid velocity is the highest for a 2-row Curtis stage and lowest in the 50% reaction stage, so the energy loss due to friction in the reaction stage is the least while that in the Curtis stage is the highest. The energy loss in the impulse stage will be in between these two values. Therefore, the efficiency of the reaction stage will be the highest and that of the Curtis stage will be the lowest, while that of impulse stage will lie in between.

\[ \eta_{\text{reaction (50%) stage}} > \eta_{\text{simple impulse stage}} > \eta_{\text{2-row Curtis}} \]

Figure 7.34 gives a comparison of diagram efficiencies with respect to the velocity ratio. Reaction turbines are more efficient but are more costly.

In modern turbines, a 2-row Curtis stage is generally used as the first stage. It is followed by a series of reaction stages. Sometimes impulse stages are also used after the Curtis stage. The use of the initial Curtis stage reduces the length of the rotor, i.e. the number of stages required (since a 2-row Curtis stage can
are simplified and the overall length of the rotor is reduced. Thus, it is a good compromise to use one 2-row Curtis stage initially, sacrificing some loss in efficiency. A large drop in enthalpy occurs in this Curtis stage. The remaining enthalpy drop occurs in the subsequent stages, the number of which can be estimated from

\[
n = \frac{(\Delta h_s)_{\text{total}} - (\Delta h_s)_{2\text{-row Curtis stage}}}{(\Delta h_s)_{\text{stage}}} \quad (7.85)
\]

where \(\Delta h_s\) represents the isentropic enthalpy drop.

The arrangement of stages for a 15-stage turbine is given below:

\[
\begin{array}{cccccccccc}
\text{N MB GB MB} & \text{FB MB} & \text{FB MB} & \text{FB MB} & \text{FB MB} & \cdots & \text{FB MB} & \text{FB MB} & \text{FB MB}
\end{array}
\]

where N: nozzle; MB moving blades, GB guide blades, and FB fixed blades.

These stages are distributed in a number of cylinders or casings, as discussed later. Simple layouts of an impulse and a reaction turbine are shown in Fig. 7.35. Figure 7.36 shows typical single cylinder designs of impulse and reaction turbines. In practice, the number of impulse and reaction turbines in operation are approximately equal, the choice being dependent on the preference of the turbine manufacturer. In some designs the stage may have some degree of reaction, 10% or less to produce a small pressure drop across the bucket. This small drop can be very useful in keeping the buckets running full of steam, with the shaft thrust positive in one direction.

### 7.3.3 Variation of Blade Velocity Along Blade Height

In the low pressure region, the blade heights are quite large. The velocity at the blade root \([(\pi D_{\text{root}} N)/60]\) will be much smaller than that at the mid-point, and still smaller than that at the tip \([(\pi D_{\text{tip}} N)/60]\). Hence, for a good efficiency, the blade angles should vary with the diameter (Fig. 7.37). For this reason, twisted (or warped) blades are used in the later stages of the turbine. The vertical line in Fig. 7.37 represents the radius of the wheel. The vertical distance between the tip and the root radii is the blade height. Steam enters and leaves the blades with velocities \(V_1\) and \(V_2\), respectively. Since \(V_b\) is proportional to the radius, angle \(\beta_1\) will vary accordingly. It is seen that the vector diagram at the root looks like one for an impulse blade, while the diagram at the tip looks like one for a reaction blade. The blade work, \(V_b \Delta V_{\text{or}}\), is constant.
Fig. 7.35 Layouts of impulse and reaction steam turbines
Fig. 7.36  Typical designs of impulse and reaction steam turbines

Fig. 7.37  Variation in low-pressure blade angles as influenced by vector diagrams for root and tip velocity
7.3.4 Nozzle and Blade Heights

Nozzle or blade height depends on the total annular area required to pass the desired flow of fluid. From Fig. 7.38, it can be seen that the area available for flow at exit of one nozzle passage is approximately.

\[ A = Oh_n \]

where \( O \) = width of flow passage at exit at mean nozzle height, and \( h_n \) = nozzle height.

It was already discussed in section, 7.2.9 how to estimate the nozzle annular area and its height. For coherence, it is being done here too. From Fig. 7.38(c),

\[ p \sin \alpha = 0 + t \]

where \( p \) = pitch of nozzles at mean nozzle height, and \( t \) = nozzle wall thickness, as shown.

\[ \therefore A = (p \sin \alpha - t)h_n \]

Assuming full peripheral admission, i.e. the nozzle diaphragm is completely occupied by nozzles, the number of nozzles, \( z \), is given by

\[ z = (\pi D_m)/p \]

Total nozzle area is

\[ A_n = p \sin \alpha \left( 1 - \frac{t}{p \sin \alpha} \right) h_n \frac{\pi D_m}{p} \]

or

\[ A_n = pD_m h_n k_{tn} \sin a \]  \hspace{1cm} (7.86)

where \( k_{tn} \) is the nozzle thickness factor.

---

Fig. 7.38 Flow through a nozzle passage

\[ z = (\pi D_m)/p \]
Similarly, it can be shown that the area available for flow at the entrance to the blades is

\[ A_b = \pi D_m h_b k_{tb} \sin \beta_1 \]  

(7.87)

where \( k_{tb} \) is the edge thickness factor for blades.

From the continuity equation, it follows that

\[ \omega_b = \frac{A_n V_1}{v_1} = \frac{A_b V_{r_1}}{v_1} \]

or

\[ \pi D_m h_n k_{tn} \sin \alpha V_1 = \pi D_m h_b k_{tb} \sin \beta_1 V_{r_1} \]  

(7.88)

For all practical purposes, it can be assumed that \( k_{tn} = k_{tb} \), so that

\[ h_b/h_n = (V_1 \sin \alpha)/(V_{r_1} \sin \beta_1) = 1 \]  

(7.89)

Although Eq. (7.89) indicates that the blade height at entrance is equal to the nozzle height at exit, it is customary to increase the blade entrance height slightly. This is done in order to avoid spilling of the fluid issuing from the nozzles. This increase in the blade entrance height is called “over lap” and is given equally at root and tip of the blade. This \( h_b \) exceeds \( h_n \) by 1.6 mm in high pressure stages and by about 20 mm in low pressure stages of large turbines, i.e. the over lap varies from 1.6 mm to 20 mm.

1. **First stage nozzles and blades**  
From the continuity equation, the nozzle area required for a given flow rate \( \omega_b \) is given by

\[ A_n = \frac{\omega_b V_1}{v_1} \]

At high pressure of steam, \( V_1 \) is small and \( V_1 = V_b/cos \alpha \) for 50% reaction blading, and \( V_1 = 2V_b/cos \alpha \) for impulse blading if the blades are assumed to operate at the maximum efficiency. Thus, at the initial stage of turbine, the flow area \( (A_n) \) required is small. Now,

\[ A_n = \pi D_m h_n \sin \alpha k_{tn} \]

where \( D_m \) is estimated from the given value of \( V_b = (\pi D_m N)/60 \). Since \( A_n \) is small and for given values of \( \alpha \) and \( k_{tn} \), the estimated value of \( h_n \) is often found to be too small to manufacture. If the nozzles are conical, the diameter of each nozzle also is found to be very small. It has been experimentally found that small diameter nozzles are less efficient than nozzles of larger diameter.

Furthermore, the blades following the nozzles are thus very short, which is difficult to fabricate. Such short blades are also less efficient. It is advisable to use a minimum height of 20 mm for the initial blades. Therefore, the nozzle height in the first stage cannot be much less than 20 mm, or

\[ (h_b)_{min} \equiv (h^*_n)_{min} \equiv 20 \text{ mm} \]

The required nozzle height \( h_n \) for a given flow is much less than this. Thus,

\[ A_n = x\pi D_m (h_n)_{min} \sin \alpha k_{tn} \]  

(7.90)
where \( x \) is the fraction of the total arc of nozzles which is open for steam flow. This is called *partial admission of steam*. Some nozzles in the arc are blocked by steel plates (Fig. 7.39). Thus, by increasing the estimated \( h_n \) to its minimum practicable value (~ 20 mm) and simultaneously blocking part of the periphery of the nozzle diaphragm, the required flow area \( (A_n) \) for a certain mass flow of steam at the given condition is obtained. For full admission, the entire arc or periphery of the nozzle diaphragm \( (\pi D_m) \) is open for steam flow. For partial admission, the arc or periphery open for steam flow is \((x\pi D_m)\), where \( x \) is less than unity.

![Diagram](image)

**Fig. 7.39 Partial admission of steam in the initial stage**

With partial admission, a given blade passage will not receive flow from the nozzles at all times. Thus exposed to alternate flows of high velocity steam, blades are subjected to vibrations which may prove dangerous in long blades.

**2. Last stage blade height** The last stage is very important in steam turbine design. As the pressure of steam decreases during expansion, the specific volume increases. The volume flow of steam increases necessitating an increased flow area. Consequently, \( h_b \) and \( D_m \) increase with decrease of pressure. At the last stage, the pressure is the minimum, and so \( h_b \) and \( D_m \) should have maximum values.

Blades are held at one end with the rotor while the other end is free. So they act as cantilevers with distributed load of steam on them. They are subjected to bending stresses. Since they are rotating at a high rpm, so they are subjected to centrifugal stresses also. As the blade height increases, both bending and centrifugal stresses increase. Due to these stresses, both blade height and blade
diameter get restricted. The maximum blade velocity is also limited depending on the material of the blades, which is about 350–400 m/s.

\[
(V_b)_{\text{max}} = 350 \text{ m/s (say)} = \frac{\pi (D_m)_{\text{max}} N}{60}
\]

The rpm of the rotor is fixed from the generator side. For a 2-pole, 50 Hz alternator,

\[
N = \frac{120 f}{p} = \frac{120 \times 50}{2} = 3000 \text{ rpm}
\]

\[
(D_m)_{\text{max}} = \frac{350 \times 60}{\pi \times 3000} = 2.23 \text{ m}
\]

For straight blades, the maximum blade height is about 20% of the mean blade ring diameter, or

\[
(h_b/D_m)_{\text{max}} = 0.20
\]

Where it is necessary to exceed this ratio because of flow requirements, the blades may be tapered or twisted (Fig. 7.40), thereby reducing both bending and centrifugal stresses. With these modifications, the blade height may approach about 30% of the mean blade ring diameter. Therefore, for twisted or tapered blades,

\[
(h_b/D_m)_{\text{max}} = 0.3
\]

\[
(h_b)_{\text{max}} = 2.23 \times 0.3 = 0.67 \text{ m}
\]

The flow area \((A_b)\) could also be increased by increasing the blade angles, which would, in turn, increase the nozzle angle \(\alpha\). This would, however, decrease the blade efficiency. By reducing the rpm, both \(h_b\) and \(D_m\) can be increased. But, this will also increase the weight and bulk of the rotor along with the cost. So, this is not justified.

When the blade height becomes a significant part of the total stage diameter, the ratio of steam to blade speed changes over the height of the blade. Figure 7.41 shows velocity diagrams at the root and tip of the twisted blade that receives a steam jet moving in a vortex flow \((V_1 r_1 = V_2 r_2\)). The blade root has been designed for impulse flow, when the tip is exposed to a pure reaction force.

### 3. Parallel exhausts: number of last stages

The maximum flow area is provided in the last stage where the volume flow is maximum.

\[
(A_b)_{\text{max}} = \pi (D_m h_b)_{\text{max}} \sin \beta_1 k_{tb}
\]

The maximum volume flow that the last stage can accommodate
Fig. 7.40  Tapered and twisted blades

\[ \pi (D_m h_b)_{\text{max}} \sin \beta_1 k_{tb} V_{r1} \]

\[ = \pi (D_m h_b)_{\text{max}} k_{tb} V_1 \sin \alpha \quad \text{[since } V_{r1} \sin \beta_1 = V_1 \sin \alpha \text{]} \quad (7.91) \]

If impulse blading is used,

\[ \frac{V_b}{V_1} = \frac{\cos \alpha}{2} \quad \therefore \quad V_1 = \frac{700}{\cos \alpha} \]

From Eq. (7.91), for \( \alpha = 20^\circ \) and \( k_{tb} = 0.9 \) (assumed),

Maximum volume flow = \( \pi \times 2.23 \times 0.67 \times \frac{700}{\cos 20^\circ} \sin 20^\circ \times 0.9 = 1075 \text{ m}^3/\text{s} \)

The maximum mass flow of steam that the last stage can accommodate (Fig. 7.42).

\[ (\omega_s)_{\text{max}} = \frac{1075}{v_2} \text{ kg/s} \]

For the turbine exhaust condition at 0.075 bar, 0.88 quality,
\[ v_2 = 0.001 + 0.88 \times 19.24 = 16.93 \text{ m}^3/\text{kg} \]

\[ (\omega_s)_{\text{max}} = \frac{1075}{16.93} = 63.5 \text{ kg/s} \]

\[ 1 \quad 2 \]

**Fig. 7.41** A twisted blade with impulse flow at root and reaction at tip

**Fig. 7.42** Specific volume at turbine exhaust

The number of parallel exhausts or last stages required for a given steam flow rate of \( \omega_s \) is

\[ n = \frac{\omega_s}{(\omega_s)_{\text{max}}} = \frac{\omega_s}{63.5} \quad (7.92) \]
4. Casing arrangement  If the number of parallel exhausts is estimated to be 4, i.e. \( n = 4 \), the casing arrangement of the turbine may be as shown in Fig. 7.43.

Steam first expands in the HP (high pressure) turbine (1–2), the exhaust from which is taken back to the steam generator for reheating (2–3). The reheated steam (3) then expands in the IP (intermediate pressure) turbine (3–4).

![Diagram of a steam turbine with casing arrangement](image_url)

**Fig. 7.43** Casing arrangement of a steam turbine having four parallel exhausts

The steam exiting the IP turbine (4) is split into four equal streams expanding in two DFLP (double flow low pressure) turbine (4–5). The four parallel exhaust streams from the LP turbines (5) then enter the condenser.

A double flow low pressure turbine (DFLPT) not only provides two parallel exhausts, but also helps the turbine contain the axial thrust. Equal and opposite axial thrusts operating in the two similar turbines of the DFLPT neutralize each other.

The HP and IP turbines are installed in the way as shown in order to reduce the axial thrust. Steam at states 1 and 3 is more or less at the same temperature. This will cause no thermal stress at HP and IP turbine inlets. If the IP turbine is oriented in the opposite way, there will be a thermal gradient along the shaft, which would cause considerable thermal stresses.

The HP turbine outer casing is often made of double shell construction, with intermediate pressure steam filling the annular space so that the pressure difference across the casing wall is reduced.

To reduce the temperature gradient around and periphery and hence, the thermal stress at turbine entrance, steam is admitted at two or three feed points in both HP and IP turbines, instead of having single entry.

If \( n = 3 \), one DFLP turbine and one single flow LP turbine will be used to accommodate the required flow rate of steam (Fig. 7.44).

Figures 7.43 and 7.44 are called tandem compounded steam turbines with all turbine cylinders mounted on the same shaft. If the number of cylinders are large, or the cylinders are of heavy weights, then the cylinders may be mounted on two shafts, each coupled to a separate electric generator (Fig. 7.45). Such an arrangement of turbine cylinders on two shafts is called a cross-compounded steam turbine.
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1. Internal losses, which are connected with the flow of steam.
2. External losses, which occur outside the turbine casing.

The internal losses may be enumerated as follows:

(a) Losses in regulating valves
(b) Nozzle friction losses
(c) Blade friction losses
(d) Disc friction losses
(e) Partial admission losses
(f) Gland leakage losses
(g) Residual velocity losses
(h) Carry-over losses

**Table 7.1** Turbine-Generator Configurations

<table>
<thead>
<tr>
<th>Fossil</th>
<th>Fossil</th>
<th>Nuclear</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC-2F LSB 26, 30 and 33.5 in Two casings 3600 r/min</td>
<td>TC-6F LSB 26, 30 and 33.5 in Five casings 3600 r/min</td>
<td>TC-4F LSB 38 and 43 in Three casings 1800 r/min</td>
</tr>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>125-400 MW</td>
<td>550-1000 MW</td>
<td>450-1000 MW</td>
</tr>
<tr>
<td>TC-4F LSB 26, 30 and 33.5 in Three casings 3600 r/min</td>
<td>TC-6F LSB 30 and 33.5 in Five casings 3600 r/min (double reheat)</td>
<td>TC-6F LSB 38 and 43 in Four casings 1800 r/min</td>
</tr>
<tr>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>250-650 MW</td>
<td>450-725 MW</td>
<td>600-1100 MW</td>
</tr>
<tr>
<td>TC-4F LSB 26, 30 and 33.5 in Four casings 3600 r/min</td>
<td>CC-4F LSB 38 and 43 in Four casings 3600/1800 r/min</td>
<td></td>
</tr>
<tr>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>550-850 MW LP LP G</td>
<td>3600 r/min HP IP LP</td>
<td>1800 r/min HP LP LP</td>
</tr>
<tr>
<td>LP LP G</td>
<td>600 - 1250 MW</td>
<td></td>
</tr>
</tbody>
</table>

Data provided by General Electric Company, TC = tandem compound, CC = Cross Compound, F = number of flow ducts to condenser, LSB = last-stage blade (1 in = 25.4 mm).
1. **Losses in regulating valves** Steam, before entering the turbine, passes through the main valve and the regulating valves, the flow through these being accompanied by pressure losses. Steam gets throttled adiabatically with constant enthalpy. However, the enthalpy drop in the turbine decreases yielding less specific output. Thus, some available energy of steam is lost due to the irreversible process of throttling. The pressure drop varies from 3 to 5% of the inlet steam pressure $p_0$.

2. **Nozzle friction losses** The friction losses in nozzles were mentioned earlier. The effect of friction is taken care of by the nozzle efficiency. Losses are due to the growth of boundary layer and the formation of eddies in the wake, apart from the frictional resistance of walls, which varies with the height and length of passage. Losses are higher in a turbulent boundary layer than in a laminar one. In reaction turbine where pressure or enthalpy drop per stage is less due to lower velocity, the laminar condition persists over a greater length of passage. So, the friction loss is less than the impulse stage. However, due to the large number of stages, the total surface area exposed to flow is more, which increases the friction loss. Thus, the nozzle loss depends on its size, surface roughness, its length, roundness of entrance, divergence angle, space between nozzles, moisture and trailing edge.

3. **Blade friction losses** Losses in moving blades are caused by various factors as enumerated below:
   
   (a) **Impingement losses:** Steam issuing out from the nozzles meets the leading edges of the blades and energy may be lost if the entry is not smooth enough and eddies are formed.
   (b) **Frictional losses:** Steam encounters these losses in the blade passages, which depends on the roughness of the blade surface.
   (c) **Turning losses:** These occur as the steam turns in the blade passage.
   (d) **Wake losses:** These occur at blade exit, depending on its shape and tip thickness.

   The moving blade losses are taken care of by the blade friction coefficient $k_b = V_{r2}/V_{r1}$ representing the reduction of relative velocity of steam from $V_{r1}$ to $V_{r2}$ due to friction.

4. **Disc friction losses** When the turbine disc rotates in the viscous steam, there is surface friction loss due to relative motion between the disc and steam particles. Due to centrifugal force, steam is thrown radially outward. The moving disc surface exerts a drag on the steam, sets it in motion from root to tip, and produces a definite circulation. Some part of the kinetic energy of steam is lost due to this friction.

5. **Partial admission loss** An impulse stage operating with partial admission, or an early stage in such a turbine with nozzles provided only over a part of the blade periphery, will have blades idle during part of the revolution. Some portion of kinetic energy of the incoming steam is spent in clearing away the steam existing within the blade passage. These are called “scavenging losses” which together with disc friction losses are often referred to as “windage
losses” in which some kinetic energy is imparted to the fluid at the expense of the kinetic energy of the blades. Since reaction turbines are designed for full peripheral admission, the windage loss, as well as disc friction can be neglected.

6. Gland leakage losses  Leakage of steam can occur between stages and along the shaft at inlet and exit ends of the casing. Diaphragm leakage takes place in both impulse and reaction stages through the radial clearance between the stationary nozzle diaphragm and the shaft or drum. Tip leakage occurs in reaction stages through the clearance between the outer periphery of the moving blades and the casing because of the pressure difference existing across the blades. Shaft leakage occurs through the radial clearance between the shaft and casing at both high and low pressure ends of turbines. At the HP end, steam leaks out to the atmosphere, while at the LP end, the pressure being less than atmospheric, air leaks into the shell.

Since the leaked steam does not work on the blades, it represents energy loss. Both diaphragm and tip leakages can be minimized by reducing the radial clearances, but it must avoid rubbing or metal-to-metal contact. The clearance may be as low as 0.5 mm. However, proper balancing of the rotor, both static and dynamic, is a must to avoid any such rubbing. It is necessary to use seals or packing to further reduce the leakage flow. These seals may be labyrinths, carbon rings, water or steam seals, or gland leak-off. To prevent shaft leakage, labyrinths may be used with carbon rings and gland leak-off. Labyrinth seals consist of a series of thin strips fixed with the casing which maintain the smallest possible clearance with the shaft (Fig. 7.46). The small constrictions make the steam throttle to lower pressures many times, till only a very little quantity leaks out. Carbon ring seals, which consist of a ring of carbon divided into segments, have the rings fit snugly to the shaft by springs so as to prevent leakage, and may be used along with labyrinth glands in series in large turbines.

![Fig 7.46 Stepped labyrinth seals](image)

7. Residual velocity loss  Steam leaving the last stage of the turbine has a certain velocity which represents an amount of kinetic energy that cannot be imparted to the turbine shaft and is thus wasted.

8. Carry-over losses  Some energy loss takes place as steam flows from one stage to the next. The kinetic energy leaving one stage and available to the next is given by \( \eta_{CO}(\frac{V^2}{2}) \), where \( \eta_{CO} \) is the carry-over efficiency.

In addition, there are some losses of energy due to wetness of steam (where the water particles are dragged along with steam at the expense of some K.E. of
steam). If the quality of steam is less than 0.88, erosion and also corrosion can take place. Since the velocity of steam leaving the last stage of turbine is quite large (100–120 m/s), there will be energy losses due to friction in the exhaust hood of the turbine. Exhaust hoods to the condenser gradually increase in area like a diffusor and thus, there is a further decrease in velocity of steam and an increase in pressure as steam enters the condenser. Such hoods allow the turbine to operate down to a slightly lower pressure than that required by the condenser (depending on temperature and flow of cooling water, and air extraction from its shell), thus increasing the turbine work.

9. External losses There are some energy losses in the bearings and governing mechanisms which can be reduced by improving the lubrication systems. Some energy is consumed by oil pumps. Since the turbines are adequately insulated the surface heat loss by radiation and convection is small. Modern large electric generators are hydrogen cooled, well designed, and very efficient, where the energy losses are within 2 to 3 per cent.

7.3.6 Reheat Factor and Condition Line

Figure 7.47 shows the typical stage efficiencies of simple impulse, Curtis and reaction stages in contrast to Fig. 7.34 which demonstrates the variation of blading efficiency. As discussed in the earlier article, there are various losses in the stage and the portion of the available energy not converted to work and remaining in the fluid is termed as "reheat". A single-stage expansion with reheat is shown in Fig. 7.48.

![Typical stage efficiencies](image1)

**Fig. 7.47** Typical stage efficiencies

![Expansion in a single stage with reheat](image2)

**Fig. 7.48** Expansion in a single stage with reheat
\[ \Delta h_s = h_1 - h_{2s} = \text{isentropic enthalpy drop} \]
\[ \Delta h = h_1 - h_2 = \text{actual enthalpy drop} \]

\[ \therefore \text{Reheat} = h_2 - h_{2s}. \]

Figure 7.49 shows the expansion in a 4-stage turbine, taking into account the effects of reheat. The following conclusions are drawn:

1. Reheat takes place with increase in entropy.
2. The reheat in a given stage is available to do work in the succeeding stage except the last stage where the reheat is a loss.
3. The constant pressure lines diverge from one another, thereby increasing the enthalpy drop for the same pressure drop.
4. Because of (2) and (3) the sum of the available energies (isentropic enthalpy drops) for each stage is greater than the available energy (isentropic enthalpy drop) for the whole turbine.
5. The condition representing the actual expansion in the turbine is approximately the locus of points indicating the actual conditions of steam at the exit of each stage.

The effect of item 4 may be expressed, assuming equal available energy per stage, by a term called "reheat factor" (RF). The total isentropic enthalpy drop, \((h_1 - h_{6s})\), is divided equally into four parts in the four stages (Fig. 7.49) in the Mollier diagram, from which the interstage pressures \(p_1, p_2,\) and \(p_3\) are noted, \(p_0\) and \(p_4\) being the boiler and condenser pressures, respectively. The isentropic enthalpy drop \((h_1 - h_{2s})\) in the first stage is multiplied by the stage efficiency to obtain the actual enthalpy drop \((h_1 - h_2)\), which is cut-off from 1 on the isobar at \(p_1\) in the Mollier diagram. From 2, a vertical line is drawn which cuts the isobar at \(p_2\) at state 3s. The isentropic enthalpy drop \((h_2 - h_{3s})\) is noted and the actual drop \((h_2 - h_3)\) is estimated by multiplying it with stage efficiency. In the same way, the enthalpy drops \((h_3 - h_{4s})\) and \((h_4 - h_{5s})\) are obtained from the Mollier diagram, and the state 5 is fixed. The states 2, 3, 4 and 5 record the actual conditions of steam at exit from each stage and the locus through these states is called the "condition line". The reheat factor is defined as
RF = \frac{(h_1 - h_{2s}) + (h_2 - h_{3s}) + (h_3 - h_{4s}) + (h_4 - h_{5s})}{h_1 - h_{6s}} \quad (7.93)

If the stage efficiency is assumed to be the same in all stages,

\eta_{st} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{h_4 - h_5}{h_4 - h_{5s}} \quad (7.94)

By substituting \( h_1 - h_{2s} = (h_1 - h_2)/\eta_{st} \), \( h_2 - h_{3s} = (h_2 - h_3)/\eta_{st} \) and so on in Eq. (7.93),

RF = \frac{\eta_{st}}{\eta_{st} \frac{h_1 - h_2}{h_1 - h_{2s}} \frac{h_2 - h_3}{h_2 - h_{3s}} \frac{h_3 - h_4}{h_3 - h_{4s}} \frac{h_4 - h_5}{h_4 - h_{5s}}}

or

RF = \frac{1}{\eta_{st}} \times \frac{h_1 - h_2}{h_1 - h_{6s}} = \frac{1}{\eta_{st}} \eta_{\text{internal}} \quad (7.95)

where the internal (or isentropic) efficiency of the turbine,

\eta_{\text{internal}} = \frac{h_1 - h_5}{h_1 - h_{6s}} \quad (7.96)

Thus, from Eq. (7.95)

\eta_{\text{internal}} = R.F. \times h_{\text{stage}} \quad (7.97)

Due to divergence of constant pressure lines, the reheat factor is greater than unity. The value of RF lies between 1.04 to 1.08. Therefore,

\eta_{\text{internal}} > \eta_{\text{stage}}

In other words, due to reheat, the expression \((h_2 - h_{2s}) + (h_3 - h_{3s}) + (h_4 - h_{4s}) + (h_5 - h_{5s})\) is greater than \((h_5 - h_{6s})\).

### 7.3.7 Design of Multi-Stage Turbines

The design of a steam turbine involves a judicious combination of theory with the results of experience, governed to a great extent by cost. The method of design outlined below is only illustrative of the theories discussed above. The following are specified to the designer: initial steam conditions, exhaust pressure, and the capacity in MW or kW. The turbine requires many stages which increase in diameter from inlet to the exit end. All wheels turn at the same speed (rpm), but \( V_p, V_1, V_2, k_t, k_w, \alpha, \beta, \gamma \), leakage efficiency, disc friction and windage loss may all vary from stage to stage. The condition line, which is the logical starting point, can only be approximated until all the stage efficiencies are known.

The calculation for the casing arrangement of a multi-stage impulse turbine is made according to the Section 7.3.4.

The first stage is most often a two-row Curtis stage. In order to increase the height of the nozzles, the stage is usually given a partial admission. In large condensing steam turbines, where the specific volume at the end of expansion in the turbine becomes very large, long blades of special design are selected. The design of multi-stage turbines is usually started with initial design considerations of first, second and last stages, while the intermediate stages are designed later.
Hidden page
\[ h_{b0} = h_n \times \frac{V_{1n}}{V_{4a}} \]

The ratio of axial components \( V_{1n}/V_{2a} \) and so on can be determined from the velocity diagrams. A constant blade diameter, \( D_m \), has been assumed for the Curtis stage.

2. Second stage  For impulse stages, the optimum velocity ratio

\[ \rho_{opt} = \cos \alpha/2 = V_b/V_1 \]

where \( \alpha \) is the nozzle angle. For the actual stage, a value somewhat less than this optimum value may be assumed. If the average blade velocity is assumed, then the average nozzle exit velocity can be estimated.

Now

\[ V_1 = 44.72 [\eta_n (\Delta h_{s\text{stage}})^{1/2}] \]

Assuming suitable value of \( \eta_{th} \), the average drop of enthalpy per stage can be computed.

Number of stages required = \( \frac{(\Delta h_s)_{\text{total}}}{(\Delta h_s)_{\text{stage}}} \)

The absolute velocity of steam leaving the second row of moving blades \( (V_4) \) of the Curtis stage is known. From the energy balance across the nozzles of the second stage (impulse), the absolute velocity of steam \( (V_1) \) entering the blades can be estimated. By assuming suitable values of relevant parameters, the velocity diagrams can be plotted, from which \( \eta_{b1} \) can be estimated, and

\[ \eta_{\text{stage}} = \eta_n \times \eta_{b1} \times (1 - h_{df}) \]

can similarly be determined.

In the stage,

\[ \Delta h_{act} = \eta_{\text{stage}} \times (\Delta h_s)_{\text{stage}} \]

Now,

\[ \omega_s = \frac{\pi D_m h_b \times k_{mb} V_1 \sin \alpha}{V_1} \]

Assuming a suitable value of \( (h_b/D_m) \), \( h_b \) and \( D_m \) can be estimated. The nozzle height will then be

\[ h_n = h_b - 1.60 \text{ mm} \]

3. Last stage  Assuming the maximum blade velocity consistent with the blade material (350–420 m/s), the last stage blade diameter is estimated. From stress consideration, the maximum height to diameter ratio for twisted or tapered blades can be assumed to be 0.3. If the blades are assumed to operate close to the maximum efficiency, the jet velocity of steam and hence, \( (\Delta h_b) \) can be determined. The velocity diagrams can be drawn from which \( \eta_{b1} \) can be estimated.

Now,

\[ \omega_s = \frac{\pi D_m h_b \times k_{mb} V_1 \sin \alpha}{v_1} \]

from which both \( h_b \) and \( D_m \) are estimated.

The procedure has also been illustrated while discussing the casing arrangements earlier.
4. **Intermediate stages** The isentropic enthalpy drops that would take place in HP, IP and LP cylinders are known. Since the enthalpy drop per stage ($\Delta h_s$) is also known, the number of stages in each cylinder can be determined. The blade and nozzle dimensions can similarly be estimated.

By knowing the stage efficiencies of all the stages, the condition line can be drawn on the Mollier diagram from which the final condition ($x$, $\nu$) of steam can be noted. The turbine internal efficiency is then determined, and the steam flow rate ($\omega_s$) is estimated. If this $\omega_s$ does not tally with the value used earlier, calculations have to be repeated.

**7.3.8 Turbine Governing and Control**

The function of a governor is to maintain the shaft speed constant as the load varies.

The simplest type of governor is the centrifugal flyball type (Fig. 7.50). The power available at the shaft is equal to $2\pi TN/60$, where $T$ is torque and $N$ is the rpm. As load (or torque) decreases, speed increases. Consequently, with the increase of centrifugal force, the flyballs fly apart and raise the sleeve which, operating through a lever and a fulcrum, actuates the main valve to close and reduce the mass flow of steam admitted to the turbine.

![Centrifugal Flyball Governor Diagram](image-url)
An oil-operated servo system in addition may be used to enhance the sensitivity of the governor (Fig. 7.51). The governor force is amplified to move a light and almost frictionless pilot valve which controls the flow of high pressure oil to a piston. The piston powered by the oil can thus operate the governor valve as desired. The steady-state speed regulation \( R_s \) is given by

\[
R_s = \frac{N_o - N}{N_r} \times 100
\]

(7.98)

where

\( N_o \) = speed at no load,
\( N \) = speed at rated load, and
\( N_r \) = rated speed.

**Fig. 7.51** Centrifugal governor with hydraulic power amplifier

Usually, the speed change from full load to no load is limited to 3–4% of the speed at rated load. If the rated speed is 3000 rpm, the speed at full load would be 3060 rpm and that at no load would be 2940 rpm. Therefore, in this case

\[
R_s = \frac{3060 - 2940}{3000} \times 100 = 4\%
\]

1. **Throttle governing** It was mentioned in the earlier Section that as the load decreases and shaft speed increases, the stop valve is partially closed to admit less steam to the turbine and to produce less power according to the demand. Due to restriction of passage in the valve, steam is throttled, say, from \( p_0 \) to \( p_{\text{throttle}} \) (Fig. 7.52). The specific ideal output of turbine thus reduces from \((h_1 - h_{2s})\) to \((h_3 - h_{4s})\). With further closure of the valve, \( p_{\text{throttle}} \) will still be less to produce a still lower output.
The steam consumption plotted against the turbine load shows a linear relationship, which is called the Willan's line (Fig. 7.53), and given by

\[ \omega_s = a + bL \]  

where \( a \) = no load steam consumption, kg/s;  
\( b \) = steam rate (or specific steam consumption), kg/kWs; and  
\( L \) = load, kW.

The throttle and stop valves are located in the steam supply line to the turbine. The stop valve is a hydraulically operated quick opening and shutting valve designed to be either fully open or shut. For small turbines, the stop valve may be manually operated. The throttle valve is used to regulate steam flow during starting or stopping.

**2. Nozzle governing** If throttle governing is done at low loads, the turbine efficiency is considerably reduced. The nozzle control may then be a better method of governing. The nozzles are made up in sets, each set being controlled by a separate valve (Fig. 7.54). With the decrease of load, the required number of nozzles may be shut off.
3. Bypass governing  To produce more power (when on overload), additional steam may be admitted through a by-pass valve to the later stages of the turbine (Fig. 7.55). By-pass regulation operates in a turbine which is throttle governed.

4. Emergency governor  Every turbine is provided with some form of an emergency governor which trips the turbine (closes the stop valve and stops the steam supply) when
   (a) shaft exceeds 110% of its rates value, i.e. 3300 rpm
   (b) lubrication system fails
   (c) balancing (static as well as dynamic) of turbine is not proper
condenser becomes hot (due to inadequate cooling water circulation) or vacuum is less.

One common type of overspeed trip employs a pin or weight on the turbine shaft. Centrifugal force acting on the pin is opposed by a spring until about 10% overspeed is reached, whereupon, the centrifugal force overcomes the spring force and the pin flies out and strikes a trigger which, in turn, releases a spring to close the stop valve immediately.

7.3.9 Control and Supervisory Instruments

Certain control and supervisory instruments are provided for the safe and effective operation of a turbine given as follows.

1. *Pressure gauges* are provided to record the pressure of main steam at the stop valve, in the steam chest, at the first stage and the exhaust, the oil pressure to the bearings, the governor mechanism and the pressure of steam or water to the gland seals. For a condensing turbine, a vacuum gauge and a barometer are installed.

2. *Thermometers* are provided to record steam temperatures at the stop valve, in the steam chest, at the first stage, and at the gland. The oil temperatures entering and leaving the bearings are noted.

3. *A speed and cam-shaft position recorder* is required to record the turbine speed in rpm. During operation, the turbine speed is obtained from the generator frequency recorder. Thus, the speed recorder is used to record the cam-shaft position, which determines the opening of the valve and the load on the turbine.

4. *An eccentricity recorder* is provided to indicate and record the eccentricity of the shaft at the high pressure end of the turbine.

5. *A vibration amplitude recorder* is provided to record vibration of the rotor.

6. *An expansion indicator* is provided on the turbine control board to show the axial expansion of the turbine casing.

7. *A noise meter* on the control board is used to pick up and amplify the noise made by the moving parts of the turbine.

8. *Flow meters* are mounted on the turbine control board to indicate, record and integrate the mass rate of flow to the turbine, the steam bled at various points, and the flow to the condenser.

9. *Wattmeters, voltmeters and ammeters* are also provided on the turbine control board, which along with the flowmeters are used to determine the steam and heat rates of the unit.

10. Handwheels to operate the various drain valves are located at the turbine or on the turbine board.

11. Governor controls are located at the turbine or on the turbine control board for proper regulation of valves.

12. A trip lock lever for testing the overspeed trip is usually mounted on the turbine control board.
7.3.10 Dummy or Balance piston

In a pure impulse turbine the axial thrust on the blades is entirely due to the change in momentum of the steam in the axial direction and is usually very small. In the reaction turbine, however, there is a pressure drop across the blades of each ring, and so there can be a large force exerted on the blades in the axial direction. If it is not contained, the entire rotor may come out of the shaft and cause a serious accident.

As shown in Fig. 7.56, the axial thrust produced is

\[ T_1 = (p_1 - p_2) A - \omega_s (V_{a1} - V_{a2}) \]  \hspace{1cm} (7.100)

where \( A \) is the annular area of the blade ring. Similarly, for all the rings of the rotor, the thrusts \( T_2, T_3, \ldots \) can be estimated, the total thrust will be the sum of all these thrusts.

Normally, the change in axial velocity components \( V_{a1} - V_{a2} \) in Eq. (7.100) is usually small and it is zero for 50% reaction blading. Therefore, the total thrust will be

\[ T = \sum [(p_1 - p_2) A_1 + (p_2 - p_3) A_2 + (p_3 - p_4) A_3 + \ldots] \]  \hspace{1cm} (7.101)

**Fig. 7.56** Axial thrust on blades of reaction turbines

To balance this thrust, a few stages are provided on the other side of the cylinder (like a double-flow LP cylinder), which are designed in such a way that an equal and opposite thrust is produced. These stages which are used to balance the axial thrust constitute what is called a dummy or balance piston (Fig. 7.57).

**Fig. 7.57** Dummy or balance piston for reaction turbine
7.3.11 Blade Stresses

The determination of blade stresses is a critical factor in the design of blades. The severest stresses are imposed by the centrifugal forces due to high rotative speeds. Bending stresses are also imposed by centrifugal forces, fluid-pressure differences, and vibration.

Centrifugal stresses are a function of the mass of material in the blade, blade length and speed. The component of centrifugal force acting radially outward exerts a tensile stress at the root. Sufficient cross-sectional area must be provided in the blade at the root and a material capable of withstanding the stress without fatigue must also be provided.

The centrifugal force on an element \( dr \) at radius \( r \) (Fig. 7.58) is given by

\[
dF = (\gamma a \, dr) \, w^2 r
\]

(7.102)

where \( \gamma \) = specific weight of blade material, kg/m\(^3\); 
\( a \) = blade cross-sectional area, m\(^2\); and
\( w \) = angular velocity, rad/s

Total centrifugal force exerted at the blade root is

\[
F_c = \int_{r_t}^{r_i} \gamma a \, dr \, \alpha \, r = \frac{\gamma a \omega^2}{2} \left( r_i^2 - r_r^2 \right)
\]

(7.103)

where \( r_t \) = tip radius, and \( r_r \) = root radius.

or

\[
F_c = \frac{\gamma a \omega^2}{2\pi} A = \frac{\gamma a A (2\pi N)^2}{2\pi}
\]

(7.104)

where \( A \) = annular area = \( \pi \left( r_i^2 - r_r^2 \right) \)

The centrifugal or the tensile stress at the blade root is thus

\[
S_c = F_c / a = \gamma A \left( \frac{N}{23.94} \right)^2
\]

(7.105)

If the blade is tapered, the mass of material is reduced, thereby reducing the centrifugal stress. Since the stress exerted at any section of the blading decreases
radially, reaching a minimum near the tip, a constant cross-sectional area is not required for strength. Hence, where the centrifugal stresses are severe the blade is tapered by decreasing both its thickness and width.

Impulse blades are subject to bending from centrifugal stress and the tangential force exerted by the fluid. Reaction blades have an additional bending stress due to large axial thrust because of the pressure drop which occurs in the blades. All turbine blades may be subjected to bending because of vibration. The total stress at a given point on a turbine blade may be found by adding the centrifugal stress at that point to the bending stress.

### 7.3.12 Blade Fastenings

There are a number of methods for fixing turbine blades to the disc or drum. The selected type of fastening must be able to resist the centrifugal and bending forces to which the blades may be subjected. Blades which are loosely fastened to the disc or drum will amplify any vibrations induced in the blades, causing fatigue failure. The failure of one blade may lead to the destruction of the entire turbine.

![Bulb and Shank Fastening](image)

**Fig. 7.59** Bulb and shank fastening

Figure 7.59 shows the bulb and shank type of fastening suitable for low pressure impulse steam turbines. Figure 7.60 illustrates a type of fastening known as straddle-T which is widely used on low pressure impulse turbines. Its modification, simply a straddle fastening (Fig. 7.61) is used for the long low-pressure blades of large steam turbines. The straddle fastening is slipped radially into the slot provided on the disc or drum at a point widened for this purpose. The blades are then pushed along in the slot until the last blade is inserted, whereupon a special stop piece is put in place.

![Straddle-T Attachment](image)

**Fig. 7.60** T-shaped attachment
Fig. 7.61  *Straddle fastening*

A T-shaped attachment is used in the high pressure impulse stages of large turbines (Fig. 7.62). Figure 7.63 shows a fir-tree or Christmas tree attachment which is inserted on the disc axially.

Fig. 7.62  *Straddle-T fastening*

Fig. 7.63  *Fir-tree attachment*
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7.3.15 Critical Speeds

Inspite of all the care taken in the construction and balancing of the turbine shaft and discs, due to some reason or other, the mass centre of the rotor does not coincide with the geometrical axis of the shaft and the distance between the two is known as eccentricity. During the shaft rotation even a small eccentricity gives rise to a transverse force that increases with shaft rpm and tends to deflect the shaft.

![Diagram showing position of centre of gravity of shaft depending on its rpm](image)

(a) $N = 0$  (b) $N < N_{cr}$  (c) $N = N_{cr}$  (d) $N > N_{cr}$  (e) $N = \infty$

**Fig. 7.66** Position of centre of gravity of shaft depending on its rpm

Let us consider a slightly out of balance shaft with an eccentricity $\delta$ (Fig. 7.66). The mass centre of the shaft is at a distance $\delta$ from the geometrical axis of the shaft. We shall consider the shaft to be supported in the vertical direction to avoid the shaft deflection due to its own weight.

The centrifugal force due to rotation is given by

$$F_c = m\omega^2(y + \delta)$$  \hspace{1cm} (7.106)

where $y =$ deflection of rotor, mm;

$m =$ mass of rotor, kg; and

$\omega =$ angular velocity, rad/s.

If $F$ is the stiffness of shaft, i.e. the force that causes the shaft to deflect by 1 mm, then by force balance,

$$F_c = F_y$$  \hspace{1cm} (7.107)

$$\therefore \quad F_y = m\omega^2(y + \delta) \quad \text{or} \quad y = \frac{\delta}{F/(m\omega^2) - 1}$$  \hspace{1cm} (7.108)

From Eq. (7.108) it follows that each value of $\omega$ conforms to a definite deflection $y$, e.g., at $F/m\omega^2 = 1$, $y = \infty$. The angular velocity of the shaft at $m\omega^2 = F$, when $y = \infty$, is known as the *critical velocity*. 

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\[ \omega_{cr} = \left[ \frac{F}{m} \right]^{1/2} \]  

(7.109)

Again,

\[ \omega_{cr} = \frac{2\pi N_{cr}}{60} = \left[ \frac{F}{m} \right]^{1/2} \]

or

\[ N_{cr} = 9.55 \left[ \frac{F}{m} \right]^{1/2} \]  

(7.110)

where \( m \) is the mass of the rotor in kg.

The rpm which numerically coincides with the natural frequency of transverse vibrations of the shaft is known as critical speed. Theoretically, at the critical speed the deflection of the shaft tends to infinity. Thus, operation of turbines at the critical speeds is to be avoided. The normal speed of rotation must not coincide with the critical speed of the shaft, i.e. with its natural frequency. For safe working of the turbine the critical speed should not differ from the normal speed by more than 20%.

Turbine shafts having critical speeds less than normal operating speed are known as flexible shafts, and those with critical speeds higher than the normal operating speed are known as rigid shafts. Rotors of impulse turbines are of both these types, flexible and rigid. If the critical speed is less than the operating speed, then while starting the turbine this speed must be passed over quickly so that there will be hardly any time for the deflection to grow, which, if allowed, could result in a bent shaft and damaged bearings.

At critical speed, the centre of gravity \( S \) coincides with the geometrical axis of the bent shaft (Fig. 7.66c). It is found from theoretical as well as practical considerations that when \( N > N_{cr} \), the c.g. \( S \) would be between the vertical axis and the bent shaft (Fig. 7.66d). In this case, the equilibrium conditions for the forces will be expressed as

\[ m\omega^2 (y - \delta) - F_y = 0 \]

(7.111)

From Eq. (7.109), \( \omega_{cr}^2 = \frac{F}{m} \)

\[ y = \frac{\delta}{1 - \frac{F}{m\omega^2}} \]  

(7.112)

From this equation it is seen that with increase in \( \omega \), \( y \) has a value less than what it has at the critical speed, i.e., as \( \omega \) increases, \( y \) decreases. When \( \omega = \omega_{cr} \), \( y = \infty \) and when \( \omega = \infty \), \( y = \delta \). Therefore, at infinite speed of rotation, the c.g. of the shaft \( S \) (Fig. 7.66e) coincides with the axis of rotation.
It is also seen from Eqs (7.108) and (7.112) that the deflection of the shaft \( y \) is a function of eccentricity \( \delta \) and hence, while balancing the rotor it is advisable to reduce the eccentricity \( \delta \) to the minimum possible.

If the shaft is in horizontal supports (Fig. 7.67), even under static conditions there will be some amount of deflection \( \Delta \) caused by the weight of the shaft and the discs mounted on it. Thus, the shaft will always be slightly bent. Consequently, while in rotation there will be an additional amount of deflection \( y \) and the shaft begins to vibrate relative to static geometric axis (Fig. 7.67). The static deflection is given from

\[
\Delta = \frac{W}{F}
\]

\( (7.113) \)

The static deflection depends on the stiffness of the shaft, the distance between the two supports and the load distribution. For simply supported shafts loaded at the centre, the deflection will be

\[
\Delta = \frac{Wl^3}{48EI}
\]

and for shafts with fixed ends and loaded at the centre

\[
\Delta = \frac{Wl^3}{192EI}
\]

where \( l \) = distance between the two supports
\( E \) = modulus of elasticity of the material of the shaft
\( I \) = moment of inertia of the shaft section, \( \pi d^4/64 \)
\( d \) = shaft diameter.

We may rewrite the expression for shaft deflection as

\[
\Delta = c \frac{Wl^3}{EI}
\]

\( (7.114) \)

where \( c \) is a coefficient depending upon the type of support and the point at which the load is applied (in the cases referred above, \( c = 1/48 \) and \( 1/192 \)).

Having determined the deflection \( \Delta \) the critical speed can be easily determined from Eqs (7.110) and (7.113).
\[ N_{cr} = \frac{9.55}{(\Delta)^{1/2}} \]  

(7.115)

The diameter of the shaft is determined from the critical speed considerations and then checked for mechanical strength. From Eqs (7.114) and (7.115).

\[ \Delta = \left( \frac{9.55}{N_{cr}} \right)^2 = \frac{cWt^3}{EI} \]

\[ I = \frac{cWt^3}{E} \left( \frac{N_{cr}}{9.55} \right)^2 = \pi \frac{d^4}{64} \]

\[ \therefore \quad d = \left[ \frac{64cWt^3}{\pi E} \left( \frac{N_{cr}}{9.55} \right)^2 \right]^{1/4} \]

(7.116)

The critical speed may be assumed for the calculation of shaft diameter. The coefficient \( c \) is obtained from the equations of strength of materials for various types of loading.

1. Other critical speeds  
   At speeds higher than the first critical speed as shown in Fig. 7.68(a), Eq. (7.115), the shaft settles down until the second critical speed is approached when it commences to bend in the curve shown in Fig. 7.68(b) with a nodal point at the centre of its length. Half the length of the shaft is now under the same conditions as the whole shaft when passing through its first critical speed (Fig. 7.68a).

Since deflection \( y \propto l^4 \) and \( N_{cr} \propto 1/\sqrt{y} \)

\[ \therefore \quad N_{cr} \propto 1/l^2 \]

For second critical speed, the length of shaft is \( l/2 \).

\[ \therefore \quad N_{cr2} : N_{cr1} = 4 : 1 \]

Similarly, for the third critical speed, the shaft length is \( l/3 \).

\[ \therefore \quad N_{cr3} : N_{cr1} = 9 : 1 \]

Hence,

\[ N_{cr1} : N_{cr2} : N_{cr3} : N_{cr4} = 1 : 2^2 : 3^2 : 4^2 = 1 : 4 : 9 : 16 \]

Fig. 7.68  Form of whirling shaft at second, third and fourth critical speeds
For further details of critical speed, the book by Kearton (1958) may be consulted.

### 7.3.16 Turning Gear

During shut-down or when the turbine is tripped, there is no steam supply to the turbine. If the turbine which was rotating at 3000 rpm is suddenly stopped, due to the inertia of motion, the rotor may get bent and distorted. There will also be thermal stresses developed due to non-uniform cooling. The reverse happens when the turbine is started. The turbine requires to be heated slowly and its rated speed is reached gradually in several steps.

The turning gear or the barring gear is a mechanism which keeps the turbine shaft rotating at about 1 to 20 rpm to avoid springing the shaft because of unequal expansions and contractions when warming or cooling the turbine. It consists of a gear integral with the turbine shaft which is driven by an electric motor through the necessary speed reduction equipment (Fig. 7.69).

![Diagram of Turning Gear Operation](image)

**Fig. 7.69 Turning gear operation during start-up and shut-down**

During shut-down or when tripped, the turbine is put on turning gear automatically and it keeps on rotating for about two days to cool down gradually and absorb the inertia of motion. Similarly, while starting the turbine, it is put on turning gear first to keep it in rotation at low rpm and then steam is admitted slowly by opening the stop valve till the rated speed is reached. When the first critical speed is approached, steam is admitted quickly to avoid this speed.

### 7.4 ELECTRICAL ENERGY GENERATION

The rotational mechanical energy of the turbine is converted to electrical energy in the generator by the rotation of the rotor’s magnetic field.

The rotation of the turbine turns the rotor of the generator, producing electrical energy in the stator of the generator. The generator rotor consists of a steel forging with slots for conductors that are called the field windings. An electrical direct current is passed through the windings, causing a magnetic field to be formed in the rotor, as shown in Fig. 7.70. This magnetic field is rotated by the turbine. The rotor is surrounded by the stator that includes copper
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The power factor of a generator is defined as the ratio of kW to kVA and is the cosine of the angle describing the angular difference between phase current and phase voltage.

Heat is produced in a generator as a result of resistive losses caused by current flow in the stator and field windings, stator core magnetic losses and windage losses. This heating effect which depends on the load and power factor, is the limiting factor in generator rating. By providing forced cooling of the rotating and stationary components, the generator rating may be increased and the physical size of the components may be made smaller.

Large generators are cooled with hydrogen. The thermal properties of hydrogen (like specific heat and thermal conductivity) are superior to those of air and allow for reduced windage and better cooling. Windage and ventilating losses are lower because of the low density of hydrogen.

The generator losses may be reduced by using hydrogen at higher pressures, say 2 bar. The specific heat of hydrogen is the highest, since its molecular weight is the least,

$$c_p = \frac{\gamma R}{\gamma - 1}$$

Since hydrogen is a diatomic gas, $\gamma = 1.4$. The characteristic gas constant

$$R = \frac{\text{universal gas constant}}{\text{molecular weight}} = \frac{8.3143}{2} = 4.157 \text{ kJ/kg K}$$

$$\therefore (c_p)_H_2 = \frac{1.4 \times 1.457}{0.4} = 14.55 \text{ kJ/kg K}$$

It is 3.47 times the $c_p$ of water.

The rate of heat removal from the armature = $\omega_g c_p (t_e - t_i)$ kW

where $\omega_g$ is the mass flow rate of hydrogen, and $t_e$ and $t_i$ are the exit and inlet temperature of hydrogen, respectively. The warm exiting hydrogen is cooled by water in a heat exchanger.

Since hydrogen gas explodes if it comes in contact with air, CO$_2$ gas, being heavier, is first used to purge the armature, i.e., drive away the air. When all the air has been removed, then only hydrogen gas is fed into the armature to take away the heat generated ($l^2R$).

**Example 7.1** Steam is expanded in a set of nozzles from 10 bar, 300 °C to 2 bar. Are the nozzles convergent or convergent–divergent? Neglecting the initial velocity, find the minimum area of the nozzles to flow 1 kg/s of steam. Assume isentropic expansion.

**Solution**

$$p_0 = 10 \text{ bar}, t_0 = 300^\circ \text{C}$$

Since the state of steam at nozzle inlet is in the superheated state, so the critical pressure $p^* = 0.546 \times 10 = 5.46$ bar.
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\[ h_{3s} = 2605 \text{ kJ/kg} \]

\[ V_1 = 44.72 \left[ (h_1 - h_{2s}) + 0.85(h_{2s} - h_{3s}) \right]^{1/2} \]

\[ h_{3s} = 417.46 + 0.962 \times 2258.01 = 2589.67 \text{ kJ/kg} \]

\[ h_{2s} - h_3 = 0.85(2916.2 - 2589.67) = 277.53 \text{ kJ/kg} \]

\[ h_3 = 2638.67 \text{ kJ/kg} = 417.46 + x_3 \times 2258.01 \]

\[ x_3 = 0.984 \]

\[ v_3 = 0.001043 + 0.984 \times 1.694 = 1.688 \text{ m}^3/\text{kg} \]

By interpolation (at 5.46 bar, 226 °C),

\[ v^* = v_{2s} = 0.416 \text{ m}^3/\text{kg} \]

\[ w_s = 1 \text{ kg/s} = (A^* V^*)/v^* = (A^* \times 521.52)/0.416 \]

\[ A^* = 7.98 \text{ cm}^2 \text{ (minimum area) } \quad \text{Ans.} \]

Also,

\[ 1 \text{ kg/s} = A_1 V_1/v_3 = \frac{A_1 \times 894.79}{1.688} \]

\[ A_1 = 18.64 \text{ cm}^2 = n \times \frac{\pi}{4} (2.5)^2 \]

\[ n = 3.797 = 4 \quad \text{Ans.} \]

**Example 7.3** Air at 7.8 bar and 180 °C expands through a convergent–divergent nozzle into a space at 1.03 bar. The flow rate of air is 3.6 kg/s. Assuming isentropic flow throughout and neglecting the inlet velocity, calculate the throat and exit areas of the nozzles.

**Solution** The critical pressure ratio (Fig. E7.3) for air (\( \gamma = 1.4 \)) is
\[
\frac{p^*}{p_0} = \left( \frac{2}{\gamma + 1} \right)^{\gamma/(\gamma - 1)} = \left( \frac{2}{2.4} \right)^{1.4/0.4} = 0.528
\]

\[p_0 = 7.8 \text{ bar} \quad t_0 = 180 \degree C\]

\[p^* = 0.528 \times 7.8 = 4.12 \text{ bar}\]

\[\frac{T^*}{T_0} = \frac{2}{\gamma + 1} = \frac{1}{1.2}\]

\[T^* = \frac{180 + 273}{1.2} = 377.5 \text{ K}\]

\[v^* = \frac{RT^*}{p^*} = \frac{287 \times 377.5}{4.12 \times 10^5} = 0.263 \text{ m}^3/\text{kg}\]

\[v^* = (\gamma RT^*)^{1/2} = (1.4 \times 287 \times 377.5)^{1/2} = 389.46 \text{ m/s}\]

[Also,]
\[V^* = 44.72 (h_0 - h^*)^{1/2} = 44.72 \times [c_p (T_0 - T^*)]^{1/2}\]
\[= 44.72 \times [1.005 (75.5)]^{1/2} = 390.5 \text{ m/s}\]

\[A^* = \frac{wV^*}{V^*} = \frac{3.6 \times 0.263}{389.46} = 0.002431 \text{ m}^2\]

\[\therefore \text{ Area of throat} = 2431 \text{ mm}^2 \quad \text{Ans.}\]

Now,
\[\frac{T_0}{T_1} = (p_0/p_1)^{(\gamma - 1)/\gamma} = \left( \frac{7.8}{1.03} \right)^{0.4/1.4} = 1.784\]

\[T_1 = \frac{453}{1.784} = 253.9 \text{ K}\]

\[v_1 = \frac{RT_1}{p_1} = \frac{287 \times 253.9}{1.03 \times 10^5} = 0.7075 \text{ m}^3/\text{kg}\]

\[V_1 = 44.72 \times [1.005 (453 - 253.9)]^{1/2} = 634.17 \text{ m/s}\]

\[\therefore \quad A_1 = \frac{wV_1}{V_1} = \frac{3.6 \times 0.7075}{634.17} = 0.004016 \text{ m}^3\]

\[\therefore \quad \text{Exit area} = 4016 \text{ mm}^2 \quad \text{Ans.}\]
Hidden page
\[ T_{1s} = \frac{723}{1.3962} = 517.84 \text{ K} \]

\[ h_n = \frac{(T_0 - T_i)}{(T_0 - T_{1s})} = \frac{723 - T_i}{723 - 517.84} = 0.93 \]

\[ T_1 = 532.2 \text{ K} \]

\[ v_1 = \frac{RT_1}{p_1} = \frac{277.5 \times 532.2}{10^5} = 1.48 \text{ m}^3/\text{kg} \]

\[ V_1 = 44.72 \times [1.11(723 - 532.2)^{1/2}] = 650.8 \text{ m/s} \]

\[ A_1 = \frac{\omega v_1}{V_1} = \frac{16.327 \times 1.48}{650.8} = 0.0371 \text{ m}^2 \]

\[ \therefore \text{ Exit area} = 0.0371 \text{ m}^2 \quad \text{Ans.} \]

**Example 7.5** Steam at 20 bar and 300 °C enters a convergent–divergent nozzle at the rate of 0.3 kg/s with negligible inlet velocity and expands into a space at 3 bar.

(a) Assuming that the steam expands isentropically according to a law \( p v^{1.3} \) = constant, estimate the throat and exit areas of the nozzles without using \( h–s \) chart.

(b) Re-calculate the throat and exit areas of the nozzle using the \( h–s \) chart and taking a coefficient of discharge of 0.98 and a coefficient of velocity as 0.92.

**Solution** At 20 bar, 300 °C, \( v_0 = 0.1255 \text{ m}^3/\text{kg} \)

Since the steam is superheated, \( \frac{p^*}{p_0} = 0.546 \).

\[ \therefore \quad p^* = 0.546 \times 20 = 10.92 \text{ bar} \]

(a) \( p^* \, v^{*k} = p_0 v_0^k \)

\[ v^* = \left( \frac{p_0}{p^*} \right)^{1/k} v_0 = \left( \frac{1}{0.546} \right)^{1/1.3} \times 0.1255 = 0.2 \text{ m}^3/\text{kg} \]
\[ \nu^* = (k p^* \nu^*)^{1/2} = (1.3 \times 10.92 \times 10^5 \times 0.2)^{1/2} = 532.8 \text{ m/s} \]

\[ A^* = \frac{\omega \nu^*}{\nu^*} = \frac{0.3 \times 0.2}{532.8} = 0.0001126 \text{ m}^2 = 112.6 \text{ mm}^2 \]

\[ \nu_1/\nu_0 = (p_0/p_1)^{1/k} = \left(\frac{20}{3}\right)^{1/1.3} = 4.309 \]

\[ \nu_1 = 0.54 \text{ m}^3/\text{kg} \]

\[ \nu_1^2 = \frac{k}{k-1} (p_0\nu_0 - p_1\nu_1) \]

\[ = \frac{1.3 \times 10^5}{0.3} (20 \times 0.1255 - 3 \times 0.54) = 3.857 \times 10^5 \]

\[ \therefore \nu_1 = 878.25 \text{ m/s} \]

\[ \therefore \text{ Exit area, } A_1 = \frac{\omega \nu_1}{\nu_1} = \frac{0.3 \times 0.54}{878.25} = 0.0001844 \text{ m}^2 \]

\[ = 184.4 \text{ mm}^2 \quad \text{Ans.} \]

From Mollier chart, (Fig. E7.5),

\[ h_0 = 3050 \text{ kJ/kg}, \ h^* = 2920 \text{ kJ/kg}, \ h_{1s} = 2650 \text{ kJ/kg} \]

\[ c_d = \omega / \omega_s = 0.98 = 0.3 / \omega_s \]

\[ \omega_s = 0.3061 \text{ kg/s} \]

\[ \nu_s^*, \text{ as before, is } 0.2 \text{ m}^3/\text{kg} \]

\[ V_s^* = 44.72 \sqrt{(3050 - 2920)}^{1/2} = 510 \text{ m/s} \]

\[ V_{1s} = 44.72 \sqrt{(3050 - 2650)}^{1/2} = 894.4 \text{ m/s} \]

Coefficient of velocity, \( \phi = V_1/V_{1s} = V_1/894.4 = 0.92 \)

\[ V_1 = 822.8 \text{ m/s} \]

\[ V_1 = 44.72 \sqrt{(h_0 - h_1)}^{1/2} = 822.8 \text{ m/s} \]
\[(h_0 - h_1) = \left(\frac{822.8}{44.72}\right)^2 = 338.52 \text{ kJ/kg}\]

\[h_1 = 3050 - 338.52 = 2711.48 \text{ kJ/kg}\]

Now,
\[h_1 = h_t + x_i h_{fg}\]
\[2711.48 = 561.47 \times x_i \times 2163.8\]
\[x_i = 0.9936\]
\[v_1 = 0.001073 + 0.9936 \times 0.6047 = 0.6019 \text{ m}^3/\text{kg}\]

The exit area of nozzles is
\[A_1 = \frac{\omega_s v_1}{V_1} = \frac{0.3061 \times 0.6019}{822.8} = 0.000224 \text{ m}^2\]
\[= 22.4 \text{ mm}^2 \quad \text{Ans.}\]

Taking the same velocity coefficient up to the throat
\[\phi = V^*/V_s^* = 0.92\]
\[V^* = 510 \times 0.92 = 469.2 \text{ m/s} = 44.72 (h_0 - h^*)^{1/2}\]
\[h_0 - h^* = (469.2/44.72)^2 = 110.0\]
\[h^* = 3050 - 110 = 2940 \text{ kJ/kg}\]

From Mollier chart, \(v^* = 0.22 \text{ m}^3/\text{kg}\)

The throat area is
\[A^* = \frac{\omega_s v^*}{V^*} = \frac{0.3061 \times 0.22}{469.2}\]
\[= 0.0001435 \text{ m}^2 = 143.5 \text{ mm}^2 \quad \text{Ans.}\]

**Example 7.6** Dry saturated steam at 5 bar enters a convergent–divergent nozzle at a velocity of 100 m/s. The exit pressure is 1.5 bar. The throat and exit areas are 1280 mm\(^2\) and 1600 mm\(^2\), respectively. Assuming isentropic flow up to the throat and taking the critical pressure ratio as 0.58, estimate the mass flow rate and nozzle efficiency.

**Solution** In Fig. E7.6,
\[ h_0 = 2749 \text{ kJ/kg}, \ s_0 = 6.822 \text{ kJ/kg K}, \ p^* = 0.58 \times 5 = 2.9 \text{ bar} \]
\[ s_0 = s^* = 6.822 = 1.660 + 5.344 \ x^* \]
\[ x^* = 0.966 \]
\[ \therefore \quad h^* = 556 + 0.966 \times (2168) = 2650 \text{ kJ/kg} \]

The energy equation gives
\[ h_0 + \frac{V_0^2}{2} = h^* + \frac{V^*^2}{2} \]
\[ 2749 + \frac{100^2}{2} \times 10^{-3} = 2650 + \frac{V^*^2}{2} \times 10^{-3} \]
\[ \therefore \quad V^* = 456 \text{ m/s} \]
\[ v^* = 0.966 \times (0.6253) = 0.6040 \text{ m}^3/\text{kg} \]
\[ \therefore \quad \omega = \frac{A \times V^*}{v^*} = \frac{1280 \times 10^{-6} \times 456}{0.6040} = 0.966 \text{ kg/s} \quad \text{Ans.} \]

Now,
\[ s_{1s} = s^* = s_0 = 6.822 = 1.434 + x_{1s} \times (5.789) \]
\[ \therefore \quad x_{1s} = 0.931 \]
\[ h_{1s} = 467 + 0.931(2226) = 2539 \text{ kJ/kg} \]

Again,
\[ \frac{V_1^2}{2} + h_1 = \frac{V^*^2}{2} + h^* \]
\[ 10^{-3} \times \frac{V_1^2}{2} + h_1 = \frac{456^2}{2} \times 10^{-3} + 2650 = 2754 \text{ kJ/kg} \]
\[ \frac{\omega}{v_1} = \frac{A V_1}{v_1} = \frac{1600 \times 10^{-6} \times V_1}{v_1} = 0.966 \text{ kg/s} \]
\[ V_1/v_1 = 603.8 \]

The exit pressure is 1.5 bar. The solution of Eqs (1) and (2) has to be obtained by an iteration scheme, by assuming \( x_1 \). Let \( x_1 = 0.94, \ v_1 = v_f + x_1 \ v_{fg} \) (at 1.5 bar) = 1.0895 m\(^3\)/kg, \( h_1 = h_f + x_1 \ h_{fg} = 2559 \text{ kJ/kg}, \ V_1 = 657.8 \text{ m/s}. \)

\[ h_1 + (\frac{V_1}{2}) \times 10^{-3} = 2559 + \frac{(657.8)^2}{2} \times 10^{-3} \]
\[ = 2775.35 \text{ kJ/kg (2754 kJ/kg)} \]

Assuming
\[ x_1 = 0.92, \ v_1 = 0.92 \times (1.1583) + 0.001 = 1.067 \text{ m}^3/\text{kg} \]
\[ h_1 = 467.11 + 0.92 \times 2226.5 = 2515.5 \text{ kJ/kg} \]
\[ V_1 = 644.3 \text{ m/s} \]
\[ \therefore \quad h_1 + (\frac{V_1}{2}) \times 10^{-3} = 2723.1 \text{ kJ/kg (2754 kJ/kg)} \]

Finally, when
\[ x_1 = 0.932, \ v_1 = 1.080 \text{ m}^3/\text{kg}, \ h_1 = 2542 \text{ kJ/kg} \]
\[ V_1 = 652.2 \text{ m/s}, \ h_1 + (\frac{V_1}{2}) \times 10^{-3} \]
\[ h_{\text{nozzle}} = \frac{h^*-h_1}{h^*-h_{ts}} = \frac{2650 - 2542}{2650 - 2539} = \frac{108}{111} = 0.973 \quad \text{Ans.} \]

**Example 7.7** A convergent–divergent nozzle receives steam at 5 bar, 200 °C and expands isentropically into a space at 2 bar. Neglecting the inlet velocity, calculate the exit area required for a mass flow of 0.3 kg/s in the following cases.

(a) when the flow is in equilibrium throughout,
(b) when the flow is supersaturated with \( p_0^{1/3} = \text{constant} \). Calculate also for part (b):
(i) the degree of supercooling
(ii) the degree of supersaturation.

**Solution** (a) From steam tables, with reference to Fig. E7.7.

![Fig. E7.7](image)

\[ v_0 = 0.4249 \text{ m}^3/\text{kg}, \quad h_0 = 2855.4 \text{ kJ/kg} \]
\[ s_0 = 7.0592 \text{ kJ/kg K} = s_1 = 1.4336 + x_1 \text{ (5.79)} \]
\[ x_1 = 0.972, \quad h_1 = 504.7 + 0.972(2201.9) = 2645 \text{ kJ/kg} \]
\[ v_1 = 0.972 \times 0.8857 = 0.86 \text{ m}^3/\text{kg} \]
\[ V_1 = 44.72 \text{ (} h_0 - h_1 \text{)}^{1/2} = 44.72 \text{ (2855.4} - 2645 \text{)}^{1/2} \]
\[ = 648.67 \text{ m/s} \]
\[ \therefore \quad \omega_s = \frac{A_1 V_1}{v_1} = \frac{A_1 \times 648.67}{0.86} = 0.3 \text{ kg/s} \]

The exit area is

\[ A_1 = 0.000398 \text{ m}^2 = 398 \text{ mm}^2 \quad \text{Ans. (a)} \]

(b)
\[ dh = Tds + v \, dp = v \, dp \quad (1) \]
\[ h_0 = h + \frac{V^2}{2} = \text{constant} \]
\[ dh = -V \, d \, V \quad (2) \]
Equating (1) and (2),
\[ V \, dV = -v \, dp \]
From \( pv^{1.3} = p_1 v_1^{1.3} = \text{constant} = C = pv^n, n = 1.3 \)
\[
\int_{V_0}^{V_R} V \, dV = \int_{p_0}^{p} \left( \frac{C}{p} \right)^{1/n} \, dp = \int_{p_0}^{p} (C^{1/n}) p^{-1/n} \, dp
\]
\[
\frac{V_R^2 - V_0^2}{2} = -C^{1/n} \left( \frac{n}{n-1} \right) (p_1^{1-1/n} - p_0^{1-1/n})
\]
Since \( V_0 = 0, \)
\[
\frac{V_R^2}{2} = \frac{n}{n-1} (p_0 v_0 - p_1 v_R)
\]
Now,
\[
\frac{v_R}{v_0} = \left( \frac{p_0}{p_1} \right)^{1/n} = \left( \frac{5}{2} \right)^{1/1.3} = \left( \frac{5}{2} \right)^{0.769} = 2.023
\]
\[
\therefore \quad v_R = 2.023 \times 0.4249 = 0.86 \, \text{m}^3/\text{kg}
\]
\[
\frac{v_R^2}{2} = \frac{1.3}{0.3} [500 \times 0.4249 - 200 \times 0.86]
\]
\[= 175.28 \, \text{kJ/kg} \times 1000 \, \text{J/kJ} \]
\[V_R = 592 \, \text{m/s} \]
\[A_1 = \text{exit area} = \frac{\omega v_R}{V_R} = \frac{0.3 \times 0.86}{592} \]
\[= 0.0004358 \, \text{m}^2 = 435.8 \, \text{mm}^2 \quad \text{Ans.} \]
\[
\frac{T_0}{T_R} = \left( \frac{p_0}{p_1} \right)^{(n-1)/n} = \left( \frac{5}{2} \right)^{0.3/1.3} = (2.5)^{0.23} = 1.2346
\]
\[
T_R = 473/1.2346 = 383.12 \, \text{K} \]
\[t_R = 110.12 \, ^\circ\text{C} \]
\[(t_{sat})_{p_1} = 120.23 \, ^\circ\text{C} \]
\[
\therefore \quad \text{Degree of subcooling} = 120.23 - 110.12 = 10.11 ^\circ\text{C} \quad \text{Ans.}
\]
\[
(p_{sat})_{T_R} = 110 ^\circ\text{C} = 1.4327 \, \text{bar}
\]
\[
\therefore \quad \text{Degree of supersaturation} = \frac{2 \, \text{bar}}{1.4327 \, \text{bar}} = 1.396 \quad \text{Ans.}
\]

**Example 7.8** Recalculate the exit area, assuming a nozzle efficiency of 0.92 and a mean \( c_p = 1.925 \, \text{kJ/kg K} \) for supersaturated steam. Check the answer by assuming \( pv \times 10^3 = 2.308 \, (h - 1943) \), where \( p \) is in bar, \( v \) in \( \text{m}^3/\text{kg} \) and \( h \) in \( \text{kJ/kg} \).

**Solution** As shown in Fig. 7.8,
\[ h_n = \frac{h_0 - h_Q}{h_0 - h_1} \]

Now,
\[ h_0 - h_1 = 2855.4 - 2645 = 210.4 \]
\[ h_0 - h_Q = 0.92 \times 210.4 = 193.57 \text{ kJ/kg} \]
\[ V_Q = 44.72 \times (193.57)^{1/2} = 622.2 \text{ m/s} \]
\[ h_0 - h_Q = c_p(t_0 - t_Q) = 193.57 \]

**Fig. E7.8**

\[ t_0 - t_Q = \frac{193.57}{1.925} = 100.56 \text{ °C} \]
\[ t_Q = 200 - 100.56 = 99.44 \text{ °C} = 372.44 \text{ K} \]
\[ \frac{p_0 v_Q}{T_Q} = \frac{p_0 v_0}{T_0} = \frac{500 \times 0.4249}{473} \]

\[ v_Q = \frac{500 \times 0.4249 \times 372.44}{473 \times 200} = 0.8364 \text{ m}^3/\text{kg} \]

\[ A_1 = \frac{0.3 \times 0.8364}{622.2} = 0.0004033 \text{ m}^2 = 403.3 \text{ mm}^2 \quad \text{Ans.} \]

Using approximate formula (Eastop and McConkey, 1986)

\[ v_Q = \frac{2.308(h_Q - 1943)}{10^3 \times p_Q} \text{ m}^3/\text{kg} \]

where \( p_Q \) is in bar and \( h_Q \) is in kJ/kg

\[ v_Q = \frac{2.308(2855.4 - 193.6 - 1943)}{10^3 \times 2} \text{ m}^3/\text{kg} = 0.8295 \text{ m}^3/\text{kg} \]

\[ A_1 = \frac{0.3 \times 0.8295}{622.2} = 0.00039995 \text{ m}^2 = 400 \text{ mm}^2 \quad \text{Ans.} \]
Example 7.9 The velocity of steam entering a simple impulse turbine is 1000 m/s, and the nozzle angle is $20^\circ$. The mean peripheral velocity of blades is 400 m/s and the blades are symmetrical. If the steam is to enter the blades without shock, what will be the blade angles?

(a) Neglecting the friction effects on the blades, calculate the tangential force on the blades and the diagram power for a mass flow of 0.75 kg/s. Estimate also the axial thrust and diagram efficiency.

(b) If the relative velocity at exit is reduced by friction to 80% of that at inlet, estimate the axial thrust, diagram power and diagram efficiency.

**Solution**

Given: $V_1 = 1000$ m/s, $V_b = 400$ m/s, $\alpha = 20^\circ$, $\beta_1 = \beta_2$, $\omega_s = 0.75$ kg/s (Fig. E7.9)

![Fig. E7.9](image)

(a) $k_b = 0$

$V_{r1} \sin \beta_1 = V_1 \sin \alpha$

$V_{r2} \cos \beta_1 = V_1 \cos \alpha - V_b$

$\beta_1 = \tan^{-1}\left[\frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b}\right]$

$\beta_1 = \tan^{-1}\left[\frac{342}{940 - 400}\right] = 32.35^\circ = \beta_2$

Ans.

$V_{r1} \sin 32.35^\circ = 342$

$V_{r1} = 639.25$ m/s = $V_{r2}$

$\Delta V_w = V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2 = 2V_{r1} \cos \beta_1 = 2 \times 639.25 \times \cos 20^\circ = 1080.07$ m/s

$\Delta V_a = V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2 = 0$

Tangential thrust is $P_t = \omega_s \Delta V_\alpha = 0.75 \times 1080.07 = 810.05$ N

Diagram power, $\dot{W}_D = P_t \times V_b = 810.05 \times 400 = 324.02$ kW

Ans.
Diagram efficiency, \( \eta_D = \frac{324.02 \text{ kW}}{\frac{1}{2} \times 0.75 \times 1000^2 \times 10^{-3} \text{ kW}} \)

\[ = 0.864 \quad \text{or} \quad 86.4\% \quad \text{Ans.} \]

Axial thrust, \( P_a = \omega_s \Delta V_a = 0 \quad \text{Ans.} \)

(b) \( k_b = 0.8 \)

\( V_{r2} = 0.8 \ V_{r1} = 0.8 \times 639.25 = 511.4 \text{ m/s} \)

\( \Delta V_w = 639.25 \cos 32.35^\circ + 511.4 \cos 32.35^\circ = 972.06 \text{ m/s} \)

Axial thrust, \( P_a = \omega_s (V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2) \)

\[ = 0.75 \times 127.85 \sin 32.35^\circ = 51.3 \text{ N} \quad \text{Ans.} \]

Diagram power, \( \dot{W}_D = 0.75 \times 972.06 \times 400 = 291.62 \text{ kW} \quad \text{Ans.} \)

Diagram efficiency, \( \eta_D = \frac{291620}{\frac{1}{2} \times 0.75 \times 1000^2} \)

\[ = 0.7776 \quad \text{or} \quad 77.76\% \quad \text{Ans.} \]

**Example 7.10** An impulse steam turbine has a number of pressure stages, each having a row of nozzles and a single ring of blades. The nozzle angle in the first stage is 20° and the blade exit angle is 30° with reference to the plane of rotation. The mean blade speed is 130 m/s and the velocity of steam leaving the nozzles is 330 m/s.

(a) Taking the blade friction factor as 0.8 and a nozzle efficiency of 0.85, determine the work done in the stage per kg of steam and the stage efficiency.

(b) If the steam supply to the first stage is at 20 bar, 250 °C and the condenser pressure is 0.07 bar, estimate the number of stages required, assuming that the stage efficiency and the work done are the same for all stages and that the reheat factor is 1.06.

**Solution**

(a) Given: \( \alpha = 20^\circ, \beta_2 = 30^\circ, V_b = 130 \text{ m/s}, V_1 = 330 \text{ m/s} \) (Fig. E7.10a)

![Fig. E7.10(a)](image-url)
\[
\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{330 \sin 20^\circ}{330 \cos 20^\circ - 130} = 0.627
\]

\[\beta_1 = 32.075^\circ\]

\[V_{r1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = \frac{330 \times 0.342}{0.531} = 212.53 \text{ m/s}\]

\[V_{r2} = 0.8 \times 212.53 = 170.025 \text{ m/s}\]

\[\Delta V_w = V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2\]

\[= 212.53 \cos 32.075^\circ + 170.025 \cos 30^\circ = 372.334 \text{ m/s}\]

\[W_D = \omega_s \Delta V_w V_b = 1 \times 327.334 \times 130 = 42.55 \text{ kJ/kg} \quad \text{Ans.}\]

\[\eta_{b1} = \eta_D = \frac{2 \Delta V_w V_b}{V_1^2} = \frac{2 \times 327.334 \times 130}{330 \times 330} = 0.7815 \quad \text{or} \quad 78.15\%\]

\[\eta_{\text{stage}} = \eta_h \times \eta_{b1} = 0.85 \times 0.7815 = 0.664 \quad \text{or} \quad 66.4\% \quad \text{Ans.}\]

\[\eta_{\text{internal}} = \eta_{\text{stage}} \times \text{reheat factor} = 0.664 \times 1.06\]

\[= 0.7041 \quad \text{or} \quad 70.41\%\]

\[h_1 = 2902.3 \text{ kJ/kg (Fig. E7.10b)}\]

---

**Fig. E7.10(b)**

\[s_1 = 6.5466 = s_{2s} = 0.5582 + x_{2s} 7.7198\]

\[x_{2s} = 0.7757\]

\[h_{2s} = 163.16 + 0.7757 (2409.54) = 2032.29 \text{ kJ/kg}\]

\[h_1 - h_2 = 0.7041 (h_1 - h_{2s}) = 0.7041 (2902.3 - 2032.3)\]

\[= 612.57 \text{ kJ/kg}\]

\[\therefore \text{ Number of stages, } n = \frac{(\Delta h)_{\text{total}}}{(\Delta h)_{\text{stage}}} = \frac{612.57}{42.55}\]

\[= 14.39 \quad \text{or} \quad 15 \text{ stages} \quad \text{Ans. (b)}\]
Example 7.11 In a stage of an impulse turbine provided with a single row wheel, the mean diameter of the blade ring is 800 mm and the speed of rotation is 3000 rpm. The steam issues from the nozzles with a velocity of 300 m/s and the nozzle angle is 20°. The rotor blades are equiangular and the blade friction factor is 0.86. What is the power developed in the blading when the axial thrust on the blades is 140 newtons?

Solution

\[ V_b = \frac{\pi D_mN}{60} = \frac{\pi \times 0.80 \times 3000}{60} = 125.6 \text{ m/s} \]

\[ V_1 = 300 \text{ m/s}, \alpha = 20\degree \]

\[ \tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{300 \sin 20\degree}{300 \cos 20\degree - 125.6} = \frac{102.61}{281.91 - 125.6} \]

\[ = 0.6565 \]

\[ \beta_1 = 33.3\degree = \beta_2 \]

\[ V_1 \sin \alpha = V_{r1} \sin \beta_1 \]

\[ 102.61 = V_{r1} \sin 33.3\degree \]

\[ \therefore \]

\[ V_{r1} = 187 \text{ m/s} \]

\[ V_{r2} = 0.86 \times 187 = 161 \text{ m/s} \]

Axial thrust,

\[ P_a = \omega_s (V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2) \]

\[ = \omega_s V_{r1} \sin \beta_1 (1 - k_b) \]

\[ = \omega_s \times 187 \sin 33.3\degree (1 - 0.86) \]

\[ = \omega_s \times 14.3654 = 140 \text{ N} \]

\[ \therefore \]

\[ \omega_s = 9.7456 \text{ kg/s} \]

\[ \Delta V_w = V_{r2} \cos \beta_2 + V_{r1} \cos \beta_1 = V_{r1} \cos \beta_1 (1 + k_b) \]

\[ = 187 \cos 33.3\degree \times 1.86 = 290.71 \text{ m/s} \]

\[ \therefore \]

\[ \text{Power developed} = 9.7456 \times 290.71 \times 125.6 \times 10^{-3} \]

\[ = 355.84 \text{ kW} \]

Ans.

Example 7.12 The nozzles of the impulse stage of a turbine receive steam at 15 bar and 300 °C and discharge it at 10 bar. The nozzle efficiency is 95% and the nozzle angle is 20°. The blade speed is that required for maximum work, and the inlet angle of the blades is that required for entry of the steam without shock. The blade exit angle is 5° less than the inlet angle. The blade friction factor is 0.9. Calculate for a steam flow of 1350 kg/h, (a) the axial thrust. (b) the diagram power, and (c) the diagram efficiency.

Solution

\[ h_1 = 3038.9 \text{ kJ/kg} \text{ (Fig. E7.12(a) & (b))} \]
Fig. E7.12(a)

\[ s_1 = 6.9224 \text{ kJ/kg K} \]

\[ (s_g)_{10 \text{ bar}} = 6.5828 \text{ kJ/kg K} \]

\[ s_{2s} > (s_g)_{10 \text{ bar}} \]

Fig. E7.12(b)

The state "2s" is in the superheated region

\[ t_{2s} = 250 \degree C, \ h_{2s} = 2943.1 \text{ kJ/kg} \]

\[ V_1 = 44.72 \left[ (3038.9 - 2943.1)0.95 \right]^{1/2} = 426.63 \text{ m/s} \]

\[ \frac{V_b}{V_1} = \frac{\cos \alpha}{2} = \frac{\cos 20^\circ}{2} = 0.4699 \]

\[ V_b = 426.63 \times 0.4699 = 200.45 \text{ m/s} \]

\[ \tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{145.916}{200.45} = 0.7279 \]

\[ \beta_1 = 36^\circ \]

\[ \beta_2 = 36 - 5 = 31^\circ \]

\[ V_{r1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = 248.25 \text{ m/s} \]
\[ V_{r2} = k_b V_{r1} = 223.42 \text{ m/s} \]
\[ \Delta V_{\omega} = V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2 = 392.35 \text{ m/s} \]
\[ \Delta V_a = V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2 = 30.85 \text{ m/s} \]

(a) Axial thrust, \[ P_a = \frac{1350}{3600} \times 30.85 = 11.57 \text{ N} \quad \text{Ans.} \]

Tangential thrust, \[ P_t = \frac{1350}{3600} \times 392.35 = 147.13 \text{ N} \]

(b) Diagram power, \[ \dot{W}_D = 147.13 \times 200.45 \times 10^{-3} \]
\[ = 29.492 \text{ kW} \quad \text{Ans.} \]

(c) Diagram efficiency \[ \eta_D = \frac{29492}{\frac{1}{2} \times \frac{1350}{3600} \times (426.63)^2} \]
\[ = 0.864 \quad \text{or} \quad 86.4\% \quad \text{Ans.} \]

**Example 7.13** The following particulars refer to a two-row velocity-compounded impulse wheel:

Steam velocity at nozzle exit = 600 m/s
Nozzle angle = 16°
Mean blade velocity = 120 m/s
Exit angles: first row moving blades = 18°, fixed guide blades = 22°,
second row moving blades = 36°
Steam flow = 5 kg/s
Blade friction coefficient = 0.85

Determine (a) the tangential thrust, (b) the axial thrust, (c) the power developed, and (d) the diagram efficiency.

**Solution**
\[ V_b = 120 \text{ m/s}, \ V_1 = 600 \text{ m/s}, \ \alpha = 16^\circ, \ \beta_2 = 18^\circ, \ k_b = 0.85, \ \alpha_1 = 22^\circ, \ \beta_4 = 36^\circ \]
(Fig. E7.13)
\[
\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{600 \sin 16^\circ}{600 \cos 16^\circ - 120} = \frac{165.382}{576.757 - 120} = 0.362 \\
\beta_1 = 19.9^\circ \\

V_{r_1} = V_1 \frac{\sin \alpha}{\sin \beta_1} = \frac{165.382}{0.34} = 485.78 \text{ m/s} \\
V_{r_2} = 0.85 \times 485.78 = 412.91 \text{ m/s} \\

\tan \delta_1 = \frac{V_{r_2} \sin \beta_2}{V_{r_2} \cos \beta_2 - V_b} = \frac{412.9 \sin 18^\circ}{412.9 \cos 18^\circ - 120} = \frac{127.593}{272.69} = 0.468 \\
\therefore \delta_1 = 25.1^\circ \\

V_2 = \frac{127.593}{\sin 25.1^\circ} = 301.06 \text{ m/s} \\
V_3 = 0.85 \times 301.06 = 255.9 \text{ m/s} \\
\Delta V_{w_1} = V_{r_1} \cos \beta_1 + V_{r_2} \cos \beta_2 = V_1 \cos \alpha + V_2 \cos \delta_1 \\
= 576.757 + 301.06 \cos 25.1^\circ = 849.44 \text{ m/s} \\
\Delta V_{a_1} = V_1 \sin \alpha - V_2 \sin \delta_1 = 165.382 - 301.06 \sin 25.1^\circ \\
= 37.79 \text{ m/s} \\

\tan \beta_3 = \frac{V_3 \sin \alpha_1}{V_3 \cos \alpha - V_b} = \frac{255.9 \sin 22^\circ}{255.9 \cos 22^\circ - 120} \\
= \frac{95.86}{117.266} = 0.8175 \\
\beta_3 = 39.26^\circ \\

V_{r_3} = V_3 \frac{\sin \alpha_1}{\sin \beta_3} = \frac{95.86}{\sin 39.26^\circ} = 151.46 \text{ m/s} \\
V_{r_4} = 0.85 \times 151.46 = 128.74 \text{ m/s} \\
\Delta V_{w_2} = V_{r_3} \cos \beta_3 + V_{r_4} \cos \beta_4 \\
= 151.46 \cos 39.26^\circ + 128.74 \cos 36^\circ = 221.42 \text{ m/s} \\
\Delta V_{a_2} = V_3 \sin \alpha_1 - V_{r_4} \sin \beta_4 = 93.86 - 75.67 = 20.19 \text{ m/s} \\
\Sigma \Delta V_w = \Delta V_{w_1} + \Delta V_{w_2} = 849.44 + 221.42 = 1070.86 \text{ m/s} \\
\Sigma \Delta V_a = \Delta V_{a_1} + \Delta V_{a_2} = 37.79 + 20.19 = 57.98 \text{ m/s} \\

(a) Tangential thrust, \\
P_1 = \omega_s \Sigma \Delta V_w = 5 \times 1070.86 \times 10^{-3} \text{ kN} \\
= 5.354 \text{ kN} \quad \text{Ans.}
(b) Axial thrust,
\[ P_a = \omega_a \Sigma \Delta V_a = 5 \times 57.98 \times 10^{-3} \text{ kN} \]
\[ = 0.29 \text{ kN} \quad \text{ Ans.} \]

(c) Power developed
\[ \dot{W}_D = P_t \times V_b = 5.354 \times 120 \]
\[ = 642.48 \text{ kW} \quad \text{ Ans.} \]

(d) Diagram efficiency,
\[ \eta_D = \frac{2 \Delta V_a V_b}{V_1^2} = \frac{2 \times 1070.86 \times 120}{600 \times 600} \]
\[ = 0.7139 \quad \text{ or } \quad 71.39\% \quad \text{ Ans.} \]

Example 7.14 The following particulars apply to a two-row velocity compounded impulse stage of a turbine: nozzle angle 17°, mean blade speed 125 m/s; exit angles of the first row moving blades, the fixed blades, and the second row moving blades 22°, 26° and 30°, respectively; blade friction factor for each row 0.9. Assume that the absolute velocity of steam leaving the stage is in the axial direction. Draw the velocity diagrams for the stage and obtain (a) the absolute velocity of steam leaving the stage, (b) the diagram work, and (c) the diagram efficiency.

**Solution**
\[ \alpha = 17^\circ, \ V_b = 125 \text{ m/s}, \ \beta_2 = 22^\circ, \ \alpha_1 = 26^\circ, \ \beta_4 = 30^\circ, \ k_b = 0.9, \ \delta_2 = 90^\circ \]

At first, the velocity diagrams for the second row of moving blades are drawn to scale (say, 1 cm = 50 m/s). Then those for the first row are drawn with the given particulars. (Fig. E7.14).

\[ \Delta V_{w_1} \quad \text{and} \quad \Delta V_{w_2} \quad \text{are measured from the diagrams.} \]

\[ \Delta V_{w_1} + \Delta V_{w_2} = (16 \text{ cm} + 4.8 \text{ cm}) \times 50 \frac{\text{m/s}}{\text{cm}} = 1040 \text{ m/s} \]
\[ V_1 = 11.5 \text{ cm} \times 50 \frac{\text{m/s}}{\text{cm}} = 575 \text{ m/s} \]

(a) Velocity of steam exiting the stage, \( V_4 \) as read from the diagram

\[ = 1.5 \text{ cm} \times 50 \frac{\text{m/s}}{\text{cm}} = 75 \text{ m/s} \quad \text{Ans.} \]

(b) Diagram work,

\[ W_D = 1040 \times 1255 = 130,000 \text{ J/kg} \]

\[ = 130 \text{ kJ/kg} \quad \text{Ans.} \]

(c) Diagram efficiency,

\[ \eta_D = \frac{130,000}{\frac{1}{2} \times 575 \times 575} \]

\[ = 0.7863 \quad \text{or} \quad 78.63\% \quad \text{Ans.} \]

**Example 7.15** An impulse steam turbine is supplied with steam at 35 bar, 350 °C, the condenser pressure being 0.07 bar. The first stage of the turbine is velocity compounded with two rings of moving blades separated by a ring of fixed guide blades. The isentropic enthalpy drop for this stage is \( 1/4 \) of that for the whole turbine. The nozzle angle is 20° and the nozzle efficiency is 88%. The mean blade velocity of both the moving rings of blades is 0.2 of the velocity of steam leaving the nozzle. The exit blade angles for both fixed and moving blades are 30° and the blade friction coefficient for all blades is 0.9. If the internal efficiency of the turbine is 75%, calculate the efficiency of the first stage and the percentage of the total power developed by the turbine in this stage.

**Solution** From steam tables, (Fig. E7.15), \( h_1 = 3106.4, \ s_1 = 6.6643 \text{ kJ/kg} \)

\[ K = 0.5582 + x_{2s} \times 7.7198 \]
\[ x_{2s} = 0.791 \]
\[ h_{2s} = 163.16 + 0.791 \times 2409.54 = 2069.11 \text{ kJ/kg} \]
\[ (\Delta h_s)_{\text{total}} = h_1 - h_{2s} = 1037.29 \text{ kJ/kg} \]

Enthalpy drop in the 2-row velocity or Curtis stage is

\[ h_1 - h_{3s} = \frac{1}{4} (\Delta h_s)_{\text{total}} = \frac{1}{4} \times 1037.29 = 259.32 \text{ kJ/kg} \]

\[ h_1 - h_3 = 0.88 \times 259.32 = 228.2 \text{ kJ/kg} \]

\[ \therefore \text{Velocity of steam leaving the nozzles}, \]
\[ V_1 = 44.72 \ (228.2)^{1/2} = 675.56 \text{ m/s} \]
\[ V_b = 0.2 \times 675.56 = 135.112 \text{ m/s} \]
\[ \alpha = 20^\circ, \beta_2 = \beta_4 = 30^\circ, k_b = 0.9 \]
\[ \tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{231.055}{499.709} \]
\[ \therefore \beta_1 = 24.815^\circ \]
\[ V_{r_1} = \frac{231.055}{\sin 24.815^\circ} = 550.54 \text{ m/s} \]
\[ V_{r_2} = 0.9 \times 550.54 = 495.486 \text{ m/s} \]
\[ \Delta V_{w_1} = V_{r_1} \cos \beta_1 + V_{r_2} \cos \beta_2 = 497.709 + 429.10 \]
\[ = 928.81 \text{ m/s} \]
\[ V_2 \cos \delta_1 = V_{r_2} \cos \beta_2 - V_b = 429.10 - 135.11 = 294 \text{ m/s} \]
\[ V_2^2 = (V_{r_2} \sin \beta_2)^2 + (V_{r_2} \cos \beta_2 - V_b)^2 \]
\[ = 61376.59 + (294)^2 \]
\[ \therefore \]
\[ V_2 = 384.46 \text{ m/s} \]
\[ V_3 = 0.9 \times 384.46 = 346 \text{ m/s} \]
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\[ V_b = \frac{\pi D_m N}{60} = \frac{\pi \times 0.67 \times 1500}{60} = 52.62 \text{ m/s} \]

By sine law,
\[ \frac{V_1}{\sin 145^\circ} = \frac{V_b}{\sin 15^\circ} = \frac{V_{r_1}}{\sin 20^\circ} \]

\[ V_1 = 52.62 \times \frac{0.5736}{0.2588} = 116.63 \text{ m/s} = V_{r_2} \]
\[ V_{r_1} = 52.62 \times \frac{0.342}{0.2588} = 69.54 \text{ m/s} = V_2 \]

\[ \Delta V_w = V_1 \cos \alpha + V_2 \cos \delta = 116.63 \cos 20^\circ + 69.54 \cos 35^\circ \]
\[ = 109.6 + 56.964 = 166.564 \text{ m/s} \]
\[ v_1 = 0.001052 + 0.96 \times 1.15937 = 1.114 \text{ m}^3/\text{kg} \]

Now,
\[ \omega_s = 3.6 \text{ kg/s} \]
\[ = \frac{\pi D_m h_b V_1 \sin \alpha}{v_1} = \frac{\pi \times 0.67 \times h_b \times 116.63 \sin 20^\circ}{1.114} \]
\[ \therefore \quad h_b = 0.0478 \text{ m} = 47.8 \text{ mm} \quad \text{Ans.} \]

Power developed by the ring
\[ = \omega_s \Delta V_w V_b = 3.6 \times 166.564 \times 52.62 \times 10^{-3} \]
\[ = 31.552 \text{ kW} \quad \text{Ans.} \]

**Example 7.17** A Parsons reaction (50%) turbine running at 400 rpm develops 5 MW using 6 kg/kWh of steam flow. The exit angle of the blades is 20° and the velocity of steam relative to the blades at exit is 1.35 times the mean blade speed. At a particular stage in the expansion the pressure is 1.2 bar and the steam quality is 0.95. Calculate for this stage (a) a suitable blade height, assuming the ratio of \( D_m/h_b \) as 12, and (b) the diagram power.

**Solution**
\[ V_{r_1} = V_2, \quad V_1 = V_{r_2} = 1.35 \quad V_b, \quad \beta_1 = \delta, \quad \beta_2 = \alpha = 20^\circ \quad \text{(Fig. E7.17)} \]

**Fig. E7.17**
\[ \omega_s = 6 \text{ kg/kWh} = \frac{6 \times 5000}{3600} = \frac{25}{3} \text{ kg/s} \]

\[ V_b = \frac{\pi D_m N}{60} = \frac{\pi \times 12 h_b \times 400}{60} = 80 \pi h_b \]

\[ V_1 = 1.35 \times 80 \pi h_b = 108 \pi h_b \]

\[ v_1 \text{ at 1.2 bar, 0.95 dry} = 0.0010468 + 0.95 (1.454) = 1.381 \text{ m}^3/\text{kg} \]

Volume flow of steam,

\[ \omega_s v_1 = \pi D_m h_b V_1 \sin \alpha \eta \]

\[ \frac{25}{3} \times 1.381 = \pi (12 h_b) h_b (108 \pi h_b) \sin 20^\circ \]

\[ \therefore \quad h_b = 0.138 \text{ m} = 138 \text{ mm} \quad \text{Ans. (a)} \]

\[ V_b = 80 \pi \times 0.138 = 34.67 \text{ m/s} \]

\[ V_1 = 1.35 \times 34.67 = 46.8 \text{ m/s} \]

\[ \Delta V_w = 2V_1 \cos \alpha - V_b = 2 \times 46.8 \cos 20^\circ - 34.67 = 53.28 \text{ m/s} \]

Diagram power,

\[ \dot{W}_D = \omega_s \Delta V_w V_b \]

\[ = \frac{25}{3} \times 53.28 \times 34.67 \times 10^{-3} \text{ kW} \]

\[ = 15.39 \text{ kW} \quad \text{Ans. (b)} \]

**Example 7.18** The speed of rotation of a blade group of a 50% reaction turbine is 3000 rpm. The mean blade speed is 100 m/s. The velocity ratio is 0.56 and the exit angle of the blades is 20°. If the mean specific volume of the steam is 0.65 m³/kg and the mean height of the blade is 25 mm, calculate the mass flow of steam through the turbine in kg/h. Neglect the effect of blade thickness on the annulus area.

If there are five pairs of blades in the group, calculate the useful enthalpy drop required and the diagram power.

**Solution**

Given: \( V_{r2} = V_1, V_{r1} = V_2, \alpha = \beta_2 = 20^\circ \) (Fig. E7.18)
\[ V_b = 100 \text{ m/s}, \quad V_b/V_1 = 0.56 \]

\[ V_1 = \frac{100}{0.56} = 178.57 \text{ m/s} = V_{r_2} \]

\[ h_b = 25 \text{ mm}, \quad v = 0.65 \text{ m}^3/\text{kg}, \quad N = 3000 \text{ rpm} \]

\[ V_b = \frac{\pi D_m N}{60} = \frac{\pi D_m \times 3000}{60} = 100 \; \text{m/s} \]

\[ D_m = \frac{2}{\pi} \text{ m} \]

\[ \omega_b = \pi D_m h_b k_{ib} V_1 \sin \alpha \]

\[ \therefore \quad \omega_b = \frac{2 \times 0.025 \times 1 \times 178.57 \sin 20^\circ}{0.65} = 4.698 \text{ kg/s} \]

\[ = 16,912.8 \text{ kg/h} \quad \text{Ans.} \]

\[ \tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{178.57 \sin 20^\circ}{178.57 \cos 20^\circ - 100} = 61.075 \]

\[ \beta_1 = 42^\circ \]

\[ V_{r_1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = \frac{61.075}{\sin 42^\circ} = 91.25 \text{ m/s} \]

\[ \Delta h_{mb} = \frac{1}{2} \left( V_{r_2}^2 - V_{r_1}^2 \right) \left( V_{r_2} + V_{r_1} \right) \left( V_{r_2} - V_{r_1} \right) \frac{1}{2} \]

\[ = \frac{(178.57 + 91.25)(178.57 - 91.25)}{2} \]

\[ \therefore \quad \Delta h_{stage} = \Delta h_{th} + \Delta h_{mb} = 2 \Delta h_{mb} = (269.82 \times 87.32) \times 10^{-3} \]

\[ = 23.56 \text{ kJ/kg} \]

For 5 pairs of blades,

\[ (\Delta h)_{total} = 5 \times 23.56 = 117.80 \text{ kJ/kg} \]

\[ \therefore \quad \text{Diagram power} = 4.698 \times 117.8 = 553.4 \text{ kW} \quad \text{Ans.} \]

**Example 7.19** A steam turbine is to develop 8 MW at 5000 rpm for driving a compressor. The steam enters at 40 bar, 500 °C and exhausts at 0.1 bar. The internal efficiency of the turbine is 0.85 and its mechanical efficiency is 0.96. Estimate (a) the number of impulse stages required, if similar impulse stages are used throughout, (b) the nozzle height for the first stage with full admission. Assume nozzle efficiency as 0.92, nozzle angle 15°, limiting blade velocity 300 m/s, and the blades operating at maximum efficiency.

**Solution** For maximum blading efficiency,
\[ \frac{V_b}{V_1} = \cos \frac{a}{2} \]

\[ V_1 = \frac{2 \times 300}{\cos 15^\circ} = 621.2 \text{ m/s} \]

Now,
\[ V_1 = 44.72 \left[ (\Delta h)_s, h_n \right]^{1/2} = 621.2 \text{ m/s} \]

\[ (\Delta h)_s = \left( \frac{621.2}{44.72} \right)^2 \frac{1}{0.92} = 209.7 \text{ kJ/kg} \]

\[ h_1 = 3445.3 \text{ kJ/kg} \text{ (Fig. E7.19)} \]

\[ s_1 = 7.0901 \text{ kJ/kg K} \]

\[ K = s_2 = 0.6493 + x_{2s} \times 7.5009 \]

\[ x_2 = 0.86 \]

\[ h_{2s} = 191.83 + 0.86 \times 2392.8 = 2246.5 \text{ kJ/kg} \]

\[ h_1 - h_{2s} = 1198.8 \text{ kJ/kg} \]

\[ \therefore \text{ Number of stages required} \]

\[ \frac{h_1 - h_{2s}}{(\Delta h)_s} = \frac{1198.8}{209.7} = 5.72 \text{ or } 6 \text{ stages} \]

\[ \text{Ans. (a)} \]

(b) Isentropic output

\[ \frac{8000}{0.85 \times 0.96} = \omega_s \times 1198.8 \]

\[ \therefore \omega_s = \frac{8000}{0.85 \times 0.96 \times 1198.8} = 8.178 \text{ kg/s} = 29.44 \text{ t/h} \]

\[ h_1 - h_3 = 0.92 \times 209.7 = 193 \text{ kJ/kg} \]

\[ h_3 = 3252.3 \text{ kJ/kg} \]

From Mollier chart,
\[ v_3 = 0.17 \text{ m}^3/\text{kg} \]
\[
\omega_k = \frac{(A_1 V_1)}{v_1} = \frac{A_1 \times 621.2}{0.17} = 8.178 \text{ kg/s}
\]

\[
A_1 = 0.002238 \text{ m}^2 = \pi D_m h_n \sin \alpha
\]

\[
V_b = 300 \text{ m/s} = \frac{\pi D_m \times 5000}{60}
\]

\[\therefore \pi D_m = 3.6 \text{ m}\]

\[\therefore 3.6 \times h_n \times \sin 15^\circ = 0.002238\]

\[h_n = 0.0024 \text{ m} = 2.4 \text{ mm} \quad \text{Ans.}\]

**Example 7.20** At a certain point in a 50% reaction turbine, the steam leaving a moving blade row is at 1.5 bar, 0.90 dry. The steam flow rate is 7 kg/s and the turbine speed is 3000 rpm. At entry to the moving blade row, the axial velocity of flow is 0.7 times and at exit from the row 0.75 times the mean blade velocity. The exit angles of both fixed and moving blades are 20°, measured from the plane of rotation, and the height of moving blades at exit is 1/10 of the mean diameter. Determine the height of the moving blades at exit and the power developed in the blade row.

**Solution**

\[\alpha = \beta_2 = 20^\circ, V_1 \sin \alpha = 0.7 V_b, V_2 \sin \delta = 0.75 V_b \quad (\text{Fig. E7.20})\]

\[
\pi D_m h_b V_2 \sin \delta = \omega_k v_{\text{exit}}
\]

\[
v_{\text{exit}} = 0.001052 + 0.9 \times 1.15937 = 1.045 \text{ m}^3/\text{kg}
\]

\[
\pi D_m (0.1 D_m) 0.75 \frac{\pi D_m \times 3000}{60} = 7 \times 1.045
\]

\[
D_m^3 = \frac{7 \times 1.045 \times 60}{\pi \times 0.1 \times 0.75 \times \pi \times 3000} = 0.19764
\]

\[
\Delta V_w
\]

\[V_{r_1} \quad V_1 \quad V_2 \quad V_{r_2}
\]

\[\beta_1 \quad \alpha \quad \beta_2 \quad V_b
\]

\[D_m = 0.582 \text{ m}
\]

\[\therefore h_b = 0.0582 \text{ m} = 58.2 \text{ mm} \quad \text{Ans.}\]
\[ V_b = \frac{\pi \times 0.582 \times 3000}{60} = 91.42 \text{ m/s} \]

\[ \Delta V_w = 2V_1 \cos \alpha - V_b = 2 \frac{0.7V_b \cos \alpha}{\sin \alpha} - V_b \]

\[ = \left( \frac{1.4}{\tan 20^\circ} - 1 \right) V_b = 260.22 \text{ m/s} \]

Power developed = \( \omega_s \Delta V_w V_b = 7 \times 260.22 \times 91.42 \times 10^{-3} \]

\[ = 166.53 \text{ kW} \quad \text{Ans.} \]

**Example 7.21** Steam expands in a turbine from 40 bar, 500 °C to 0.10 bar isentropically. Assuming ideal conditions, determine the mean diameter of the wheel if the turbine were of (a) single impulse stage, (b) single 50% reaction stage, (c) four pressure (or Rateau) stages, (d) one two-row Curtis stage, and (e) four 50% reaction stages. Take the nozzle angle as 16 ° and \( N \) as 300 rpm.

**Solution**

\[ h_1 = 3445.3 \text{ kJ/kg}, \]

\[ s_1 = 7.0901 \text{ kJ/kg K} \quad K = s_2 = 0.6493 + x_{2s} \times 7.5009 \]

\[ x_{2s} = 0.86 \]

\[ h_{2s} = 191.83 + 0.86 \times 2392.8 = 2246.5 \text{ kJ/kg} \]

\[ h_1 - h_{2s} = 1198.8 \text{ kJ/kg} \]

\[ (a) \]

\[ V_1 = 44.72 \left( \frac{1198.8}{1198.8} \right)^{1/2} = 1548.37 \text{ m/s} \]

\[ \frac{V_b}{V_1} = \cos \alpha = \frac{\cos 16^\circ}{2} = 0.4806 \]

\[ V_b = 744.2 \text{ m/s} = \frac{\pi D_m N}{60} \]

\[ D_m = 4.73 \text{ m} \quad \text{Ans.} \]

\[ (b) \]

\[ V_1 = 44.72 \left[ \frac{\Delta h_{\text{stage}}}{2} \right]^{1/2} = 44.72 \left[ \frac{1198.8}{2} \right]^{1/2} \]

\[ = 1094.86 \text{ m/s} \]

\[ \frac{V_b}{V_1} = \cos \alpha = \cos 16^\circ \]

\[ V_b = 1052.4 \text{ m/s} = \frac{\pi D_m N}{60} \]

\[ \therefore \]

\[ D_m = 6.7 \text{ m} \quad \text{Ans.} \]
(c) 
\[ V_1 = 44.72 \left[ \frac{1198.8}{4} \right]^{1/2} = 774.19 \text{ m/s} \]

\[ \frac{V_b}{V_1} = \frac{\cos \alpha}{2}, \quad V_b = 372.07 \text{ m/s} = \frac{\pi D_m N}{60} \]

\[ D_m = 2.37 \text{ m} \quad \text{Ans.} \]

(d) 
\[ \frac{V_b}{V_1} = \frac{\cos \alpha}{4} = 0.2403, \quad V_1 = 1548.37 \text{ m/s} \]

\[ V_b = 372.1 \text{ m/s} = \frac{\pi D_m N}{60} \]

\[ D_m = 2.368 \text{ m} \quad \text{Ans.} \]

(e) Four 50% reaction stages

\[ \therefore \quad \Delta h_{\text{stage}} = \frac{1198.8}{4} = 299.7 \text{ kJ/kg} \]

\[ V_1 = 44.72 \left[ \frac{\Delta h_{\text{stage}}}{2} \right]^{1/2} = 547.43 \text{ m/s} \]

\[ \frac{V_b}{V_1} = \cos \alpha \]

\[ \therefore \quad V_b = 526.22 = \frac{\pi D_m N}{60} \]

\[ \therefore \quad D_m = 3.35 \text{ m} \quad \text{Ans.} \]

**Example 7.22** A steam turbine is to operate between 150 bar, 600 °C and 0.1 bar. The bucket velocity is limited to 300 m/s and the average nozzle efficiency is expected to be 95%, except for a 2-row Curtis stage for which it will be 90%. Nozzle angles will be assumed as 15° for impulse stages and 25° for reaction stages. All stages operate close to the speed of maximum efficiency. Estimate the number of stages required for each of the following arrangements:

(a) all simple impulse stages
(b) all 50% reaction stages
(c) a 2-row Curtis stage followed by simple impulse stages
(d) a 2-row Curtis stage followed by 50% reaction stages.

**Solution** With reference to Fig. E7.22.
Fig. E7.22

\[ h_1 = 3582.3 \text{ kJ/kg} \]

\[ s_1 = 6.6776 \text{ kJ/kg K} \]

\[ K = s_{2s} = 0.6493 + x_{2s} \times 7.5009 \]

\[ x_{2s} = 0.804, h_{2s} = 191.83 + 0.804 \times 2392.8 = 2114.9 \text{ kJ/kg} \]

\[ h_1 - h_{2s} = (\Delta h_s)_{\text{total}} = 1467.4 \text{ kJ/kg} \]

(a) All simple impulse stages

\[ \frac{V_b}{V_1} = \frac{\cos \alpha}{2} = \frac{\cos 15^\circ}{2} \]

\[ V_1 = \frac{300 \times 2}{\cos 15^\circ} = 621.2 \text{ m/s} \]

\[ V_1 = 44.72 \times [((\Delta h_s)_{\text{stage}} \times h_n)^{1/2}] = 621.2 \text{ m/s} \]

\[ (\Delta h_s)_{\text{stage}} = \left( \frac{621.2}{44.72} \right)^2 \times \frac{1}{0.95} = 203.1 \text{ kJ/kg} \]

\[ \therefore \text{ Number of simple impulse stages required} \]

\[ \frac{1467.4}{203.1} = 7.22 \text{ or } 8 \text{ stages} \quad \text{Ans.} \]

(b) All 50% reaction stages

\[ \frac{V_b}{V_1} = \cos a = \cos 25^\circ, \eta_r = 0.90 \]

\[ V_1 = \frac{300}{\cos 25^\circ} = 331 \text{ m/s} \]
\[ V_1 = 44.72 \left[ \frac{(\Delta h_s)_{\text{stage}}}{2} \times \eta_s \right]^{1/2} = 331 \text{ m/s} \]

\[(\Delta h_s)_{\text{stage}} = 115.34 \text{ kJ/kg}\]

\[\therefore \text{ Number of 50\% reaction stages required} = \frac{1467.4}{115.34} = 12.72 \text{ or } 13 \text{ stages} \quad \text{Ans.} \]

(c) A 2-row Curtis stage followed by simple impulse stages

\[\frac{V_b}{V_1} = \frac{\cos \alpha}{4}\]

\[V_1 = \frac{300 \times 4}{\cos 15^\circ} = 1242.4 \text{ m/s}\]

\[V_1 = 44.72 \left[ (\Delta h_s)_{\text{stage}} \times \eta_s \right]^{1/2}\]

\[(\Delta h_s)_{\text{stage}} = \left( \frac{1242.4}{44.72} \right)^2 \times \frac{1}{0.90} = 857.58 \text{ kJ/kg}\]

\[\therefore (\Delta h_s)_{\text{impulse}} = 1467.4 - 857.6 = 609.8 \text{ kJ/kg}\]

Number of impulse stages required = \[\frac{609.8}{203.1} = 3 \text{ or } 3 \text{ stages}\]

1, 2-row Curtis + 3 simple impulse stages \quad \text{Ans.}

(d) A 2-row Curtis followed by 50\% reaction stages

\[(\Delta h_s)_{\text{reaction}} = 609.8 \text{ kJ/kg}\]

\[\therefore \text{ Number of 50\% reaction stages required} = \frac{609.8}{115.34} = 5.28 \text{ or } 6 \text{ stages}\]

\[\therefore 1, 2 \text{- row Curtis + 6 50\% reaction stages} \quad \text{Ans.}\]

**Example 7.23** Steam at 20 bar, 400 °C expands in a steam turbine to 0.1 bar. There are four stages in the turbine and the total enthalpy drop is divided equally among the stages. The stage efficiency is 75\% and it is the same in all the stages. Determine the interstage pressures, the reheating factor and the turbine internal efficiency.

**Solution** From Mollier chart (Fig. E7.23),

\[h_1 - h_{6s} = 3250 - 2282 = 968 \text{ kJ/kg}\]

\[h_1 - h_{2s} = 968/4 = 242 \text{ kJ/kg}\]

The interstage pressures, \(p_2, p_3\) and \(p_4\) as read from the Mollier chart are 8 bar, 2.6 bar and 0.60 bar, respectively.

\[h_1 - h_2 = 0.75 \times 242 = 181.5 \text{ kJ/kg}\]
\[ h_2 - h_{3s} = 3060 - 2800 = 260 \text{ kJ/kg} \]
\[ h_2 - h_3 = 0.75 \times 260 = 195 \text{ kJ/kg} \]
\[ h_3 - h_{4s} = 2870 - 2605 = 265 \text{ kJ/kg} \]

Fig. E7.23

\[ h_3 - h_4 = 0.75 \times 270 = 202.5 \text{ kJ/kg} \]
\[ h_4 - h_{5s} = 2680 - 2410 = 270 \text{ kJ/kg} \]
\[ h_4 - h_5 = 0.75 \times 270 = 202.5 \text{ kJ/kg} \]
\[ h_5 = 2470 \text{ kJ/kg}, x_5 = 0.958 \]

Reheat factor \[ R = \frac{(h_1 - h_{2s}) + (h_2 - h_{3s}) + (h_3 - h_{4s}) + (h_4 - h_{5s})}{h_1 - h_{6s}} \]
\[ = \frac{242 + 260 + 265 + 270}{984} = \frac{1037}{968} = 1.071 \]

\[ \eta_{\text{internal}} = \frac{h_1 - h_3}{h_1 - h_{6s}} = \frac{780}{968} = 0.805 \text{ or } 80.5\% \]

Also, \( \eta_{\text{internal}} = h_{\text{stage}} \times \text{RF} = 0.75 \times 1.071 = 0.803 \text{ or, } 80.3\% \text{ Ans.} \]

**Example 7.24** Steam which is initially dry and saturated at an absolute temperature \( T_1 \), expands in a turbine to an absolute temperature \( T_2 \), the stage efficiency being \( \eta_s \). Assuming a very large number of stages and that the condition curve on the \( T-s \) diagram is a straight line, show that the reheat factor,

\[ R = \frac{T_1 + T_2}{2T_2 + \eta_s(T_1 - T_2)} \]

Is the actual reheat factor greater or less than this approximate value?

**Solution** By definition (Fig. E7.24)
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(a) the flow rate of steam required, assuming that all stages develop equal work,
(b) the mean blade diameter,
(c) the speed of the rotor.

**Solution**

\[
\eta_{\text{internal}} = \eta_{st} \times RF = 0.75 \times 1.04 = 0.78
\]

From Mollier chart, \( \Delta h_s = 855 \text{ kJ/kg} \) (Fig. E7.25)

\[
\Delta h_{\text{act}} = 0.78 \times 855 = 667 \text{ kJ/kg}
\]

\[
\omega_s(\Delta h)_{\text{act}} = 12 \text{ MW}
\]

![Diagram](image)

**Fig. E7.25**

\[
\therefore \quad \omega_s = \frac{12000}{667} = 17.99 \text{ kg/s} \quad \text{Ans. (a)}
\]

Work done per kg of steam flow,

\[
W_D = V_b (2V_1 \cos \alpha - V_b)
\]

Now,

\[
\frac{V_b}{V_1} = 0.7, \quad \alpha = 20^\circ
\]
\[ W_D = V_b \left( \frac{2V_b}{0.7} \cos 20^\circ - V_b \right) = 1.6848 \ V_b^2 \text{ J/kg} \]

Enthalpy drop per stage = work done per stage = \( \frac{667}{20} \) = 33.35 kJ/kg

\[ \therefore \ 1.6848 \ V_b^2 \times 10^{-3} = 33.35 \]

\[ \therefore \ \ V_b = 141.4 \text{ m/s} \]

\[ \sin \alpha = \frac{V_b}{0.7} \sin \alpha = \frac{141.4}{0.7} \sin 20^\circ = 69.1 \text{ m/s} \]

Volume flow rate per sec at 1 bar

\[ = \pi D_m h_b \times V_1 \sin \alpha = \pi D_m \frac{D_m}{12} \sin 69.1 = 18.09 \ D_m^2 \text{ m}^3/\text{s} \]

At 1 bar,

\[ v_g = 1.694 \text{ m}^3/\text{kg} \]

\[ \omega_s = \frac{18.09 \ D_m^2}{1.694} = 17.99 \text{ kg/s} \]

\[ \therefore \ D_m = 1.298 \text{ m} \quad \text{Ans. (b)} \]

\[ V_b = 141.4 = \frac{\pi \times 1.298 \times \pi}{60} \]

\[ \therefore \ N = 2081 \text{ rpm} \quad \text{Ans. (c)} \]

**Example 7.26** The first stage of a steam turbine is a two-row velocity compounded impulse wheel. The steam velocity at inlet is 600 m/s and the mean blade velocity is 120 m/s. The nozzle angle is 16° and the exit angles of the first row of moving blades, fixed blades, and second row of moving blades are 18°, 21° and 35°, respectively. The steam flow rate is 5 kg/s and the nozzle height is 25 mm. Neglecting the nozzle wall thickness, estimate the length of the nozzle arc. The specific volume of steam leaving the nozzles is 0.375 m³/kg. Assuming that all the blades have a pitch of 25 mm and an exit tip thickness of 0.5 mm, calculate the blade height at exit from each row. Take \( k_b \) as 0.9 for all blades.

**Solution** By continuity equation,

\[ \omega_s = \frac{x \pi D_m h_b V_1 \sin \alpha k_{m_i}}{v_1} \]

where \( x \pi D_m \) is the length of the nozzle arc.

\[ x \pi D_m = \frac{5 \times 0.375}{\sin 16^\circ \times 600 \times 0.02} = 0.454 \text{ m} \quad \text{Ans.} \]

\[ 0 + t = p \sin \beta_2 \text{ (Fig. E7.26)} \]

\[ 0 = p \sin \beta_2 - t = \text{opening for steam flow} \]

\[ A_b = 0 \times h_b \times z \]
\[ A_b = \frac{\pi D_m h_b}{p} (p \sin \beta_2 - t) \]

By continuity equation,

\[ \omega v_2 = \frac{\pi D_m}{p} h_b (p \sin \beta_2 - t)V_{r_2} \]

Fig. E7.26

where \( z \) = number of such openings = \( (\pi D_m)/p \)

Given: \( \alpha = 16^\circ, \beta_2 = 18^\circ, \alpha_1 = 21^\circ, \beta_4 = 35^\circ, V_b = 120 \text{ m/s}, \)

\[ V_1 = 600 \text{ m/s} \]

\[ \tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{600 \sin 16^\circ}{600 \cos \alpha - 120} = \frac{165.38}{456.76} \]

\[ \beta_1 = 19.9 = 20^\circ \]

\[ V_{r_1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = \frac{600 \sin 16^\circ}{\sin 20^\circ} = 483.54 \text{ m/s} \]

\[ V_{r_2} = 435.2 \text{ m/s} \]
\[ V_2^2 = V_{r2}^2 + V_b^2 - 2V_{r2}V_b \cos \beta_2 \]
\[ = (435.2)^2 + (120)^2 - 2(435.2)(120) \cos 18^\circ \]
\[ = 189399 + 14400 - 99336 = 104463 \]
\[ V_2 = 323.2 \text{ m/s}, \ V_3 = 291 \text{ m/s} \]

\[ \tan \beta_3 = \frac{V_3 \sin \alpha_1}{V_3 \cos \alpha_1 - V_b} = \frac{291 \sin 21^\circ}{291 \cos 21^\circ - 120} = 104.29 \]
\[ \beta_3 = 34.5^\circ \]

\[ V_{r3} = \frac{V_3 \sin \alpha_1}{\sin \beta_3} = \frac{291 \sin 21^\circ}{\sin 34.5^\circ} = 184.13 \text{ m/s} \]

\[ V_{r4} = 166 \text{ m/s} \]

First row of moving blades: From Eq. (1),

\[ 5 \times 0.375 = \frac{0.454}{0.025} \times (0.025 \sin 18^\circ - 0.0005) \times h_{b1} \times 435.2 \]

\[ h_{b1} = \frac{5 \times 0.375 \times 0.025}{0.454 \times 0.00723 \times 435.2} = 0.0331 \text{ m} = 33.1 \text{ mm} \quad \text{Ans.} \]

Fixed row of guide blades: From Eq. (1),

\[ 5 \times 0.375 = \frac{0.454}{0.025} \times (0.025 \sin 21^\circ - 0.0005) \times h_n \times 291 \]

\[ h_n = \frac{5 \times 0.375 \times 0.025}{0.454 \times 0.00846 \times 291} = 0.0420 \text{ m} \quad \text{Ans.} \]

Second row of moving blades: From Eq. (1),

\[ 5 \times 0.375 = \frac{0.454}{0.025} \times (0.025 \sin 35^\circ - 0.0005) \times h_{b2} \times 166 \]

\[ \therefore \quad h_{b2} = 0.0446 \text{ m} = 44.6 \text{ mm} \quad \text{Ans.} \]

**Example 7.27** Give the casing arrangements for turbines having the following ratings:

(a) 200 MW, steam condition at inlet – 180 bar, 550 °C condenser pressure–0.07 bar.

(b) 600 MW, steam condition at inlet – 300 bar, 580 °C condenser pressure–0.10 bar.

Assume 90% turbine efficiency in each case. Find also the intercasing steam condition.

**Solution** Steam expands in the HP turbine from state 1 to state 2, when it is reheated in the boiler till the original temperature of 550 °C is reached. The reheat pressure is assumed to be 1/4 th the boiler pressure (180 bar), i.e. 45 bar (Fig. E7.27a)
From Mollier chart,

\[ h_1 = 3430 \text{ kJ/kg}, \quad h_{2s} = 3040 \text{ kJ/kg} \]

\[ h_1 - h_{2s} = 390 \text{ kJ/kg} \]

\[ h_1 - h_2 = 0.9 \times 390 = 351 \text{ kJ/kg} \]

\[ h_2 = 3070 \text{ kJ/kg}, \quad v_2 = 0.06 \text{ m}^3/\text{kg} \]

\[ h_4 = 3560 \text{ kJ/kg}, \quad h_{3s} = 2000 \text{ kJ/kg} \]

\[ h_1 - h_{3s} = 3430 - 2000 = 1430 \text{ kJ/kg} \]

\[ h_1 - h_3 = 0.9 \times 1430 = 1287 \text{ kJ/kg}, \quad v_3 = 21 \text{ m}^3/\text{kg} \]

\[ \omega_s (h_1 - h_3) = 200 \times 10^3 \text{ kW} \]

\[ \omega_s = 155.4 \text{ kg/s} \]

Assuming \((V_b)_{\text{max}} = 350 \text{ m/s and } \alpha = 25^\circ,\)

\[ V_b = \frac{\pi D_m N}{60}, \quad D_m = \frac{350 \times 60}{\pi \times 3000} = 2.215 \text{ m} \]

Assuming for the last stage,

\[(h_b/D_m)_{\text{max}} = 0.3 \text{ (with tapered/twisted blades)} \]

\[ h_b = 0.3 \times 2.215 = 0.6645 \text{ m} \]

Flow area,

\[ A_b = \pi D_m h_b k_{th} \sin \alpha \]

\[ = \pi \times 2.215 \times 0.6645 \times 0.9 \times 0.4226 = 1.76 \text{ m}^2 \]

Assuming reaction blading,

\[ \frac{V_b}{V_1} = \cos \alpha, \]
\[ V_1 = 386.2 \text{ m/s} \]

Volume flow of steam that one last stage can accommodate

\[ = A_b \cdot V_1 = 1.76 \text{ m}^2 \times 386.2 \text{ m/s} = 679.7 \text{ m}^3/\text{s} \]

Steam expands in the IP turbine from state 4 to state 5, such that the last stage can accommodate the required volume flow.

Specific volume at the exhaust of a single casing should not exceed

\[ = \frac{697.7 \text{ m}^3/\text{s}}{155.4 \text{ kg/s}} = 4.37 \text{ m}^3/\text{kg} \]

\[ h_{5s} = 2456 \text{ kJ/kg}, p_5 = 0.36 \text{ bar} \]

Steam expands in the IP turbine to a pressure of 0.36 bar.

\[ h_4 - h_{5s} = 3560 - 2456 = 1104 \text{ kJ/kg} \]

\[ h_4 - h_5 = 993.6 \text{ kJ/kg} \]

\[ h_5 = 2566.4 \text{ kJ/kg}, x_5 = 0.952 \]

\[ h_5 - h_{6s} = 2566.4 - 2340 = 226.4 \text{ kJ/kg} \]

\[ h_5 - h_6 = 203.76 \text{ kJ/kg} \]

\[ h_6 = 2362.64 \text{ kJ/kg}, v_6 = 18 \text{ m}^3/\text{kg} \]

Maximum mass flow that one last stage can accommodate

\[ = \frac{679.7}{18} = 37.76 \text{ kg/s} \]

\[ \therefore \text{ Number of parallel exhausts} = \frac{155.4}{37.76} = 4.11 \]

By suitable adjustment of exit blade angles, the number of parallel exhausts can be made to be 4.

The conditions of steam in between the casings are shown in the diagram (Fig. E7.27(b)).
(b) The state of steam at turbine inlet: 300 bar, 580 °C.

Reheat pressure (assumed) \( = \frac{1}{4} \times 300 = 75 \) bar

\[
\begin{align*}
    h_1 &= 3410, \ h_{2s} = 3015 \text{ kJ/kg} \\
    h_1 - h_{2s} &= 395 \text{ kJ/kg}, \\
    h_1 - h_2 &= 0.9 \times 395 = 355.5 \text{ kJ/kg}, \ v_2 = 0.035 \text{ m}^3/\text{kg} \\
    h_4 &= 3060 \text{ kJ/kg}, \ h_{3s} = 1960 \text{ kJ/kg}, \ h_1 - h_{3s} = 1450 \text{ kJ/kg} \\
    h_1 - h_3 &= 1305 \text{ kJ/kg}, \ h_3 = 2105 \text{ kJ/kg} \\
    \omega_s &= \frac{600 \times 1000}{1305} = 459.8 \text{ kg/s}
\end{align*}
\]

Steam expands in the HP turbine up to the reheat pressure.

The maximum specific volume to which steam can expand in the IP turbine, the last stage of which has the maximum blade dimensions,

\[
    v_{\text{max}} = \frac{679.7}{459.8} = 1.478 \text{ m}^3/\text{kg}
\]

Volume flow rate at the HP turbine exhaust

\[
    = \omega_s v_2 = 459.8 \times 0.035 = 16.09 \text{ m}^3/\text{s}
\]

The pressure up to which steam can expand in the IP turbine cylinder as read from the Mollier Chart is 1.2 bar.

\[
\begin{align*}
    h_{5s} &= 2300 \text{ kJ/kg}, \ h_4 - h_{5s} = 3060 - 2300 = 760 \text{ kJ/kg} \\
    h_4 - h_5 &= 684 \text{ kJ/kg}, \ h_5 = 2376 \text{ kJ/kg}, \ v_5 = 1.25 \text{ m}^3/\text{kg}, \\
    x_5 &= 0.86, \ h_{6s} = 2050, \ h_5 - h_{6s} = 2376 - 2050 = 326 \text{ kJ/kg} \\
    h_5 - h_6 &= 0.9 \times 326 = 293.4, \ h_6 = 2082.6 \text{ kJ/kg}, \\
    v_6 &= 12 \text{ m}^3/\text{kg}, \ x_6 = 0.792
\end{align*}
\]

Maximum mass flow that one last stage can accommodate

\[
    = \frac{679.7}{12} = 56.64 \text{ kJ/kg}
\]

\[
\therefore \quad \text{Number of parallel exhausts} = \frac{459.8}{56.64} = 8.11
\]

By adjusting blade exit angles and since the required mass flow will be somewhat less than estimated, the number of streams to which steam will be split in the LP turbines can be made eight. The maximum number of turbine cylinders that can be mounted on a single shaft in tandem arrangement is six. Therefore, the present turbine is cross-compounded with two shafts and two electric generators as shown (Fig. E7.27(c)).

Since the quality at turbine exhaust is only 0.792, there will be considerable erosion of turbine blades in later stages due to impact of entrained water particles. The cycle parameters need to be adjusted (two reheats may require to be adopted) so that the quality at turbine exhaust does not fall below 0.88.
Example 7.28  A 100 MW turbine generator unit is supplied with steam at 90 bar, 550 °C and the condenser pressure is 0.1 bar. At rated load the steam supplied is 500,000 kg/h and at zero load it is 25,000 kg/h. Determine (a) the steam rate in kg/kWh at 1/4, 1/2, 3/4 and full load, (b) the Rankine cycle efficiency, (c) the actual efficiency at full load of the plant assuming 100% boiler efficiency, (d) the turbogenerator efficiency at full load based on generator output.

Solution  From Willan’s line law, \( \omega_s = a + bL \)

where \( \omega_s \) is the steam consumption rate in kg/h and \( L \) is the load in kW.

At no load,

\[ 25,000 = a + b \times 0 \]

\[ a = 25,000 \text{ kg/h} \]

At full load,

\[ 500,000 = 25,000 + b \times 100,000 \]

\[ b = 4.76 \text{ kg/kWh} \]

\[ \omega_s = 25,000 + 4.75L \]

\[ \text{Steam rate} = \frac{\omega_s}{L} = \frac{25000}{L} + 4.75 \text{ kg/kWh} \]
(a) At one-fourth load,
\[ L = 25,000 \text{ kW} \]
\[ \therefore \text{Steam rate} = \frac{25,000}{25,000} + 4.75 = 5.75 \text{ kg/kWh} \]
At half the load,
\[ L = 50,000 \text{ kW} \]
\[ \therefore \text{Steam rate} = \frac{25,000}{50,000} + 4.75 = 5.25 \text{ kg/kWh} \]
At three-fourth load,
\[ L = 75,000 \text{ kW} \]
\[ \therefore \text{Steam rate} = \frac{25,000}{75,000} + 4.75 = 5.08 \text{ kg/kWh} \]
At full load,
\[ L = 100,000 \text{ kW} \]
\[ \therefore \text{Steam rate} = \frac{25,000}{100,000} + 4.75 = 5.0 \text{ kg/kWh} \]

(b) \[ h_1 = 3511 \text{ kJ/kg}, s_1 = 6.8142 \text{ kJ/kg K} \]
\[ s_2s = 0.6493 + x_{2s} \times 7.5009 \]
\[ x_{2s} = 0.822 \]
\[ h_{2s} = 191.83 + 0.822 \times 2392.8 = 2158.44 \text{ kJ/kg} \]
Neglecting pump work,
\[ \eta_{\text{Rankine}} = \frac{3511 - 2158.44}{3511 - 191.83} = \frac{1352.56}{3319.17} = 0.407 \text{ or } 40.7\% \]

(c) \[ \eta_{\text{actual}} = \eta_{\text{cycle}} \times \eta_{\text{boiler}} \times \eta_{\text{turbogenerator}} \]
\[ = \frac{\omega_s (h_1 - h_{2s})}{\omega_s (h_1 - h_4)} \times 1 \times \frac{100 \times 1000 \text{ kW}}{100 \times 1000} = \frac{100 \times 1000}{\omega_s (h_1 - h_3)} \]
\[ = \frac{100 \times 1000 \times 3600}{500,000 \times 3319.17} = 0.217 \text{ or } 21.7\% \text{ Ans.} \]

(d) \[ \eta_{\text{TG}} = \frac{100,000}{\omega_s (h_1 - h_{2s})} = \frac{100,000 \times 3600}{500,000 \times 1352.56} \]
\[ = 0.532 \text{ or } 53.2\% \text{ Ans.} \]

**SHORT-ANSWER QUESTIONS**

7.1 How does a steam turbine convert energy in steam to shaft work?
7.2 What do you understand by (a) a nozzle, (b) a diffusor?
7.3 What is a compressible fluid?
7.4 What is the velocity of a pressure pulse in an isentropic flow?
7.5 What is the velocity of sound in an ideal gas? On what factors does it depend?
7.6 What is a stagnation state?
7.7 What are stagnation pressure and stagnation temperature?
7.8 Explain (a) supersonic nozzle, (b) subsonic nozzle, (c) subsonic diffuser, (d) supersonic diffuser.
7.9 What do you understand by (a) critical pressure ratio, (b) choked flow?
7.10 What is critical discharge?
7.11 What is the critical pressure ratio for (a) air, (b) dry saturated steam, and (c) superheated steam?
7.12 What is Zeuner’s relation?
7.13 Explain (a) nozzle efficiency, (b) velocity coefficient, (c) coefficient of discharge.
7.14 What is shock? Where does it occur?
7.15 What do you understand by (a) Supersaturated flow, (b) metastable state?
7.16 What is Wilson line?
7.17 What is (a) degree of supersaturation, (b) degree of supercooling?
7.18 How does energy conversion occur in (a) impulse blades (b) reaction blades?
7.19 Define (a) diagram power, (b) diagram efficiency.
7.20 What is the optimum velocity ratio for (a) an impulse stage, (b) a two-row Curtis stage, (c) a 50% reaction stage?
7.21 Why are steam turbines compounded? What are the different methods of compounding?
7.22 How is the number of stages in a turbine estimated?
7.23 Why does the effectiveness of a Curtis stage decrease as the number of rows of moving blades increases?
7.24 How is degree of reaction defined? What is a 50% reaction turbine?
7.25 Why are reaction blades unsymmetrical?
7.26 Compare the diagram efficiencies of impulse, two-row Curtis and 50% reaction stages.
7.27 Compare the stage enthalpy drops of impulse, two-row Curtis and 50% reaction stages.
7.28 Why is a two-row Curtis stage most often used as the first stage in large steam turbines?
7.29 Show that the diagram work per unit mass of steam for maximum blading efficiency of a 50% reaction stage is $V_b^2$, where $V_b$ is the mean blade velocity.
7.30 What do you understand by carryover efficiency?
7.31 How would you estimate nozzle and blade heights?
7.32 What do you understand by partial admission of steam?
7.33 How are the last stage blade dimensions fixed up?
7.34 What are parallel exhausts? Why are these needed?
7.35 Enlist the various losses taking place in a steam turbine.
7.36 What do you understand by throttle governing and nozzle governing?
7.37 What is bypass governing?
7.38 What do you understand by Willan’s line?
7.39 What is the function of a governor?
7.40 Why is an oil operated servo system added to a governor?
7.41 What is an emergency governor? When does it operate?
7.42 What is disc friction loss?
7.43 What is windage loss?
7.44 Why is there some energy loss due to wetness of steam?
7.45 What is leakage loss? How is it restricted?
7.46 What are labyrinth glands? Where are they used?
7.47 Name some supervisory and control instruments used in steam turbines.
7.48 What do you understand by reheat factor and condition line? Why is RF greater than unity?
7.49 What is a dummy piston? Where is it used?
7.50 What are the stresses to which turbine blades are subjected?
7.51 Why are the turbine blades tapered towards the tip?
7.52 How are blades fastened to the disc or drum?
7.53 What are shrouds? Where are they used?
7.54 What are lacing wires? Why are they used?
7.55 What is critical speed?
7.56 What is (a) flexible shaft, (b) rigid shaft?
7.57 What is a turning gear? When is it used?
7.58 Why is hydrogen gas used for cooling of the generator? How is hydrogen filled into it?

**PROBLEMS**

7.1 Steam at 30 bar, 350 °C expands through a convergent–divergent nozzle. The exit plane pressure is 3 bar. The flow rate is 0.5 kg/s and the nozzle efficiency is 0.8. Assuming that the velocity at inlet is negligible, determine the throat and exit areas, steam velocity at the exit, and the quality of steam at the exit plane. The critical pressure ratio can be taken as 0.546.

[Ans. 127.6 mm², 323.5 mm², 922 m/s, \( x_2 = 0.985 \)]

7.2 Dry saturated steam at 10 bar is expanded in a convergent-divergent nozzle. The velocity of steam at exit is 685 m/s, the flow rate is 7 kg/s and the nozzle efficiency is 85%. Assume the flow to be isentropic up to the throat. The critical pressure ratio can be taken as 0.54. Determine the throat and exit areas of the nozzle and the pressure at exit. Neglect the velocity of steam at inlet to the nozzle.

[Ans. 4870 mm², 7000 mm², 2.4 bar]

7.3 Dry saturated steam at 26 bar expands isentropically in a convergent-divergent nozzle to 12 bar. Determine the mass flow rate per cm² of throat area and the steam quality at the nozzle exit if the expansion is assumed to be

(a) in equilibrium, \( n = 1.135 \)
(b) supersaturated, \( n = 1.3 \)

[Ans. (a) 0.37 kg/s, 0.935, (b) 0.368 kg/s, superheated]
7.4 A convergent–divergent nozzle receives dry saturated steam and discharges it at a velocity of 800 m/s into a chamber at a pressure of 1.4 bar. The nozzle efficiency is 85% and \( n = 1.135 \). Estimate the pressure of steam supply. Neglect inlet velocity. If the mass flow rate of steam is 10 kg/s, determine the throat and exit areas of the nozzle.

[Ans. 13 bar, 0.0054 m\(^2\), 0.133 m\(^2\)]

7.5 Steam passes through a convergent–divergent nozzle from a pressure of 8 bar. The steam is initially dry saturated \( (n = 1.135) \). The nozzle efficiency is 90%. Given that the exit area = 2 \( \times \) throat area, determine the pressure at exit. The inlet velocity is negligible.

[Ans. 1.2 bar]

7.6 Show that the maximum mass flow rate through a convergent nozzle passing air is given by

\[
m = 4.04 \times 10^{-3} \frac{Ap_0}{\sqrt{T_0}} \text{ kg/s}
\]

where, \( A \) = nozzle area (mm\(^2\)), \( p_0 \) = reservoir pressure (bar), and \( T_0 \) = reservoir temperature.

Air is discharged from a large container through a convergent nozzle of 10 mm diameter. The conditions in the container are 10 bar, 20 °C. The pressure at the nozzle is 6 bar. Calculate the mass flow rate and the flow Mach number at the nozzle.

[Ans. 0.184 kg/s, 0.89]

7.7 Steam at 7 bar, 290 °C expands in a convergent-divergent nozzle to a pressure of 0.55 bar. Assuming a negligible inlet velocity, calculate the mass flow and the nozzle exit area. The area of the nozzle throat is 970 mm\(^2\) and the coefficient of discharge is 0.95. Take a velocity coefficient of 0.92 and the critical pressure ratio of 0.546.

[Ans. 0.875 kg/s, 2790 mm\(^2\)]

7.8 A convergent nozzle receives steam at 4 bar, 150°C and negligible inlet velocity, and expands it into a space at atmospheric pressure. Assuming supersaturated expansion and a nozzle efficiency of 0.9, calculate the nozzle throat area required for a mass flow of 1.2 kg/s.

[Ans. 2010 mm\(^2\)]

7.9 Calculate the throat and exit diameters of a convergent–divergent nozzle which will discharge 0.25 kg/s of steam from a pressure of 8 bar superheated to 250 °C into a chamber having a pressure of 1.5 bar. Friction loss in the divergent part of the nozzle may be taken as 0.15 of the enthalpy drop. The convergent part is sharp and frictionless. Neglect the inlet velocity of steam.

[Ans. 16.5 mm, 21.8 mm]

7.10 Show that the critical velocity at the throat is given by

\[
V^* = (n \frac{p^*}{\nu^*})^{1/2}
\]

where \( n \) is the polytropic index of expansion.
7.11 Steam at 7 bar, 200 °C expands isentropically in a convergent-divergent nozzle into a space at 3 bar. Neglecting the inlet velocity, estimate the exit area required for a mass flow of 0.1 kg/s when
(a) the flow is in equilibrium throughout,
(b) the flow is supersaturated with $pv^{1.2} = C$.
Find (b) the degree of supercooling and the degree of supersaturation.

[Ans. (a) 103.7 mm$^2$, (b) 101.5 mm$^2$, 17.5 °C, 1.713]

7.12 A stage of an impulse steam turbine operates close to the maximum blading efficiency. The blades are equiangular, and the friction effects in blades may be neglected. The mean blade velocity is 200 m/s and the steam flow rate is 0.75 kg/s. Find (a) the discharge angle at which the steam leaves the blades, (b) the diagram power.

[Ans. (a) 90°, (b) 60 kW]

7.13 The velocity of steam leaving the nozzle of an impulse turbine is 900 m/s and the nozzle angle is 20°. The blade velocity is 300 m/s and the blade friction factor is 0.7. Calculate for a mass flow rate of 1 kg/s and symmetric blading (a) the blade inlet angle, (b) the driving force on the wheel, (c) the axial thrust, (d) the diagram power, (e) the diagram efficiency.

[Ans. (a) 29°24', (b) 927.7 N (c) 92.3 N, (d) 278.3 kW, (e) 68.7%]

7.14 In a stage of an impulse steam turbine the mean diameter of the blade ring is 800 mm and the speed of rotation is 3000 rpm. The direction of final absolute velocity of steam is axial. The inlet and exit angles of the blades are 30°. Assuming a blade friction factor of 0.85 and a steam flow rate of 1 kg/s, determine (a) the nozzle angle, (b) the absolute velocity of steam leaving the nozzle, (c) the enthalpy drop in the stage, (d) the tangential thrust, (e) the axial thrust, (f) the blading work, and (g) the blading efficiency.

Ans. (a) 17.33°, (b) 72.56 m/s, (c) 41.06 kJ/kg, (d) 273.5 N, (e) 12.8 N, (f) 34.37 kW, (g) 83.7%.

7.15 A single stage impulse turbine rotor has a mean blade ring diameter of 500 mm and rotates at a speed of 10,000 rpm. The nozzle angle is 20° and the steam leaves the nozzles with a velocity of 900 m/s. The blades are equiangular and the blade friction factor is 0.85. Construct the velocity diagrams for the blades and determine the inlet angle of the blades for shockless entry of steam. Determine (a) the diagram power for a steam flow of 750 kg/h, (b) the diagram efficiency, (c) the axial thrust, and (d) the loss of kinetic energy due to friction.

[Ans. 28°, (a) 56.8 kW, (b) 0.70, (c) 11.98 kW]

7.16 In an impulse turbine, the nozzle angle is $\alpha$; the blade inlet and outlet angles are equal; the blade friction factor is $k$; and the steam velocity at nozzle outlet is $V'$. Show that the optimum blade speed is given by $(V' \cos \alpha)/2$ and that the optimum blade efficiency is $\frac{1 + k}{2} \cos^2 \alpha$.

A turbine rotor has the mean diameter of 250 mm and the blade angles are equal. The nozzle angle is 20°, the steam speed at nozzle outlet is 930 m/s and the blade friction factor is 0.85. Find the best angle of the blades, the
turbine speed in rpm, the steam consumption to generate 10 kW, and the blade efficiency.

\[\text{Ans. } 36.1^\circ, 33,300 \text{ rpm, } 102.1 \text{ kg/h, } 0.816\]

7.17 A single stage simple impulse turbine generator set is to operate under the following conditions:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steam at inlet</td>
<td>8 bar, dry saturated</td>
</tr>
<tr>
<td>Exhaust</td>
<td>0.2 bar</td>
</tr>
<tr>
<td>Nozzle efficiency</td>
<td>0.90</td>
</tr>
<tr>
<td>Nozzle angle</td>
<td>$15^\circ$</td>
</tr>
<tr>
<td>Internal efficiency</td>
<td>0.75</td>
</tr>
<tr>
<td>Mechanical efficiency</td>
<td>0.92</td>
</tr>
<tr>
<td>Generator efficiency</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Find (a) the mean blade speed for maximum blading efficiency, (b) the steam flow rate for developing 200 kW.

\[\text{Ans. (a) } 490 \text{ m/s, (b) } 0.564 \text{ kg/s}\]

7.18 Deduce a general expression for the blade efficiency of a stage of an impulse turbine with single row wheels, assuming equiangular blades, a nozzle angle $\alpha$, and a blade friction factor $k_b$. What is the maximum efficiency if $\alpha = 20^\circ$ and $k_b = 0.83$? What is the velocity ratio? If the blade efficiency is 90% of the maximum value, what are the values of the velocity ratios? Draw the velocity diagrams for each case and state the blade angles.

\[\text{Ans. } 80.7\%, 0.47, 0.32 \text{ and } 0.62, 36^\circ \text{ and } 48^\circ 6\]

7.19 A stage of an impulse steam turbine is velocity compounded with two rows of moving blades. The isentropic enthalpy drop from the stage is 320 kJ/kg, the nozzle angle is $16^\circ$, blade speed is 150 m/s, velocity coefficient is 0.95 and the blade friction factor is 0.9 (all blades). All blades are symmetrical and the steam flow rate is 20 kg/s. Determine the blade angles, power output, stage efficiency and kinetic energy of the steam leaving the stage.

\[\text{Ans. } 19.8^\circ, 26.8^\circ, 42.4^\circ, 4264 \text{ kW, } 0.667, 234 \text{ kW}\]

7.20 An impulse steam turbine has nozzles inclined at 20$^\circ$ to the plane of rotation. The inlet and exit angles of the moving blades are equal, the blade friction factor is 0.8 and the mean diameter of the blades is 0.5 m. The steam leaves the nozzle with a velocity of 750 m/s. Determine the optimum value of the blade angles, the steam flow rate required to produce 20 kW and the blading efficiency.

\[\text{Ans. } 36^\circ 4', 323 \text{ kg/h, } 0.795\]

7.21 A stage of an axial-flow impulse steam turbine has two rows of moving blades separated by a row of fixed guide blades. The inlet and exit angles of all moving blades are $30^\circ$, measured from the plane of rotation. The blade speed of each moving blade row is 130 m/s. The direction of the final absolute velocity of steam leaving the second row of moving blades is axial. Assuming a blade friction coefficient of 0.85 for both fixed and moving blades,
determine the velocity of steam leaving the nozzles, the work done per kg of steam and the blade efficiency.

\[ \text{Ans.} 650 \text{ m/s, } 149.5 \text{ kJ/kg, } 0.707 \]

7.22 The following particulars refer to a two-row velocity-compounded impulse wheel which forms the first stage of a combination turbine:

- Steam velocity at nozzle outlet: 630 m/s
- Mean blade velocity: 125 m/s
- Nozzle angle: 16°
- Outlet angle, first row of moving blades: 18°
- Outlet angle, fixed guide blades: 22°
- Outlet angle, second row of moving blades: 36°
- Steam flow rate: 2.6 kg/s

The ratio of the relative velocity at outlet to that at inlet is 0.84 for all the blades. Determine (a) the velocity of whirl, (b) the tangential thrust on the blades, (c) the axial thrust on the blades, (d) the power developed, and (e) the blading efficiency.

\[ \text{Ans.} (a) 1127.5 \text{ m/s, (b) 2.93 kN, (c) 188.5 N, (d) 366.4 kW, (e) 0.71} \]

7.23 Show that in a 50% reaction steam turbine stage, the maximum stage efficiency is

\[ \frac{2 \cos^2 \alpha}{1 + \cos^2 \alpha} \]

where \( \alpha \) is the nozzle angle.

In a particular stage the mean diameter is 500 mm and the blade height is 30 mm. The blade angles are 60° at inlet and 160° at outlet. The density of steam is 2.7 kg/m³, the speed is 3000 rpm. Calculate the mass flow rate of steam, power developed and stage efficiency.

\[ \text{Ans. 4.61 kg/s, 43.57 kW, 0.915} \]

7.24 In a reaction stage of a steam turbine the nozzle angle is 20° and the absolute velocity of steam at inlet to the moving blades is 240 m/s. The blade velocity is 210 m/s. If the blading is designed for 50% reaction, determine (a) the blade height at inlet and exit, (b) the enthalpy drop per kg of steam in the moving blades and in the complete stage, (c) the diagram power for a steam flow of 1 kg/s, and (d) the diagram efficiency.

7.25 A 50% reaction turbine is supplied with steam at 60 bar, 600 °C. The condenser pressure is 0.07 bar. If the reheat factor is assumed to be 1.04 and the stage efficiency is constant throughout at 80%. Calculate the steam flow required for a diagram power of 25 MW.

\[ \text{Ans. 21 kg/s} \]

7.26 Steam at 20 bar, 400 °C expands in a 50% reaction turbine to a pressure 0.2 bar. The steam leaving the turbine is dry saturated. The reheat factor is 1.05 and the isentropic efficiency of each stage is the same throughout. There are 14 stages and the enthalpy drop is the same in each. All the blades have an exit angle of 22° and the mean value of blade velocity ratio is 0.82. The mass flow rate of steam is 34,000 kg/h. The turbine speed is 2400 rpm. Calculate
(a) the stage efficiency, (b) the diagram power, (c) the drum diameter, (d) the blade height for the last row of moving blades, and (e) the pressure at the entry to the last stage.

\[ \text{Ans. (a) 67.7\% (b) 6025 kW, (c) 1.34 m, (d) 175 mm, (e) 0.32 bar} \]

7.27 A stage of a 50\% reaction turbine delivers dry saturated steam at 2.7 bar from the fixed blades at 90 m/s. The mean blade height is 40 mm, and the moving blade exit angle is 20°. The axial velocity of steam is 3/4 of the mean blade velocity. Steam flow rate is 9000 kg/h. The effect of blade tip thickness on the annulus area can be neglected. Calculate (a) the wheel speed in rpm, (b) the diagram power, (c) the diagram efficiency, and (d) the enthalpy drop of steam in this stage.

\[ \text{Ans. (a) 1824 rpm, (b) 13.14 kW, (c) 0.787 and (d) 5.26 kJ/kg} \]

7.28 The nozzles of a single stage impulse turbine discharge the working fluid at an angle of 25° to the plane of rotation of the blades. The fluid leaves the blades with an absolute velocity of 290 m/s and a trailing angle of 60° to the plane of rotation. The blades have equal inlet and outlet angles and there is 5\% reduction in the axial velocity component over the stage.

(a) Determine: (i) the blade angles, (ii) the blade work per kg fluid, (iii) the blade velocity coefficient, (iv) the blading efficiency.

(b) If the fluid is steam and the nozzle inlet conditions are 1.5 bar, 150 °C, determine the pressure at the moving blades and express as a percentage of the isentropic enthalpy drop (i) the nozzle loss, (ii) the loss in blade channels, (iii) the disc friction loss, (iv) the residual velocity loss, (v) the net work available.

The nozzle efficiency is 87\% and the disc friction loss is 3\% of the work done on the blades. Show the losses on an \( h-s \) diagram for the stage.

7.29 A steam turbine is to operate between 140 bar, 560 °C and 0.075 bar. The maximum blade velocity is 320 m/s and the nozzle efficiency in all stages is 0.90. Nozzle angles will be 15° for impulse stages and 25° for reaction stages. All stages operate close to the maximum efficiency. Estimate the number of stages required for each of the following arrangements:

(a) all simple impulse stages,

(b) all 50\% reaction stages,

(c) a two-row Curtis stage followed by simple impulse stages

(d) a two-row Curtis stage followed by 50\% reaction stages.

7.30 A five-stage steam turbine working between 20 bar, 350 °C and 2 bar has equal enthalpy drops in all stages, and the stage efficiency is 75\% in all stages. Plot on the Mollier diagram the condition line. Find the interstage pressures, reheat factor, and turbine internal efficiency using properties from the diagram.

7.31 In a four-stage pressure-compounded impulse turbine steam at 23 bar, 345 °C expands to 0.07 bar. The internal efficiency of the turbine is 0.72. Assuming that the work is shared equally among the stages and the condition line is
straight, estimate the stage pressures, efficiency of each stage and reheat factor.

[Ans. 7.25 bar, 1.9 bar, 0.39 bar; 61.68%, 68.14%, 70.41%, 72.84%; 1.0574]

7.32 Give the casing arrangements for the turbines having the following ratings
(a) 75 MW, steam condition at inlet – 100 bar, 550 ºC, condenser pressure – 0.1 bar
(b) 300 MW, steam condition at inlet – 180 bar, 550 ºC, condenser pressure – 0.07 bar
Assume 90% turbine efficiency in each case. Find also the intercasing condition of steam.

7.33 A 3 MW steam turbine consumes 4.611 kg/s at full load and 2.53 kg/s at half load. Using Willan’s line estimate the steam consumption at no load and the consumption at a load of 2.5 MW.

[Ans. 0.444 kg/s, 3.917 kg/s]

BIBLIOGRAPHY

8.1 NEED OF A CONDENSER

A condenser where the exhaust steam from the turbine is condensed operates at a pressure lower than atmosphere. There are two objects of using a condenser in a steam plant:

1. To reduce the turbine exhaust pressure so as to increase the specific output of the turbine (Fig. 8.1). If the circulating cooling water temperature in a condenser is low enough (say 30 °C), it creates a low back pressure (vacuum) for the turbine. This pressure is equal to the saturation pressure corresponding to the condensing steam temperature (say 0.074 bar at 40 °C, Fig. 8.1), which, in turn, is a function of the cooling water temperature. It is known that the enthalpy drop or turbine work per unit pressure drop is much greater at the low pressure end than

![Diagram]

Fig. 8.1 The use of condenser increases the specific work output of turbine from \( (h_1 - h_3) \) to \( (h_1 - h_2) \)
at the high pressure end of a turbine. A condenser by lowering the back pressure, say, from 1.013 to 0.074 bar, thus increases the plant efficiency and reduces the steam flow for a given output. The lower the pressure, the greater the output and efficiency. Hence, it is important to use the lowest possible cooling water temperature. This restricts the temperature rise of cooling water in the condenser tubes to 5–8 °C so that the tube outer surface temperature remains low and consequently, the condensing steam temperature is low and vacuum is high.

2. To recover high quality feedwater in the form of condensate and feed it back to the steam generator without any further treatment.

As a result, only the makeup water to replenish the water losses in the cyclic plant needs be treated.

### 8.1.1 Types

There are two broad classes of condensers:

(a) *Direct contact type condensers*, where the condensate and cooling water directly mix and come out as a single stream.

(b) *Surface condensers*, which are shell-and-tube heat exchangers where the two fluids do not come in direct contact and the heat released by the condensation of steam is transferred through the walls of the tubes into the cooling water continuously circulating inside them.

### 8.2 DIRECT CONTACT CONDENSERS

These can be of three types:

(a) Spray condenser
(b) Barometric condenser
(c) Jet condenser

In a spray condenser, the cooling water is sprayed into the steam. Steam by mixing directly with cold water gets condensed. The exhaust steam from the turbine at state 2 mixes with cooling water at state 5 to produce saturated water at state 3, which is pumped to state 4 (Fig. 8.2).

Part of the condensate (ω₄), equal to the turbine exhaust flow (ω₂) is sent back to the plant as feedwater. The remainder is cooled in a dry cooling tower to state 5, and is then sprayed on to the turbine exhaust. Since the cooling water mixes with the steam and part of the condensate is used as feedwater, the water must be of high purity.

In a geothermal or OTEC (ocean thermal energy conversion) plant, only vacuum is required to be maintained in the condenser and no feedwater is needed. Hence, the mixture at state 4 is discarded.

For the spray condenser (Fig. 8.2), a mass balance and an energy balance give the following equations

\[ ω_2 = ω_4, \quad ω_3 = ω_2 + ω_5 \]
\[ \omega_2 h_2 + \omega_3 h_5 = \omega_3 h_3 \]

from which

\[
\frac{\omega_3}{\omega_2} = \frac{h_2 - h_3}{h_3 - h_5}
\] (8.1)

**Fig. 8.2** Schematic flow diagram of a direct-contact spray condenser (a) and the corresponding T–S diagram (b)

Since \( h_2 - h_3 \) is much greater than \( h_3 - h_5 \), the circulating water flow (\( \omega_3 \)) is much larger than the steam flow (\( \omega_2 \)).

In a barometric condenser (Fig. 8.3a), the cooling water is made to fall in a series of baffles to expose large surface area for the steam fed from below to come in direct contact. The steam condenses and the mixture falls in a tail pipe to the hot well below. By virtue of its static head, the tail pipe compresses the mixture to atmospheric pressure. Thus,

\[
p_{\text{atm}} - p_{\text{cond}} + \Delta p_f = \rho g H
\] (8.2)

where \( \rho = \) density of mixture, \( H = \) height of tail pipe, and \( \Delta p_f \) is the pressure drop due to friction.
For low values of $\Delta p$, $H$ is around 9.5 m. Higher is the value of $H$, higher is the friction. Friction is lowered by increasing the tail pipe diameter, which results in a tall and heavy system.

In the jet condenser (Fig. 8.3b), the height of the tail pipe is reduced by replacing it with a diffusor. The diffusor helps raising the pressure in a short distance than a tail pipe.

**Fig. 8.3 Schematics of direct-contact condensers:**
(a) barometric, (b) diffuser or jet

In spray type condensers, the non-condensable gases are usually removed with a steam jet air ejector (SJAE), as discussed later.

### 8.3 SURFACE CONDENSERS

Surface condensers are mostly used in power plants. They are essentially shell-and-tube heat exchangers. For the convenience of cleaning and maintenance, cooling water flows through the tubes and steam condenses outside the tubes. Figure 8.4 shows a surface condenser with two passes on the water side. It consists of a steel shell with water boxes on each side. The right water box is divided to allow for two water passes. At each end there are tube sheets into which the water tubes are rolled. This prevents leakage of circulating water into the steam. An expansion joint allows for the different rates of expansion between the tubes and shell. There are vertical plates at intermediate points between the two tube sheets to provide support to the long tubes and to prevent tube vibration. The hot well acts as a reservoir of the condensate with a capacity equal to the total condensate flow for a certain period of time, say 5 min.
**Fig. 8.4 Schematics of a two-pass surface condenser**

In a single pass condenser (Fig. 8.5), cooling water flows through the tubes once, from one end to the other. Compared to a two-pass condenser, a single-pass condenser with the same number and size of the tubes and with the same water velocity requires twice as much water flow but results in half the water temperature rise and therefore, lower condenser pressure. Thus, a single-pass condenser is good for overall plant efficiency and reduces thermal pollution, but requires more than twice the water flow and hence, four times the pumping power.

**Fig. 8.5 Single-pass condenser with divided water boxes**
Apart from the division required to allow for a number of water passes, the water boxes are often further divided. For example, in a single-pass condenser, both the inlet and outlet water boxes are divided by a partition. This permits to take off half the condensing surface out of service for cleaning while water flows through the other half to keep the unit running at half load (Fig. 8.5).

The tube arrangement in the condenser shell controls its effectiveness. Older designs tended to crowd a maximum number of tubes inside the shell to make available as much surface area as possible. Modern designs have steam lanes between tube banks to get maximum steam flow with least pressure drop and uniform distribution of steam in the shell. Tube lengths of 9–15 m are now quite common. Such long tubes result in a large rise in temperature of cooling water and the condensing ability decreases at the exit ends of the tubes. Thus, the tubes are closer at the cold end than at the hot end. To minimise the unequal distribution of steam flow from the turbine exhaust duct to condenser tubes, a well-tapered steam dome is added above the tube bundles. An expansion joint provided between the turbine exhaust and the condenser inlet permits the condenser to be rigidly mounted on the floor.

Figure 8.6 shows a typical modern two-pass surface condenser for a large steam power plant. Steam enters the tube bundles in two separate sections from the top, sides and bottom, and flows toward the centre of the tube nest in each section. At that point most of steam has condensed leaving only air and other non-condensable gases which are cooled and removed by SJAE, as explained later.

The tube material can be (a) cupronickel (70% copper, 30% nickel) (b) aluminium brass (76% copper, 22% zinc and 2% aluminium), (c) aluminium bronze (95% copper and 5% aluminium), (d) muntz metal (60% copper, 40% zinc), (e) admiralty alloy (71% copper, 28% zinc and 1% tin), or (f) stainless steel. The outside diameter of tubes is either 22 mm, 23 mm or 25.4 mm. The length varies from 9 to 15 m.

### 8.3.1 Design Calculations

When wet steam comes in contact with a cold surface, the temperature of which is below the saturation temperature at the exhaust pressure of steam, it cannot but condense rejecting the latent heat of condensation (Fig. 8.7). For filmwise condensation, the average heat transfer coefficient for a horizontal tube as given by Nusselt is

\[
h_{av} = 0.725 \left[ \frac{k_t \rho_f^2 g h_{tg}}{N \mu_t d_0 \theta} \right]^{1/4}
\]

(8.3)

where \(N = \) number of horizontal tubes in a vertical tier and \(\theta = t_{sat} - t_w\). So, \(h_{av} \propto 1/N^{1/4}, h_{av} \propto 1/\theta^{1/4}\) and \(h_{av} \propto h_{tg}^{1/4}\).
Fig. 8.6 Sections through a typical two-pass surface condenser for a large steam power plant.
Fig. 8.7 Film condensation of steam on a horizontal tube

The bulk temperature of the condensate lies between $t_{\text{sat}}$ and $t_{\omega}$. So, it is less than $t_{\text{sat}}$ and subcooled. This subcooling is not desired as it increases the heat supply in the boiler. By making some exhaust steam flow upward through the condensate, its temperature is increased. Nusselt's equation gives only a conservative value for the condensing film coefficient of heat transfer, which is also influenced by vapour superheat, vapour velocity, turbulence, and the inside air.

The inside heat transfer coefficient on the water side may be obtained with the help of Dittus–Boelter equation,

$$Nu_d = 0.023 \, \text{Re}_d^{0.8} \, \text{Pr}^{0.4}$$

(8.4)

Where $\text{Re}_d$ is the Reynolds number ($\frac{V_d}{v}$) and $\text{Pr}$ is the Prandtl number ($\frac{c_p \mu}{k}$).

For water, $h_i \propto V^{0.8}$, $V$ being the water velocity. Higher water velocity will improve heat transfer but increases pumping power also. The optimum water velocity varies between 2.0 to 2.5 m/s.

The overall heat transfer coefficient for a condenser tube is

$$\frac{1}{U_0A_0} = \frac{1}{h_iA_i} + \frac{1}{h_sA_i} + \frac{x_w}{k_w A_{\text{tm}}} + \frac{1}{h_0A_0}$$

(8.5)

Since condenser tubes are thin and made of good thermal conductivity, the tube wall resistance can be ignored, then

$$\frac{1}{U_0} = \frac{1}{h_i} + \frac{1}{h_s} + \frac{1}{h_0}$$

(8.6)

Here $h_0$ is much larger than $h_i$, and $U_0$ mainly depends on water velocity as given by

$$\frac{1}{U_0} = A + B \frac{1}{V^{0.8}}$$

(8.7)

where

$$A = \frac{1}{h_s} + \frac{1}{h_0} \quad \text{and} \quad B = \frac{1}{0.023 k_f \text{Pr}^{0.4}} \frac{0.023 k_f \text{Pr}^{0.4}}{d_i^{0.2} v_f^{0.8}}$$
By estimating the overall heat transfer coefficient at different water velocities, a plot (Wilson plot) can be made on log–log coordinates from which the intercept $A$ and slope $B$ can be obtained (Fig. 8.8). The rate of heat transfer from the condensing vapour to the cooling water is

$$Q = \omega_s (h_2 - h_3) = \omega_c c_{pc} (t_{C_2} - t_{C_1}) = U_0 A_0 \Delta t_{1,m}$$

(8.8)

where $\Delta t_{1,m} = (\Delta t_i - \Delta t_c) \left( \ln \frac{\Delta t_i}{\Delta t_c} \right)$ is the logarithmic mean temperature difference (Fig. 8.9). The temperature difference at exit, $\Delta t_c$, is the terminal temperature difference (TTD) of the condenser. A small TTD results in a large condenser but reduced water flow and higher exit water temperature. It increases the capital cost but reduces the operating cost. The cooling water inlet temperature should be sufficiently low to have a good vacuum in the condenser shell. It is usually recommended that $\Delta t_i$ should lie between 11 to 17 °C and that $\Delta t_c$ or TTD should not be less than 3 °C.

Fig. 8.9 Temperature profiles in a condenser
From Eq. (8.8), the water mass flow rate is

\[ \omega_c = \frac{\omega_s (h_2 - h_3)}{c_{p_s} (t_{c_2} - t_{c_1})} \]  

(8.9)

The rise in water temperature is limited to about 8–10 °C. So, for every kg of steam condensed, 75 to 100 kg of cooling water is required. Thus, to meet the huge water demand, the power plant is located where water is available in plenty.

The surface area needed by the condenser is obtained from Eq. (8.8).

\[ A_0 = \frac{\omega_s (h_2 - h_3)}{U_0 \Delta_{l,m}} = n \pi d_0 l \]  

(8.10)

where \( n \) = number of tubes, and \( l \) = length of one tube (for a single-pass condenser). The water flows through the tubes. Therefore,

\[ \omega_s = \left( \frac{n \pi d_i^2}{4} \right) \rho V \]  

(8.11)

where \( \rho \) is the density of water (1000 kg/m³) and \( V \) is the water velocity (1.8–2.5 m/s). Therefore, the length and number of tubes can be estimated from the above two equations. For a modern condenser of a utility plant the number of tubes may be as high as 50,000 (22–23 mm o.d.).

The pressure drop in the condenser consists of (1) the pressure drop in the water boxes and (2) the pressure drop due to friction in the tubes \( \left( \frac{fL}{d} \times \frac{\rho V^2}{2} \right) \).

The pumping power required is then given by

\[ P = \omega_c v \Delta p = \frac{\omega_c \Delta p}{\rho} \]  

(8.12)

where \( \Delta p \) = total pressure head to be developed by the pump in overcoming the losses.

### 8.3.2 Air Removal

Air leaks into the condenser shell through flanges. Some air also comes along with steam, which has leaked into the exhaust end of the turbine along the shaft. This air affects the condenser performance badly because of the following reasons.

1. It reduces the heat transfer considerably.
2. It reduces the condenser vacuum and increases the turbine exhaust pressure thus reducing the turbine output.

As air–water vapour mixture approaches the cold tube surface, water vapour condenses. Air, being non-condensable, forms an air film around the condensate film. Since air has a low thermal conductivity, the heat transfer is greatly reduced.

The amount of air infiltrating into the shell may be estimated by Dalton’s law of partial pressures.

\[ p_{sh} = p_{air} + p_{st} \]  

(8.13)
where \( p_{st} \) is the saturation pressure at the measured shell temperature, \( t_{sh} \), \( P_{air} \) is the partial pressure of air inside the shell, and \( p_{sh} \) is the measured total pressure of the shell (Fig. 8.10). Assuming that air behaves as an ideal gas at such a low pressure, we have

\[
p_{air} \omega_a \nu_2 = \omega_a R_a (t_{sh} + 273) \tag{8.14}
\]

where \( \nu_2 \) = specific volume of exhaust steam, \( \omega_a \) = rate of air leakage, \( R_a \) = characteristic gas constant of air = 0.287 kJ/kg K. With \( p_{air} \) being known from Eq. (8.13), the rate of air leakage can be estimated from Eq. (8.14). This air has to be continuously removed from the condenser shell.

![Diagram](Fig. 8.10 Condenser shell pressure and temperature)

The pressure in the condenser (\( p_{sh} \)) is approximately constant, and steam and air enter the condenser in fixed proportions when steady conditions prevail. As some of the steam is condensed, the partial pressure of the remaining steam (\( p_{st} \)) decreases and hence, the partial pressure of the air increases to maintain the same total pressure (Eq. 8.13). At reduced partial pressure, the steam has a saturation temperature which is below that of the incoming steam. Hence, condensation proceeds at progressively lower temperatures.

For the convenience of air removal an air cooler section is provided in the condenser shell. In Fig. 8.11(a), most of the condensation is carried out on the main bank of tubes and the air is drawn over another smaller bank which is shielded from the main bank by a baffle and is called the aircooler. Here, further condensation takes place at a lower temperature and thus, there is saving in feedwater as well as in air ejection load. In Fig. 8.11(b), the air cooling tubes are in the centre of the condenser and air is removed from this section. The incoming steam passes all around the bank of tubes and some is drawn upward to the centre. In doing so, it meets the subcooled condensate falling below and thus reduces the amount of subcooling by heating it.

A steam jet air ejector, SJAE, is mostly used to remove air from the condenser shell (Fig. 8.12). As explained above, air is extracted from the coldest part of the condenser to reduce the loss of vapour accompanying it and hence, to reduce the ejector load. Baffles keep off the air-cooler section from the main
Fig. 8.12 Two-stage steam jet air ejector with inter-condenser and after-condenser
8.4 FEEDWATER HEATERS

As discussed in Chapter 2 regenerative feedwater heaters are always used in steam power plants to improve the cycle efficiency. They raise the temperature of the feedwater before it enters the economiser. Both open and closed type heaters are used. In small industrial plants, only one open feedwater heater may be used. But in large industrial and utility plants, five to seven closed heaters and one open heater are used. The open heater acts as a deaerator.

8.4.1 Closed Feedwater Heater

Closed feedwater heaters are shell-and-tube heat exchangers. They are basically small condensers which operate at higher pressures than the main condenser because bled steam is condensed on the shell side, whereas the feedwater, acting like circulating cooling water in the condenser, is heated on the tube side.

It was shown in Chapter 2 that the temperature rise in each heater and economiser is equal for maximum cycle efficiency. Thus the heaters receive bled steam from the turbine at pressures determined roughly by equal temperature rise from the condenser to the boiler saturation temperature. They are classified as low pressure (LP) and high pressure (HP) heaters depending upon their locations in the cycle. The LP heaters are usually located between the condensate pump and the deaerator, which is followed by the boiler feed pump (BFP). The HP heaters are located between the BFP and the economiser.

When bled steam entering a feedwater heater is superheated, as in a HP heater, the heater includes a desuperheating zone where steam is cooled to its saturation temperature. It is followed by a condensing zone where the steam is condensed to a saturated liquid rejecting the latent heat of condensation. This liquid, called heater drain, is then cooled below its saturation temperature in a subcooling zone or a drain cooling zone before the drain is cascaded backward or pumped forward.

Figure 8.13 shows the schematic diagram and the temperature profiles of a three-zone closed feedwater heater. There are, however, two-zone heaters that include a desuperheating and a condensing zone or a condensing and a subcooling zone. There are also single-zone heaters that include only a condensing zone. A drain-cooling zone, instead of being a part of the shell, may be located outside it. It is then called a drain cooler.

Closed feedwater heaters may be either horizontal or vertical, depending upon space availability. Vertical heaters occupy less space. Fig. 8.14 shows a typical horizontal three-zone closed feedwater heater. The feedwater tubes are usually in the form of U-tube bundles. The feedwater enters a divided water box and flows through the subcooling zone and then through the condensing zone, and leaves to the water box through the desuperheating zone. The bled steam first flows through the desuperheating zone separated by a shroud. The vertical baffles provide good heat transfer and tube support. The condensing zone is the
major portion of the heater. The subcooling zone is separated from the rest of the heater by an end plate. Pressure drops of the feedwater in heaters are usually large due to friction in long small diameter tubes of the heaters.

**Fig. 8.13(a)** *A three-zone horizontal closed-type feedwater heater*

**Fig. 8.13(b)** *Temperature profiles along the path length in a three-zone feedwater heater*
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8.4.2 Open Feedwater Heater

An open feedwater heater or deaerator is one in which the feedwater is heated by direct mixing with the steam bled from the turbine. It is used to remove dissolved gases in feedwater (Chapter 2). It is located at a sufficient height (20–25 m) above the boiler feed pump so that the suction pressure does not fall below saturation pressure to prevent cavitation.

There are three types of deaerating heaters.

(a) **Spray-type deaerators**: Here, feedwater is sprayed through nozzles into the heater from the top and bled steam is fed from the bottom. Water is heated and scrubbed to release the dissolved gases.

(b) **Tray-type deaerators**: Feedwater here falls through a series of cascading horizontal trays. As water falls from tray to tray, it comes in direct contact with the upflowing bled steam, and gets heated and scrubbed to release the dissolved gases.

(c) **Combined spray-tray deaerators**: Feedwater is first sprayed and then made to cascade down a series of trays and bled steam flows upward. These types are now preferred in power plants. A typical heater of this type is shown in Fig. 8.15.

*Fig. 8.15 A typical open-type deaerating feedwater heater*
8.5 CIRCULATING WATER SYSTEM

The circulating water system supplies cooling water to the turbine condensers and thus acts as a medium through which heat is rejected from the steam cycle to the environment. Cooling water can flow through the condenser in two methods: (1) Once-through system, and (2) Closed loop system.

Once-through system (Fig. 8.16) is used when there is a large source of water available. Water is taken from a natural body of water like a lake, river, or ocean and pumped through the condenser, where it is heated, and then discharged back to the source.

![Diagram of once-through circulating water system](image)

**Fig. 8.16 Schematic of a once-through circulating water system**

In closed loop systems, warm water from the condenser is passed through a cooling device like a cooling tower or a spray pond and the cooled water is then pumped back for condenser circulation (Fig. 8.17). However, a natural body of water is still necessary nearby to supply the makeup water to replace the loss due to evaporation, blowdown and so on.

![Diagram of closed loop cooling system](image)

**Fig. 8.17 Schematic usual view of a wet cooling tower operating in the closed mode**
The once-through system, though more efficient, causes thermal pollution. In addition, availability of huge quantity of water is shrinking. Closed loop systems are now almost universally preferred.

### 8.6 COOLING TOWERS

Cooling towers cool the warm water discharged from the condenser and feed the cooled water back to the condenser. They, thus, reduce the cooling water demand in the power plant. They can be either wet type or dry type.

#### 8.6.1 Wet Cooling Towers

Wet cooling towers have a hot water distribution system that showers or sprays water evenly over a lattice of horizontal slats or bars called fill or packing (Fig. 8.18). The fill thoroughly mixes the falling water with air moving through the fill as the water splashes down from one fill level to another by gravity. Outside air enters the tower through louvres on the side of the tower. Intimate mixing of water and air enhances heat and mass transfer (evaporation), which cools the water. More the water evaporates, more will be the cooling since the latent heat of evaporation is taken from water itself (evaporative cooling). Cold water is collected in a concrete basin at the bottom of the tower, from where it is pumped back to the condenser. Hot and moist air leaves the tower from the top.

Air entering the tower is unsaturated and as it comes in contact with the water spray, water continues to evaporate till the air becomes saturated. So, the minimum temperature to which water can be cooled is the adiabatic saturation or wet bulb temperature of the ambient air. At this temperature (WBT), air is 100% saturated and cannot absorb any more water vapour. Hence, there will be no further evaporation and cooling. The humid air while moving up comes in contact with warm water spray and so the air temperature rises.

A cooling tower is specified by (a) approach, (b) range, and (c) cooling efficiency. The approach \((A)\) is defined as the difference between the exit temperature of cooling water and the wet bulb temperature of the ambient air, or

\[
A = t_{C_2} - t_{wb}
\]  

(8.15)

Warm water from the condenser enters the cooling tower at temperature \(t_{C_1}\), and is cooled to temperature \(t_{C_2}\), higher than the minimum value, the wet bulb temperature, \(t_{wb}\), and this unattainable temperature difference is the approach. The approach varies from 6 °C to 8 °C.

The cooling range or simply range \((R)\) is defined as the difference in temperatures of the incoming warm water \((t_{C_1})\) and the exiting cooled water \((t_{C_2})\), or
\[ R = t_{C_1} - t_{C_2} \]  \hspace{1cm} (8.16)

It is the range by which warm water from the condenser is cooled. The range varies from 6 °C to 10 °C.

The cooling efficiency is defined as the ratio of the actual cooling of water to the maximum cooling possible, or

\[ \eta_{\text{cooling}} = \frac{\text{actual cooling}}{\text{maximum cooling possible}} = \frac{t_{C_1} - t_{C_2}}{t_{C_1} - t_{\text{wb}}} \]  \hspace{1cm} (8.17)

The approach, range and cooling efficiency are the performance parameters of cooling towers.

Wet cooling towers can be either mechanical draught or natural draught cooling towers. In mechanical draught cooling towers, air is moved through the fill by one or more fans driven by motors. As in steam generators, the fans could be of the forced draught (FD) type or induced draught (ID) type. The FD fan is mounted on the lower side of the tower (Fig. 8.19). Since it operates on cooler air, it consumes less power. However, it has the disadvantages of (a) air distribution problems in the fill, often causing channeling of air flowing through paths of less flow resistance, (b) leakage and (c) recirculation of the hot and moist air back to the tower.

![Diagram of cooling towers](image)

**Fig. 8.19**  *Induced draught counterflow cooling towers*

Most of the mechanical draught cooling towers for utility applications are of the induced draught type. The ID fan is located at the top of the tower Fig. 8.19(a). Air enters the sides of the tower through large openings at low velocity and passes through the fill. Hot humid air is exhausted by the fan at the top to the atmosphere. It maintains the tower at a negative pressure thereby reducing leakage. The ID fans are large, having 0.6 to 10 m in diameter. They are driven by electric motors at low speeds through reduction gearing. They are of the propeller type which deliver large volume flow of air at lower static
pressures. The blades are usually made of cast aluminium, stainless steel, or fibre glass so as to protect them from corrosion.

The air flow into the tower is more or less horizontal. However, in the fill the flow can be horizontal or vertical, in which case it is either a cross-flow or a counter-flow cooling tower. The main advantages of mechanical draught cooling towers are:

1. Low capital and construction costs.
2. Assured supply of the required quantity of air at all loads and climatic conditions,
3. Small physical structure.

In natural draught cooling towers, the flow of air occurs due to the natural pressure head caused by the difference in density between the cold outside air and the hot humid air inside (Fig. 8.20(a) and (b)). Thus, the pressure head developed is

$$\Delta p_{d} = (\rho_{0} - \rho_{i}) g H$$  \hspace{1cm} (8.18)

where $H$ = height of the tower above the fill, $\rho_{0}$ = density of outside air, and $\rho_{i}$ = density of inside air.

Because of relatively small density difference, $\rho_{0} - \rho_{i}$, $H$ must be large so as to result in the desired $\Delta p_{d}$, which must

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**Fig. 8.20(a) Natural draught cooling tower**

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**Fig. 8.20(b) Counterflow hyperbolic natural draught cooling tower**

balance the air pressure losses in the tower. Natural draught cooling towers are, therefore, very tall. The tower body, above the water distribution system and the fill, is an empty shell of circular cross-section, but with a hyperbolic vertical profile. The hyperbolic profile offers superior strength and the greatest resistance to outside wind loading compared to other forms. Natural draught cooling towers are, therefore, often termed as hyperbolic towers. Made of reinforced concrete, they are an imposing sight and are conspicuous from a distance.
Mechanical draught cooling towers are preferred when the approach is low and a broad range of water flow is expected. The broad range is possible since they are made of multicell units with a variable air-flow fan. These towers are, therefore, more versatile and respond readily to changes in cooling parameters and demands.

Natural draught cooling towers are chosen (1) in cool, humid climates (low wet bulb temperature and high relative humidity), (2) when the wet bulb temperature is low and high condenser water inlet and outlet temperature, i.e. a broad range and a long approach, or (3) in heavy winter loads. However, their initial capital cost is high and occupy more space.

There is always some water loss in the cooling tower due to

(a) evaporation
(b) drift
(c) blowdown

Water that evaporates leaves the tower along with air in the form of water vapour. The evaporation loss rate is 1–1.5 per cent of the total circulating water flow rate. Drift is fine water droplets entrained and carried by the air. This water is thus lost to the circulating water system. Drift eliminators are provided at exit to minimise the drift loss (Fig. 8.21). The baffles force the air to make a sudden change in direction. Heavier water particles separate out by gravity. Thus the drift loss is much less, about 0.03 per cent.

To maintain a certain solid concentration, blowdown is necessary from the cold water basin at the bottom of the tower. The blowdown loss of water is also 1–1.5 per cent of the total water flow. To replenish these losses, makeup water (2–2.2% of water flow) is added.

8.6.2 Dry Cooling Towers

Dry cooling towers are employed where cooling water is not available in plenty, even for the use of makeup. In a dry tower, warm water from the condenser flows through finned tubes over which the cooling air is passed. Heat is rejected to the air as water is cooled.

There are two basic types of dry cooling towers: direct and indirect. In a direct dry cooling tower (Fig. 8.22), turbine exhaust steam flows into a large
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flows through finned tubes and is cooled by atmospheric air blown over the tubes. There are two heat exchangers in series, one between steam and water in the condenser and the other between water and air in the tower.

The second design uses a direct contact spray condenser (Fig. 8.24). The turbine exhaust steam enters the open condenser and the cold circulating water is sprayed into the steam for intimate mixing. The condensate falls into a bottom receiver from which a part is fed to the plant as feedwater, and the remainder is pumped into finned tubes cooled by air flow to return to the condenser sprays. Condensate polishers may be used to maintain the desired quality of feedwater.

![Diagram of a direct contact spray condenser](image)

**Fig. 8.24** *Schematic of an indirect dry-cooling tower with an open-type condenser*

The third design of an indirect dry cooling tower employs ammonia as the coolant for the condenser (Fig. 8.25). In the steam condenser ammonia evaporates. In the dry tower ammonia condenses, rejecting the latent heat of condensation to the air which is heated. Saturated liquid ammonia is then pumped back to the condenser.

Dry cooling towers have attracted much attention now-a-days. Plants can be erected without regard for large supplies of cooling water. Typical sites are near the sources of abundant fuel where there is no sufficient water. Their disadvantages are that they are not so efficient as evaporative cooling, and that there is an increase in turbine exhaust pressure and a decrease in cycle efficiency. However, as power plants grow bigger and water sources are dwindling, they are likely to receive greater attention in future.


**Fig. 8.25** Schematic of an indirect dry-cooling tower with a surface condenser having ammonia as the coolant

### 8.7 COOLING TOWER CALCULATIONS

In a wet cooling tower, ambient air is used to cool the warm water exiting the condenser. Properties associated with air–water vapour mixture may now be discussed.

Atmospheric air is considered to be a mixture of dry air and water vapour. If $p_a$ and $p_w$ are the partial pressures of dry air and water vapour, respectively, then by Dalton’s law of partial pressures,

$$ p_a + p_w = p $$

where $p$ is the atmospheric pressure. Since $p_w$ is very small, the saturation temperature of water vapour at $p_w$ is less than atmospheric temperature, $t_{atm}$ (Fig. 8.26). So, the water vapour in air exists in the superheated state and air is said to be unsaturated. Saturated air holds the maximum water vapour at the given temperature. If the temperature is increased, then only it can accept more water vapour (till it is again saturated). If the temperature of saturated air is decreased, some water vapour will condense and the new cooler air would also be saturated. At 15 °C, the partial pressure of water vapour in saturated air ($p_d$) is equal to 1.705 kPa from steam tables. So, the partial pressure of dry air ($p_a$) is $(101.325 - 1.705) = 99.62$ kPa.
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Dew point temperature ($t_{dp}$) is the saturation temperature at the partial pressure of water vapour. When air is cooled at constant pressure, the temperature at which water vapour starts condensing is called the dew point temperature (Fig. 8.26).

Dry bulb temperature ($t_{db}$) is the temperature recorded by a thermometer with a dry bulb.

Wet bulb temperature ($t_{wb}$) is the temperature recorded by a thermometer when the bulb is enveloped by a cotton wick saturated with water. A psychrometer measures both dbt and wbt. If air flowing over the bulb is relatively dry, water in the wick would evaporate at a rapid rate, cooling the bulb and resulting in a much lower reading than if the bulb were dry. If the air is humid, the evaporation rate is slow, the lowest temperature recorded by the moistened bulb is the wbt. If air is saturated, i.e. $\phi = 100\%$, wbt = dbt. The wbt is also called adiabatic saturation temperature.

Psychrometric chart (Fig. 8.27) is a graphical plot with specific humidity and partial pressure of water vapour as ordinates and dbt as abscissa. The volume of the mixture, wbt, relative humidity and enthalpy of the mixture appear as parameters. Any two of these parameters fix the condition of the mixture. The chart is plotted for one barometric pressure, say 760 mm Hg. The constant wbt line represents the adiabatic saturation process and coincides with the constant enthalpy line.

The cooling tower utilizes the phenomenon of evaporative cooling to cool the warm water below the dbt of air. However, water never reaches the minimum temperature, i.e., the wbt since an excessively large cooling tower would then be required. Also, since warm water is continuously introduced to the tower (Fig. 8.28), the equilibrium conditions are not achieved, and the dbt of air is increased. Hence, while the water is cooled, the air is heated and humidified.

If $x$ is the make-up water supplied to replenish the evaporation loss (Fig. 8.28), then

$$x = G(W_2 - W_1) \tag{8.21}$$

where $G$ = the mass flow rate of dry air, kg/s, $W$ = specific humidity, kg water vapour per kg dry air.

By energy balance,

$$G h_1 + w_c h_3 + x h_w = G h_2 + w_{c4} h_{w4}$$

$$w_c (h_3 - h_{w4}) = G (h_2 - h_1) - G (W_2 - W_1) h_w \tag{8.22}$$

where $\omega_c = \omega_{c4} = \omega_c$ = circulating water flow rate, kg/s; $h_w$ = enthalpy of circulating water, kJ/kg; and $h$ = enthalpy of dry air, kJ/kg

$$\text{Range (R)} = t_{w_3} - t_{w_4} = \frac{G}{c_{pw} \omega_c} [(h_2 - h_1) - (W_2 - W_1) h_w] \tag{8.23}$$

where $c_{pw}$ is the specific heat of water and $h_w$ is the enthalpy of makeup water.

$$\text{Approach (A)} = t_{w_3} - t_{wb_1} \tag{8.24}$$
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Most towers are rated in practice for wet-bulb approach between 6 °C and 8 °C. Values of 6 to 10 °C are common for the cooling range. The relationships among $t_{c1}$, $t_{c2}$, $R$ and $A$ are shown in Fig. 8.29. Vaporization, hence cooling, takes place as long as the partial pressure of water vapour at the cooling water surface exceeds the partial pressure of water vapour in the bulk air stream.

**Fig. 8.29** Temperature relationship in a counterflow cooling tower
Hidden page
\[ \Delta t_{1,m} = \frac{\Delta t_i - \Delta t_e}{\ln(\Delta t_i/\Delta t_e)} = \frac{8 - 2}{\ln 8/2} = 4.33 \, ^\circ C \]

\[ Q = U_0 A_0 \Delta t_{1,m} = \omega_c (h_2 - h_3) \]

\[ 2.6 \times A_0 \times 4.33 = \frac{250 \times 1000}{3600} \times 2118.16 \]

\[ A_0 = 13066 \, m^2 \]

\[ \omega_c = \left( n \frac{\pi}{4} d_i^2 \right) \rho V = 5855.2 \, kg/s \]

\[ n \frac{\pi}{4} (25.4 - 2.5)^2 \times 10^{-6} \times 1000 \times 1.8 = 5855.2 \]

\[ n = \frac{5.8552 \times 4 \times 10^6}{524.41 \times \pi \times 1.8} = 7898 \quad \text{Ans. (d)} \]

Again,

\[ A_0 = n \pi d_0^2 = 13066 \, m^2 \]

\[ l = \frac{13066}{7898 \times \pi \times 25.4 \times 10^{-3}} = 20.73 \, m \quad \text{Ans. (c)} \]

**Example 8.2** Steam enters the condenser at 35°C. The condenser vacuum is 70 cm of mercury when the barometer reads 75.5 cm of Hg. Determine the vacuum efficiency. Estimate the mass of air present in the condenser per kg of steam.

**Solution** Saturation pressure of steam at 35°C = 0.05622 bar and \( \nu_g = 25.24 \, m^3/kg \)

1std. atm. pr = 76 cm Hg = 1.013 bar.

\[ P_{\text{sat}} = 0.05622 \, \text{bar} = 4.2717 \, \text{cm Hg} \]

\[ P_{\text{abs}} = 75.5 - 70 = 5.5 \, \text{cm Hg} \]

Vacuum gauge corrected to standard atmosphere

\[ = 76 - 5.5 = 70.5 \, \text{cm Hg} \]

\[ \text{Vacuum efficiency} = \frac{70.5}{76 - 4.2717} = 0.9829 \quad \text{or} \quad 98.29\% \quad \text{Ans.} \]

Absolute pressure inside the condenser = 5.5 cm Hg

Partial pressure of steam at 35°C = 4.2717 cm Hg

\[ \therefore \text{Partial pressure of air} = 5.5 - 4.2717 \]

\[ = 1.2283 \, \text{cm Hg} \]

\[ \therefore \text{Mass of air associated with 1 kg steam} = \frac{p\nu}{Rt} = \frac{1.2283 \times 1.013 \times 10^5 \times 25.24}{76 \times 287 \times (273 + 35)} \]

\[ = 0.467 \, \text{kg air/kg steam} \quad \text{Ans.} \]
Example 8.3  Exhaust steam having a quality of 0.9 enters a surface condenser at an absolute pressure of 0.13 bar and comes out as water at 45°C. The circulating water enters at 30°C and leaves at 40°C. Estimate the quantity of circulating water and the condenser efficiency.

Solution  At 0.13 bar, \( t_{sat} = 51.06°C \)

\[
h_2 = h_f + x_2 h_{fg} = 213 + 0.9 \times 2380.3 = 2355.97 \text{ kJ/kg}
\]

\( h_f \) at 45°C = 188.35 kJ/kg

By energy balance,

\[
\dot{m}_w c_{pw} (t_{w2} - t_{w1}) = \dot{m}_s (h_2 - h_3)
\]

\[
\therefore \quad \frac{\dot{m}_w}{\dot{m}_s} = \frac{2355.97 - 188.35}{4.182 \times (40 - 30)} = 51.77 \quad \text{kg water/kg steam} \quad \text{Ans.}
\]

Condenser efficiency = \[
\frac{40 - 30}{51.06 - 30} = 0.475 \text{ or } 47.5% \quad \text{Ans.}
\]

Example 8.4  The following readings were taken during a test on a surface condenser:

Mean condenser temperature = 35°C, Hot well temperature = 30°C, condenser vacuum = 69 cm Hg, barometer reading 76 cm Hg. Condensate collected 16 kg/min. Cooling water enters at 20°C and leaves at 32.5°C, flow rate being 37,500 kg/h. Calculate (a) mass of air present per cubic metre of condenser, (b) quality of steam at condenser inlet, (c) vacuum efficiency, and (d) condenser efficiency.

Solution  Absolute pressure inside the condenser = 76 - 69 = 7 cm Hg = 0.0933 bar

\[ p_{sat} \text{ at } 35°C = 0.05622 \text{ bar} \]

\[ \therefore \quad \text{Partial pressure of air, } p_a = 0.0933 - 0.05622 \]

\[ = 0.03708 \text{ bar} \]

\[ \therefore \quad \text{Mass of air present, } m_a = \frac{p_a}{RT} \]

\[ \text{or, } m_a = \frac{0.03708 \times 10^5 \times 1}{287 \times (273 + 35)} = 0.042 \text{ kg/m}^3 \quad \text{Ans. (a)} \]

Let \( x \) be the quality of steam at condenser inlet

\[
\dot{m}_s \left[ xh_{fg} + c_p (t_{sat} - 30) \right] = \dot{m}_w c_{pw} (t_{w2} - t_{w1})
\]

\[
16 \times 60 \left[ x + 2418.8 + 4.182 (35 - 30) \right] = 37,500 \times 4.182 \times (32.5 - 20)
\]

\[ = 1.96 \times 10^6 \]
\[ x = 0.836 \quad \text{Ans. (b)} \]

Vacuum efficiency = \[
\frac{69}{79 - 4.22} = 0.9613 \text{ or } 96.13\% \quad \text{Ans. (c)}
\]

Condenser efficiency = \[
\frac{12.5}{35 - 20} = 0.833 \text{ or } 83.33\% \quad \text{Ans. (d)}
\]

**Example 8.5** A surface condenser receives 20 t/h of dry saturated steam at 40 °C. The air leakage to the condenser is estimated to be 0.35 kg per 1000 kg of steam. The condensate leaves at a temperature of 38 °C. Makeup water is supplied at 10 °C. The cooling water enters at 32 °C and leaves at 38 °C. A separate air extraction pump (SJA) is added and from the air cooler section air along with some steam leaves at 27 °C. The pressure in the condenser is assumed to remain constant. Calculate (a) the rate of saving of condensate and the rate of saving in the heat supply in the boiler due to separate air extraction pump, (b) the percentage reduction in air ejector load due to this separate air extraction method, (c) the rate of cooling water flow.

**Solution** At 40 °C, \( p_{\text{sat}} = 0.07384 \text{ bar} \) and \( v_g = 19.52 \text{ m}^3/\text{kg} \) (Fig. E8.5a)

From Eq. (8.14),

![Diagram](image)

**Fig. E8.5**
\[ p_{\text{air}} \omega_s v_1 = \omega_a R_a (t_{\text{sh}} + 273) \]

\[
p_{\text{air}} = \frac{0.35 \times 0.287 \times 313 \times 10^3}{1000 \times 19.52} \text{ N/m}^2
\]

\[ = 1.61 \text{ N/m}^2 = 1.61 \times 10^{-5} \text{ bar} \]

This is very small and can be neglected.

Condensate leaves at 38 °C. At 38 °C, \( p_{\text{sat}} = 0.06624 \text{ bar} \) and \( v_g = 21.63 \text{ m}^3/\text{kg} \). The total pressure in the condenser is 0.07384 bar

\[ p = p_{\text{sat}} + p_{\text{air}} \]

\[ 0.07384 = 0.06624 + p_{\text{air}} \quad \text{or} \quad p_{\text{air}} = 0.00760 \text{ bar} \]

Mass of air removed per hour,

\[ \omega_a = 20,000 \times \frac{0.35}{1000} = 7 \text{ kg/h} \]

Volume of air removed per hour,

\[ V = \frac{\omega_a R_a (273 + 38)}{p_{\text{air}}} = \frac{7 \times 287 \times 311}{760} = 822.1 \text{ m}^3/\text{h} \]

The mass of steam accompanying this air is

\[ \omega_s = \frac{822.1}{21.63} = 38 \text{ kg/h} \]

Separate air extraction pump: Air along with some steam leaves the air cooler section at 27 °C.

At 27 °C, \( p_{\text{sat}} = 0.03564 \text{ bar} \) and \( v_g = 38.81 \text{ m}^3/\text{kg} \)

\[ p_{\text{air}} = 0.07384 - 0.03564 = 0.03820 \text{ bar} \]

Therefore, the volume of air removed,

\[ V = \frac{\omega_a R_a T}{p_{\text{air}}} = \frac{7 \times 287 \times 300}{3820} = 157.77 \text{ m}^3/\text{h} \]

Mass of steam accompanying this air,

\[ \omega_s = \frac{157.77}{38.81} = 4.07 \text{ kg/h} \]

Hence, the saving in condensate by using separate extraction is

\[ 38 - 4.07 = 33.93 \text{ kg/h} \]

Ans. (a)

Saving in heat supply in the boiler

\[ = 33.98 \times 4.187 (38 - 10) \]

\[ = 3977.8 \text{ kJ/h} = 1.105 \text{ kW} \]

Ans. (a)

Air ejector capacity without air cooler = 822.1 \text{ m}^3/\text{h}

Air ejector capacity with the air cooler = 157.77 \text{ m}^3/\text{h}

Percentage reduction in air ejector load
\[
\begin{align*}
&= \frac{822.1 - 157.77}{822.1} \times 100 = 80.8\% \quad \text{Ans. (b)}
\end{align*}
\]

By making an energy balance for the condenser (Fig. E8.2b), with subscripts s, a and c being used for steam, air and condensate respectively,

\[Q_2 = \omega_{s_1} h_{s_1} + \omega_{a_1} h_{a_1} - (\omega_{s_2} h_{s_2} + \omega_{a_2} h_{a_2}) - \omega_c h_c\]

Here,

\[\omega_{a_1} = \omega_{a_2} = 7 \text{ kg/h}, \quad \omega_{s_2} = 4.07 \text{ kg/h}, \]

\[\omega_c = 20,000 - 4.07 = 20,000 \text{ kg/h} = \omega_{s_1}\]

\[h_c = h_f \text{ at } 38 \text{ °C} = 159.3 \text{ kJ/kg},\]

\[h_{s_1} = h_g \text{ at } 40 \text{ °C} = 2574.3 \text{ kJ/kg},\]

\[h_{s_2} = h_g \text{ at } 27 \text{ °C} = 2550.3 \text{ kJ/kg},\]

\[Q_2 = \omega_{s_1} (h_{s_1} - h_c) - \omega_{a_1} (h_{a_1} - h_{a_2}) - \omega_{s_2} h_{s_2}\]

\[= 20,000 (2574.3 - 159.3)\]

\[- 7 \times 1.005(40 - 27) - 4.07 \times 2550.3\]

\[= 48,300,000 - 91.5 - 10379.7\]

\[= 48389529 \text{ kJ/h} = 13441.5 \text{ kW}\]

Now,

\[Q_2 = \omega_c c_p (t_{c_2} - t_{c_1})\]

\[13441.5 = \omega_c \times 4.187 \times 6\]

\[\omega_c = 535 \text{ kg/s} \quad \text{Ans. (c)}\]

If we neglect the energy leaving with the flow to the ejector,

\[Q_2 = \omega_{s_1} (h_{s_1} - h_c) = \omega_c c_p (t_{c_2} - t_{c_1})\]

\[48,300,000 = w_c \times 4.187 \times 6\]

\[\omega_c = 534 \text{ kg/s} \quad \text{Ans. (c)}\]

Example 8.6 Water at 30 °C flows into a cooling tower at the rate of 1.15 kg per kg air. Air enters the tower at the dbt of 20 °C and a relative humidity of 60 % and leaves it at a dbt of 28 °C and 90% relative humidity. Makeup water is supplied at 20 °C. Determine (a) the temperature of water leaving the tower, (b) the fraction of water evaporated, and (c) the approach and range of the cooling tower.

Solution Properties of air entering and leaving the tower (Fig. 8.28) are obtained from psychrometric chart.

\[t_{wb_1} = 15.2 \text{ °C}, \quad t_{wb_2} = 26.7 \text{ °C},\]

\[h_1 = 43 \text{ kJ/kg dry air}, \quad h_2 = 83.5 \text{ kJ/kg dry air},\]

\[W_1 = 0.0088 \text{ kg water vapour/kg dry air}\]

\[W_2 = 0.0213 \text{ kg water vapour/kg dry air}\]

Enthalpies of water entering the tower and the makeup water are

\[h_{w_3} = 125.8 \text{ kJ/kg}, \quad h_w = 84 \text{ kJ/kg}\]

From the energy balance Eq. (8.22),

\[h_{w_3} - h_{w_4} = \frac{G}{\omega_c} \left[ (h_2 - h_1) - (W_2 - W_1) h_w \right]\]
\[
\frac{1}{1.15} \left[ (83.5 - 43) - (0.0213 - 0.0088)84 \right]
= 34.2 \text{ kJ/kg}
\]

\[
t_w^3 - t_w^4 = \frac{34.2}{4.19} = 30 - t_w^4
\]

\[t_w^4 = 21.8 ^\circ C\]  \(\text{Ans. (a)}\)

\[\text{Approach} = t_w^4 - t_{wb_1} = 21.8 - 15.2 = 6.6 ^\circ C\]  \(\text{Ans. (c)}\)

\[\text{Range} = t_w^3 - t_w^4 = 30 - 21.8 = 8.2 ^\circ C\]  \(\text{Ans. (c)}\)

Fraction of water evaporated.

\[x = \frac{G(W_2 - W_1)}{= 1(0.0231 - 0.0088)}\]

\[= 0.0125 \text{ kg/kg dry air} \quad \text{Ans. (b)}\]

**Example 8.7** Warm water at 45 °C enters a cooling tower at the rate of 6 kg/s. An ID fan draws 10 m³/s of air through the tower and absorbs 4.90 kW. The air entering the tower is at 20 °C dbt and 60% relative humidity. The air leaving the tower is assumed to be saturated and its temperature is 26 °C. Calculate the final temperature of the water and the amount of makeup water required per second. Assume that the pressure remains constant throughout the tower at 1.013 bar.

**Solution** At inlet, RH \(\phi = p_w/p_s = 0.6\), At 20 °C, \(p_s = 0.0234\) bar

\[p_{s_1} = 0.6 \times 0.0234 = 0.01404 \text{ bar}\]

\[p_{a_1} = 1.013 - 0.01404 = 0.99896 \text{ bar}\]

Dry air flow,

\[G_1 = \frac{10^5 \times 0.99896 \times 10}{0.287 \times 10^{-3} \times 293} = 11.8795 \text{ kg/s} = 11.88 \text{ kg/s}\]

Moisture flow,

\[\omega_1 = \frac{0.01404 \times 10^5 \times 10}{0.4619 \times 10^3 \times 293} = 0.1037 \text{ kg/s}\]

\[W_1 = \frac{0.1037}{11.8795} = 0.00874 \text{ kg vap/kg dry air}\]

At exit, at 26 °C,

\[p_s = 0.0336 \text{ bar}, \phi = 100\%\]

\[p_{w_2} = 0.0336 \text{ bar}\]

\[W_2 = W_s = 0.622 \frac{p_{w_2}}{p - p_{w_2}} = 0.622 \frac{0.0336}{1.013 - 0.0336} = 0.02133 \text{ kg vap./kg dry air}\]
Now, \[ W_2 = \frac{G_2}{\omega_2} = 0.02133 \text{ kg vap/kg dry air} \]

\[ G_1 = G_2 \]

Moisture flow at exit,
\[ \omega_2 = 0.02133 \times 11.88 = 0.2534 \text{ kg/s} \]

Makeup water required
\[ = \omega_2 - \omega_1 = 0.2534 - 0.1037 \]
\[ = 0.1497 \text{ kg/s} \quad Ans. \]
\[ \omega_{c_1} = 6 \text{ kg/s} = \text{cooling water inflow} \]
\[ \omega_{c_2} = 6 - 0.1497 = 5.8503 \text{ kg/s} = \text{cooling water outflow} \]

Applying steady flow energy equation (Fig. 8.22),
\[ \omega_{c_1} h_{w_3} + G_1(h_1 + W_1 h_{w_1}) + W_s = \omega_{c_2} h_{w_4} + G_2(h_2 + W_2 h_{w_2}) \]
where \( W_s = \text{shaft work input to the fan} = 4.90 \text{ kW} \).
\[ h_{w_3} = 4.187 \times 45 = 188.4 \text{ kJ/kg} \]
\[ h_{w_1} = h_g + c_p(t-t_{sat}) \]

At \( 20^\circ C \), \( h_g = 2538.1 \text{ kJ/kg} \)

At \( p_{w} = 0.01404 \text{ bar}, t_{sat} = 12^\circ C \)
\[ h_{w_1} = 2538.1 + 1.88(20 - 12) = 2553.14 \text{ kJ/kg} \]
\[ h_{w_2} = h_g \text{ at } 26^\circ C = 2548.4 \text{ kJ/kg} \]
\[ \omega_{c_1} h_{w_3} - \omega_{c_2} h_{w_4} = G[h_2 - h_1] + W_2 h_{w_2} - W_1 h_{w_1} - W_s \]
\[ 6 \times 188.4 - 5.85 h_{w_4} = 11.88 [1.005 (26 - 20) + 0.02133 \times 2548.4] \]
\[ - 0.00874 \times 2553.14] - 4.90 \]
\[ h_{w_4} = 116.752 \text{ kJ/kg} \]

Exit water temperature \[ = \frac{116.752}{4.187} = 27.88^\circ C \quad Ans. \]

---

**SHORT-ANSWER QUESTIONS**

8.1 What are the functions of a condenser in a steam power plant?
8.2 Why is the temperature rise of cooling water restricted?
8.3 What are the different classes of condensers?
8.4 What is a spray condenser? Where is it used?
8.5 Explain a barometric condenser. What is the function of the tail pipe?
8.6 How is a jet condenser different from a barometric condenser?
8.7 What is a surface condenser? Why does cooling water flow inside the tubes and steam condense outside the tubes?
8.8 What are single-pass and two-pass condensers?
8.9 What is a hot well?
8.10 Explain the importance of tube arrangement in the condenser shell. What are steam lanes?
8.11 Why is an air-cooling section provided in the condenser?
8.12 Why is the bulk temperature of the condensate less than the saturation temperature?
8.13 Why is the condensate subcooling not desired? How is it overcome?
8.14 Which parameter affects the overall heat transfer coefficient most?
8.15 What is TTD? How does it influence the condenser design?
8.16 How is the water velocity in a condenser tube optimized?
8.17 How is the pumping power required by a condenser estimated?
8.18 How does air leakage affect the condenser performance?
8.19 Explain how the rate of air leakage into the condenser can be estimated.
8.20 Explain the objective of the steam jet air ejector.
8.21 How does an SJAE operate?
8.22 What are intercondenser and aftercondenser?
8.23 Why are feedwater heaters used?
8.24 Why is one of the heaters always an open heater?
8.25 Where are LP and HP heaters located?
8.26 What is a 3-zone heater? What are the three zones?
8.27 What is a drain cooler?
8.28 Why is the deaerator located at a sufficient height above the BFP?
8.29 What are the different types of deaerators?
8.30 What do you understand by once-through and closed loop circulating water systems?
8.31 Why is open loop system not used?
8.32 What is the function of a cooling tower?
8.33 How does a cooling tower operate?
8.34 What is the need of fill in a tower?
8.35 What do you mean by evaporative cooling?
8.36 What is the minimum temperature to which water can be cooled?
8.37 Define (a) approach, (b) range, and (c) cooling efficiency of a cooling tower.
8.38 What are the different types of cooling towers?
8.39 What is an FD cooling tower? Mention its merits and demerits.
8.40 Why are ID cooling towers preferred in utility plants?
8.41 What are the main advantages of mechanical draught cooling towers?
8.42 What are cross-flow and counter-flow cooling towers?
8.43 What is a natural draught cooling tower? What is the reason of its hyperbolic shape?
8.44 When is a natural draught cooling tower a good choice?
8.45 How does water loss occur in a cooling tower?
8.46 What is drift? How is the drift eliminated?
8.47 Define a dry cooling tower. When is it recommended?
8.48 What are the different types of dry cooling towers?
8.49 Explain an indirect dry cooling tower where a direct contact spray type (open) condenser is used.
8.50 Explain an indirect dry cooling tower where ammonia is used as the coolant in the condenser.
Hidden page
8.6 Water at 60 °C leaving the condenser at the rate of 22.5 kg/s is sprayed into a natural draught cooling tower and leaves it at 27 °C. Air enters the tower at 1.013 bar, 13 °C and 50% relative humidity and leaves it at 38 °C, 1.013 bar and saturated, Calculate (a) the air flow rate required in m³/s, and (b) the makeup water required in kg/s.

[Ans. (a) 21 m³/s, (b) 1 kg/s]

8.7 Water from a cooling system is itself to be cooled in a cooling tower at a rate of 2.78 kg/s. The water enters the tower at 65 °C and leaves a collecting tank at the base at 30 °C. Air flows through the tower, entering the base at 15 °C, 0.1 MPa, 55% RH and leaving the top at 35 °C, 0.1 MPa, saturated. Makeup water enters the collecting tank at 14 °C. Determine the air flow rate into the tower in m³/s and the makeup water flow rate in kg/s.

[Ans. 3.438 m³/s, 0.129 kg/s]

8.8 Cooling water enters a cooling tower at a rate of 1000 kg/h and 70 °C. Water is pumped from the base of the tower at 24 °C and some makeup water is added afterwards. Air enters the tower at 15 °C, 50% RH, 1.013 bar, and is drawn from the tower saturated at 34 °C, 1 bar. Calculate the flow rate of the dry air in kg/h and the makeup water required per hour.

[Ans. 2088 kg/h, 62.9 kg/h]

BIBLIOGRAPHY

9

Nuclear Power Plants

The unit cost per kWh of electricity generation in nuclear power plants is now comparable to or even lower than the unit cost in coal-fired power plants in most parts of the world. In addition, the problems associated with environmental pollution, mine safety, fuel transportation and so on are much less severe in nuclear power stations. Nuclear power utilization can help save a considerable amount of fossil fuels which can be used in other areas of utility.

In recent years, a strong public opinion has grown against the use of nuclear energy for power generation due to the problems related to nuclear safety, radioactive waste disposal and nuclear weapons proliferation. Despite these difficulties, the future of large capacity electricity generation includes nuclear energy as one of the main sources. In many countries like France, Japan, the U.K. and Russia, the bulk of the electricity is produced in nuclear power plants. In India also, the Nuclear Power Corporation has been forging ahead with installation of new plants all around the country.

9.1 STRUCTURE OF THE ATOM

All matter is composed of unit particles called atoms. An atom consists of a relatively heavy, positively charged nucleus and a number of much lighter negatively charged, electrons orbiting around the nucleus. The nucleus consists of protons and neutrons, which together are called nucleons. Protons are positively charged, while the neutrons are electrically neutral. The electric charge on the proton is equal in magnitude but opposite in sign to that on an electron. The atom as a whole is electrically neutral, since the number of protons is equal to the number of electrons in orbit.

The number of protons in the nucleus is called the atomic number, Z. The total number of nucleons in the nucleus is called the mass number, A. Nuclear symbols are written conventionally as

\[ _Z^A X \]

or

\[ ^Z_X \]

where X is the usual chemical symbol. Most of the weight of an atom is concentrated in the nucleus. The radius of a nucleus is of the order of \(10^{-16}\) m and that of a atom is \(10^{-11}\) m. Figure 9.1 illustrates the atomic structure of some simple atoms. The masses of the three primary atomic subparticles are:
Neutron mass, $m_n = 1.008665$ amu = $1.674 \times 10^{-27}$ kg

Proton mass, $m_p = 1.007277$ amu = $1.673 \times 10^{-27}$ kg

Electron mass, $m_e = 0.0005486$ amu = $9.109 \times 10^{-31}$ kg

The atomic mass unit, amu, is a unit of mass approximately equal to $1.66 \times 10^{-27}$ kg.

Hydrogen ($^1\text{H}$) has a nucleus composed of one proton, no neutron, and one orbital electron ($Z = 1, A = 1$). It is the only atom that has no neutron. Deuterium ($^2\text{H}$) has one proton and one neutron in its nucleus and one orbital electron ($Z = 1, A = 2$). Helium ($^4\text{He}$) has two protons, two neutrons and two electrons ($Z = 2, A = 4$).

Atoms with nuclei having the same number of protons have similar chemical and physical properties and differ mainly in their masses. They are called isotopes. For example, deuterium, often called heavy hydrogen, is an isotope of hydrogen. When combined with oxygen, ordinary hydrogen and deuterium form ordinary water ($\text{H}_2\text{O}$) and heavy water ($\text{D}_2\text{O}$) respectively.

Natural uranium is composed of 99.282 % $^{238}\text{U}$, 0.712 % $^{235}\text{U}$ and 0.006 % $^{234}\text{U}$, the atomic number being 92 in all cases. Many isotopes do not appear in nature and are synthesized in the laboratory or in nuclear reactors.

Two other particles are of importance, viz., the positron and neutrino. The positron is a positively charged electron having the symbols $^0_1\text{e}^+$, $e^+$ or $\beta^+$, the symbol for electron being $^0_{-1}\text{e}^-$, $e^-$ or $\beta^-$. The neutrino is a tiny, electrically neutral particle, ejected along with $\beta$ particle during nuclear fission. The ejected neutrinos ($\nu$) carry some 5% of the total energy produced in fission.
Electrons that orbit in the outermost shell of an atom are called valence electrons which decide the chemical properties of an element.

### 9.2 CHEMICAL AND NUCLEAR REACTIONS

Atoms are combined or separated in a chemical reaction. In the reaction,

\[ C + O_2 = CO_2 \]

an energy of 4 electron volt (eV) is released. In nuclear engineering, the unit of energy is electron volt, 1 eV = \(1.6021 \times 10^{-19}\) J = \(4.44 \times 10^{-26}\) kWh. It is the energy acquired by an electron when it is accelerated across a potential difference of 1 volt.

In chemical reactions, although the molecules change, each atom participates as a whole and retains its identity. Only the valence electrons are shared or exchanged. The nuclei do not change. In a chemical reaction, the number of atoms of each element in the products is equal to the number in the reactants.

In nuclear reactions, the products do not have the reactant nuclei but some other nuclei. The number of nucleons in the products are the same as those in the reactants. If \(A, B, C\) and \(D\) represent the chemical symbols, the corresponding nuclear equation may be written as

\[ Z_1A^{A_1} + Z_2B^{A_2} \rightarrow Z_3C^{A_3} + Z_4D^{A_4} \]

where \(Z_1 + Z_2 = Z_3 + Z_4\) and \(A_1 + A_2 = A_3 + A_4\).

Sometimes, electromagnetic radiation (\(\gamma\)-rays) and neutrino (\(\nu\)) are often emitted but they do not affect the above balance, since they carry only energy and have zero \(Z\) and \(A\).

### 9.3 NUCLEAR STABILITY AND BINDING ENERGY

The sum of the masses of the protons and neutrons that comprise the nucleus exceeds the mass of the atomic nucleus. This difference in mass is called the mass defect. The mass defect \((\Delta m)\) is found by adding up all the individual particle weights and subtracting the actual mass \((m)\) of the atom:

\[ \Delta m = n_n m_n + (m_p + m_e)Z - 2m^A \quad (9.1) \]

where \(n\) refers to the number and \(m\) the mass of particles. The mass defect is converted to energy in a nuclear reaction as given by Einstein's law:

\[ \Delta E = \Delta m \cdot C^2 \quad (9.2) \]

where \(E\) = energy, J; \(C\) = velocity of light = \(3 \times 10^8\) m/s; and \(\Delta m\) = mass defect, kg.

The energy associated with the mass defect is known as the binding energy (BE) of the nucleus. It acts as a “glue” which binds the protons and neutrons together in the nucleus. The energy equivalent of 1 g of mass is
\[ \Delta E = 1 \times 10^{-3} \text{ kg} \times (3 \times 10^8 \text{ m/s})^2 = 9 \times 10^{13} \text{ J} \]

Similarly, the energy equivalent of 1 amu of mass is

\[ \Delta E = 1.66 \times 10^{-27} \text{ kg} \times (3 \times 10^8 \text{ m/s})^2 \]
\[ = 14.94 \times 10^{-11} \text{ J} = 9.31 \times 10^8 \text{ eV} \]
\[ = 931 \text{ MeV} \quad (9.3) \]

Therefore, if 1 amu of mass could be completely converted to energy, 931 MeV would be yielded.

The binding energy per nucleon (i.e., proton and neutron) determines the stability of the nucleus. Let us consider a helium nucleus as a simple example.

He nucleus: \( 2_1^1\text{p} + 2_1^1\text{p} \longrightarrow 2_2^4\text{He} \)

Experimental mass (by mass spectrography) of helium atom – mass of two orbital electrons = 4.00387 – 2 \times 0.00055 = 4.00277 amu

Calculated mass = \( 2m_p + 2m_n \) = 2 \times 1.00759 + 2 \times 1.00898 = 4.03314 amu

\[ \Delta m = \text{mass defect} = 0.03037 \text{ amu} \]

and

\[ \Delta E = 0.03037 \times 931 = 28.2 \text{ MeV} \]

This is the energy released when two protons and two neutrons are bound together. If we were to change the helium nucleus back into its constituents, we would have to give back this 28.2 MeV to the nucleus. The binding energy per nucleon is then

\[ \text{BE/nucleon} = 28.2/4 = 7.05 \text{ MeV} \]

For deuterium it is 1.115 MeV/nucleon. In this way, the binding energy per nucleon can be calculated for all the isotopes. Higher the binding energy per nucleon, higher is the stability of the nucleus (Fig. 9.2). The binding energy curve shows that the most stable elements (like iron, cobalt, nickel etc.) are in the intermediate mass number range. If elements of low mass number are fused together, it would lead to more stable elements. The elements of higher mass number are less stable if they are fissioned, they would form elements of less mass number, which are more stable. Thus light isotopes like hydrogen, deuterium and so on are good for fusion reactions, while the heavier isotopes like uranium are suitable for fission reaction.

For most medium and heavy nuclei, the binding energy per nucleon falls roughly between 7.5 and 8.7 MeV. Thus, if a nucleus is to expel one nucleon, say a neutron, it should first have a minimum excitation energy of between 7.5 and 8.7 MeV. Only in such an excited state a nucleus can emit a neutron.

It was found that the nuclei of the even-even type, i.e., having an even number of protons and even number of neutrons, are very stable. Therefore, a \( _{92}^\text{U}^{238} \) atom having 92 protons and 146 neutrons is quite stable and requires very high energy neutrons for fission, whereas a \( _{92}^\text{U}^{235} \) atom having 92 protons and 143 neutrons can be fissioned even by low energy neutrons.

Except for light nuclei, where \( n_p = n_n \), the number of neutrons \( (n_n) \) of known isotopes exceeds the number of protons \( (n_p) \). Thus, for heavier atoms, more neutrons are necessary to shield the protons and overcome the electrical repulsive forces between them in the nucleus.
**9.4 RADIOACTIVE DECAY AND HALF LIFE**

Most isotopes that occur in nature are stable. Some isotopes of heavy elements like thallium (Z = 81), lead (Z = 82) and bismuth (Z = 83), and all isotopes of heavier elements starting with polonium (Z = 84) are not stable (the binding energy per nucleon being small) and emit radiation till a more stable nucleus is reached. Thus, a spontaneous disintegration process, called *radioactive decay*, occurs. The resulting nucleus is called the *daughter* and the original nucleus is called the *parent*. The daughter product may be stable or radioactive. A few lower mass isotopes are also naturally radioactive, such as K\(^{40}\), Rb\(^{87}\) and In\(^{115}\).

Radioactive isotopes, both natural and man-made, are commonly called *radioisotopes*. An example of radioactivity is

\[
_{49}\text{ln}^{115} \rightarrow _{50}\text{Sn}^{115} + _{-1}e^0
\]  

(9.4)

where \(\text{ln}^{115}\) is a naturally occurring radioisotope and its daughter, \(\text{Sn}^{115}\), is stable. Radioactivity is always accompanied by a decrease of mass or liberation of energy. The energy thus liberated shows up in the form of KE of the emitted particles and as electromagnetic radiation (γ-rays).

Naturally occurring radioisotopes emit (1) \(α\) particles, (2) \(β\) particles, (3) \(γ\) radiation; undergo, (4) Positron decay, (5) orbital electron absorption, called \(K\) capture, and also emit (6) neutrons and neutrinos.

1. **Alpha decay**  Alpha particles are helium nuclei, commonly emitted by heavier radioactive nuclei and are accompanied by \(γ\)-radiation. For example,

\[
_{94}\text{Pu}^{239} \rightarrow ^{α}_{92}\text{U}^{235} + _{2}\text{He}^4
\]  

(9.5)

2. **Beta decay**  It is equivalent to the emission of an electron and raises the atomic number by one, while the mass number remains the same. It is usually accompanied by the emission of neutrino (ν) and \(γ\) radiation.
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In neutron decay the daughter is an isotope of the parent. Though it occurs rarely, it, however, comes about in nuclear reactors yielding delayed fission neutrons which greatly influence the reactor control.

The rate of decay is a function only of the number of radioactive nuclei present at a time, provided that the number is large. It does not depend on temperature, pressure or the physical and chemical states of matter, i.e. whether it is in solid, liquid or gaseous phase, or in chemical combination with other atoms.

If \( N \) be number of radioactive nuclei of one species at any time \( \theta \), the rate of decay

\[
- \frac{dN}{d\theta} = \lambda N \tag{9.12}
\]

where \( \lambda \) is a constant of proportionality, called the decay constant, having different values for different isotopes, with the dimension \( s^{-1} \). By integrating the above, we obtain a simple exponential relation,

\[
N = N_0 e^{-\lambda \theta} \tag{9.13}
\]

where \( N_0 \) = radioactive atoms present at time \( \theta = 0 \), and \( N \) = radioactive atoms present at time \( \theta \).

The rate of decay (\(- dN/d\theta\)) is also called activity, \( A \), and has the dimension of disintegration per second or dis/s or \( s^{-1} \). Thus, from Eqs (9.12) and (9.13),

\[
A = - \frac{dN}{d\theta} = \lambda N = \lambda N_0 e^{-\lambda \theta} = A_0 e^{-\lambda \theta} \tag{9.14}
\]

The decay rate is often expressed in the form of half-life, \( \theta_{1/2} \), i.e. the time during which one-half of the number of radioactive species decays. Thus,

\[
\frac{N}{N_0} = \frac{A}{A_0} = \frac{1}{2} = e^{-\lambda \theta_{1/2}}
\]

or

\[
\theta_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{\lambda} \tag{9.15}
\]
Thus, the half-life is inversely proportional to the decay constant. Starting with \( N_0 \), half of \( N_0 \) decay after one half-life; one-half of the remaining atoms or \( 1/4 \) of \( N_0 \) decay during the second half-life and so on (Fig. 9.4).

**Fig. 9.4** Radioactive decay rate and half-life

Half-lives of radioactive isotopes differ by a wide range, varying from fractions of a microsecond to billions of years (Table 9.1). No. two radioisotopes

<table>
<thead>
<tr>
<th>Isotope</th>
<th>( \theta_{1/2} )</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tritium (H³)</td>
<td>12.26 yr</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Carbon 14</td>
<td>5730 yr</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Krypton 87</td>
<td>76 min</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Strontium 90</td>
<td>28.1 yr</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Xenon 135</td>
<td>9.2 h</td>
<td>( \beta ) and ( \gamma )</td>
</tr>
<tr>
<td>Barium 139</td>
<td>82.9 min</td>
<td>( \beta ) and ( \gamma )</td>
</tr>
<tr>
<td>Radium 223</td>
<td>11.43 days</td>
<td>( \alpha ) and ( \gamma )</td>
</tr>
<tr>
<td>Radium 226</td>
<td>1600 yr</td>
<td>( \alpha ) and ( \gamma )</td>
</tr>
<tr>
<td>Thorium 232</td>
<td>( 1.41 \times 10^{10} ) yr</td>
<td>( \alpha ) and ( \gamma )</td>
</tr>
<tr>
<td>Thorium 233</td>
<td>22.1 min</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Protactinium 233</td>
<td>27.0 days</td>
<td>( \beta ) and ( \gamma )</td>
</tr>
<tr>
<td>Uranium 233</td>
<td>( 1.65 \times 10^{5} ) yr</td>
<td>( \alpha ) and ( \gamma )</td>
</tr>
<tr>
<td>Uranium 235</td>
<td>( 7.1 \times 10^{6} ) yr</td>
<td>( \alpha ) and ( \gamma )</td>
</tr>
<tr>
<td>Uranium 238</td>
<td>( 4.51 \times 10^{9} ) yr</td>
<td>( \alpha ) and ( \gamma )</td>
</tr>
<tr>
<td>Neptunium 239</td>
<td>2.35 days</td>
<td>( \beta ) and ( \gamma )</td>
</tr>
<tr>
<td>Plutonium 239</td>
<td>( 2.44 \times 10^{4} ) yr</td>
<td>( \alpha ) and ( \gamma )</td>
</tr>
</tbody>
</table>

have exactly the same half-lives. Thus, half-lives are considered "finger-prints" to identify a radioisotope.
Readily fissionable isotopes U-233, U-235 and Pu-239 have extremely long half-lives, showing that they can be stored practically indefinitely.

The unit of radioactivity is curie (Ci),

\[ 1 \text{ curie} = 3.615 \times 10^{10} \text{ dis/s} \]

It was based on measurement of the activity of 1 g of radium 226. Curie has now been superseded by the SI unit, becquerel (Bq), which is defined as one disintegration per second. Since this is very small, the levels of radioactivity are expressed in kBq or MBq. Another unit, called roentgen (r), is used to provide some measure of the extent of biological injury, say due to X-rays and γ-rays.

## 9.5 Nuclear Fission

Fission can be caused by bombarding with high energy α-particles, protons, deuterons, X-rays as well as neutrons. However, neutrons are most suitable for fission, They are electrically neutral and thus require no high KE to overcome electrical repulsion from positively charged nuclei. Two or three neutrons are usually released for each one absorbed in fission, and can thus keep the reaction going (Fig. 9.5). Isotopes like U-233, U-235 and Pu-239 can be fissioned by neutrons of all energies, whereas isotopes U-238, Th-232 and Pu-240 are fissionable by high energy (14 MeV) only. As shown in Fig. 9.5,

![Fission reaction diagram](image)

**Fig. 9.5 A fission reaction**

\[ { }_{92}^{235}U + { }_0^1n \longrightarrow { }_{54}^{140}Xe + { }_{38}^{94}Sr + 2{ }_0^1n \]  

(9.16)

The immediate (prompt) products of a fission reaction, such as Xe\(^{140}\) and Sr\(^{94}\) are called fission fragments, which along with other decay products (α, β, γ etc.) are called fission products.

When a neutron collides with and is absorbed by a fissionable nucleus, the latter is transformed into a compound nucleus in an excited state, e.g.,

\[ { }_{92}^{235}U + { }_0^1n \longrightarrow \left[ { }_{92}^{236}U \right]^* \longrightarrow \text{fission} \]

which then undergoes fission. If the excitation energy is not sufficiently large, the nucleus may not undergo fission and may emit only γ-radiation or eject a particle. Such absorption of a neutron in a non-fission reaction occurs about 16% of the time in all neutron absorptions by U-235.
The two fission fragments in Eq. (9.16) are not equal in size and are radioactive. The original nuclei of U-235 have neutron–proton ratio of 1.55. Their fission fragments have also similar \( n-p \) ratios, which for stable nuclei are, however, 1.2 to 1.4. The fission fragments, therefore, undergo several stages of \( \beta \) decay (converting neutrons into protons) until a stable product is formed in each case.

\[
\begin{align*}
_{54}\text{Xe}^{140} & \xrightarrow{\beta} \_{55}\text{Cs}^{140} \xrightarrow{\beta} \_{56}\text{Ba}^{140} \xrightarrow{\beta} \_{57}\text{La}^{140} \xrightarrow{\beta} \_{58}\text{Ce}^{140} \\
\text{16\text{s}} & \quad \text{60\text{s}} & \quad 12.8 \text{ days} & \quad 40 \text{ hrs} & \text{ (stable)}
\end{align*}
\]

and

\[
\begin{align*}
_{38}\text{Sr}^{94} & \xrightarrow{\beta} \_{39}\text{Y}^{94} \xrightarrow{\beta} \_{40}\text{Zr}^{94} \text{ (stable)} \\
\text{2 min} & \quad 17 \text{ min}
\end{align*}
\]

(9.17)

This series is called a fission chain. Since \( \beta \)-decay is usually accompanied by \( \gamma \)-radiation, suitable shielding against \( \gamma \)-rays as well as neutrons must be provided in a reactor. Figure 9.6 shows fission product data for U-235 by thermal and fast (14 MeV) neutrons and for U-233 and Pu-239 by thermal neutrons. The most probable fission products have mass numbers in the ranges 85 to 105 and 130 to 150, meaning that the products are not equal in size.

**Fig. 9.6** Fission product data for (a) U-235 by thermal and 14 MeV neutrons, and (b) U-233 and Pu-239 by thermal neutrons

### 9.5.1 Energy from Fission and Fuel Burnup

There are many fission reactions which release different amounts of energy. For the reaction,
\[ \text{has the mass balance} \]
\[ \text{235.0439 + 1.00867} \rightarrow 138.9061 + 96.9212 + 2 \times 1.00867 \]
or,
\[ 236.0526 \rightarrow 235.8446 \text{ amu} \]

There is thus a reduction in mass, which appears in the form of energy (exothermic). The mass defect is
\[ \Delta m = 235.8446 - 236.0526 = -0.2080 \text{ amu} \]

Therefore,
\[ \Delta E = -0.2080 \times 931 = -193.6 \text{ MeV} \]

The fission of U-235 yields on an average about 193 MeV. which is the same for the fission of U-233 and Pu-239. This amount of energy is prompt, i.e. released during the fission process. More energy is, however, produced due to (i) the slow decay of the fission fragments, and (ii) the non-fission capture of excess neutrons. The total energy released per fission reaction is about 200 MeV. The complete fission of 1 g of U-235 nuclei thus produces

\[
\text{Avogadro constant} \times 200 \text{ MeV} = \frac{6.023 \times 10^{23}}{235.0439} \times 200 \text{ MeV} = 5.126 \times 10^{23} \text{ MeV}
\]
\[ = 8.19 \times 10^{10} \text{ J} = 2.276 \times 10^{24} \text{ kWh} = 0.984 \text{ MW-day} \]

Thus, a reactor burning 1 g of U-235 generates nearly 1 MW-day of energy. This is referred to by the term “fuel burnup”, which is the amount of energy in MW-days produced of each metric ton of fuel.

The complete fission of all U-235 nuclei in a fuel mass is impossible, since many of the fission products capture neutrons in a non-fission reaction. In time, the number of neutrons so captured becomes great enough because of the accumulation of products, and the fission chain can no longer be sustained. Depending upon fuel enrichment, this happens when only a small percentage, often less than 1% of the fissionable nuclei in the fuel has been consumed. Further use of this poisoned fuel can only be made by removing the fission products and reprocessing.

### 9.6 CHAIN REACTION

The number of newly born fission neutrons in a single fission for U\textsuperscript{235} nuclei is either 2 or 3 and on an average 2.47. In a reactor where controlled and sustained energy production is desired, conserving neutrons is a vital matter.

There are two reasons why not all the fission neutrons cause further fission.

1. Non-fission capture or absorption of some neutrons by the fission products, non-fissionable nuclei in the fuel, structural material, coolant, moderator and so on.
2. Leakage of neutrons escaping from the core.

The smaller the surface-volume ratio of the core, i.e. the larger its size, the lower is the percentage of leakage of neutrons. The core size is increased to the point where a chain reaction is possible. This size is called the critical size. The mass of fuel in such a core is called the critical mass.

In a reactor using U\(^{235}\) as fuel, 100/2.47 or about 40.5 of each 100 fission neutrons must ultimately engage in fission to keep the reactor critical. However, only about 84% of the neutrons that get absorbed in U\(^{235}\) cause fission. The remaining 16% neutrons reacting with it produce U\(^{236}\) (non-fission capture), an isotope of no particular importance. Therefore, a total of about 40.5/0.84 or 48 neutrons must be absorbed in U\(^{235}\) to cause fission. Thus, a maximum of about 52 neutrons may be allowed to leak out of the core and be absorbed in other core materials.

### 9.7 NEUTRON ENERGIES

The kinetic energy of a neutron, KE\(_n\) or simply \(E_n\), is given by

\[
E_n = \frac{1}{2} m_n V^2
\]

where \(m_n\) is the mass of neutron (1.008665 amu), and \(V\) the speed of neutron. Thus,

\[
E_n = \frac{1}{2} \times 1.008665 \times \frac{1}{0.965 \times 10^{18}} V^2 = 5.227 \times 10^{-19} V^2 \text{ MeV} = 5.227 \times 10^{-13} V^2 \text{ eV}
\]

(9.19)

where \(V\) is in cm/s.

The newly born fission neutrons have energies varying between 0.075 to 17 MeV. As these neutrons travel through matter, they collide with other nuclei and get slowed down. This process is called scattering. The neutron gives up some of its energy with each successive collision.

Neutrons are classified into three general categories according to their energy as fast, intermediate and slow.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Neutron energy (eV)</th>
<th>Corresponding velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast</td>
<td>&gt; 10^5</td>
<td>&gt; 4.4 \times 10^6</td>
</tr>
<tr>
<td>Intermediate</td>
<td>1–10^5</td>
<td>(1.38 to 4.4) \times 10^6</td>
</tr>
<tr>
<td>Slow</td>
<td>&lt; 1</td>
<td>&lt; 1.38 \times 10^4</td>
</tr>
</tbody>
</table>

Newly born fission neutrons carry, on an average, about 2% of a reactor fission energy in the form of KE. As stated earlier, they can be (1) Prompt neutrons, emitted within 10\(^{-14}\) s after fission occurs from the fission fragments, and (2) Delayed neutrons, produced in radioactive decay reactions of the fission fragments and their products. Though the energies of delayed neutrons are relatively small, they play a vital role in nuclear reactor control.
9.7.1 Neutron Scattering

In the scattering process, the energy balance of colliding particles before and after collision gives

\[(E_n + KE_c)_1 = (E_n + KE_c + E^*_c)_2\]  \hspace{1cm} (9.20)

where the subscripts \(n\) and \(c\) denote neutron and nucleus, and 1 and 2 denote before and after collision. \(E^*_c\) is the excitation energy of the struck nucleus. Scattering can be of two types:

1. **Inelastic scattering**, in which momentum and total energy of the particles before and after collision are conserved. However, \(KE\) is not conserved. Part of the \(KE\) before collision is absorbed by the nucleus to have the excitation energy \(E^*_c\). Thus, for a neutron to engage in inelastic scattering, it has to possess an initially high \(KE\) (> \(E^*_c\)).

2. **Elastic scattering**, in which a neutron does not possess the necessary minimum \(KE\) and \(E^*_c\) is zero. Both momentum and \(KE\) of the colliding particles are conserved.

In each scattering process, a part of the \(KE\) of the neutron is transferred to the initial relatively stationary and heavier nucleus, thereby slowing down the neutron. The amount of energy lost by a neutron in each collision depends upon the mass of the nucleus and the angle of scatter. The maximum energy is lost in a head-on collision. If a neutron possesses an initial \(KE\) \(E_{n,i}\), its \(KE\) after a head-on collision \(E_{n,min}\) is given by

\[E_{n,min} = E_{n,i} \left( \frac{M - m_n}{M + m_n} \right)^2\]  \hspace{1cm} (9.21)

where \(M\) and \(m_n\) are the nucleus and neutron masses, respectively. It can be approximately expressed as

\[\frac{E_{n,min}}{E_{n,i}} = \left( \frac{A - 1}{A + 1} \right)^2\]  \hspace{1cm} (9.22)

where \(A\) is the mass number of the nucleus. A neutron may lose a maximum of less than 2% in a collision with \(U^{238}\) nucleus, but about 28% with a carbon nucleus and all its energy in a single collision with a hydrogen nucleus \((A = 1)\).

The average neutron energy lost per elastic collision is expressed in terms of a quantity called the logarithmic energy decrement, \(\xi\), defined by

\[\xi = \ln E_{n,i} - \ln E_{n,av} = \ln \frac{E_{n,i}}{E_{n,av}}\]  \hspace{1cm} (9.23)

where \(E_{n,av}\) is the average energy of the neutron after a single collision. \(\xi\) is given by

\[\xi = 1 - \left[ \frac{(A - 1)^2}{2A} \ln \frac{A + 1}{A - 1} \right]\]  \hspace{1cm} (9.24)

where \(A\) is the mass number of the struck nucleus (moderator). From the above equation, it is seen that as \(A \rightarrow 1\), \(\xi = 1\). Thus, if a neutron collides with a
hydrogen nucleus \((A = 1)\), the average neutron energy after one collision is \(1/e\) of its initial energy. This is the maximum possible decrease of neutron energy.

Thus, the number of collisions, \(n\), required to slow down a neutron from initial energy \(E_{n, i}\) to a final energy \(E_{n, f}\) in elastic scatter is given by

\[
n = \frac{\ln \frac{E_{n, i}}{E_{n, f}}}{\xi}
\]

(9.25)

A moderator is used to slow down the neutron in a reactor. Thus, smaller the nucleus, better the moderator. Table 9.2 gives the values of \(n\) to bring down the neutron energies from 2 MeV to 0.025 eV in elastic collisions. However, \(n\) is not the sole criterion of moderator effectiveness. Other aspects, such as the probability of collision, the probability of absorption and scattering, as well as the number of moderator nuclei in a given volume also influence the moderator effectiveness.

Table 9.2  Number of elastic collisions \((n)\) between 2 MeV and 0.025 eV

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>A</th>
<th>(\xi)</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1</td>
<td>1.000</td>
<td>18</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>0.725</td>
<td>25</td>
</tr>
<tr>
<td>Be</td>
<td>9</td>
<td>0.208</td>
<td>86</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>0.158</td>
<td>114</td>
</tr>
<tr>
<td>Al</td>
<td>27</td>
<td>0.074</td>
<td>246</td>
</tr>
<tr>
<td>Fe</td>
<td>56</td>
<td>0.038</td>
<td>472</td>
</tr>
<tr>
<td>Zr</td>
<td>91</td>
<td>0.021</td>
<td>866</td>
</tr>
<tr>
<td>U</td>
<td>238</td>
<td>0.004</td>
<td>4480</td>
</tr>
</tbody>
</table>

9.7.2 Thermal Neutrons

When a large number of neutrons are slowed down in a medium, such as a moderator, the lowest energies that they can attain are those that put them in thermal equilibrium with the molecules of that medium. In this state (ground state) they become thermalized and are called thermal (or slow) neutrons. A reactor primarily utilizing thermal neutrons for fission is called a thermal reactor.

From kinetic theory of gases, it is known that at a certain temperature the molecules of a gas have a wide range of velocity varying from zero to infinity (or velocity of light?) demonstrating Maxwell–Boltzmann velocity distribution (Nag, 1981). The thermalized neutrons will have similar distributions of velocity and also of energy. The most probable velocity of a neutron is thus,

\[
V_m = \left[ \frac{2 \, KT}{m_n} \right]^{1/2}
\]

(9.26)

where \(K\) = Boltzmann constant = \(1.38 \times 10^{-23}\) J/ molecule-K; \(m_n\) = mass of neutron = \(1.674 \times 10^{-27}\) kg, and \(T\) = absolute temperature. K.
On substitution, 
\[ V_m = 128.4 \ T^{1/2} \text{ m/s} \]  
(9.27)

The most probable kinetic energy of a neutron will be

\[ KE_m = \frac{1}{2} m_n V_m^2 \]
\[ = \frac{1}{2} \times \frac{1.674 \times 10^{-27} \times (128.4)^2 T}{1.6021 \times 10^{-19}} \text{ eV} \]
\[ = 8.613 \times 10^{-5} T \text{ eV} \]  
(9.28)

At 20 °C, \( KE_m = 0.02524 \text{ eV} \) and \( V_m = 2198 \text{ m/s} \equiv 2200 \text{ m/s} \)

Table 9.3 gives thermal neutron energies and speeds at different temperatures.

**Table 9.3** Thermal neutron energies and speeds

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>( KE_m (\text{eV}) )</th>
<th>( V_m (\text{m/s}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.0252</td>
<td>2200</td>
</tr>
<tr>
<td>260</td>
<td>0.0459</td>
<td>2964</td>
</tr>
<tr>
<td>537.8</td>
<td>0.0699</td>
<td>3656</td>
</tr>
<tr>
<td>1000</td>
<td>0.0097</td>
<td>4580</td>
</tr>
</tbody>
</table>

Neutrons having energies greater than thermal such as those slowing down in a thermal reactor are called epithermal neutrons.

### 9.8 NUCLEAR CROSS-SECTIONS

If a group of neutrons travel with the same \( KE \) and the corresponding speed is \( v \) cm/s and if their volume density in the beam at a particular point is \( n \) neutrons/cm\(^3\), then the product \( nv \) is equal to the number of neutrons crossing a unit target area of 1 cm\(^2\) per second and is called the neutron flux. \( \phi \). Therefore,

\[ \phi = nv \]  
(9.29)

Its dimension is neutrons/cm\(^2\)-s. However, the speeds of neutrons vary widely (Maxwell-Boltzmann velocity distribution). Then,

\[ \phi = \int_{v}^{\infty} n(v) \ dv \]  
(9.30)

where \( n(v) \ dv \) is the number of neutrons per unit volume whose speeds vary between \( v \) and \( v + dv \).

The interaction rate between a beam of neutrons and the nuclei in a target material has been experimentally observed to be proportional to (i) the neutron flux, and (ii) the number of atoms (or nuclei) in the target.
Let us consider a beam of neutrons, all of speed $V$ cm/s and density $n$ neutrons/cm$^3$, incident on a target area of $A$ cm$^2$ and thickness $dx$ and containing $N$ nuclei/cm$^3$ (Fig. 9.7). The interaction rate, being proportional to the neutron flux, $\phi$, and the number of nuclei on the target material, $NA$ dx, is thus

$$\text{Interaction rate} = \sigma \phi NA \, dx$$

$$= \sigma \phi N V_0 \quad (9.31)$$

where $\sigma$ is the constant of proportionality, called the microscopic cross-section of the isotope concerned, and $V_0$ the volume of the target ($A \, dx$). $\sigma$ has the unit of cm$^2$/nucleus, and it can be regarded as the area presented by each nucleus to neutrons to cause a reaction. The radius of a nucleus is given by

$$r_c = 1.4 \times 10^{-13} \, A^{1/3} \quad (9.32)$$

where $A$ is the mass number. The radii of nuclei vary roughly between $1.4 \times 10^{-13}$ and $10^{-12}$ cm. Therefore, the cross-sectional area of an average nucleus ($\pi r_c^2$) is equal to $10^{-24}$ cm$^2$. This value is taken as the unit of microscopic cross-section and is called one barn.

$$1 \text{ barn} = 10^{-24} \, \text{cm}^2 \quad (9.33)$$

The total cross-section of all nuclei in unit volume of a material is called the macroscopic cross-section, $\Sigma$, and has units cm$^2$/cm$^3$ or cm$^{-1}$.

$$\Sigma = N \sigma \frac{\text{nuclei}}{\text{cm}^3} \, \text{cm}^2 \quad \text{or} \quad \text{cm}^{-1} \quad (9.34)$$

and the interaction rate per unit volume,

$$F = \Sigma \phi \frac{\text{neutrons}}{\text{cm}^3 \text{s}} \quad (9.35)$$

The probability that a neutron entering the target will collide or interact within a distance $dx$,

$$\frac{\sigma \phi N A dx}{\phi A} = \sigma N dx = \Sigma dx \quad (9.36)$$

Thus, the macroscopic cross-section can be explained as the probability per unit length that a neutron will collide, i.e. the collision cross-section.

As the neutrons collide with target nuclei, they get removed from the group. For neutrons which survive collision in an element of thickness $dx$,

$$\text{Rate of collision} = \text{neutron flux (in–out)} \, A$$

$$\sigma \phi NA \, dx = -A \, d\phi$$

The negative sign indicates that the flux is decreasing. Rearranging,

$$\frac{d\phi}{\phi} = -\sigma N \, dx = -\Sigma \, dx$$

$$\phi(x) = \phi_0 e^{-\Sigma x} \quad (9.37)$$
where $\phi_0$ is the incident neutron flux (at $x = 0$). This is referred to as the “survival equation”, i.e., the neutron flux which has survived collision after traveling a distance $x$ in the target.

The average distance that a neutron travels without making a collision or interaction with a target nucleus is called the mean free path, $\lambda$,

$$\lambda = \frac{\int x \phi(x) \, dx}{\phi_0}$$

Using Eq. (9.37)

$$\lambda = \frac{\int x \left( -\phi_0 e^{-\Sigma x} \right) \, dx}{\phi_0} = \frac{1}{\Sigma}$$

(9.38)

Thus the mean free path is the reciprocal of the macroscopic cross-section.

Neutrons have as many cross-sections as there are reactions. The reactions can be scattering, capture and fission. Thus, the rates at which elastic scattering, inelastic scattering, capture and fission take place are characterized by the elastic scattering cross-section, $\sigma_e$, the inelastic scattering cross-section, $\sigma_i$, the capture cross-section $\sigma_c$, and the fission cross-section $\sigma_f$. For non-fissionable isotopes $\sigma_f$ is zero. The total cross-section, $\sigma_t$, is the sum of these cross-sections.

$$\sigma_t = \sigma_e + \sigma_i + \sigma_c + \sigma_f$$

(9.39)

The absorption cross-section, $\sigma_a$, is the sum of the capture and fission cross-sections, or

$$\sigma_a = \sigma_c + \sigma_f$$

If an isotope has capture and elastic scattering cross-sections of 0.1 and 10 barns, respectively, it is obvious that elastic scattering is the most probable reaction and capture is almost negligible, occurring in less than 1% of all reactions.

Macroscopic cross-sections for mixtures and compounds can be calculated from a knowledge of the cross-sections and numbers of atoms per unit volume of the constituents. If a mixture or compound contains $N_1, N_2, N_3$, etc. atoms per cm$^3$ of elements whose microscopic cross-sections are $\sigma_1, \sigma_2, \sigma_3$ and so on, the macroscopic cross-section of the mixture or compound, $\Sigma$, is given by

$$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + \ldots$$

(9.40)

### 9.9 NEUTRON FLUX AND REACTION RATES

The neutron flux $\phi$ is given by the equation

$$\phi = nV$$

(9.41)

where $n$ is the volume density of neutrons in the beam (neutrons/m$^3$) and $V$ the average velocity (m/s) of the neutrons.
The neutron flux $\phi$ varies within the reactor core depending on the energy of neutrons. It is maximum at the core geometric centre and minimum near the core edges. In a thermal reactor, where high-energy fission neutrons are born in the fuel and thermalized in the moderator, fast-neutron fluxes peak in the fuel and thermal-neutron fluxes peak in the moderator. At any position in the fuel, the neutron flux is proportional to the power. It is maximum during full power operation and zero during shutdown.

If a target medium containing nuclei of density $N$ is subjected to a neutron flux $\phi$, the reaction rate between the nuclei and neutrons is given by $\sigma N (nV)$, where $\sigma$ is the microscopic cross-section of the particular reaction (i.e., scatter, absorption etc.). It is equal to $\Sigma \phi$ and hence, is inversely proportional to the mean free path of the neutrons.

### 9.10 MODERATING POWER AND MODERATING RATIO

It was shown earlier that the logarithmic energy decrement factor $\xi$ represents the effect of nucleus size on the average number of collisions required to slow down a neutron over a prescribed energy range. The hydrogen moderator would slow a neutron from 2 MeV to 0.025 eV in 18 collisions, deuterium in 25 collisions and so on. However, aspects like the probability of scattering and absorption, and the number of moderator nuclei per unit volume, $N$, are also important. These along with $\xi$ are grouped together in two parameters, called

$$\text{Moderating power} = \xi N \sigma_s = \xi \Sigma_s$$

$$\text{Moderating ratio} = \xi \frac{\sigma_s}{\sigma_a} = \xi \frac{\Sigma_s}{\Sigma_a}$$

(9.42)

Since $\xi$ is the average loss in the $\ln E_n$ per collision and $\Sigma_s$ is the probability of scatter collision/m, the slowing-down power is equal to the average loss in $\ln E_n$ per m. It has the unit (m$^{-1}$) and should be as large as possible for good moderation. It may be noticed that a high nuclear density $N$ is essential since there will be more reactions if there are more nuclei to react. Thus, hydrogen and deuterium are not suitable as moderators, in gaseous forms, instead they are used in light and heavy water. Similarly, when CO$_2$ is a coolant, graphite is the moderator.

The moderating ratio is a relative measure of the ability of a moderator to scatter neutrons without appreciably absorbing them. It should also be as high as possible for good moderation.

The selection of a moderator also depends on cost, chemical and structural considerations. Heavy water is an excellent moderator, but is extremely costly. Light water is cheap, but has a small neutron absorption cross-section. It is used as a coolant and a moderator when enriched uranium is used as fuel. Graphite is low in cost but is structurally weak. Liquid metal-cooled fast reactors need no moderator.
9.11 VARIATION OF NEUTRON CROSS-SECTIONS WITH NEUTRON ENERGY

The neutron cross-section for any nucleus depends upon the energy of the neutron reacting with it. Plots of $\sigma$ vs. $E_n$ are usually made on log-log coordinates. In most cases, scattering cross-sections, $\sigma_s$, are so small compared with absorption cross-sections, $\sigma_a$, that the total cross-section $\alpha$ is

$$\sigma_t \equiv \sigma_a$$

Also, for most nuclei, $\sigma_s$ does not change much with neutron energy $E_n$.

$$\sigma_a = \sigma_t + \sigma_c \quad \text{and} \quad \sigma_s = \sigma_t - \sigma_a$$

Variations of absorption cross-sections, $\sigma_a$, with neutron energy, $E_n$, can be divided into three regions: (1) $1/V$ region, (2) resonance region, and (3) fast neutron region (Figs 9.8 and 9.9).

![Graph showing variations of absorption cross-sections](image)

**Fig. 9.8** Neutron cross-section for U-235

1. **$1/V$ region** In the low energy region, $\sigma_a$ is inversely proportional to the square root of the neutron energy $E_n$.

$$\sigma_a = C_1 \left( \frac{1}{E_n} \right)^{0.5} \quad \text{(9.43)}$$

Thus,

$$\sigma_a = C_1 \left( \frac{1}{V^2} \right)^{0.5} = C_2 \frac{1}{V} \quad \text{(9.44)}$$
where $C_1$ and $C_2$ are constants, $m_n$ is the neutron mass and $V$ is the neutron velocity.

This relationship is known as the $1/V$ law. It indicates that lower is the velocity of the neutron, longer will be the time a neutron spends in the vicinity of the nucleus, and higher will be the probability of its absorption by the nucleus. It is a straight line with a slope of about – 0.5 (Fig. 9.8 and 9.9).

2. **Resonance region** Most neutron absorbers, after the $1/V$ region, show one or more $\sigma_a$ peaks occurring at definite neutron energies. These are called resonance peaks. Indium has only one peak, whereas U-235 and U-238 have many. U-238 has very high resonance $\sigma_a$, with the highest peak, about 4000 barns, occurring at about 7 eV. The design of thermal reactors is affected by this fact, since U-238 absorbs many of the neutrons passing through the region and affects the reactor neutron balance. Many elements, particularly those of low mass numbers and low $\sigma_a$, do not exhibit resonance absorption and hence, can be used as reactor construction materials.

3. **Fast neutron region** As the neutron energies increase beyond the resonance region, the absorption cross-sections gradually decrease. At very high values of $E_n$,

$$
\sigma_t = \sigma_a + \sigma_s \rightarrow 2 \times \text{cross-sectional area of the target nucleus}
$$

$$
\sigma_t = 2 \pi r_c^2
$$

Combining with Eq. (9.32),
\[ \sigma_i = 2\pi (1.4 \times 10^{-13})^2 \ A^{2/3} \text{ cm}^2 \]
\[ = 0.125 \ A^{2/3} \text{ barns} \] (9.45)

where 1 barn = \(10^{-24}\) cm\(^2\) and \(A\) is the mass number of the target nucleus.

Thus, in the very high neutron energy range, \(\sigma_i\) is very low, usually less than 5 barns for heavier nuclei.

### 9.12 NEUTRON LIFE CYCLE

In a reactor core, neutrons are born at all times and in all places having fissionable material and diffuse in all directions. We will examine the life cycle of a group of neutrons, all assumed to be born at the same time, which undergo scatter, leakage, absorption and other reactions, and finally cause fission and attain the same energy levels simultaneously. This group of neutrons is called a generation. The series of events or processes that such a group of neutrons undergoes from birth until a new generation is born by fission is called a life cycle of neutrons.

Figure 9.10 shows the events that the generation of \(N\) fission neutrons produced in U-235 undergoes in a thermal reactor.

**Fig. 9.10** Neutron life cycle in a thermal reactor
Hidden page
explosive rate as in an atom bomb. If $k_{\text{eff}} < 1$, the reactor is subcritical and the chain reaction decreases and eventually dies out.

Let us consider a chain reaction in an infinitely large system by adding more and more fuel such that the neutron leakage becomes negligibly small and $P = 1$, Therefore,

$$k_{\text{eff}} = k_{\infty} = \varepsilon p f \eta$$

(9.47)

where $k_{\infty}$ is called the infinite multiplication factor. This is a four-factor formula, where $\varepsilon, p$ and $f$ are functions of the fuel and the internal configuration of the core, called the lattice constants, and $\eta$ depends on the kind of fuel and is a fuel constant.

In a critical reactor, the rate at which neutrons are used up or lost must be exactly equal to the rate at which they are produced by fission. The theory of nuclear reactors is concerned with the analysis of all the processes which take place in the core of a reactor, and in particular with the slowing down, diffusion and absorption of neutrons. Only by analyzing these processes in detail can accurate calculations be made to determine the multiplication factor, critical mass of fuel or size of a reactor (permitting a certain amount of neutron leakage).

The reactivity $\rho$ is defined as

$$\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$$

(9.48)

The neutron life time, $\tau^*$, is defined as the average time between successive neutron generations (prompt neutrons). The number of neutrons existing in a unit volume is $n$. Then, the time rate of change of the neutron density is

$$\frac{dn}{d\tau} = n\rho \tau^*$$

or

$$\frac{dn}{n} = \rho \tau^* d\tau$$

or

$$\ln \frac{n}{n_0} = \frac{\rho \tau}{\tau^*}$$

where $n$ and $n_0$ represent the neutron density at time $\tau$ and $\tau = 0$. It can also be expressed as

$$n = n_0 e^{z}$$

(9.49)

where,

$$z = \frac{\rho}{\tau^*} \tau$$

9.13 REFLECTORS

Surrounding the reactor core exists a reflector, which is a medium of low neutron absorption and high neutron scattering cross-sections. Some of the neutrons leaking out of the core are scattered back into it by the reflector nuclei. This
Hidden page
conductivity, good corrosion resistance, good mechanical strength at high temperatures and a high limiting temperature for operation. The cladding serves three functions.

1. To provide structural support and strength for the fuel and prevent distortion.
2. To prevent the release of radioactive fission products into the coolant stream.
3. In certain types of reactors (mainly gas-cooled) to provide extended surfaces in the form of fins to promote more heat transfer to the coolant.

Materials suitable for cladding should have low neutron capture cross-section, high thermal conductivity, good mechanical strength at high temperatures and chemical compatibility with the fuel and coolant. The most common cladding materials are aluminium, magnesium alloys (Magnox), stainless steel and alloys of zirconium (zircaloy).

The rate of heat release by fission per unit volume of fuel is called the volumetric thermal source strength, \( q_G \), given by

\[
q_G = G \phi \frac{\sigma_f}{N} \text{MeV/m}^3\text{s}
\]  
(9.50)

where \( G = \) energy per fission (~180–190 MeV), \( N = \) number of fissionable nuclei/\( m^3 \), \( \sigma_f = \) microscopic fission cross-section of that fuel, \( m^2 \), and \( \phi = \) neutron flux per \( m^2s \).

### 9.14.1 Heat Conduction in Fuel Elements

Fourier’s heat conduction equation in three dimensions is given by [7],

\[
\nabla^2 T + \frac{q_G}{k} = \frac{1}{\alpha} \times \frac{\partial T}{\partial t}
\]  
(9.51)

where \( T \) is the temperature (°C), \( q_G \) the volumetric source strength (W/m\(^3\)), \( k \) the thermal conductivity of the solid (W/mK), \( \alpha \) the thermal diffusivity of the solid (m\(^2\)/s) and \( t \) the time (s).

For a plate type fuel element (Fig. 9.12), in which the fuel of thickness \( 2a \) is enclosed on both sides by a cladding of thickness \( b \), the dimensions of the plate in the \( y \) and \( z \) directions are large compared with the values of \( a \) and \( b \). Consequently, heat conduction may be assumed to be in the direction of the x-axis only. In the fuel, Eq. (9.51) reduces to

\[
\frac{d^2T}{dx^2} = -\frac{q_G}{k_F}
\]

or

\[
\frac{dT}{dx} = -\frac{q_G}{k_F}, \quad x + C,
\]

Fig. 9.12 Temperature distribution in a plate type fuel element
\( k_F \) being the thermal conductivity of fuel.

When \( x = 0 \), \( \frac{dT}{dx} = 0 \), therefore \( C = 0 \). Integrating again between \( x = 0 \) and \( x = a \), we get

\[
\Delta T_F = \frac{q_G a^2}{2k_F}
\]  

(9.52)

where \( \Delta T_F \) is the temperature drop from the centre to the surface of the fuel. The temperature distribution within the fuel is parabolic.

In the cladding there is no energy release, and the heat conducted per unit area through the cladding on each side of the fuel is \( q_G \cdot a \) W/m\(^2\). Therefore,

\[
q = -k_{cl} \frac{dT}{dx} = q_G \cdot a
\]

or

\[
\frac{dT}{dx} = \frac{-q_G \cdot a}{k_{cl}}
\]

Integrating from \( x = a \) to \( x = a + b \), we get

\[
\Delta T_{cl} = \frac{q_G ab}{k_{cl}}
\]  

(9.53)

where \( \Delta T_{cl} \) is the temperature drop through the cladding. The total temperature drop from the centre of the fuel to the surface of the cladding is

\[
\Delta T_F + \Delta T_{cl} = q_G \cdot a \left( \frac{a}{2k_F} + \frac{b}{k_{cl}} \right)
\]  

(9.54)

If a coolant is now considered to be flowing along the cladding, heat will be transferred from the cladding surface to the coolant. Let the cladding surface temperature be \( T_s \) and the bulk fluid temperature be \( T_c \). Then the heat flow per unit area,

\[
q = q_G \cdot a = h(T_s - T_c)
\]

or

\[
T_s - T_c = \frac{q_G a}{h} = \Delta T_c
\]  

(9.55)

= temperature drop in the coolant.

Therefore, the total temperature drop from the centre of the fuel to the bulk fluid is

\[
\Delta T_F + \Delta T_{cl} + \Delta T_c = q_G a \left[ \frac{a}{2k_F} + \frac{b}{k_{cl}} + \frac{1}{h} \right]
\]  

(9.56)

For a cylindrical fuel rod of radius \( a \) surrounded by a cladding of thickness \( b \), the heat conduction along the rod is negligible (Fig. 9.13) and the heat conduction equation in cylindrical coordinates [7] is

\[
\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = -\frac{q_G}{k_F}
\]
Fig. 9.13 Temperature distribution in cylindrical fuel element

\[ \frac{d}{dr} \left[ r \frac{dT}{dr} \right] = -\frac{q_G r}{k_F} \]

or

\[ \frac{dT}{dr} = -\frac{q_G r}{2k_F} + \frac{C_1}{r} \quad (9.57) \]

When \( r = 0 \), \( \frac{dT}{dr} = 0 \), therefore, \( C_1 = 0 \).

Integrating between \( r = 0 \) and \( r = a \), the temperature drop from the centre to the surface of the fuel is given by

\[ \Delta T_F = \frac{q_G a^2}{4k_F} \quad (9.58) \]

The heat conducted through the cladding per unit length of fuel element = \( \pi a^2 q_G \).

The radial heat conduction at any radius is

\[ Q = -k 2\pi r L \frac{dT}{dr} \]

or

\[ dT = -\frac{Q}{2\pi k L} \times \frac{dr}{r} \]

Integrating between the limits of \( r = a \) to \( r = a + b \) in the cladding and substituting \( Q/L \) as \( \pi a^2 q_G \), we get the temperature drop through cylindrical wall of cladding as

\[ \Delta T_{cl} = \frac{q_G a^2 \ln \left( \frac{a + b}{a} \right)}{2k_{cl}} \quad (9.59) \]

The total temperature drop from the centre of the fuel to the surface of the cladding is:
\[ \Delta T_F + \Delta T_{c1} = \frac{q_G a^2}{2} \left[ \frac{1}{2k_F} + \frac{\ln \left( \frac{a+b}{b} \right)}{k_{cl}} \right] \] (9.60)

If we consider the coolant flowing with bulk temperature \( T_C \), the heat transfer to the fluid per unit length is given by
\[ Q = \pi a^2 q_G = h2\pi (a + b) (T_s - T_c) \] (9.61)
where \( h \) is the heat transfer coefficient (W/m\(^2\)K) and \( T_s \) the cladding surface temperature.
The temperature drop from the cladding surface to the bulk fluid is
\[ \Delta T_c = T_s - T_c = \frac{q_G a^2}{2h(a + b)} \] (9.62)
The total temperature drop from the centre of the fuel to the bulk fluid is then
\[ \Delta T_F + \Delta T_{c1} + \Delta T_c = \frac{q_G a^2}{2} \left[ \frac{1}{2k_F} + \frac{\ln \left( \frac{a+b}{b} \right)}{k_{cl}} + \frac{1}{h(a + b)} \right] \] (9.63)

### 9.14.2 Axial Temperature Distribution of Coolant and Fuel Element

The temperatures expressed by the relationship of the previous section are valid if \( q_G \) is uniform. Let us consider a single fuel element whose core height is equal to the length \( L \) (Fig. 9.14). The variation of neutron flux \( \phi \) in the axial direction is assumed to be a cosine function of \( z \) such that \( \phi = 0 \) at \( z = \pm L/2 \), \( L \) being the extrapolated height and \( z = 0 \) being the core centre plane and

---

**Fig. 9.14**  Coolant flow and temperatures in the central fuel channel
\[ \phi = \phi_c \cos \frac{\pi z}{L} \]

where \( \phi_c \) is the maximum neutron flux at the core centre. Therefore, the volumetric thermal source strength \( q_G \) may be assumed to vary in a similar way,

\[ q_G = (q_G)_{\text{max}} \cos \frac{\pi z}{L} \quad (9.64) \]

where \( (q_G)_{\text{max}} \) is the source strength at the core centre. An energy balance for a differential section of the fuel element of length \( dz \) at height \( z \) is given by

\[ m_c c_c \, dT_c = q_G \cdot A_c \, dz \]

where \( m_c \) = mass flow rate of coolant, kg/s; \( c_c \) = specific heat of the coolant, kJ/kg K; \( dT_c \) = temperature rise of coolant in the length \( dz \), °C; and \( A_c \) = cross-sectional area of the fuel element, m².

The temperature of the coolant at any point in the channel can now be found by integration

\[ T_c(z) - T_{c_i} = \frac{(q_G)_{\text{max}} \pi a^2}{m_c c_c} \int_{-L/2}^{z} \cos \frac{\pi z}{L} \, dz \]

Therefore,

\[ T_c(z) - T_{c_i} = \frac{(q_G)_{\text{max}} \pi a^2 L}{\pi m_c c_c} \left( \sin \frac{\pi z}{L} + \sin \frac{\pi L}{2L} \right) \quad (9.65) \]

The temperature rise of the coolant in the whole channel is

\[ T_{c_2} - T_{c_1} = \frac{2 (q_G)_{\text{max}} \pi a^2 L}{\pi m_c c_c} \sin \frac{\pi L}{2L} \]

\[ = \Delta T_c \quad (9.66) \]

using this result, \( T_c(z) \) can be expressed in the following form

\[ T_c(z) = T_{c_i} + \frac{\Delta T_c}{2} \left( 1 + \frac{\sin \beta z}{\sin \beta L/2} \right) \quad (9.67) \]

where \( \beta = \pi/L \).

The temperatures at the surface of the cladding, \( T_s \), and at the centre of the fuel, \( T_f \), can be found at any position \( z \) as follows:

\[ T_s(z) = T_c(z) + \theta_s(z) \]

\[ = T_{c_i} + \frac{\Delta T_c}{2} \left( 1 + \frac{\sin \beta z}{\sin \beta L/2} \right) + \theta_{CO} \cos \beta z \quad (9.68) \]

\[ T_f(z) = T_{c_i} + \frac{\Delta T_c}{2} \left( 1 + \frac{\sin \beta z}{\sin \beta L/2} \right) \]
\[ + \theta_{CO} \cos \beta z \left[ \frac{bh \ln \left( \frac{a + b}{a} \right)}{1 + \frac{K_{el}}{K_{cl}} + \frac{bh}{2k_{E}}} \right] \]  

(9.69)

where \( \theta_{CO} = \frac{(q_G)_{max} a^2}{2h(a + b)} \), the temperature drop from the surface of the cladding to the coolant at the midpoint of the channel.

Figure 9.14 shows the variation of \( T_c \), \( T_s \) and \( T_F \) along the coolant channel and shows the existence of maximum values in \( T_s \) and \( T_F \). Differentiating Eq. (9.66) and equating to zero, the location of the maximum cladding surface temperature is given by the equation:

\[ z = \frac{1}{\beta} \tan^{-1} \left( \frac{\Delta T_c/2}{\theta_{CO} \sin(\beta L/2)} \right) \]  

(9.70)

Substituting this expression for \( z \) into Eq. (9.68), the maximum cladding surface temperature is given by:

\[ T_s (\text{max}) = T_{ci} + \frac{\Delta T_c}{2} \left[ 1 + \left( \csc^2 \frac{\beta L}{2} + \frac{\theta_{CO}}{2} \left( \frac{\Delta T_c}{\theta_{CO}} \right) \right) \right]^{1/2} \]  

(9.71)

By the same procedure, the maximum fuel temperature may be found as

\[ T_F (\text{max}) = T_{ci} + \frac{\Delta T_c}{2} \left[ 1 + \left( \csc^2 \frac{\beta L}{2} + \frac{C}{\left( \frac{\Delta T_c}{\theta_{CO}} \right)} \right) \right]^{1/2} \]  

(9.72)

where,

\[ C = \theta_{CO} \left[ \frac{bh \ln \left( \frac{a + b}{a} \right)}{1 + \frac{K_{el}}{K_{cl}} + \frac{bh}{2k_{E}}} \right] \]

These temperatures fix the maximum coolant temperature that can be allowed.

The heat transfer coefficient for turbulent flow can be estimated from the Dittus–Boelter equation,

\[ \text{Nu}_d = 0.023 \left( \text{Re}_d \right)^{0.8} \text{Pr}^{0.4} \]  

(9.73)

where the fluid properties are evaluated at the bulk temperature \( T_b \) given by

\[ T_b(z) = T_{ci} + \frac{1}{m c_p \text{inlet}} \int_{z} q_L \, dz \]  

(9.74)

where \( q_L \) is the heat transferred to the fluid per unit length of the duct. For non-circular ducts, the equivalent diameter should be used for Reynolds and Nusselt numbers. For liquid metals, the following correlation is often used

\[ \text{Nu} = 7 + 0.025 \left( \text{Pe} \right)^{0.8} \]  

(9.75)

where \( \text{Pe} = \text{Peclet number} = (\text{Re})(\text{Pr}) \)
Table 9.4 gives typical values of heat transfer coefficients for gases, water and liquid metals under reactor operating conditions.

**Table 9.4 Typical values of Prandtl number and heat transfer coefficient of reactor coolants**

<table>
<thead>
<tr>
<th>Coolants</th>
<th>Prandtl Number</th>
<th>$h$ (W/m²K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gases</td>
<td>0.8</td>
<td>50 to 500</td>
</tr>
<tr>
<td>Water</td>
<td>1 to 7</td>
<td>2000 to 20,000</td>
</tr>
<tr>
<td>Liquid metals</td>
<td>0.01</td>
<td>5000 to 50,000</td>
</tr>
</tbody>
</table>

Since $h$ for gases is low, large surface area is required for a certain heat transfer duty in gas cooled reactors and cladding is most often finned. The criteria for the choice of coolants for reactors are: (1) Low neutron capture cross-section, (2) High specific heat, density, thermal conductivity and heat transfer coefficient, (3) Good chemical stability, and (4) Low neutron induced radioactivity.

### 9.14.3 Pumping Power

The pumping work, $W$, required by the circulating coolant to overcome pressure losses through the loop is given by

$$W = \Delta p \cdot A_c V$$  \hspace{1cm} (9.76)

where, $\Delta p =$ pressure drop through the loop, N/m²; $A_c =$ cross-sectional area of coolant passage, $m^2$; and $V =$ coolant speed, m/s. In turbulent flow,

$$\Delta p = \frac{fL}{D_e} \times \frac{\rho V^2}{2}$$  \hspace{1cm} (9.77)

where $f =$ friction factor, $D_e =$ equivalent diameter, $4A_c/P$, $P$ being the wetted perimeter, $L =$ channel length , and $\rho =$ density of coolant. The well-known Moody chart can be used to find $f$.

### 9.15 TYPES OF REACTORS

Reactors can be heterogeneous or homogeneous. A heterogeneous reactor has a large number of fuel rods with the coolant circulating around them and carrying away the heat released by nuclear fission. In a homogeneous reactor, the fuel and moderator are mixed, e.g. a fissionable salt of uranium like uranium sulphate (or nitrate) dissolved in the moderator like H₂O or D₂O. The solution is critical in the core. Due to difficulties in component maintenance, induced radioactivity, erosion and corrosion, homogeneous reactors are not common. Present day nuclear reactors are of the heterogeneous class. These reactors are again classified according to the type of fuel used, the neutron flux spectrum, the coolant, and the moderator, if used (Table 9.5).
Table 9.5 Reactor classification

<table>
<thead>
<tr>
<th>Neutron flux spectrum</th>
<th>Moderator</th>
<th>Coolant</th>
<th>Fuel material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal</td>
<td>Light water</td>
<td>Light water</td>
<td>Enriched uranium</td>
</tr>
<tr>
<td></td>
<td>Heavy water</td>
<td>Heavy water</td>
<td>Natural uranium</td>
</tr>
<tr>
<td></td>
<td>Graphite</td>
<td>Gas (CO₂)</td>
<td>Natural or enriched uranium</td>
</tr>
<tr>
<td>Fast</td>
<td>Nil</td>
<td>Liquid metal (Na, K)</td>
<td>Plutonium, thorium</td>
</tr>
</tbody>
</table>

Light water-cooled and moderated reactors (LWR) using slightly enriched uranium fuel are the type most commonly used for power production. These reactors are further divided into:

1. Pressurized water reactor (PWR)
2. Boiling water reactor (BWR)

High temperature gas-cooled reactors (HTGCR) have been used in countries like the UK, France and Germany. Fast reactors which use high energy neutrons for fission and require no moderator utilize a liquid metal as a coolant with either plutonium or a plutonium–uranium mixture for fuel. Liquid metal fast breeder reactors (LMFBR) are likely to be the source of electrical power for the future. A breeder reactor produces more fissionable isotope than what it consumes. The characteristics of different reactor systems are provided in Table 9.6. The schematics for nuclear steam supply systems (NSSS) of five reactor types are shown in Fig. 9.15.

Table 9.6 Characteristics of typical power reactors

<table>
<thead>
<tr>
<th></th>
<th>PWR</th>
<th>BWR</th>
<th>LMFBR</th>
<th>HTGR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric power (MWe)</td>
<td>1300</td>
<td>1050</td>
<td>1000</td>
<td>330</td>
</tr>
<tr>
<td>Thermal power (MWth)</td>
<td>3800</td>
<td>3000</td>
<td>2750</td>
<td>842</td>
</tr>
<tr>
<td>Specific power (kWth/kg)</td>
<td>33</td>
<td>26</td>
<td>575</td>
<td>50</td>
</tr>
<tr>
<td>Power density (kWth/m³)</td>
<td>100</td>
<td>60</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>Core height (m)</td>
<td>4.25</td>
<td>3.75</td>
<td>1.50</td>
<td>5.0</td>
</tr>
<tr>
<td>Core diameter (m)</td>
<td>3.50</td>
<td>4.90</td>
<td>3.25</td>
<td>5.9</td>
</tr>
<tr>
<td>Coolant</td>
<td>H₂O</td>
<td>H₂O</td>
<td>liq.Na</td>
<td>He</td>
</tr>
<tr>
<td>Pressure (MPa)</td>
<td>15.5</td>
<td>7.2</td>
<td>0.8</td>
<td>4.8</td>
</tr>
<tr>
<td>Inlet temperature (°C)</td>
<td>280</td>
<td>275</td>
<td>330</td>
<td>400</td>
</tr>
<tr>
<td>Outlet temperature (°C)</td>
<td>310</td>
<td>285</td>
<td>500</td>
<td>770</td>
</tr>
<tr>
<td>Coolant flow rate (Mg/s)</td>
<td>20</td>
<td>12</td>
<td>11</td>
<td>0.45</td>
</tr>
<tr>
<td>Average linear heat rate (kW/m)</td>
<td>22.5</td>
<td>20</td>
<td>30</td>
<td>—</td>
</tr>
</tbody>
</table>

9.16 PRESSURIZED WATER REACTOR (PWR)

The excellent properties of water as a moderator and coolant make it a natural choice for power reactors, and the PWR has been extensively developed in the
USA. The most important limitation on a PWR is the critical temperature of water, 374°C. This is the maximum possible temperature of the coolant in the reactor, and in practice it is considerably less, possibly about 300 °C, to allow a margin of safety. In a PWR, the coolant pressure must be greater than the saturation pressure at, say, 300 °C (85.93 bar) to suppress boiling. The pressure is maintained at about 155 bar so as to prevent bulk boiling.

Fig. 9.15  Schematics for nuclear steam supply systems
A PWR power plant is composed of two loops in series, the coolant loop, called the primary loop, and the water–steam or working fluid loop (Fig. 9.16). The coolant picks up heat in the reactor and transfers it to the working fluid in the steam generator. The steam is then used in a Rankine type cycle to produce electricity.

The fuel in PWRs is slightly enriched uranium in the form of thin rods or plates. The cladding is either of stainless steel or zircaloy. Because of very high coolant pressure, the steel pressure vessel containing the core must be about 20 to 25 cm thick. A typical PWR contains about 200 fuel assemblies, each assembly being an array of rods. In a typical fuel assembly, there are 264 fuel rods and 24 guide tubes for control rods. Grid spacers maintain a separation between the fuel rods to prevent excessive vibration and allow some axial thermal expansion.

![Diagram of a PWR power plant]

**Fig. 9.16 Schematic of a PWR power plant**

The coolant leaving the reactor enters the steam generator which can be either shell and tube type with U-tube bundles or once-through type, the former being more common. In the U-tube steam generator, the hot coolant enters an inlet channel head at the bottom, flows through the U-tubes, and reverses direction to an outlet at the bottom. It can produce only saturated steam. In the once-through design, the primary coolant enters at top, flows downward through tubes and exits at the bottom to the main pumps. Feedwater is on the shell side. A dry or low degree of superheat steam is possible.

The first land-based PWR for power generation was built at Shippingport, USA in 1957. Its thermal output is 231 MW, the pressure in the primary circuit is 141 bar, and the water temperature at outlet from the reactor is 282 °C. Dry saturated steam is generated in the heat exchangers at 41 bar, 252 °C. For a gross electrical output of 68 MW, the thermal efficiency is 29.4%.

The Shippingport cycle has been modified in the Indian Point (USA) PWR by the inclusion of an oil-fired superheater between the main heat exchangers and
the turbines (Fig. 9.17). There is also an economiser along with some feedwater heaters. The steam condition improves to 25.5 bar and 538 °C at turbine inlet, and so the cycle efficiency increases.

![Diagram](image)

**Fig. 9.17** The Indian Point (USA) PWR with oil-fired superheater

### 9.16.1 Pressurizer

The coolant in the PWR primary loop is maintained at a pressure (about 155 bar) greater than the saturation pressure corresponding to the maximum coolant temperature in the reactor to prevent bulk boiling. Because liquids are incompressible, small changes in volume due to changes in coolant temperatures because of either load variation or sudden nuclear reactivity insertions cause severe or oscillatory pressure changes, due to which pressure may increase or decrease. If the pressure increases, some water will flash into steam and it will affect the reactor performance, often leading to its burnout. If the pressure decreases, there may be cavitation. It is thus necessary to provide a surge chamber that will accommodate the coolant volume changes while maintaining pressure within permissible limits. Such a chamber is called a pressurizer.

Figure 9.18 shows a vapour pressurizer which is essentially a small boiler where the liquid, the same as the coolant, is kept at a constant temperature by controlled electric heating. There is, thus, a constant vapour pressure above its surface. This pressure is the same as that of the primary coolant at the junction between the pressurizer and hot leg of the primary loop. Thus, the pressurizer temperature is higher than the primary coolant temperature because the latter is subcooled. A spray nozzle located at the top of the pressurizer is connected to the cold leg of the primary coolant system after the pump. The pressurizer is half full with water and half full with vapour.

When there is a positive surge, the volume of coolant increases and the vapour in the top is compressed. Some coolant is then sprayed to condense some of the
vapour and thus check the pressure rise. If there is a negative surge, coolant volume decreases, there is a momentary decrease of pressure as a result of which some liquid flashes into vapour. Also, the electric heaters operate. A relief valve is installed at the top to protect against pressure surges beyond the capacity of the pressurizer. It discharges steam into a pressurizer relief tank containing water in which it condenses.

![Diagram of a PWR primary loop with a vapour-type pressurizer system](image)

**Fig. 9.18** *A PWR primary loop with a vapour-type pressurizer system*

### 9.17 BOILING WATER REACTOR (BWR)

A BWR differs from the PWR in that the steam flowing to the turbine is produced directly in the reactor core. Steam is separated and dried by mechanical devices located in the upper part of the pressure vessel assembly. The dried steam is sent directly to the high pressure turbine thus eliminating the need for steam generators (Fig. 9.19). The coolant thus serves the triple function of coolant, moderator and working fluid. Since the coolant boils in the reactor itself, its pressure is much less than that in a PWR and it is maintained at about 70 bar with steam temperature around 285 °C. However, an increase in the boiling rate displaces water (moderator) in the core and reduces the ability of the moderator to thermalize neutrons and hence, reduces the reactor power level. At power levels above 60% of the nominal, the fraction of steam in the core can be kept nearly constant by varying the coolant circulation rate.
Fig. 9.19  *Schematic of a direct-cycle BWR plant*

The saturated liquid that separates from the vapour at the top of the reactor in a steam separator flows downward either internally within the reactor or externally outside the reactor and mixes with the return condensate (Fig. 9.20). This recirculating coolant again either flows naturally due to density difference or by a forced circulation pump. The ratio of the recirculated coolant to the saturated vapour produced is called the circulation (or recirculation) ratio (as defined earlier in Chapter 6). It is a function of the core average exit quality. The BWR core exit quality varies from 10 to 14%, so that circulation ratio is of the range 6–10. This is necessary to avoid large void fractions in the core, which would reduce the moderating power of the coolant resulting in low heat transfer coefficient or vapour blanketing and burnout.

---

**Fig. 9.20**  *A BWR system with (a) internal and (b) external circulation*

A slightly subcooled liquid enters the core bottom at a rate of $m_i$ and rises through the core and chimney, if any. The chimney is an unheated section above the core that helps increase the pressure for natural circulation. The resulting vapour separates and goes to the turbine at a rate of $m_g$, while saturated liquid
recirculates via the downcomer at the rate of \( m_f \). There it mixes with the cold return feedwater \( m_d \) to form the subcooled inlet liquid \( m_i \).

An overall mass balance in the reactor core gives

\[
\begin{align*}
    m_d &= m_g \\
    m_g + m_f &= m_i
\end{align*}
\]

The quality of the liquid–vapour mixture at the core exit \( x_c \) is given by

\[
x_c = \frac{m_g}{m_f + m_g} = \frac{m_d}{m_d + m_f} = \frac{m_d}{m_i}
\]

The circulation ratio \( R \) is then

\[
R = \frac{m_f}{m_g} = \frac{1 - x_c}{x_c}
\]

Neglecting any heat loss and KE and PE changes, an energy balance of the core gives

\[
m_i h_i = m_i h_f + m_d h_d
\]

where \( h_i \), \( h_f \) and \( h_d \) are the enthalpies of the inlet, recirculated and incoming feedwater, respectively. On rearrangement,

\[
    h_i = (1 - x_c) h_f + x_c h_d
\]

or

\[
x_c = \frac{h_f - h_i}{h_f - h_d}
\]

The enthalpy of subcooling of liquid entering the bottom of the core is

\[
\Delta h_{sub} = h_f - h_i = x_c (h_f - h_d)
\]

or the degree of subcooling is given by

\[
\Delta t_{sub} = t_f - t_i
\]

The total heat generation \( Q_t \) then becomes

\[
Q_t = m_i \left[ h_f + x_c h_{ig} \right] - h_i
\]

\[
= m_g \left( h_g - h_d \right)
\]

A number of BWR power plants have used a dual pressure, direct cycle arrangement, e.g. Dresden 1 (USA) as shown in Fig. 9.21. There is a decrease in the temperature of the water entering the reactor and thus, the power output increases with the unaltered exit condition. However, there is the disadvantage of increased complexity of the plant.

The active or fueled core region of a BWR consists of about 800 fuel assemblies. Each typically contains an 8 by 8 array of fuel rods. The zircaloy channel around the fuel rods prevents cross flow in the core. BWR fuel rods are slightly larger than PWR fuel rods. A typical pellet diameter is 10.6 mm with an outside cladding diameter of 12.5 mm. The average fuel enrichment varies from 1.9 to 2.6 %.  

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9.18 GAS-COOLED REACTORS

The first gas-cooled reactors with CO₂ gas (at a pressure of 16 bar) as coolant and graphite as moderator were developed in Britain during 1956–69. The fuel was a natural uranium, clad with an alloy of magnesium called Magnox (Fig. 9.22).

Several types of gas-cooled reactors have been designed and built, with England developing an advanced gas-cooled reactor (AGR) system, and Germany and the USA developing helium-cooled, graphite-moderated systems (HTGR). The AGR uses UO₂ as the fuel clad in stainless steel tubes with CO₂ gas as coolant and graphite as moderator.

The graphite moderated helium-cooled HTGR is designed to use U-233 as the fissile material and thorium as fertile material. Initially, the system would have to be fuelled with U-235, until sufficient U-233 is available for makeup fuel. Because of the very high melting point of graphite, these fuel elements can operate at very high temperatures, and it is possible to generate steam at conditions
equivalent to those in modern coal-fired power plant. The basic fuel forms are small spheres of fissile and fertile material as carbides, UC₃ or ThC₂. The fissile spheres are 0.35 to 0.50 mm in diameter and the fertile spheres are 0.6 to 0.7 mm in diameter. Each sphere is coated with two to three layers of carbon and silicon carbide to prevent fission products from escaping from the particles. Helium is a suitable coolant in the sense that it is chemically inert, has good heat transfer characteristics and low neutron absorption. Being a monatomic gas, it can produce more power for given temperatures in the Brayton cycle and higher efficiency.

A direct cycle HTGR gas turbine plant is shown in Fig. 9.23. It incorporates a regenerator and multi-stage compression with intercooling. Typical figures for such a cycle are: pressure ratio 4, turbine inlet pressure 50 bar, turbine inlet (reactor outlet) temperature 900 °C, compressor inlet temperature 50 °C. The temperatures at other points are shown in the figure, With U-233/Th-232 fuel, the HTGR functions as a thermal breeder reactor.

![Diagram of HTGR direct cycle gas turbine plant using helium](image)

**Fig. 9.23** A-HTGR direct cycle gas turbine plant using helium

### 9.19 LIQUID METAL FAST BREEDER REACTOR

Fast breeder reactors are designed to create or breed new fissile material, while producing useful electric power. Most produce fissile plutonium from fertile uranium 238. The fuel rods in the core region thus contain a mixture of fissile Pu-239 and U-238. The active core region is surrounded by a blanket of fertile U-238. This blanket region captures neutrons that would otherwise be lost through leakage, thus producing additional fissile material. A fast neutron reaction with U-238 producing Pu-239 is shown as follows.
Hidden page
Fig. 9.25 Liquid metal fast breeder reactor (a) Pool type, (b) Loop type

Heavy water-moderated and cooled reactors have been extensively developed in Canada, and form the basis of the nuclear power programme in that country. They are called CANDU-PHW (Canadian Deuterium Uranium Pressurized Heavy Water). The CANDU reactors have several features that distinguish them from other types. The moderator is contained in a cylindrical steel vessel, called the calandria, with a large number of zircaloy tubes through it parallel to its axis, which is horizontal (Fig. 9.26). The active core region is approximately 6 m high with a diameter of 7 to 8 m. The D₂O coolant enters the regular array of pressure tubes at 260 °C and 110 bar, flows through the fuel elements, and leaves the
pressure tubes at 320 °C, and the net efficiency is about 29%. Like PWR, there is no bulk boiling of coolant.

The heavy water coolant pressure in the reactor is 88.3 bar, and the inlet and outlet temperatures are 250 °C and 290 °C, respectively. In heat exchangers, steam is generated at 41 bar pressure, 251 °C. The thermal power of each reactor (there are 8 at Pickering at Canada) is 1744 MW, and the net electrical output is 515 MW, giving a thermal efficiency of 29.5% (Fig. 9.26).

Fig. 9.26  Layout of the calandria, heat exchangers and simplified steam cycle of the CANDU reactor

The calandria contains up to 380 horizontal pressure tubes, called calandria tubes, which are welded to the tube sheets at each end of the vessel. The moderator temperature is maintained at about 70 °C and low pressure to reduce heavy water losses. The fuel assembly contains 37 fuel rods, as shown in Fig. 9.27. Each rod contains natural uranium dioxide (UO₂) fuel pellets with 0.38 zircaloy cladding. Each rod bundle is about 0.1 m in diameter and 0.5 m long.

9.21 India’s Nuclear Power Programme

From the several reactor types available, India has selected Pressurized Heavy Water Reactors (PHWR) because of several inherent advantages. A PHWR uses natural uranium as fuel with heavy water as moderator and coolant. Natural uranium being easily available in India, helps cut heavy investments for enriched uranium, the import of which is very difficult due to restrictive international trade practices. Besides, the PHWR core containing natural uranium is safer with its lesser reactivity and has on-power refuelling facility. Due to its excellent neutron economy, a PHWR has a greater yield of plutonium.
India has planned a three stage programme to benefit from the atom:

1st stage: PHWRs use natural uranium fuel to produce electricity and plutonium fuel with 96% unused uranium (U-238).

2nd stage: The use of fast breeder reactors with plutonium as fuel will generate electricity and more plutonium from U-238, kept as a blanket. It will also produce U-233 fuel from thorium, used as a blanket material.

3rd stage: U-233 will be used as fuel and thorium as blanket, producing more U-233 fuel than the fuel consumed in fast and thermal reactors.

India’s nuclear power programme up to 2000 A.D. as envisaged by the Nuclear Power Corporation (Government of India) is given in Table 9.7.

### Table 9.7

<table>
<thead>
<tr>
<th>Operational units</th>
<th>Station capacity MWe</th>
<th>Cumulative capacity MWe</th>
<th>Year of commissioning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tarapur (BWR) 1 and 2</td>
<td>2 × 210</td>
<td>420</td>
<td>1969</td>
</tr>
<tr>
<td>Rajasthan 1 and 2</td>
<td>2 × 220</td>
<td>860</td>
<td>1973, 1981</td>
</tr>
<tr>
<td>Madras 1 and 2</td>
<td>2 × 235</td>
<td>1330</td>
<td>1983, 1985</td>
</tr>
<tr>
<td>Narora 1 and 2</td>
<td>2 × 235</td>
<td>1800</td>
<td>1990, 1991</td>
</tr>
<tr>
<td>Kakrapar 1 and 2</td>
<td>2 × 235</td>
<td>2270</td>
<td>1991, 1992</td>
</tr>
<tr>
<td>Under construction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaiga 1 and 2</td>
<td>2 × 235</td>
<td>2740</td>
<td>1995, 1996</td>
</tr>
<tr>
<td>Rajasthan 3 and 4</td>
<td>2 × 235</td>
<td>3210</td>
<td>1995, 1996</td>
</tr>
<tr>
<td>Under sanction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaiga 3, 4, 5, and 6</td>
<td>4 × 235</td>
<td>4150</td>
<td>1996, 1997</td>
</tr>
<tr>
<td>Tarapur 3 and 4</td>
<td>2 × 500</td>
<td>5150</td>
<td>1997, 1998</td>
</tr>
<tr>
<td>Rajasthan 5, 6, 7 and 8</td>
<td>4 × 500</td>
<td>7150</td>
<td>1998, 2000</td>
</tr>
<tr>
<td>Planned</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kudankulam (PWR) 1 and 2</td>
<td>2 × 1000</td>
<td>9150</td>
<td>1998, 1999</td>
</tr>
</tbody>
</table>
Since the resources of fossil fuels are fast depleting, India has to depend more on nuclear power. Besides, it is economical in regard to the cost of generation of electricity. An optimal mix of thermal, hydel and nuclear sources is required to help India achieve self-sufficiency in power generation for years to come.

9.22 FUSION POWER REACTORS

Fusion of light nuclei to form a heavy nucleus also releases energy. To cause fusion, it is necessary to accelerate the positively charged nuclei to high kinetic energies, in order to overcome electrical repulsive forces, by raising their temperature to hundreds of millions of degrees resulting in a plasma. The plasma must be prevented from contacting the walls of the container, and must be confined for about tenth of a second having a density of around $10^{15}$ ions/cm$^3$.

There are several possible reactions between the nuclei of light elements that can be the basis for controlled fusion. Deuterium, a stable heavy isotope of hydrogen, present in natural water, is the main fuel for a fusion reactor. Four reactions involving deuterium are given below:

<table>
<thead>
<tr>
<th>Fusion reaction</th>
<th>Energy per reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1\text{H}^2 + ^1\text{H}^2 \rightarrow ^2\text{He}^3 + ^1\text{n}$</td>
<td>3.2 MeV</td>
</tr>
<tr>
<td>(D + D $\rightarrow$ He$^3 + n$)</td>
<td></td>
</tr>
<tr>
<td>$^1\text{H}^2 + ^1\text{H}^2 \rightarrow ^1\text{H}^3 + ^1\text{p}$</td>
<td>4.0 MeV</td>
</tr>
<tr>
<td>(D + D $\rightarrow$ T + p)</td>
<td></td>
</tr>
<tr>
<td>$^1\text{H}^2 + ^1\text{H}^3 \rightarrow ^2\text{He}^4 + ^1\text{n}$</td>
<td>17.6 MeV</td>
</tr>
<tr>
<td>(D + T $\rightarrow$ He$^4 + n$)</td>
<td></td>
</tr>
<tr>
<td>$^1\text{H}^2 + ^2\text{He}^3 \rightarrow ^2\text{He}^4 + ^1\text{p}$</td>
<td>18.3 MeV</td>
</tr>
<tr>
<td>(D + He$^3 \rightarrow$ He$^4 + p$)</td>
<td></td>
</tr>
</tbody>
</table>

where $n$, $p$, $D$ and $T$ are the symbols for neutron, proton, deuterium ($^1\text{H}^2$) and tritium ($^1\text{H}^3$), respectively.

Fusion reaction occurs most easily between deuterium and tritium (D + T), which is self-sustaining at a temperature of $50 \times 10^6$ K releasing 17.6 MeV per reactions. The first two D – D reactions occur at $500 \times 10^6$ K and release less energy (3.2 and 4.0 MeV). The fourth reaction releases very high energy (18.3 MeV), but it requires a very high temperature ($1000 \times 10^6$ K) also for fusion.

Tritium does not occur abundantly in nature. It can, however, be produced in a lithium “breeding blanket” that surrounds the plasma core of the fusion reactor.

Figure 9.28 shows the schematic diagram of a futuristic deuterium–tritium fusion reactor (Sorensen, 1983). The plasma is contained inside an evacuated tube of about 4 m. The surrounding vacuum wall through which 14 MeV neutrons from the plasma pass, is maintained at about 750 °C. Outside this wall are two concentric regions, viz., the lithium breeding moderator and the magnetic shield. Tritium is manufactured in the lithium blanket. Large cryogenic superconducting magnets of 7 to 8 m diameter maintain the magnetic shield. The binary vapour power cycle consists of a potassium topping cycle and a conventional steam cycle. It includes a tritium recovery system.
Fig. 9.28  Deuterium-tritium (D–N) fusion power plant conceptual design

Significant advantages of the fusion power plant are:

1. The supply of deuterium is almost inexhaustible.
2. Radioactive wastes are not produced.
3. It is very safe to operate.
4. High energy conversion efficiency (~60%) can be achieved.
5. Low heat rejection to the environment takes place per kW of electricity generated.

It is almost certain that large, practical fusion power plants will be built in the twenty-first century. Once this technology is developed, an almost unlimited supply of energy will be available for the world's needs ushering in a better living standard for the human kind all over the world.

Example 9.1 Calculate the mass defect and binding energy per nucleon of oxygen. Given: \( m_p = 1.007277 \text{ amu} \), \( m_n = 1.008665 \text{ amu} \), \( m_e = 0.00055 \text{ amu} \), atomic mass of oxygen \( \sim 16 \approx 15.99491 \text{ amu} \).

Solution: A molecule of oxygen has 8 protons, 8 electrons, and 8 neutrons. Therefore, the mass defect is
\[\Delta m = 8 \times 1.007277 + 8 \times 0.00055 + 8 \times 1.008665 - 15.99491\]
\[= 0.13701 \text{ amu}\]

Binding energy = \(0.13701 \times 931 = 127.6 \text{ MeV}\)

Binding energy per nucleon = \(127.6/16 = 7.97 \text{ MeV}\) \hspace{1cm} \text{Ans.}

**Example 9.2** The half-life of radium 226 (atomic mass = 226.095) is 1620 yrs. Compute (a) the decay constant, and (b) the initial activity of 1 g of radium 226.

**Solution** From Eq. (9.15),
\[\theta_{1/2} = \frac{0.6931}{\lambda}\]

(a) The decay constant
\[\lambda = \frac{0.6931}{1620 \times 365 \times 24 \times 3600} = 1.3566 \times 10^{-11} \text{ s}^{-1}\]

(b) Number of atoms per g of radium 226
\[= \frac{\text{Avogadro's constant}}{\text{Atomic mass}} = \frac{6.023 \times 10^{23}}{226.095} = 2.6645 \times 10^{21}\]

Initial activity \(A_0 = \lambda N_0 = 1.3566 \times 10^{-11} \times 2.6645 \times 10^{21}\)
\[= 3.615 \times 10^{10} \text{ dis/s}\]

**Example 9.3** Each fission of U-235 yields 190 MeV of useful energy. Assuming that 85% of neutrons absorbed by U-235 cause fission, the rest being absorbed by non-fission capture to produce an isotope U-236, estimate the fuel consumption of U-235 per day to produce 3000 MW of thermal power.

**Solution** Each fission yields 190 MeV \times 1.60 \times 10^{-13} \text{ J/MeV} or \(3.04 \times 10^{-11}\) J of useful energy.

Number of fissions required to obtain \(W\)-s of energy
\[= \frac{1}{3.04 \times 10^{-11} \text{ J}} = 3.3 \times 10^{10}\]

In one day’s operation (i.e., 86,400 s) of a reactor per MW of thermal power, the number of U-235 nuclei burned is
\[(10^6 \text{ W})(3.3 \times 10^{10} \text{ fission/W-s})(86,400 \text{ s/day})\]
\[\text{0.85 fission/absorption}\]
\[= 3.35 \times 10^{21} \text{ absorptions/day}\]

Mass of U-235 consumed to produce 1 MW power is
\[(3.35 \times 10^{21} \text{ day}^{-1})(235 \text{ g/g mol})\]
\[6.023 \times 10^{23} \text{ (nuclei/g mol)}\]
\[= 1.3 \text{ g/day}\]

Therefore, the fuel consumption of U-235 to produce 3000 MW is 3.9 kg/day. \hspace{1cm} \text{Ans.}

To produce the same energy by use of fossil fuels, millions of times as much weight would be required.
Example 9.4 A certain nucleus has a cross-section of 10 barns for 2200 m/s neutrons. Find the cross-section if the KE of the neutrons increases to 0.1 eV. The two neutron energies are within 1/V range of the nucleus.

**Solution** At 2200 m/s, \( E_n = 0.02525 \text{ eV} \)

In the low energy region,

\[
\sigma_a = C \left( \frac{1}{E_n} \right)^{0.5} = C_1 \frac{1}{V}
\]

or

\[
\frac{\sigma_{a_1}}{\sigma_{a_2}} = \frac{V_2}{V_1} = \left( \frac{E_{n_1}}{E_{n_2}} \right)^{0.5} = \left( \frac{0.1}{0.02525} \right)^{0.5} = 1.98
\]

\[
\sigma_{a_2} = \frac{10}{1.98} = 5.01 \text{ barns} \quad \text{Ans.}
\]

Example 9.5 Calculate the microscopic absorption cross-section of natural uranium, which consists of 99.285% U-238 and 0.715% U-235. The microscopic cross-sections for 0.025 eV neutrons are:

U-238: \( \sigma_c = 2.72 \) barns \( \sigma_r = 0 \) U-235: \( \sigma_c = 101 \) barns \( \sigma_r = 579 \) barns

**Solution** For natural uranium, \( \sigma_a = \sigma_c + \sigma_r \)

\[
\sigma_a = 0.99285 (2.72 + 0) + 0.00715 (101 + 579)
\]

\[
= 7.6 \text{ barns} \quad \text{Ans.}
\]

Example 9.6 Calculate the macroscopic capture cross-section of water of density 1 g/cm\(^3\). The microscopic capture cross-sections of hydrogen and oxygen are 0.332 barn and 0.0002 barn, respectively.

**Solution** Number of molecules of water per cm\(^3\) is

\[
\frac{6.023 \times 10^{23 \text{ molecules}}}{18 \text{ g/mol}} \times \frac{1 \text{ g}}{1 \text{ cm}^3} = 3.35 \times 10^{22}
\]

\( \Sigma_c \) for water (H\(_2\)O) = \( N_1 \sigma_{c_1} + N_2 \sigma_{c_2} \)

\[
= 2 \times 3.35 \times 10^{22} \times 0.332 \times 10^{-24} + 3.35 \times 10^{22} \times 0.0002 \times 10^{-24}
\]

\[
= 0.0222 \text{ cm}^{-1} \quad \text{Ans.}
\]

Example 9.7 A 230 g piece of boron (mol wt. 10) absorbs thermal neutrons at the rate of \( 9.57 \times 10^{13} \) per (cm\(^3\) \cdot s). Boron density is 2.3 g/cm\(^3\). Find (a) the thermal neutron flux, and (b) the average distance that a neutron travels before it is absorbed. For thermal neutrons, \( \sigma_a = 755 \) barns and \( \sigma_s = 4.0 \) barns.

**Solution**

\[
\sigma_t = \sigma_a + \sigma_s = 755 + 4 = 759 \text{ barns}
\]

The number density of neutrons is
\[ N = \frac{\rho}{M} N_a = \frac{2.3 \text{ g/cm}^3}{10 \text{ g/g mol}} \times 6.023 \times 10^{23} \frac{\text{molecules}}{\text{g mol}} = 1.3853 \times 10^{24} \text{ cm}^{-3} \]

\[ \Sigma_t = N \sigma_t = 1.3853 \times 10^{23} \times 759 \times 10^{-24} \text{ cm}^{-1} = 105.144 \text{ cm}^{-1} \]

Reaction rate, \[ R = \phi \Sigma = 9.57 \times 10^{13} \text{ cm}^{-3} \text{ s}^{-1} \]

Neutron flux is \[ \phi = \frac{9.57 \times 10^{13}}{105.144} \text{ cm}^{-2} \text{ s}^{-1} = 0.091 \times 10^{13} \text{ cm}^{-2} \text{ s}^{-1} \quad \text{Ans. (a)} \]

Average distance a neutron travels before it is absorbed,

\[ \lambda = \frac{1}{\Sigma} = \frac{1}{105.144} = 9.51 \times 10^{-3} \text{ cm} \quad \text{Ans. (b)} \]

**Example 9.8** A newly born neutron of 4.8 MeV is to be slowed to 0.025 eV in a graphite moderator. Assuming all collisions to be elastic, calculate the logarithmic energy decrement representing the neutron energy loss per elastic collision and the number of collisions necessary.

**Solution** From Eq. (9.24), the logarithmic energy decrement \( \xi \) is given by

\[ \xi = 1 - \left[ \frac{(A-1)^2}{2A} \ln \frac{A+1}{A-1} \right] \]

where \( A \) is the mass number of the nucleus (graphite) with which the neutron collides. For graphite (carbon), \( A = 12 \).

\[ \xi = 1 - \left[ \frac{(12-1)^2}{2 \times 12} \ln \frac{12+1}{12-1} \right] = 0.158 \]

The number of collisions required to slow down the neutron from 4.8 MeV to 0.025 eV is given by Eq. (9.25).

\[ n = \frac{\ln \frac{E_{n,i}}{E_{n,f}}}{\xi} = \frac{\ln \frac{4.8 \times 10^6}{0.025}}{0.158} = 120.72 = 121 \quad \text{Ans.} \]

**Example 9.9** A reactor is fuelled with 100 tonnes of natural uranium (atomic mass 238.05) in which the average thermal neutron (2200 m/s) flux is \( 10^{13} \text{ neutrons/cm}^2\text{s} \). The 2200 m/s cross-section of U-235 (atomic mass 235.04) are; \( \sigma_t = 579 \text{ barns} \) and \( \sigma_c = 101 \text{ barns} \). The energy release per fission is 200 MeV and 0.715% of natural uranium is U-235. Calculate (a) the rating of the reactor in MW/tonne, (b) the rate of consumption of U-235 per day.

**Solution** The number of U-235 atoms in the reactor
\[
\begin{align*}
10^5 \text{kg} \times 6.023 \times 10^{26} \frac{\text{atoms}}{\text{kg mol}} & \times 0.00715 \\
& = \frac{238.05 \text{kg/kg mol}}{238.05 \text{kg/kg mol}} \\
& = 1.81 \times 10^{27} \text{ atoms}
\end{align*}
\]

The rate of fission in the reactor is given by Eq. (9.31)
\[
\begin{align*}
\sigma \phi N V_0 \\
& = 579 \times 10^{-24} \text{cm}^2 \times 10^{13} \frac{\text{neutrons}}{\text{cm}^2 \text{s}} \times 1.81 \times 10^{27} \\
& = 1.05 \times 10^{19} \text{ fissions/s}
\end{align*}
\]

The rate of energy release or thermal power of the reactor
\[
\begin{align*}
& = 1.05 \times 10^{19} \times 200 \text{ MeV/s} \\
& = 1.05 \times 10^{19} \times 200 \times 1.602 \times 10^{-19} \text{ MW} = 336 \text{ MW}
\end{align*}
\]

Rating of the reactor \( = 336/100 = 3.36 \text{ MW/tonne} \quad \text{Ans. (a)} \)

Rate of consumption of U-235 by fission
\[
\begin{align*}
& = \frac{1.05 \times 10^{19} \times 235.04 \times 60 \times 60 \times 24}{6.023 \times 10^{26}} \\
& = 0.353 \text{ kg/day or 353 g/day} \quad \text{Ans. (b)}
\end{align*}
\]

Thus complete fissioning of 1 g of U-235 releases about 1 MWd of thermal energy. For the same amount of energy release in a combustion process, 3 tonnes of coal are required. If 15% of the neutrons absorbed result in non-fission capture to produce U-236, the total consumption rate of U-235 becomes

\[
\frac{0.353}{0.85} = 0.415 \text{ kg/day}
\]

The burnup of nuclear fuel is a measure of the total amount of energy released by fission per unit mass of fuel over a period of time. If the reactor operates at steady and continuous power for one year the burnup is \( 3.36 \times 365 = 1226 \text{ MWd/tonne} \). The fraction of U-235 consumed, both by fission and neutron capture, in one year is

\[
\frac{0.415 \times 365 \times 6.023 \times 10^{26}}{235.04 \times 1.81 \times 10^{27}} = 0.214
\]

**Example 9.10** The fuel density \( N \) for a uranium oxide fuel is given by

\[
N = 2.373 f \times 10^{22} \text{ U-235 nuclei/cm}^3
\]

where \( f \) is the mass fraction of U-235 in the fuel.

Determine for a light water moderated uranium reactor the specific energy release rate (in W/cm\(^3\)) for the following conditions:

\[
\phi = \text{neutron flux} = 10^{13}/\text{cm}^2\text{s}
\]

\[
G = \text{energy per fission} = 180 \text{ MeV}
\]
\[ f = 3.5\% \text{ U-235 enrichment, i.e. } 0.035 \]
\[ \sigma_t = 577 \text{ barns} \]

**Solution**

\[ N = 2.372 f \times 10^{22} = 2.372 \times 0.035 \times 10^{22} \]
\[ = 8.302 \times 10^{20} \text{ nuclei/cm}^3 \]

From Eq. 9.48, the volumetric source strength,

\[ q_G = \text{rate of energy release} = GN\sigma_t \phi \]
\[ = 180(8.302 \times 10^{20})(577 \times 10^{-24}) \times 10^{13} \]
\[ = 8.622 \times 10^{14} \text{ MeV/cm}^3 \text{s} \]

Since \[ 1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J} \]
\[ q_G = 138.13 \text{ W/cm}^3 \quad \text{Ans.} \]

**Example 9.11** A reactor is operating at a low power of 1 W. It then becomes supercritical with \( k_{\text{eff}} = 1.0015 \). The average neutron life is 0.0001 s for prompt neutrons. Determine the reactor power level at the end of 1 s.

**Solution**

\[ \rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} = \frac{1.0015 - 1}{1.0015} = 0.0014978 \]

From Eq. (9.49)

\[ z = \frac{\rho}{\tau} = \frac{0.0014978}{0.0001} \times 1 = 14.978 \]

\[ n/n_0 = e^z = e^{14.978} = 3.198 \times 10^6 \]

The neutron density increases by \( 3.198 \times 10^6 \) times in 1 s. Since the reactor power is proportional to neutron density, it is increased from 1 W to 3.198 MW within a period of 1 s.

### SHORT-ANSWER QUESTIONS

9.1 Briefly describe the structure of an atom.
9.2 What is the difference between atomic number and mass number?
9.3 What is amu?
9.4 What do you understand by an “isotope”? What are the isotopes of hydrogen?
9.5 Explain the difference between chemical and nuclear reactions.
9.6 What do you mean by mass defect and binding energy?
9.7 What is nuclear stability? Why are elements of higher mass number not stable?
9.8 What do you understand by radioactive decay? What are radioisotopes?
9.9 What do naturally occurring radioisotopes emit? What is \( K \) capture?
9.10 What do you mean by half-life? Why are half-lives regarded as “finger-
prints” of radioisotopes?
9.11 What is a curie? What is a roentgen?
9.12 What are fission fragments and fission products?
9.13 Explain a fission chain with an example.
9.14 What is the average energy released per fission for U-233 and Pu-239?
9.15 Explain the term “fuel burnup”. What is fuel poisoning?
9.16 What do you mean by critical size and critical mass?
9.17 How are neutrons classified according to neutron energy?
9.18 What are prompt and delayed neutrons?
9.19 Explain inelastic and elastic scattering. What is logarithmic energy
decrement?
9.20 Explain the function of a moderator. What is the criterion of its
effectiveness?
9.21 What is a thermal reactor? What is a fast reactor?
9.22 What is neutron flux? How would you define nuclear cross-section? What is
a barn?
9.23 On what factors does the nuclear reaction rate depend?
9.24 What do you mean by moderating power and moderating ratio?
9.25 Briefly explain how the neutron cross-section varies with the neutron
energy.
9.26 Define the life cycle of neutrons. What is the four factor formula?
9.27 What is multiplication factor? Explain the subcritical and supercritical
chain reactions
9.28 What are the functions of a reflector?
9.29 Explain the function of a cladding. What are the criteria of selecting a
suitable cladding?
9.30 Define the volumetric thermal source strength.?
9.31 What are homogeneous and heterogeneous reactors?
9.32 Explain the characteristic features of a PWR.
9.33 What is the function of pressurizer in a PWR?
9.34 Explain the characteristic features of a BWR. What do you mean by external
and internal circulation?
9.35 What is an HTGR? Why is it called Magnox? Explain its main features.
9.36 What is an LMFBR? Why is a liquid metal the preferred coolant in a fast
reactor? What is its drawback?
9.37 Explain the terms: (a) breeding ratio, (b) burner, (c) converter, (d) breeder,
(d) doubling.
9.38 What is a CANDU-type reactor? Explain with a sketch its main features.
What is a calandria?
9.39 What are the three stages in India’s nuclear power programme?
9.40 What is the basis of energy release by fusion power?
9.41 What are the four reactions involving deuterium in a fusion reactor? Which
one is achieved quite easily?
9.42 What are the sources of deuterium and tritium?
9.1 The number density of fuel atoms \((N)\) of U-235 is \(0.048 \times 10^{24}\text{ cm}^{-3}\) and the microscopic cross-section for absorption \((\sigma_a)\) is \(681 \times 10^{-24}\text{ cm}^2\). With a neutron flux of \(2 \times 10^{13}\text{ cm}^{-2}\text{s}^{-1}\), estimate the reaction rate for absorption, What is the average distance travelled before striking a U-235 nucleus?

\[\text{Ans. } 6.5 \times 10^{14}\text{ cm}^{-3}\text{s}^{-1}, 0.03058\text{ cm}\]

9.2 Calculate the energy yield from the reaction

\[^{91}\text{U}^{235} + _0\text{n}^1 \rightarrow ^{55}\text{Cs}^{140} + ^{37}\text{Rb}^{92} + 4_0\text{n}^1 + E\]

using atomic masses 139.91711 for cesium and 91.91914 for rubidium.

9.3 Calculate the time required for the reactor power to double for (a) assumed fission with prompt neutrons \((\tau^* = 0.0001\text{ s})\) and (b) actual fission with prompt and delayed neutrons \((\tau^* = 0.1\text{ s})\). Take \(k_{eff} = 1.002\).

\[\text{Ans. } (a) 0.0347\text{s}, (b) 34.73\text{s}\]

9.4 A nuclear power plant is operated continually for one year producing 500 MW. The reactor contained 75 tonnes of 3% enriched uranium dioxide fuel. Assuming the power plant efficiency to be 33%, calculate (a) the mass of U-235 consumed in kg, and (b) the fuel burnup in MWd/tonne.

9.5 Boron 10 is used in reactor cores as a control rod material. Natural boron has an atomic mass of 10.8110 amu, a density of 2.3 g/cm\(^3\) and contains 19.78 atomic per cent of B-10 which has an atomic mass of 10.0194 amu and a microscopic absorption cross-section for 2200 m/s thermal neutrons of 3837 barns. Calculate the number of such neutrons absorbed per sec by 1 kg of natural boron.

9.6 Calculate the power generated in MeV/cm\(^3\) and kW/m\(^2\) for a 3.5% enriched uranium dioxide fuel element in a thermal reactor if the effective fission cross-section is 350 barns and the neutron flux per (cm\(^2\)s) is \(10^{14}\). The density of UO\(_2\) is 10.5 g/cm\(^3\).

9.7 A PWR has inlet and exit water at 290 and 320 \(^\circ\text{C}\) respectively. It has a 30 m\(^3\) vapour pressuriser which is normally 60% full of water at a pressure of 140 bar. A case of an insurge occurred during which 0.25 m\(^3\) of water entered the pressuriser from the primary circuit hot leg, 0.05 m\(^3\) entered through the spray, and 50 kWh was added by electric heaters. Determine the internal energy of the contents of the pressuriser before and after the event. Neglect heat losses to the surroundings.

9.8 A BWR operating at a pressure of 70 bar produces 1200 kg/s of saturated steam from feedwater at 200 \(^\circ\text{C}\). The average core exit quality is 10%. Calculate (a) the recirculation ratio, (b) the core inlet enthalpy and temperature. (c) the degree of subcooling, and (d) the heat generated in the reactor.

9.9 A fast breeder reactor generates 3000 MW of heat. The fuel is composed of 20% Pu\(^{239}\) O\(_2\), 80% U\(^{238}\) O\(_2\) by mass. The average neutron flux is \(10^{16}\). Estimate the total mass of the fuel material in the core. Ignore fast fission in U-238 and take neutron losses by leakage and parasitic absorption as 0.25 per neutron absorbed.
6. P.K. Nag, Statistical and Irreversible Thermodynamics, Khanna Publisher, Delhi, 1981.
10.1 Introduction

In hydroelectric power plants the energy of water is utilized to drive the turbine which, in turn, runs the generator to produce electricity. Rain falling upon the earth's surface has potential energy relative to the oceans towards which it flows. This energy is converted to shaft work where the water falls through an appreciable vertical distance. The hydraulic power is thus a naturally available renewable energy source given by Eq. (10.1).

\[ P = g \rho Q H \]  

(10.1)

Here \( P \) is the hydraulic power in Watts, \( g \) is 9.81 m/s\(^2\) (the acceleration due to gravity), \( \rho \) is the water density, 1000 kg/m\(^3\), \( Q \) is the flow or discharge, m\(^3\)/s and \( H \) is the height of fall of water or head, m. The electrical energy produced in kWh can then be written in the form of Eq. (10.2).

\[ W = 9.81 \times 1000 \times Q \times H \times \eta \times t \]

\[ = 9.81 \ QH \ \eta t \ \text{kWh} \]  

(10.2)

where \( t \) is the operating time in hours (8760 h/year) and \( \eta \) is the efficiency of the turbine-generator assembly, which varies between 0.5 and 0.9. The power developed thus depends on quantity (\( Q \)) and head (\( H \)) of water.

Hydro or water power is important only next to thermal power. Nearly 20 per cent of the total power of the world is met by hydropower stations. There are some countries like Norway and Switzerland where the hydropower forms almost the total installed capacity.

Hydroelectric power was initiated in India in 1897 with a run-of-river unit near Darjeeling. However, the first major plant was the Sivasamudram Scheme in Mysore of 4.5 MW capacity commissioned in 1902. Khopoli project of 50 MW in Maharashtra was put into operation in 1914 to supply power to Mumbai city. Since independence a substantial growth in hydropower has occurred with the commissioning of large multipurpose projects like
10.2 ADVANTAGES AND DISADVANTAGES OF WATER POWER

These have been stated point by point as below.

10.2.1 Advantages of Water Power

Hydropower have some inherent advantages which make it very attractive.

1. Water source is perennially available. No fuel is required to be burnt to generate electricity. It is aptly termed as 'the white coal'. Water passes through turbines to produce work and downstream its utility remains undiminished for irrigation of farms and quenching the thirst of people in the vicinity.

2. The running costs of hydropower installations are very low as compared to thermal or nuclear power stations. In thermal stations, besides the cost of fuel, one has to take into account the transportation cost of the fuel also.

3. There is no problem with regards to the disposal of ash as in a thermal station. The problem of emission of polluting gases and particulates to the atmosphere also does not exist. Hydropower does not produce any greenhouse effect, cause the pernicious acid rain and emit obnoxious NO.

4. The hydraulic turbine can be switched on and off in a very short time. In a thermal or nuclear power plant the steam turbine is put on turning gear for about two days during start-up and shut-down.

5. The hydraulic power plant is relatively simple in concept and self-contained in operation. Its system reliability is much greater than that of other power plants.

6. Modern hydropower equipment has a greater life expectancy and can easily last 50 years or more. This can be compared with the effective life of about 30 years of a thermal or nuclear station.

7. Due to its great ease of taking up and throwing off the load, the hydropower can be used as the ideal spinning reserve in a system mix of thermal, hydro and nuclear power stations.

8. Modern hydro-generators give high efficiency over a considerable range of load. This helps in improving the system efficiency.

9. Hydro-plants provide ancillary benefits like irrigation, flood control, afforestation, navigation and aqua-culture.

10. Being simple in design and operation, the hydro-plants do not require highly skilled workers. Manpower requirement is also low.
10.2.2 Disadvantages of Water Power

Major disadvantages of water power are the following:

1. Hydro-power projects are capital-intensive with a low rate of return. The annual interest of this capital cost is a large part of the annual cost of hydro-power installations.
2. The gestation period of hydro projects is quite large. The gap between the foundation and completion of a project may extend from ten to fifteen years.
3. Power generation is dependent on the quantity of water available, which may vary from season to season and year to year. If the rainfall is in time and adequate, then only the satisfactory operation of the plant can be expected.
4. Such plants are often far away from the load centre and require long transmission lines to deliver power. Thus the cost of transmission lines and losses in them are more.
5. Large hydro-plants disturb the ecology of the area, by way of deforestation, destroying vegetation and uprooting people. Strong public opinion against erection of such plants is a deterrent factor. The emphasis is now more on small, mini and micro hydel stations.

10.3 OPTIMIZATION OF HYDRO-THERMAL MIX

A hydroelectric power plant was earlier used as an exclusive source of power. However, it suffers seasonal variation of output proportional to the variation of water flow. To meet the variable load demand, large amount of water requires to be stored. At the times of low water flow rates the hydro plants cannot meet the maximum load. Again, if the maximum capacity of the station is based on the minimum water flow, this will prove uneconomical. There will be a great wastage of water over the dam for greater part of the year. Hence, the present trend is to use hydroelectric power in conjunction with thermal power in an interconnected system. This hydro-thermal mix is optimized to achieve minimum cost of power generation, which may be 30 per cent hydro-70 per cent thermal or 35 per cent hydro-65 per cent thermal. Load sharing by hydro is maximum when the available flow of water is maximum, say during the monsoon months. As long as there is planety of water stored in the reservoir the hydro part of the system carries the base load, with thermal plants taking the peaks. When water availability is low, say during the dry months of winter and spring, the steam plants take the base load and hydro plants meet the peak load (Fig. 10.1). By interconnecting hydropower with steam, a great deal of saving in cost can be effected by way of the following.

(i) Reduction in necessary reserve capacity.
(ii) Diversity of construction programmes.
(iii) Higher utilization factors of hydro-plants.
(iv) Higher capacity factors of thermal plants.
**Fig. 10.1** Typical division of load on a hydro-steam system

### 10.4 SELECTION OF SITE FOR A HYDROELECTRIC PLANT

The following factors should be considered while selecting the site for hydroelectric power plant.

1. Availability of water
2. Water storage capacity
3. Available water head
4. Accessibility of the site
5. Distance from the load centre
6. Type of land of site

#### 1. Availability of water
The design and capacity of the hydro-plant greatly depends on the amount of water available at the site. The run-off data along with precipitation at the proposed site with maximum and minimum quantity of water available in a year should be made available to

   (a) decide the capacity of the plant,
   (b) set up the peak load plant such as steam, diesel or gas turbine plant,
   (c) provide adequate spillways or gate relief during flood period.

#### 2. Water storage capacity
Since there is a wide variation in rainfall all round the year, it is always necessary to store the water for continuous generation of power. The storage capacity can be estimated with the help of mass curve.

#### 3. Available water head
In order to generate the desired quantity of power it is necessary that a large quantity of water at a sufficient head should be available. An increase in effective head, for a given output, reduces the quantity of water required to be supplied to the turbines.

#### 4. Accessibility of the site
The site should be easily accessible by rail and road. An inaccessible terrain will jeopardize the movement of men and material.
5. Distance from the load centre  If the site is close to the load centre, the cost of transmission lines and the transmission losses will be reduced.

6. Type of the land of the site  The land of the site should be cheap and rocky. The dam constructed at the site should have large catchment area to store water at high head. The foundation rocks of the masonry dam should be strong enough to withstand the stresses in the structure and the thrust of water when the reservoir is full.

10.5 HYDROLOGICAL CYCLE

Hydrology is the science that deals with the processes governing depletion and replenishment of water resources over and within the earth's surface. With the knowledge of hydrology at a certain site it is possible to design the irrigation and flood control works, power projects, water supply schemes, navigation works, etc.

As water vapour in atmospheric air goes up it cools, condenses and falls as rain, hail, snow or sleet. When this precipitation falls on hills and mountains and converges to form streams and rivers, it can be used for power generation. Intensity of rainfall, season and topography largely determine the usefulness of rainfall for power purposes. Light falls aid the growth of vegetation but do not contribute to stream flow. When total monthly precipitation concentrates in one or more storms, the runoff will increase greatly though vegetation may suffer. Distribution of precipitation may be classified as (i) direct evaporation (ii) absorption and transpiration by vegetation. (iii) seepage and storage; and (iv) direct surface runoff, eventually forming rivers (Fig. 10.2).

(i) A major part of precipitation on land areas that reaches the soil re-evaporates to the atmosphere, the rate being large from surfaces of lakes, ponds and swamps. A rise in temperature and drop in humidity increase the evaporation rate with the wind aiding it.

(ii) Plants absorb water through their roots and transpire it as vapour through their leaves to the atmosphere.

(iii) Precipitation absorbed by the soil seeps or percolates into the ground, forming bodies of water called the water table or ground storage. It is also called “infiltration” which is a process by which water enters the surface strata of the soil and makes its way downwards to the water table. The amount of seepage or infiltration depends on the geological character of the surface and subsoil.

(iv) The remaining water flows over the ground surface as direct runoff to form brooks and rivers (Fig. 10.2). The amount of runoff from a given rainfall depends on the nature of precipitation. Short, hard showers may produce relatively little runoff, whereas long rainfall saturates the soil lowering seepage rate and slows down evaporation by increased humidity and thus produces more runoff.
Fig. 10.2 Graphic portrayal of the water cycle

The water equation summarizing the disposal of the rainfall over a certain area during a given period is given by

\[ \text{Runoff} + \text{Seepage} + \text{Evaporation} + \text{Transpiration} = \text{Precipitation} \pm \text{Change in storage} \]

The best way to study the rainfall pattern is with the help of graphical plots. The hyetographs are the rainfall intensity-time curves which indicate the variation of the rate of rainfall with respect to time. The cumulative value of rainfall plotted against time represents the mass curve of rainfall.

10.6 HYDROGRAPHS

The variation of stream flow at a given site depends on the geographical, geological and topographical features of the drainage area feeding the river as well as the magnitude of the area rainfall. Hydrographs show the variation of river flow (discharge) with time. Runoff may be plotted as flow duration curves (Fig. 10.3 a), which show the time when a stream flow rate is equalled or exceeded in any period (daily, weekly or monthly basis). The area under the flow duration curve represents the average yield from the stream. By changing the ordinate to power (kW) instead of discharge (m³/s) in Fig. 10.3 (a), the power duration curve is obtained and the area under the curve would then represent the average yield of power from the hydro-power project. It can be noted in Fig. 10.3. (b) that \( Q_m \) is the minimum flow rate that would be available.
Fig. 10.3 (a)  Flow duration curve with % time on x-axis and Run-off on y-axis

Fig. 10.3 (b)  Flow duration curve of a typical river having a low flow
for all the times (i.e. for 100 per cent of time) and the area \( OABC \) would represent the firm yield of water or power, often termed as primary power. The additional output available at higher water flows is called secondary power. If a flow rate of \( Q_n \) is required for all the times as indicated by the area under the flow demand line \( DEF \), then it would be possible to meet this uniform demand of flow rate (or power) for all the times only if storage equal to area \( BEF \) is provided. An alternative to this is to install a thermal power unit of \( BF \) capacity to work as a supplement to the hydro-power unit. The curve also shows that natural flow sufficient to meet the flow demand \( Q_n \) is available for 53.5 per cent of time or 195 days in the year of the lowest flow of the record. In the absence of any storage, area \( BCDE \) represents the secondary power that would be available from the river.

In order to facilitate the storage computation, mass curves are commonly used. A mass curve is a plot of accumulated flow (in hectare-metre) against time, made from the records of mean monthly flows of a stream (Fig. 10.4). The slope of the curve at any point indicates the rate of flow at that particular time. If the curve is horizontal, the flow is zero and if there is a high rate of flow the curve rises steeply. Relatively dry periods are indicated as concave depressions on the mass curve.
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10.7 STORAGE AND PONDAGE

As stated earlier the flow rate of a stream varies considerably with time. For example, during rainy season when the stream is in floods it carries a huge quantity of water as compared to other times of the year when the quantity of water carried by it is considerably less. However, the demands for power ordinarily do not correspond to such variations of the natural flow of the stream. As such some arrangement in the form of storage and pondage of water is required for the regulation of the flow of water so as to make it available in requisite quantity to meet the power demand at a given time.

Storage may be defined as impounding of a considerable amount of excess run off during seasons of surplus flow for use in dry seasons. This is accomplished by constructing a dam across the stream at a suitable site and building a storage reservoir on the upstream side of the dam.

Pondage may be defined as a regulating body of water in the form of a relatively small pond or reservoir provided at the plant. The pondage is used to

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Fig. 10.5 Hydro-power units with pondage and storage
regulate the variable water flow to meet power demand. It caters for short-term fluctuations which may occur due to (a) sudden increase or decrease of load on the turbine (b) sudden changes in the inflow of water, say by breaches in the conveyance channel (c) change of water demand by turbines and the natural flow (supply) of water from time to time. The turbines are often required to meet the power demand higher than the average load when the pondage supplies the excess quantity of water required during that period. Figure 10.5 shows the locations of power houses with storage and pondage. Pondage increases the capacity of a river over a short-time, such as a week. Storage, however, increases the capacity of a river over an extended period of 6 months to as much as 2 years.

10.8 ESSENTIAL ELEMENTS OF A HYDROELECTRIC POWER PLANT

Figure 10.6 gives the flow diagram of a typical hydroelectric power plant. The essential elements of such a plant are the following.

1. Catchment area  2. Reservoir
3. Dam        4. Spillways
5. Conduits  6. Surge tanks
7. Draft tubes  8. Powerhouse
9. Switch yard for transmission of power.

10.8.1 Catchment Area

The whole area behind the dam draining into a stream or river across which the dam has been constructed is called the catchment area. The characteristics of the catchment include its size, shape, surface, orientation, altitude, topography and geology. The bigger the catchment, steeper is the slope, higher is the altitude, and greater is the total runoff of water.

10.8.2 Reservoir

Storage during times of plenty for subsequent use in times of scarcity is fundamental to the efficient use of water resources. The management of
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Fig. 10.8  Cross-section of solid gravity type of masonry dam

Fig. 10.9  Buttress or hollow gravity type of masonry dam with flat deck

Fig. 10.10  Arch type of masonry dam
Earth dams  For a small project of up to 70 m in height, dams constructed of earth fill or embankment are used. A large volume of material is required and it should be available in the vicinity. The dam construction varies with the height and the side slopes are flatter (Fig. 10.11). It is cheaper than masonry dam, but has more seepage losses. There may be serious damage from erosion by water overtopping the dam or seeping through it.

![Earth dam diagram]

**Fig. 10.11  Earth dam**

Rock-fill dams  It is made of loose rock of all sizes and has a trapezoidal shape with a wide base, having a watertight section to reduce seepage. It is used in mountainous region where rock is available.

10.8.4 Spillways

When the water level in the reservoir basin rises, the stability of the dam structure is endangered. To relieve the reservoir of this excess water, a structure is provided in the body of a dam or close to it. This safeguarding structure is called a spillway. It provides structural stability to the dam under conditions of floods without raising reservoir level above H.F.L. (high flood level). Following are the various types of spillways.

1. **Overall spillway**  It is also called solid gravity spillway. It is provided in concrete and masonry dams (Fig. 10.12). Water spills and flows over the crest in the form of a rolling sheet of water. The bucket at the lower end changes the direction of the fast moving water, destroying its excess energy.

![Overall spillway diagram]

**Fig. 10.12  Overall spillway**
2. **Chute or trough spillway**  It is suitable when the valley is too narrow to accommodate the solid gravity spillway in the body of the dam. After crossing over the crest the water shoots down a channel or trough to meet the river downstream of the dam.

3. **Side channel spillway**  This is used when the valley is too narrow and in non-rigid dams where the flood water is not desired to flow over the dam. When there is no room to provide chute spillway, the side channel spillway (Fig. 10.13) is used. Here, after crossing the crest water flows parallel to it.

![Diagram of Side channel spillway](image)

**Fig. 10.13**  *Side channel spillway*

4. **Saddle spillway**  When conditions are not favourable for any of the above types of spillway, a saddle spillway is used. Some natural depression or saddle on the periphery of the reservoir basin away from the dam is used as the spillway, with the bottom of the depression being at the full reservoir level (Fig. 10.14).

![Diagram of Saddle spillway](image)

**Fig. 10.14**  *Saddle spillway*
10.8.6 Surge Tanks

A surge tank is a small reservoir in which the water level rises or falls to reduce the pressure swings so that they are not transmitted to the closed conduit. If the power house is located within a short distance of the headworks, surge tanks are not necessary. Thus for run off plants and medium head schemes no surge tank is needed. Surge tanks are required for high head plants where water is taken to the power house through tunnels and penstocks. A typical arrangement is shown in Fig. 10.17, where the surge tank is a vertical standpipe connected to the penstock.
with no overflow of water. A further discussion on surge tanks has been made in Section 10.23.

### 10.8.7 Draft Tubes

The draft tube allows the turbine to be set above the tailrace to facilitate inspection and maintenance and by diffuser action regains the major portion of the kinetic energy or velocity head at runner outlet, which would otherwise go waste as an exit loss. The draft tube can be a straight conical tube (Fig. 10.18 a) or an elbow tube (Fig. 10.18 b). The conical type is used for low power units, while the elbow type is more common. In the elbow type energy is regained in the vertical portion which flattens in the elbow section to discharge water horizontally to the tailrace.

![Diagram of Draft Tubes](image)

**Fig. 10.18 (a) Straight conical draft tube (b) Elbow type draft tube**

### 10.8.8 Powerhouse

A powerhouse should have a stable structure and its layout should be such that adequate space is provided around the equipment for convenient dismantling and repair. The equipment provided in the powerhouse includes the following.

1. Hydraulic turbines
2. Electric generators
3. Governors
4. Gate valves
5. Relief valves
6. Water circulation pumps
7. Air duct
8. Switch board and instruments
9. Storage batteries
10. Cranes
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10.9.3 Low Head Power Plants

A dam is constructed across a river and a sideline stream diverges from the river at the dam. Later this channel joins the river further downstream (Fig. 10.21). Francis turbine or Kaplan turbine is used for power generation.

10.9.4 Base Load Plants

These plants are required to supply constant power to the grid. They run continuously without any interruption and are mostly remote controlled.

10.9.5 Peak Load Plants

They only work during certain hours of a day when the load is more than the average. Thermal stations work with hydel plants in tandem to meet the base load and peak load during various seasons.
10.9.6 Run-Of-River Plants with or Without Pondage

Such a plant works daily according to the nature and limit to the flow in the river. Power generated depends on the quantity of flow. Sometimes, a small storage reservoir or pond is built, which can store a few hours’ supply of water to the plant, when the river flow exceeds the amount required by the plant. Such a scheme is called a run-of-river plant with pondage. The pondage or stored water is used in generating power during the hours when the demand is in excess of the flow of the river at the moment.

10.9.7 Hydroelectric Plants with Storage Reservoir

These plants are most common in India. During the rainy season water is stored in reservoirs so that it can be utilized during other seasons to supplement the flow of the river whenever the flow in the river falls below a specified minimum. Power can be generated directly from the reservoir. Sometimes canals are constructed to convey water from the reservoir for irrigation purposes.

10.9.8 Pumped Storage Plants

Water after working in turbines is stored in the tailrace reservoir. During low load, say night time, the water is pumped back from the tail to the head reservoir drawing excess electricity from the grid or from the nearby steam plant. During peak load, this water is used to work on turbines to produce electricity (Fig. 10.22). It is always economical to run the steam power plants all the time at full plant capacity factor. Whenever the load demand is less than the full plant capacity, the surplus energy instead of being wasted is transmitted to a pump is installed at the tailrace of the hydroelectric plant. The advantages of such a plant can be summarized as follows:

(a) Substantial increase in peak load capacity at low cost
(b) High operating efficiency
(c) Better load factor
(d) Independence of stream flow conditions.

![Diagram of Pumped Storage Plant](image-url)
About 70 per cent of power used in pumping is recovered. A motor-generator set and a reversible turbine-pump unit can be profitably used as the same machine can be operated as a motor or generator and similarly as a pump or a turbine (Deriaz turbine).

10.9.9 Mini and Micro-Hydel Plants

More emphasis is now being given on such plants. The natural water source in hilly terrain can be utilized for power generation with low-head standardized turbo-generator units. Its adverse effect on ecology is negligible. The mini-plants operate with 5 m–20 m head producing about 1 MW to 5 MW of power, while micro-plants are still smaller and work under a head of less than 5 m and generate electricity between 0.1 MW to 1 MW. The potential energy source in India in this category is around 20,000 MW.

10.10 HYDRAULIC TURBINES

Hydraulic turbines convert the potential energy of water into shaft work, which, in turn, rotates the electric generator coupled to it in producing electric power. Historically, hydraulic turbines of today are derived from the waterwheels of the middle ages used for flour mills (to grind wheat) and ore-crushing. One such waterwheel (pan-chakki) can still be seen at Aurangabad, which is, at least, four hundred years old. Modern turbines have undergone many technological advances in diverse areas like fluid mechanics, metallurgy and mechanical engineering.

10.10.1 Classification of Hydraulic Turbines

The hydraulic turbines can be classified according to the (a) head and quantity of water available (b) name of the originator (c) nature of working on the blades (d) direction of flow of water (e) axis of the turbine shaft (f) specific speed.

1. **According to the head and quantity of water available** The difference in elevation of water surface between upstream and downstream of the turbine is the head under which the turbine acts (Fig. 10.23). The turbines work under a wide range of heads varying from 2 to 2000 m. A classification of turbine based on head as follows.

<table>
<thead>
<tr>
<th>Head Type</th>
<th>Head Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low head</td>
<td>2–15 m</td>
</tr>
<tr>
<td>Medium head</td>
<td>16–70 m</td>
</tr>
<tr>
<td>High head</td>
<td>71–500 m</td>
</tr>
<tr>
<td>Very high head</td>
<td>Above 500 m</td>
</tr>
</tbody>
</table>

For low heads, only Kaplan or propeller turbines are used. For medium heads either Kaplan or Francis turbines are used. For high heads either Francis or Pelton turbines are used. For very high heads, invariably Pelton turbines are used. Deriaz turbines are used up to a head of 300 m. Their use is, however, restricted under reversible flow conditions (i.e. pumped-storage plants where the turbine also works as a pump).
Turbines can also be classified as low discharge, medium discharge and high discharge turbines, depending on the flow available. Pelton turbines are relatively low discharge turbines. Kaplan turbines are high discharge turbines, while Francis turbines occupy an intermediate position in this regard.

2. **According to the name of the originator**  
   (i) Pelton turbine—named after Lester Allen Pelton of the USA, an impulse turbine used for high head and low discharge.  
   (ii) Francis turbine—named after James B. Francis, a reaction turbine used for medium head and medium discharge.  
   (iii) Kaplan turbine—named after Dr. Victor Kaplan, a reaction turbine used for low head and large discharge.  
   (iv) Deriaz turbine—named after the Swiss engineer Deriaz, a reversible turbine-pump used up to a head of 300 m.

3. **According to the nature of working on the blades**  
   Turbines are classified as impulse and reaction turbines depending on the mode of energy conversion of potential energy of water into shaft work. In an impulse turbine all the available head of water is converted into kinetic energy in a nozzle. The water shoots out of the nozzle in a free jet into a bucket which revolves round a shaft. During this action, the water is in contact with air all the time and the water discharged from bucket falls freely through the discharge passage into the tailwater. The free jet is at atmospheric pressure before and after striking the vanes. These are pressureless or impulse turbines. Pelton wheel belongs to this category.

   In reaction turbines, the entire flow from the headwater to the tailwater takes place in a closed conduit system which is not open to the atmosphere at any point in its passage. At the entrance to the runner, only a part of P.E. is converted into K.E. and the remaining into pressure energy. The runner converts both K.E. and pressure energy into mechanical energy. Such turbines are called reaction or pressure
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Fig. 10.24  (a) Pelton wheel

Fig. 10.24  (b) Velocity diagrams and section through a bucket

The nozzle directs the flow on the wheel. It also governs the quantity of flow with the help of a spear valve controlled by the governor action. In the simple
arrangement there is a single nozzle feeding water to the turbine. However, for larger discharge there are turbines having up to six jets, all symmetrically arranged and causing rotation in the same direction. Figure 10.25 shows a multi-jet arrangement with four jets. Multi-jet machines usually have vertical shafts.

The specific speed of a multi-jet machine, i.e., $N_{s_{mj}}$ is given by the following reaction.

$$N_{s_{mj}} = \sqrt{n} N_{s_{sj}}$$  \hspace{1cm} (10.5)

where $N_{s_{sj}}$ is the specific speed for a single-jet machine and $n$ is the number of jets. Thus the specific speed of a given wheel can be increased by using multi-jet arrangement. The maximum number of jets used so far is six and the maximum speed for a single jet is of the order of 30. Thus the maximum specific speed for multi-jet machines is about 70.

![Diagram of Multi-jet Pelton wheel arrangement](figure)

**Fig. 10.25**  Multi-jet Pelton wheel arrangement

It may be noted that the bucket deflection angle is of the order of $165^\circ$ (Fig. 10.26). The slightly oblique direction of existing water allows it to escape freely without hitting the back of the next bucket. The water after leaving the bucket drops freely into the tailrace.

The jet moves in a tangential plane before and after striking the wheel and the bucket moves at a speed given by Eq. (10.6)

$$V = \omega r = \frac{\pi DN}{60}$$  \hspace{1cm} (10.6)

where $r$ and $D$ are the bucket circle radius and diameter respectively and $\omega$ is the angular velocity given by $\frac{2\pi N}{60}$, $N$ being the rpm.

With the nozzle diameter $d$, $D/d$ is a size parameter for the turbine. This is known as jet ratio, $m$, having a value in the range of 10 to 24. The net head
available at the nozzle is equal to the gross head less losses in the pipeline. If it is equal to \( H \), the velocity of jet issuing from the nozzle is as follows.

\[
V_1 = C_v [2gH]^{1/2}
\]  

(10.7)

where \( C_v \) is the coefficient of velocity (0.97 – 0.99).

![Diagram of nozzle and bucket](image)

**Fig. 10.26** Bucket deflection angle and velocity diagrams

Total energy transferred to the wheel (Fig. 10.26) is given by Euler equation (Eq. 10.8).

\[
E = (V_{w_1} V_{b_1} - V_{w_2} V_{b_2})/g = \frac{V_b}{g} (V_{w_1} - V_{w_2})
\]

(10.8)

where subscript 1 represents the condition at inlet and subscript 2 the condition of water at outlet of the bucket, \( V_w \) is the velocity of whirl (tangential component) and \( V_b \) is the bucket velocity given by \( V_{b_1} = V_{b_2} = \frac{\pi DN}{60} \).

Now, from the exit velocity diagram (Fig. 10.26), we get

\[
V_{w_2} = V_b - V_{r_2} \cos (180 - \theta) = V_b + V_{r_2} \cos \theta
\]

(10.9)

where \( \theta \) is the bucket deflection angle (~165°).

Now, \( V_{r_2} = \) relative velocity of water at exit

\[
= kV_{r_1} = k(V_1 - V_b)
\]

(10.10)

where \( k \) is the blade friction coefficient, \( V_1 \) is the absolute velocity of water from the jet, and \( V_{r_1} \) is the relative velocity of water at inlet.

\[
V_{w_2} = V_b + k(V_1 - V_b) \cos \theta
\]

(10.11)

Substituting \( V_{w_2} \) from Eq. (10.11) in Eq. (10.8) and since \( V_{w_1} = V_b \), we get

\[
E = \frac{V_b}{g} [V_1 - V_b - k (V_1 - V_b) \cos \theta]
\]
\[
\frac{V_b}{g} (V_1 - V_b) (1 - k \cos \theta) \\
= \frac{1-k \cos \theta}{g} \left( V_1 V_b - V_b^2 \right) 
\] (10.12)

For given values of \( V_1 \), \( k \) and \( \theta \), there is a certain value of \( V_b \) for which \( E \) is maximum. Differentiating \( E \) with respect to \( V_b \) and putting it equal to zero, we get

\[ \frac{dE}{dV_b} = \frac{1-k \cos \theta}{g} (V_1 - 2V_b) = 0 \]

\[ V_b = \frac{V_1}{2} \] (10.13)

Therefore, the optimum bucket velocity for maximum work output is half the jet velocity. On its substitution, we get

\[ E_{\text{max}} = \frac{1-k \cos \theta \frac{V_1^2}{4}}{g} \] (10.14)

The kinetic energy of the input jet = \( \frac{V_1^2}{2g} \).

Therefore, the blading or diagram or hydraulic efficiency of the wheel is given by

\[ \eta_B = \frac{E}{V_1^2 / 2g} = \frac{1-k \cos \theta}{g} \left( V_1 V_b - V_b^2 \right) \frac{2g}{V_1^2} \]

\[ = 2 (1 - k \cos \theta) (\rho - \rho^2) \] (10.15)

where \( \rho \) is the velocity ratio, \( \frac{V_b}{V_1} \) (Fig. 10.27)

\[ \eta \]

\[ \eta_{\text{max}} \]

\[ 0 \quad 0.5 \quad 1.0 \]

\[ \rho = V_b / V_1 \]

**Fig. 10.27** Variation of \( x \) with velocity ratio
Making \( \frac{d\eta_D}{d\rho} = 0 \), we get
\[
2(1 - k \cos \theta) (1 - 2\rho) = 0
\]
or
\[
(\rho_{opt})_{\text{max eff}} = 1/2
\]
(10.16)
Thus, for maximum hydraulic efficiency also, the wheel velocity is half the jet velocity.

Substituting results of Eq. (10.16) in Eq. (10.15), we get
\[
(\eta_D)_{\text{max}} = 2(1 - k \cos \theta) (1/2 - 1/4)
\]
\[
= \frac{1 - k \cos \theta}{2}
\]
(10.17)
Again, the diagram efficiency for maximum work is given by
\[
(\eta_D)_{\text{max work}} = \frac{E_{\text{max}}}{V_1^2 / 2g} = \frac{1 - k \cos \theta}{g} \frac{V_1^2}{4} \times \frac{2g}{V_1^2}
\]
\[
= \frac{1 - k \cos \theta}{2} = (\eta_D)_{\text{max}} = \eta_{\text{max}}
\]
(10.18)
If \( k = 1 \), i.e. there is no energy loss due to friction, then
\[
\eta_{\text{max}} = \frac{1 - \cos \theta}{2}
\]
If \( \theta = 180^\circ \), \( \eta_{\text{max}} = 1 \) or 100%.
However, \( k \) lies between 0.8 and 0.85 and \( \theta \approx 165^\circ \), so that the exiting water does not hit the following bucket.
\[
\eta_{\text{max}} = \frac{1 - 0.8 \cos 165^\circ}{2} \approx 0.886
\]
In practice, \( \rho_{\text{opt}} \approx 0.46 \), instead of 0.5.
If we plot \( \eta \) vs \( \rho \), we get Fig. 10.27. Now,
\( V_b \) is constant and \( V_1 = c_v [2gH]^{1/2} \), which depends on net head \( H \). Then the discharge is given by the following equation.
\[
Q = A c_v [2gH]^{1/2} \text{ m}^3/\text{s}
\]
(10.19)
where the flow area \( A \) is controlled by the spear to regulate \( Q \).
The velocity of wheel \( V_b \) is given by \( V_b = \phi \sqrt{2gH} \)
where \( \phi = \text{speed ratio} \), which varies from 0.43 to 0.48.
The minimum number of buckets in the wheel is approximately given by
\[
Z = \frac{m}{2} + 15
\]
(10.19a)
where \( m \) is equal to the jet ratio, \( D/d \).

The erosion of the Pelton wheel occurs (i) on the buckets due to erosive effect of flow and (ii) at the nozzle due to cavitation effect (discussed later). To
protect the buckets from wear and tear, chrome alloy steel or stainless steel is used. In India, many Pelton turbines are in operation such as at Koyna (475 m head, 4 jets), Sharavathi (570 m head, 4 jets), Kundah I (360 m head, 5 jets) and Kundah II (690 m, 3 jets). The world’s largest Pelton turbine is a 6-jet, 840 m head turbine at Aurland-2 in Brazil producing 243 MWe.

10.13 DEGREE OF REACTION

By applying Bernoulli’s equation to the inlet and outlet of a turbine, we get

\[ \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = E + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \]  \hspace{1cm} (10.20)

where \( E \) is the energy transferred from fluid to the rotor. Therefore,

\[ E = \frac{p_1 - p_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} \]  \hspace{1cm} (10.21)

The first term on the R.H.S. is the energy transfer due to drop in static pressure and the second term represents the energy transfer due to drop in velocity head.

If \( p_1 = p_2 \), i.e. if pressure is constant,

\[ E = \frac{V_1^2 - V_2^2}{2g} \]  \hspace{1cm} (10.22)

This happens in the case of impulse turbine, i.e. Pelton wheel where the pressure is atmospheric.

If \( V_1 = V_2 \),

\[ E = \frac{p_1 - p_2}{\rho g} \]  \hspace{1cm} (10.23)

This holds good for a pure reaction turbine, where the wheel rotates only due to pressure drop across it exerting a reaction by Newton’s third law of motion.

Degree of reaction, \( R \), is defined in the following manner.

\[ R = \frac{\text{Energy transfer due to pressure drop}}{\text{Total energy transfer}} \]

\[ = \frac{(p_1 - p_2)/\rho g}{E} \]

\[ = \frac{V_1^2 - V_2^2}{2g} \]

\[ = \frac{E - V_1^2 - V_2^2}{E} \]
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Fig. 10.29  Velocity triangles of a Francis turbine blade

Blading or diagram efficiency, \( \eta_D = \frac{E}{E + E_1} = 1 - \frac{E_1}{E + E_1} \)

\[ = 1 - \frac{1}{1 + 2 \cot \alpha_1 (\cot \alpha_1 - \cot \beta_1)} \]  (10.27)

Degree of reaction,

\[ R = 1 - \frac{V_{t1}^2 \cot^2 \alpha_1}{E} \]

Hydraulic efficiency,

\[ \eta_h = \frac{E}{H} = \frac{V_{w1}V_{bl}}{gH} \]  (10.28)

Overall efficiency,

\[ \eta_0 = \frac{P}{pQgH} \]  (10.29)

where \( P \) is the total power output.

10.15  PROPELLER AND KAPLAN TURBINES

The propeller turbine is a reaction turbine used for low heads (4 m – 80 m) and high specific speeds (300 – 1000). It is an axial flow device providing large flow area utilizing a large volume flow of water with low flow velocity. It consists of an axial-flow runner usually with four to six blades of airfoil shape (Fig. 10.30). The spiral casing and guide blades are similar to those in Francis turbines. In propeller turbines as in Francis turbines the runner blades are fixed and non-adjustable.
A special type of a propeller turbine is the Kaplan turbine in which the individual runner blades are pivoted on the hub (Fig. 10.31) so that their inclination may be adjusted during operation responding to changes in load. The blades are adjusted automatically rotating about pivots with the help of a governor servo-mechanism. The efficiency of a reaction turbine depends on the inlet blade angle. In fixed blade runners, it is not possible to vary the inlet blade angle for varying demands of power (load). So such turbines are designed for maximum efficiency only for a particular load. At all other loads their efficiency is less than this. In the Kaplan turbine, because of the arrangement for automatic variation of inlet blade angle with variation in load, the turbine can be run at maximum efficiency at all loads.
Hidden page
Fig. 10.31 (c) Cross-section of typical low head concrete spiral-case setting with Kaplan turbine

Fig. 10.32 Velocity triangles for propeller or Kaplan runner blade

10.16 DERIAZ TURBINE

The Deriaz turbine is also known as the 'diagonal turbine'. The flow over the runner is at an angle of 45° to the axis (Fig. 10.33). It has adjustable blades like
Kaplan turbines. At the same time the flow is diagonal or mixed as in Francis turbines. It can be described as a cross between the two turbines (Kaplan and Francis) and can be used for heads up to 200 m. The number of blades varies from 10 to 12. The guide blades and the stay vanes are also inclined.

The Deriaz runner is particularly suited for reversible flow conditions when the turbine also has to work as a pump as in pumped-storage power plants.

Fig. 10.33 Deriaz turbine

10.17 BULB TURBINE

Tubular or bulb turbines are small fixed axial flow propeller turbines operating under low heads. The turbo-generator is housed in an enclosed bulb-shaped casing, which is installed right in the middle of the flow passage. The bulb and the propeller form an integral unit followed by a straight conical flaring draft tube (Fig. 10.34). Bulb turbines are suitable for tidal power plants.

10.18 SPECIFIC SPEED

To analyse hydroelectric schemes it is economical to make a scale model and perform necessary hydraulic tests on it in order to predict what will happen in the prototype or full-sized system under similar operating conditions. The suitability of a turbine for a particular application depends on (a) head of water (b) rotational speed (c) power developed, which together fix a parameter called ‘specific speed’.
The specific speed of a turbine is defined as the speed of operation of a geometrically similar model of the turbine which is so proportioned that it produces 1 kW power when operating under 1 m head.

We know,

\[ P = \rho QgH \]

Therefore, \( P \propto QH \), since the density of water and acceleration due to gravity \( g \) are constant,

or \( P \propto (AV)H \) \hspace{1cm} (10.30)

where \( A \) is the cross-sectional flow area and \( V \) is the water velocity.

or \( P \propto D^2 [2gH]^{1/2} H \)

or \( P \propto D^2 \times H^{3/2} \) \hspace{1cm} (10.31)

where \( D \) is the wheel diameter through which the water flows axially and \( H \) is the net head of water. Again, the blade velocity is

\[ V_b \propto V \] \hspace{1cm} (10.32)

or

\[ DN \propto [2gH]^{1/2} \]

or

\[ D \propto \frac{H^{1/2}}{N} \] \hspace{1cm} (10.33)

Substituting Eq. (10.33) in Eq. (10.31), we get

\[ P \propto \frac{H}{N^2} \cdot H^{3/2} \]

\[ \because \]

\[ N^2 \propto \frac{H^{5/2}}{P} \]

\[ \therefore \]

\[ N \propto \frac{H^{5/4}}{P^{1/2}} \] \hspace{1cm} (10.34)

or

\[ N = \frac{KH^{5/4}}{P^{1/2}} \]
or

\[ K = \frac{NP^{1/2}}{H^{5/4}} \]

If \( P = 1 \text{ kW}, H = 1 \text{ m} \), then \( K \) is equal to \( N \), called the specific speed, \( N_s \).

\[ N_s = \frac{NP^{1/2}}{H^{5/4}} \tag{10.35} \]

The classification of turbines on the basis of specific speeds has been discussed in Section 10.10. The ranges of specific speeds of different turbines are given in Table 10.2. The non-dimensional form of specific speed is given by the following equation which is Eq. (10.4) derived earlier.

\[ N_s' = \frac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}} \]

It is also known as the shape number of the turbine.

### 10.18.1 Scale Ratio

The model of a turbine and its prototype are in definite geometric ratio depending on their respective heads and the rotative speeds. The ratio of blade velocity \( V_b \) and the water velocity \( V \) is called the speed ratio, which has a definite value for a particular turbine (0.42 to 0.47 for a Pelton turbine, from 0.55 to 1.00 or more for a Francis turbine and 1.5 to 3.00 or more for a propeller turbine).

\[ V_b \propto V \]

\[ DN \propto \sqrt{H} \]

Using subscript \( m \) for the model turbine and \( p \) for the prototype, we get

\[
\frac{D_m N_m}{D_p N_p} = \sqrt{\frac{H_m}{H_p}}
\]

\[
\frac{D_m}{D_p} = \sqrt{\frac{H_m}{H_p}} \cdot \frac{N_p}{N_m} \tag{10.36}
\]

This is called the scale ratio which represents the ratio of the diameters of the model turbine and the prototype turbine.

### 10.18.2 Unit Speed, Unit Power and Unit Discharge

The terms 'unit speed', 'unit power' and 'unit discharge' are frequently used to express the operational characteristics of hydraulic turbines.

The unit speed \( N_u \) is defined as the speed of a geometrically similar turbine working under a head of 1 m,
\[ V_b \propto V \]

\[ DN \propto [2gH]^{1/2} \]

\[ N \propto \sqrt{H} \]

\[ \therefore \quad N = K \sqrt{H} \]

where \( K \) is a constant.

When \( H = 1 \text{ m}, N = N_u = K \)

\[ \therefore \quad N_u = \frac{N}{\sqrt{H}} \quad (10.37) \]

The unit power \( P_u \) is the kW of power generated by a geometrically similar turbine working under a head of 1 m.

\[ P = \rho Q g H = \rho (AV) g H \]

\[ = \rho A [2gH]^{1/2} \cdot g H \]

or

\[ P \propto H^{3/2} \]

When \( H = 1 \text{ m}, P = P_u = K \)

\[ \therefore \quad P_u = \frac{P}{H^{3/2}} \quad (10.38) \]

The unit discharge, \( Q_u \) is the flow rate the turbine would have under a head of 1 m.

\[ Q = AV = A [2gH]^{1/2} \]

\[ Q = K \sqrt{H} \]

where \( K \) is a constant.

When \( H = 1 \text{ m}, Q = Q_u = K \)

\[ \therefore \quad Q_u = \frac{Q}{H^{1/2}} \quad (10.39) \]

10.19 COMPARISON OF TURBINES

The characteristic features of common types of turbine are summarized in Table 10.4.

10.20 CAVITATION

When the velocity of a fluid increases its pressure falls. In any turbine part if the pressure drops below the vapour pressure at that temperature some of the liquid flashes into vapour. The bubbles formed during vaporization are carried by the water stream to higher pressure zones, where the bubbles condense into liquid
forming a cavity or vacuum. The surrounding liquid rushes towards the cavity giving rise to a very high local pressure which may be as high as 7000 atm. The formation of such a cavity and high pressure occurs repeatedly hundreds of times in a second. This phenomenon is known as cavitation, which causes pitting on the metallic surface of runner blades and draft tube. It is accompanied by considerable vibration and noise.

<table>
<thead>
<tr>
<th></th>
<th>Pelton wheel</th>
<th>Francis turbine</th>
<th>Kaplan/Propeller turbine</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Flow</td>
<td>Tangential, single stage, impulse</td>
<td>Inward radial flow, single stage, reaction</td>
<td>Axial flow, single stage, reaction</td>
</tr>
<tr>
<td>2. Maximum capacity</td>
<td>250 MW</td>
<td>720 MW</td>
<td>225 MW</td>
</tr>
<tr>
<td>3. Number of jets/kind of blades</td>
<td>1 to 6</td>
<td>Fixed blades</td>
<td>Propeller turbines have fixed blades, while Kaplan turbines have adjustable blades</td>
</tr>
<tr>
<td>4. Head</td>
<td>100–1750 m</td>
<td>30–550 m</td>
<td>1.3–77.5 m</td>
</tr>
<tr>
<td>5. RPM</td>
<td>75–1000</td>
<td>93.8–1000</td>
<td>72–600</td>
</tr>
<tr>
<td>6. Hydraulic efficiency</td>
<td>Single jet 85–90%</td>
<td>90–94%</td>
<td>85–93%</td>
</tr>
<tr>
<td>7. Specific speed</td>
<td>6–60</td>
<td>50–400</td>
<td>280–1100</td>
</tr>
<tr>
<td>8. Regulation mechanism</td>
<td>Spear nozzle and deflector plate</td>
<td>Guide vanes</td>
<td>Blade stagger</td>
</tr>
</tbody>
</table>

This table can be used in selecting a turbine for a specific application.

Cavitation should be minimised or avoided by selecting proper material like stainless steel or alloy steel, by adequate polishing of the surface, by selecting a runner of low specific speed or by keeping the runner under water.

### 10.21 GOVERNING OF HYDRAULIC TURBINES

Hydraulic turbines are directly coupled to the electric generators. The generators are always required to run at a constant speed irrespective of the variations in the load. This constant speed (rpm) of the generator is given by

\[ N = \frac{120f}{p} \]  

\[(10.40)\]
where \( f \) is the frequency for power generated in cycles per second and \( p \) is the number of poles for the generator. The speed of the generator can be maintained at a constant level only if the speed of the turbine runner is constant as given by Eq. (10.40). It is known as the synchronous speed of the turbine runner for which it is designed.

If the load on the generator goes on varying and if the input for the turbine remains the same, then the speed of the runner tends to increase if the load goes down or it tends to decrease if the load on the generator goes up. Therefore, the speed of the generator and hence, the frequency will vary accordingly, which is not desired. Therefore, the speed of the runner is always required to be maintained at a constant level at all loads.

It is done automatically by a governor which regulates the quantity of water flowing through the runner in proportion to the load.

### 10.21.1 Governing of Impulse Turbine

In a Pelton turbine, water flow to the runner is regulated by the combined action of the spear and the deflector plate. There is a centrifugal governor, as in the case of a steam turbine, where its sensitivity to load variation is augmented by an oil-operated servo-mechanism (Fig. 10.35). When the load on the generator drops, the speed of turbine runner increases. The flyballs of the centrifugal governor fly outward due to more centrifugal force (due to higher rpm). The sleeve moves up, the portion of the lever to the right of the fulcrum moves down pushing the piston rod of the control valve downwards.

![Fig. 10.35 Governing of Pelton turbine](image-url)
With the downward motion of the piston rod, valve $V_1$ closes and valve $V_2$ opens as shown in Fig. 10.35. A gear pump pumps oil from the oil sump to the control or relay valve. Oil flows through valve $V_2$ and exerts force on the face $L$ of the piston of the relay cylinder. The piston (or spear) rod along with the spear moves to the right, thus decreasing the flow area and hence, the rate of water flow to the turbine. The speed of the turbine falls till it becomes normal when the flyballs, sleeve, lever, etc. also come to normal position. The reverse happens when the load on the generator increases, speed decreases, flyballs fly inward with less centrifugal force (due to less rpm), the sleeve moves down, the piston rod of control valve goes up, valve $V_1$ opens and valve $V_2$ closes, the oil under pressure flows through valve $V_1$ and exerts a force on the face $M$ of the piston. The piston rod and the spear move to the left as a result of which more water flows to the turbine to take up more load and the speed becomes normal, i.e. attains its rated value.

The spear or needle valve is used normally for small load fluctuations. When there is a sudden fall of load, the spear has to move rapidly to close the nozzle. This rapid closing may cause water hammer. It is quite serious in large capacity plants with long penstocks. To avoid the water hammer effects during a sudden fall of load, a deflector is introduced in the system, which is not shown in Fig. 10.35. The function of the deflector is to deflect some water from the jet advancing to the turbine runner when the load on the turbine suddenly decreases. The quantity of water flowing through the nozzle remains the same, but a certain part of water coming out from the nozzle is deflected and is not allowed to strike the buckets. The deflected water goes waste into the tailrace level.

**10.22 GOVERNING OF REACTION TURBINES**

The governing of Francis turbine is similar to the governing of Pelton wheel except that the motion of the piston in servo-motor is used to partially close or open the guide vanes gate through which the water is supplied to the turbine (instead of the spear in the nozzle of the Pelton turbine). The working diagram of the governor is shown in Fig. 10.36. The position of the control valve and the servo-motor correspond to the design load on the turbine and operate in the same way as in the case of Pelton wheel. A compensating device is, however, added to prevent the governor from overshooting. When the servo-motor piston moves to the right, the bell-crank lever EFG is rotated downward about $F$ and the arm $G$ is lowered. This pulls down the pivot $A$, which, in turn, lowers the fulcrum $B$. Thus the relay port ‘$b$’ is partially or fully closed, restricting the piston motion to the right.

The governor is always operated with a pressure relief valve (not shown). A sudden closure of wicket gates will open the relief valve due to the sudden increase in pressure and protect the conduit from inertia effects of speeding water. The relief valve consists of a spear and is held by fluid pressure to close the bypass of water from the spiral casing to the tailrace at design load. When the load decreases suddenly, a bell-crank lever opens the pilot valve of the pressure chamber so that the pressure on the spear is reduced, thereby permitting
the spear to be lifted up and allowing a portion of water to flow directly from the spiral casing to the tailrace though the bypass without striking the turbine runner. Thus, both the deflector of Pelton wheel and the relief valve of Francis or Kaplan turbine perform the same function of protecting the system from water hammer effects when the load suddenly decreases.

In the case of Kaplan turbine, in addition to guide vanes the runner vanes are also adjustable and hence the governor is required to operate both sets of vanes simultaneously. The runner vanes are also operated by a separate servo-motor and a control valve which are interconnected with those of the guide vanes to ensure that for a given guide vane opening there is a definite runner vane inclination.

**Fig. 10.36** Governing of Francis turbine

### 10.23 SURGE TANKS

A preliminary discourse on surge tanks was made in Section 10.8.6. When the load on the generator decreases the governor reduces the rate of flow of water striking the runner in order to maintain the constant speed of the runner. But the sudden reduction of the rate of flow in the penstock may build a water hammer in the pipe, which may cause excessive inertia pressure in the pipeline due to which the pipe may burst. Two devices, viz. the deflector and the relief valve, as described earlier, are provided to avoid the sudden reduction of the rate of flow in the penstock. But neither of these devices is of any help when the load on the generator increases and the turbine is in need of more water. Thus, in order to fulfil both the above objectives, in addition to the deflector or the relief valve, certain other devices such as surge tank and forebay are provided. Surge tanks are employed in the case of high head and medium head power plants where the penstock is very long and forebays are suitable for medium head and low head power plants where the length of the penstock is short.
An ordinary surge tank is a cylindrical open-topped storage reservoir, as shown in Fig. 10.37, which is connected to the penstock at a point as close as possible to the turbine. The upper lip of the tank is kept well above the maximum water level in the supply reservoir.

**Fig. 10.37** Ordinary surge tank

When the load on the turbine is normal and steady, there are no velocity variations in the pipeline and the normal pressure gradient is \( oaa_1 \) (Fig. 10.37). The water surface in the surge tank is lower than the reservoir surface by an amount equal to the friction head loss in the pipe connecting the reservoir and the surge tank. When the load on the generator decreases, the turbine gates are partially closed and the excess water moving towards the turbine is stored in the surge tank in the space between the levels \( a \) and \( b \) and a rising pressure gradient \( abb_1 \) develops. The resulting retarding head reduces the velocity of flow in the pipeline corresponding to the reduced discharge required by the turbine.

When the load on the generator increases, the governor opens the turbine gates to increase the rate of flow entering the runner. The increased demand of water by the turbine is partly met by the water stored between levels \( a \) and \( c \) in the surge tank (Fig. 10.37). As such the water level in the surge tank falls and a falling pressure gradient \( Occ_1 \) is developed. The surge tank thus provides an accelerating head which increases the velocity of flow in the pipeline corresponding to the increased demand by the turbine.

Various other types of surge tanks are also shown in Fig. 10.38. Type (a) is a conical type surge tank. Type (b) has an internal bell-mouth spillway which permits the overflow to be easily disposed of. Type (c) is a differential surge tank, which has a central riser pipe having small ports at its lower end. It
Hidden page
Fig. 10.39 Constant head characteristics of Pelton wheel and reaction turbines
(b) Constant speed characteristic curves In these tests the constant speed is attained by regulating the gate opening (i.e. discharge) as the load varies. The head may or may not remain constant. The characteristic curves of efficiency against load for different turbines are shown in Figs 10.40 (a) and (b). The efficiency increases with load and reaches the maximum at the full or rated load. It is observed that the Kaplan turbine and the Pelton wheel maintain a high efficiency over a longer range of part load as compared with either the Francis or the fixed blade propeller turbine.

![Graph showing efficiency curves for various turbines](image)

**Fig. 10.40 (a) Overall efficiency variation with load for various turbines**

![Graph showing efficiency comparison between Kaplan and propeller turbines](image)

**Fig. 10.40 (b) Efficiency variation with load for propeller and Kaplan turbines**
Figure 10.41 shows the plots of efficiency and power varying with discharge, where \( Q_0 \) is the maximum discharge required to initiate the motion of the turbine runner from the state of rest. Since the power \( (P = \rho QgH) \) is directly proportional to discharge if the head is constant, the \( P \) vs \( Q \) plot is a straight line. However, the overall efficiency increases with discharge and becomes more or less constant beyond a certain value of discharge.

![Overall efficiency and brake power varying with discharge](image)

**Fig. 10.41** Overall efficiency and brake power varying with discharge

**c** Constant efficiency curves Figure 10.42 shows the constant efficiency curves for all conditions of running, which are also called the universal characteristic curves of the turbine. The innermost curve represents the highest efficiency of the turbine and the outer curves represent lower efficiencies. If a vertical line is drawn at a certain \( Q_u \), it will intersect an efficiency curve at two points and it will also touch some other efficiency curve of higher \( \eta \) at one point. Thus, for a unit discharge (or power) the vertical line touches the curve of maximum efficiency at only one point. Now, if these points are joined together by a smooth curve, we obtain the best performance curve for the turbine. By drawing a horizontal line for a given \( N_u \) (at certain \( H \) and \( N \)) which cuts this best performance curve, the point of maximum efficiency is known, corresponding to which \( Q_u \) or \( P_u \) can be obtained and hence \( Q \) and \( P \) can be estimated at which the turbine efficiency is maximum for the given \( H \) and \( N \).

There is a term called ‘runaway speed’ which is the maximum speed of the turbine under no load and no governing action. The hydraulic design is for optimum speed, but it must also satisfy structurally the safety conditions at runaway speed, it is about 1.8 to 2.3 times the optimum speed.
Fig. 10.42 *Constant efficiency and best performance curves (universal characteristics) of a Francis turbine*

10.25 **SELECTION OF TURBINES**

The hydraulic turbine is selected according to the specific conditions under which it has to operate and attain the maximum possible efficiency. The choice depends on the head available, power to be developed and the speed at which it has to run. The following factors basically govern the selection of a suitable type of turbine.

(a) **Operating head** The present practice is to use Kaplan and Propeller type of turbines for heads up to 50 m. For head from 50 to 400 m, Francis turbines are used. For heads greater than 400 m, impulse or Pelton turbines are used. The range of heads as mentioned is not rigid and may change if other conditions dominate to achieve economy.

(b) **Specific speed** It is better to choose turbines of high specific speeds. High speed turbines mean small sizes of turbines, generators, power house, etc. and are therefore, more economical. The range of specific speeds of the turbines should correspond to the synchronous speed of the generator, \( N = \frac{120f}{p} \), where \( f \) is the frequency and \( p \) the number of poles.

(c) **Height of installation** It is better to install the turbines as high above the tail water level (TWL) as possible. This saves the cost of excavation for the draft tube. Care should be taken to ensure that cavitation does not occur.
(d) **Performance characteristics of turbine**  The performance characteristics of turbines as discussed in Section 10.24 should be studied carefully before recommending the type of turbine to be used. A turbine has the maximum efficiency at a certain load. When a turbine has to operate mostly at part loads, only those turbines whose efficiencies do not fall appreciably with part loads should be selected. Kaplan and Pelton turbines are better than Francis and propeller turbines in this respect.

(e) **Size of turbine**  It is better to go in for as large a size of turbine as possible since this results in economy of size of the power house, the number of penstocks, the generator, etc. Bigger size means less number of runners. However, the number of runners should not be less than two so that at least one unit is always available for service in the case of a plant breakdown.

**Example 10.1**  A Pelton wheel driven by two similar jets transmits 4000 kW to the shaft when running at 400 rpm. The head from the reservoir level to the nozzle is 200 m and the efficiency of power transmission through the pipelines and nozzles is 90 per cent. The jets are tangential to a 1.50 m diameter circle. The relative velocity decreases by 10 per cent as the water traverses the buckets, which are so shaped that they would, if stationary, deflect the jet by 165°. Neglecting windage losses, estimate (a) the efficiency of the runner and (b) the diameter of each jet.

**Solution**

Velocity of fluid at inlet to the bucket is,

\[
V_1 = \sqrt{2gh_0 k_v} = \sqrt{2 \times 9.81 \times 200 \times 0.9} = 59.43 \text{ m/s}
\]

\[
V_b = \frac{\pi DN}{60} = \frac{\pi \times 1.5 \times 400}{60} = 31.42 \text{ m/s}
\]

\[
\eta = \frac{(1-k \cos \theta) (V_1 - V_b) V_b}{g \times V_1^2/2g} = \frac{2(1-k \cos \theta) (V_1 - V_b) V_b}{V_1^2}
\]

\[
= \frac{2(1-0.9 \cos 165^\circ)(59.43-31.42)31.42}{(59.43)^2} = 0.9312 \text{ or } 93.12\%
\]

*Ans. (a)*

Power developed, is

\[
P = \frac{4000}{0.9312} = 4295.53 \text{ kW}
\]

Power developed per jet is,

\[
= \frac{4295.53}{2} = 2147.77 \text{ kW}
\]

\[
2147.77 = (\rho A_1 V_1) \frac{V_1^2}{2} = \rho \frac{\pi d^2}{4} \frac{V_1^3}{2}
\]
\[ = 1000 \times \frac{\pi}{8} d^2 \times (59.43)^3 \times 10^{-3} \]

\[ \therefore \quad d = \sqrt[3]{\frac{2147.77 \times 8}{\pi \times (59.43)^3}} = 0.1614 \text{ m} \]

= diameter of each jet \hspace{1cm} \text{Ans. (b)}

**Example 10.2**

A Pelton wheel has to be designed for the following specifications:
Power to be developed = 6000 kW. Net head available = 300 m. Speed = 550 rpm. Ratio of jet diameter to wheel diameter = 1/10. Hydraulic efficiency = 0.85. Assuming the velocity coefficient \( C_v = 0.98 \) and speed ratio \( f = 0.46 \), find (a) the number of jets (b) diameter of each jet (c) diameter of the wheel and (d) the quantity of water required.

**Solution**

\[ V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 300} = 75.19 \text{ m/s} \]

\[ V_b = 0.46\sqrt{2 \times 9.81 \times 300} = 35.29 \text{ m/s} \]

\[ \eta_0 = \frac{P}{\rho Q g H} = \frac{6000 \times 10^3}{1000 \times \rho \times 9.81 \times 300} = 0.85 \]

\[ \therefore \quad Q = \frac{20}{9.81 \times 0.85} = 2.4 \text{ m}^3/\text{s} \hspace{1cm} \text{Ans. (d)} \]

\[ V_b = 35.29 \text{ m/s} = \frac{\pi D \times 550}{60} \]

\[ D = \text{diameter of the wheel} \]

\[ = \frac{35.29 \times 6}{\pi \times 55} = 1.23 \text{ m} \hspace{1cm} \text{Ans. (c)} \]

\[ \frac{d}{D} = \frac{1}{10} \]

\[ \therefore \quad d = \text{diameter of each jet} = 0.123 \text{ m} \hspace{1cm} \text{Ans. (b)} \]

Number of jet's \[ = \frac{Q}{V_1 \times \frac{\pi}{4} d^2} \]

\[ = \frac{2.4 \times 4}{75.19 \times \pi \times (0.12)^2} \]

\[ = 2.822, \text{ i.e. } 3 \text{ jets} \hspace{1cm} \text{Ans. (a)} \]
Example 10.3  A single jet impulse turbine of 10 MW capacity is to work under a head of 500 m. If the specific speed of the turbine is 10, the overall efficiency is 80 per cent and the coefficient of velocity is 0.98, find the diameters of the jet and the bucket wheel. Assume the speed of the bucket wheel as 0.46 of the velocity of jet.

**Solution**

\[ N_s = \frac{N\sqrt{P}}{H^{5/4}} \]

\[ N = \frac{N_s H^{5/4}}{\sqrt{P}} = \frac{10 \times (500)^{5/4}}{\sqrt{10000}} \]

\[ = 236.4 \text{ rpm} \]

Velocity of jet, \( V = C_v \sqrt{2gH} \)

\[ = 0.98 \sqrt{2 \times 9.81 \times 500} \]

\[ = 97.06 \text{ m/s} \]

Speed of bucket wheel, \( V_b = 0.46 \times 97.06 \)

\[ = 44.65 \text{ m/s} \]

\[ V_b = \frac{\pi DN}{60} = 44.65 \]

\[ \therefore \]

\[ D = \frac{60 \times 44.65}{\pi \times 236.4} = 3.61 \text{ m} \]

Ans.

\[ \eta_0 = \frac{P}{\rho QgH} = \frac{P}{\rho \times \frac{\pi}{4} d^2 \times V \times gH} \]

\[ 0.80 = \frac{10000 \times 10^3}{1000 \times \frac{\pi}{4} d^2 \times 97.06 \times 9.81 \times 500} \]

\[ d^2 = \frac{80}{0.8 \times \pi \times 97.06 \times 9.81} = 0.0334 \text{ m}^2 \]

\[ \therefore \] Diameter of jet, \( d = 0.183 \text{ m} \)

Ans.

Example 10.4  Show that the specific speed of a single jet Pelton wheel is about 202 \((d/D)\) where \(d\) and \(D\) represent the jet and bucket wheel diameters respectively. Take \(C_v = 0.97, f = 0.45\) and \(h = 0.85\).

**Solution**

\[ Q = aV = n \frac{\pi}{4} d^2 C_v \sqrt{2gH} \]

where \(n = \text{number of jets}\).
\[ P = \rho Q g H \eta = \rho \eta \frac{\pi}{4} d^2 C_v \sqrt{2gH} \cdot g H \eta \times 10^{-3} \] where \( P \) is in kW.

The peripheral velocity \( V_b \) of the Pelton wheel of diameter \( D \), is

\[ V_b = \frac{\pi DN}{60} = \phi \sqrt{2gH}, \text{ where } \phi \text{ is the speed ratio.} \]

\[ N = \frac{60 \phi \sqrt{2gH}}{\pi D} \]

\[ N_s = N \frac{\sqrt{P}}{H^{5/4}} \]

or,

\[ N_s = \frac{60}{\pi} \cdot \phi \sqrt{2g} \cdot \frac{d}{D} \sqrt{\rho n \frac{\pi}{4} d^2 C_v \sqrt{2gH} \cdot g H \eta \times 10^{-3}} \]

Substituting \( \phi = 0.45, C_v = 0.97, h = 0.85, \rho = 1000 \text{ kg/m}^3 \), we get

\[ N_s = \frac{60}{\pi} \times 0.45 \sqrt{2g} \cdot \frac{d}{D} \sqrt{n \frac{\pi}{4} \times 0.97 \sqrt{2g} \times g \times 0.85} \]

\[ = 38.07 \frac{d}{D} \cdot 5.30 \cdot \sqrt{n} \]

\[ = 202 \sqrt{n} \frac{d}{D} \]

For a single jet turbine, \( n = 1 \). Thus the specific speed is given by

\[ N_s = 202 \frac{d}{D}. \text{ Hence proved.} \]

**Example 10.5** Four jets each of 60 mm diameter strike the buckets of an impulse wheel and each gets deflected by an angle of 165°. The speed of the bucket wheel is 45 m/s. Find the velocity of the jet for maximum efficiency, power developed and the hydraulic efficiency. Assume that the bucket moves linearly.

**Solution**

For maximum efficiency, the jet velocity is,

\[ V_1 = 2 \ V_b = 2 \times 45 = 90 \text{ m/s} \quad \text{Ans.} \]

Flow through the jet,

\[ Q = \frac{\pi}{4} d^2 \times V_1 \]
\[ \frac{\pi}{4} \times (0.06)^2 \times 90 \]
\[ = 0.2545 \text{ m}^3/\text{s} \]

Power developed, \( P \), is given by Eq. (10.14), i.e. \[ \frac{1-k \cos \theta}{g} \cdot \frac{V_1^2}{4} \text{ w.g.} \]

Taking friction coefficient \( k \) to be unity,
\[ P = (1 - \cos 165^\circ) \cdot \frac{90}{4} \times 0.2545 \times 1000 \times 10^{-3} \text{ kW} = 1013 \text{ kW} \]

For four jets
\[ P = 4 \times 4052 = 16208 \text{ kW} \quad \text{Ans.} \]

The maximum efficiency is given by Eq. (10.17), i.e.
\[ (\eta_D)_{\text{max}} = \frac{1-k \cos \theta}{2} \]
\[ = \frac{1-\cos165^\circ}{2} = 0.983 \text{ or } 98.3\% \quad \text{Ans.} \]

**Example 10.6** The peripheral velocity of the wheel of an inward flow reaction turbine is 20 m/s. The velocity of whirl of the inflowing water is 17 m/s and the radial velocity of flow is 2 m/s. If the flow is 0.7 m\(^3\)/s and the hydraulic efficiency is 80 per cent, find the head on the wheel, the power generated by the turbine and the angles of the vanes. Assume radial discharge.

**Solution**

The velocity triangles for the moving vanes are shown in Fig. E10.6. Since the discharge is radial, \( Vw_2 \) is zero.

\[ \therefore \text{Hydraulic efficiency, } \eta_h = \frac{Vw_1 \cdot V_b}{gH} \]

\[ 0.8 = \frac{17 \times 20}{9.81 \times H} \]

\[ \therefore \text{Head on the wheel, } H = 43.3 \text{ m} \quad \text{Ans.} \]

Power generated is,
\[ P = \rho Q g H \eta_h = 10^3 \times 0.7 \times 9.81 \times 43.3 \times 0.8 \times 10^{-3} \]
\[ = 238 \text{ kW} \]

Exit angle of guide vanes = \( \alpha_1 \).

Now,
\[ \alpha_1 = \tan^{-1} \left( \frac{V_{f_1}}{V_{w_1}} \right) = \tan^{-1} \left( \frac{2}{17} \right) \]
\[ = 180 - 6.71 = 173.29^\circ \quad \text{Ans.} \]
Inlet blade angle = \( \beta_1 \)

\[
\beta_1 = \tan^{-1} \left( \frac{V_{f1}}{V_{b1} - V_{w1}} \right) = \tan^{-1} \left( \frac{2}{20 - 17} \right) = \tan^{-1} 0.6667 = 33.7^\circ \\ \text{Ans.}
\]

**Example 10.7**  A runner of a Francis turbine having 1.50 m outer diameter and 0.75 m inner diameter operates under a head of 150 m with a specific speed of 120 and generates 14 MW. If the water enters the wheel at angle of 11°20′ and leaves the blade radially with no velocity of whirl, what will be the inlet and outlet blade angles? Assume the hydraulic efficiency to be 92 per cent.

**Solution**

\[ N_s = \frac{N \sqrt{P}}{H^{5/4}} \]

\[ 120 = \frac{N \sqrt{14000}}{(150)^{5/4}} \]

\[ N = 532 \text{ rpm} \]

\[ V_{b1} = \frac{\pi DN}{60} = \frac{\pi \times 1.5 \times 532}{60} = 41.76 \text{ m/s} \]

\[ \eta_h = 0.92 = \frac{V_{w1}V_{b1} - V_{w2}V_{b2}}{gH} \]

\[ = \frac{V_{w1}V_{b1}}{gH}, \text{ since } V_{w2} = 0 \]
\[ V_{w1} = \frac{0.92 \times 9.81 \times 150}{41.76} = 32.42 \text{ m/s} \]

\[ \tan \alpha_1 = \tan 11^\circ20' = 0.2 = \frac{V_{f1}}{V_{w1}} \]

\[ \therefore \quad V_{f1} = 0.2 \times 32.42 = 6.49 \text{ m/s} = V_{r2} \]

\[ \tan \beta_1 = \frac{V_{f1}}{V_{b1} - V_{w1}} = \frac{6.49}{41.76 - 32.42} = 0.6955 \]

\[ \therefore \quad \beta_1 = 34^\circ49' \quad \text{Ans.} \]

Since the inner diameter is half the outer diameter,

\[ V_{b2} = \frac{V_{b1}}{2} = \frac{41.76}{2} = 20.88 \text{ m/s} \]

\[ \tan \beta_2 = \frac{V_{r3}}{V_{b2} - V_{w2}} = \frac{6.49}{20.88 - 0} = 0.3108 \]

\[ \therefore \quad \beta_2 = 17.27 = 17^\circ16' \]

**Example 10.8**  The following data relate to a Francis turbine: Net head = 70 m; Speed = 700 rpm; Overall efficiency = 85 per cent; Shaft power = 350 kW; Hydraulic efficiency = 92 per cent; Flow ratio, \( V_1 \sqrt{2gH} = 0.22 \); Breadth ratio, \( B/D = 0.1 \); Outer diameter of the runner = 2 \times \) inner diameter of runner; Velocity of flow, \( V_1 = \) constant; Outlet discharge = radial. The thickness of vanes occupies 6 per cent of circumferential area of the runner. Determine (a) the guide vane angle (b) the runner vane angles at inlet and outlet (c) the diameters of the runner at inlet and outlet; (d) the width of the wheel at inlet.

**Solution**

The velocity triangles are shown in Fig. E10.8.
Flow ratio, \( \frac{V_{f1}}{\sqrt{2gH}} = 0.22 \)

\[ V_{f1} = 0.22 \sqrt{2 \times 9.81 \times 76} = 8.153 \text{ m/s} \]

\[ = V_{f2} \]

\[ A_b = 0.94 \pi D_1 B_1 \]

Since discharge at outlet is radial,

\[ V_{w2} = 0, \ V_{f2} = V_2 = 8.153 \text{ m/s} \]

Now \( \eta_0 = \frac{\text{shaft power}}{\text{water power}} = \frac{350 \text{ kW}}{1000 \times Q \times 9.81 \times 70 \times 10^{-3}} \)

\[ \therefore Q = \frac{350}{9.81 \times 70 \times 0.85} = 0.5996 \text{ m}^3/\text{s} \equiv 0.6 \text{ m}^3/\text{s} \]

\[ Q = 0.94 \pi D_1 B_1 \times V_{f1} \]

\[ 0.6 = 0.94 \pi D_1 \times 0.1 D_1 \times 8.153 \]

\[ \therefore D_1 = 0.4992 \text{ m} \equiv 0.5 \text{ m} \] \( \text{Ans. (c)} \)

\[ \therefore B_1 = 0.1 \times 0.5 = 0.05 \text{ m} = 5 \text{ m} \] \( \text{Ans. (d)} \)

\[ D_2 = \frac{0.5}{2} = 0.25 \text{ m} \] \( \text{Ans. (c)} \)

\[ V_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.5 \times 700}{60} = 18.33 \text{ m/s} \]

\[ \eta_h = \frac{V_{w1} V_{b1}}{2} = \frac{V_{w1} \times 18.33}{9.81 \times 70} = 0.92 \]

\[ \therefore V_{w1} = 34.47 \text{ m/s} \]

\[ \tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{8.153}{34.47} = 0.2365 \]

\[ \therefore \alpha = 13.3^\circ = 13^\circ 18' \] \( \text{Ans. (a)} \)

\[ \tan \beta_1 = \frac{V_{f1}}{V_{w1} - V_{b1}} = \frac{8.153}{34.47 - 18.33} = 0.5051 \]

\[ \therefore \beta_1 = 26.8^\circ = 26^\circ 148' \] \( \text{Ans. (b)} \)

\[ \tan \beta_2 = \frac{V_f}{V_{b2}} \]

\[ \frac{V_{b1}}{r_1} = \frac{V_{b2}}{r_2} = \omega \]

\[ \therefore V_{b2} = \frac{D_2}{D_1} \frac{V_{b1}}{2} = \frac{18.33}{2} \]

\[ = 9.17 \text{ m/s} \]
\[ V_b = \frac{\pi DN}{60} = 45.41 \]

\[ \therefore \text{Pitch circle diameter of the wheel, } D \text{ is,} \]
\[ D = \frac{45.41 \times 60}{\pi \times 300} = 2.89 \text{ m} \]
\[ \text{Ans. (d)} \]

\[ N_s = \frac{N\sqrt{P}}{H^{5/4}} = 300\sqrt{260.000}{(475)^{5/4}} = 68.98 \]
\[ \text{Ans. (e)} \]

Jet ratio,
\[ \frac{D}{d} = \frac{2.89}{0.227} = 12.73 \]

\[ \therefore \text{Number of buckets} = \frac{D}{2d} + 15 = \frac{12.73}{2} + 15 = 21.37 \text{ or } 22 \]
\[ \text{Ans. (f)} \]

Work done per kg is
\[ E = \frac{(V_1 - V_{bl})(1 - k \cos \theta)V_{bl}}{g} \]
\[ = \frac{(94.6 - 45.41)(1 - 0.98 \cos 165^\circ) \times 45.41}{9.81} \]
\[ = 443.24 \text{ kg-m/kg} \]
\[ \text{Ans. (g)} \]

\[ \eta_h = \eta_{head} \times \eta_{dis} = \frac{443.24}{475} \times 0.9975 \]
\[ = 0.93 \text{ or } 93\% \]
\[ \text{Ans. (h)} \]

**Example 10.10** Water is supplied to an axial flow turbine under a gross head of 35 m. The mean diameter of the runner is 2 m and it rotates at 145 rpm. Water leaves the guide vanes at 30° to the direction of the runner rotation and at mean radius the angle of the runner blade at outlet is 28°. If 7 per cent of the gross head is lost in the casing and guide vanes, and the relative velocity is reduced by 8 per cent due to friction in the runner, determine the blade angle at inlet and the hydraulic efficiency of the turbine.

**Solution**

Net heat \[ H = 0.93 \times 35 \]
\[ = 32.6 \text{ m} \]

\[ \therefore V_1 = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 32.6} \]
\[ = 25.3 \text{ m/s} \]

\[ V_b = \frac{\pi DN}{60} = \frac{\pi \times 2 \times 145}{60} = 15.2 \text{ m/s} \]

With Reference to Fig. E10.10,
\[ V_{rl} \sin \beta_1 = V_1 \sin \alpha \]
\[ V_{rl} \cos \beta_1 = V_1 \cos \alpha - V_b \]
\[ \tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{25.3 \sin 30^\circ}{25.3 \cos 30^\circ - 15.2} = 1.8852 \]

\[ \therefore \quad \beta_1 = 62.1^\circ \]

\[ V_{r1} = \frac{25.3 \sin 30^\circ}{\sin 62.1^\circ} = 14.31 \text{ m/s} \]

\[ V_{r2} = 0.92 \times 14.31 = 13.17 \text{ m/s} \]

\[ V_{w1} = V_1 \cos 30^\circ = 21.91 \text{ m/s} \]

\[ V_{w2} = V_b - V_{r2} \cos 28^\circ = 15.2 - 13.17 \times 0.883 = 3.57 \text{ m/s} \]

\[ E = \frac{V_b(V_{w1} - V_{w2})}{g} = \frac{15.2(21.91 - 3.57)}{9.81} = 28.42 \text{ m} \]

\[ \therefore \quad \eta_h = \frac{28.42}{35} = 0.812 \quad \text{or} \quad 81.2% \quad \text{Ans.} \]

**Example 10.11**  A Kaplan turbine develops 10000 kW under a head of 12 m when the following conditions prevail. Speed ratio = 2, flow ratio = 0.65, diameter of hub = 0.3 times the external diameter of the vane and the overall efficiency = 94 per cent. Estimate (a) the speed (b) the diameter of the runner and (c) the specific speed.

**Solution**

\[ P = \rho Q g H \eta \times 10^{-3} \]
10000 = Q \times 9.81 \times 12 \times 0.94
Q = 90.37 \text{ m}^3/\text{s}

Flow ratio,

\phi' = \frac{V_{r1}}{\sqrt{2gH}} = 0.65

V_{r1} = 0.65 \sqrt{2 \times 9.81 \times 12} = 9.97 \text{ m/s}

Area of flow,

A_b = \frac{90.37}{9.97} = 9.064 \text{ m}^2

A_b = \frac{\pi}{4} \left( D^2 - d_h^2 \right) = 9.064

D^2 - (0.3D)^2 = \frac{9.064 \times 4}{\pi} = 11.54

D^2 = \frac{11.54}{0.91} = 12.682

Runner diameter, D = 3.56 \text{ m} \quad \text{Ans. (b)}

Speed ratio

\phi = 2 = \frac{V_b}{\sqrt{2gH}} = \frac{V_b}{\sqrt{19.62 \times 12}}

V_b = 30.69 \text{ m/s} = \frac{\pi DN}{60}

N = \frac{30.69 \times 60}{\pi \times 3.56} = 164.6 \quad \text{or} \quad 165 \text{ rpm}

Synchronous speed,

N = \frac{120f}{p}

If we take \frac{120f}{p} = 165, then p = \frac{120 \times 50}{165} = 36.36

Let 36 poles or 18 pairs of poles are taken.

Then

N = \frac{120 \times 50}{36} = 166.7 \text{ rpm} \quad \text{Ans. (a)}

Specific speed

N_s = \frac{N\sqrt{P}}{H^{5/4}}

= \frac{166.7\sqrt{10000}}{25^{5/4}} = 746 \quad \text{Ans. (i)}
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\[
\frac{180\sqrt{30000}}{H_p^{5/4}} = 210
\]

\[
H_p^{5/4} = \frac{6\sqrt{30000}}{7} \therefore H_p = 54.61 \text{ m}
\]

Substituting,

\[
\frac{D_p}{D_m} = \frac{285}{180} \frac{\sqrt{54.61}}{4.5} = 5.516 \quad \text{Ans.}
\]

Flow through the turbine

\[
Q_p = \frac{P_p}{\rho g H_p \eta} = \frac{30000 \times 1000}{100 \times 9.81 \times 54.61 \times 0.88}
\]

\[
= 63.6 \text{ m}^3/\text{s} \quad \text{Ans.}
\]

Example 10.15 Tests conducted on a one-fifth scale model of a Francis turbine under a head of 1.5 m indicated that it could develop 5 kW power at 450 rpm. Determine the speed and power of a full sized turbine while working under a head of 30 m.

Solution

\[
\frac{D_m}{D_p} = \frac{N_p}{N_m} \sqrt{\frac{H_m}{H_p}}
\]

\[
1 = \frac{N_p}{N_m} \sqrt{\frac{1.5}{30}}
\]

\[
N_p = 90\sqrt{20} = 402 \text{ rpm} \quad \text{Ans.}
\]

\[
N_s = \frac{N_m \sqrt{P_m}}{H_m^{5/4}} = \frac{450\sqrt{5}}{(1.5)^{5/4}} = 606.16
\]

Again,

\[
N_s = \frac{N_p \sqrt{P_p}}{H_p^{5/4}} = 606.16 = \frac{402 \sqrt{P_p}}{30^{5/4}} = 606.16
\]

\[
\therefore P_p = \text{Power of the full sized turbine} = 11208 \text{ kW} \quad \text{Ans.}
\]

Example 10.16 A turbine works under a head of 19 m and has a maximum flow rate of 3 m\(^3\)/s and a speed of 600 rpm. If it has to work in another plant under a head of 5 m, at what speed must the turbine run in order to attain approximately the same efficiency and what will be the maximum flow rate?

Solution

\[
\frac{D_m}{D_p} = \frac{\sqrt{H_m}}{H_p} \cdot \frac{N_p}{N_m}
\]

Since the same turbine is used in both places, the diameter is the same.
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Kaplan turbines \[ 820 = \frac{250\sqrt{P}}{30^{5/4}} \]

\[ P = 53033.57 \text{ kW} \]

\[ \therefore \text{ Number of turbines} = \frac{89614.35}{53033.57} = 1.69 \]

i.e. \[ 2 \text{ turbines} \quad \text{Ans. (b)} \]

**Example 10.18** The following data refers to a proposed hydroelectric power plant:

Available head = 27 m, Catchment area 430 sq. km, Rainfall = 150 cm/year,
Percentage of total rainfall utilized = 65%, Penstock efficiency = 95%,
Turbine efficiency = 80%, Generator efficiency = 86% and Load factor = 0.45.

(a) Calculate the power developed.

(b) Suggest suitable turbines for the plant.

**Solution**

Quantity of water available per year

\[ = (430 \times 10^6) \text{ m}^2 \times 1.50 \text{ m} \times 0.65 \]

\[ = 419.25 \times 10^6 \text{ m}^3 \]

Quantity of water available per second

\[ Q = \frac{419.25 \times 10^6}{365 \times 24 \times 3600} = 13.29 \text{ m}^3 \]

Power developed

\[ P = h_p \times h_i \times \eta_G \rho QgH \]

\[ = 0.95 \times 0.8 \times 0.86 \times 1000 \times 13.29 \times 9.81 \times 27 \times 10^{-3} \]

\[ = 2300 \text{ kW} \quad \text{Ans. (a)} \]

Load factor = \[ \frac{\text{Average load}}{\text{Peak load}} = 0.45 \]

\[ \therefore \text{ Peak load capacity} = \frac{2300}{0.45} = 5111 \text{ kW} \]

If two machines of equal capacity are provided,

Capacity of each unit = \[ \frac{5111}{2 \times 0.86} = 2971.5 \text{ kW} \]

As the available head is low, Kaplan turbines are suggested. Two such turbines, each of 3000 kW capacity, may be installed.

**Example 10.19** The run off data of a river at a particular site is tabulated in Table E10.19(a).
<table>
<thead>
<tr>
<th>Month</th>
<th>Mean discharge (millions of cu.m.)</th>
<th>Month</th>
<th>Mean discharge (millions of cu.m.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>30</td>
<td>July</td>
<td>80</td>
</tr>
<tr>
<td>February</td>
<td>25</td>
<td>August</td>
<td>100</td>
</tr>
<tr>
<td>March</td>
<td>20</td>
<td>September</td>
<td>110</td>
</tr>
<tr>
<td>April</td>
<td>0</td>
<td>October</td>
<td>65</td>
</tr>
<tr>
<td>May</td>
<td>10</td>
<td>November</td>
<td>45</td>
</tr>
<tr>
<td>June</td>
<td>50</td>
<td>December</td>
<td>30</td>
</tr>
</tbody>
</table>

(a) Draw the hydrograph and find the mean flow.
(b) Draw the flow duration curve.
(c) Find the power developed if the head available is 90 m and the overall efficiency of generation is 86 per cent. Assume each month of 30 days.

**Solution**

The hydrograph of the given data is shown in Fig. E10.19 (a).

![Fig. E10.19 (a) Hydrograph](image)

The mean discharge

\[
\frac{30 + 25 + 20 + 0 + 10 + 50 + 80 + 100 + 110 + 65 + 45 + 30}{12} = \frac{565}{12} = 47.08 \text{ million m}^3/\text{s}
\]

To obtain the flow duration curve, it is necessary to find the lengths of time during which certain flows are available as given in Table E10.19 (b).
<table>
<thead>
<tr>
<th>Discharge per month (million m³)</th>
<th>Total number of months during which flow is available</th>
<th>Percentage time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>91.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>83.3</td>
</tr>
<tr>
<td>25</td>
<td>9</td>
<td>75.0</td>
</tr>
<tr>
<td>30</td>
<td>8</td>
<td>66.7</td>
</tr>
<tr>
<td>40</td>
<td>6</td>
<td>50.0</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td>41.7</td>
</tr>
<tr>
<td>60</td>
<td>4</td>
<td>33.3</td>
</tr>
<tr>
<td>70</td>
<td>3</td>
<td>25.0</td>
</tr>
<tr>
<td>80</td>
<td>3</td>
<td>25.0</td>
</tr>
<tr>
<td>90</td>
<td>3</td>
<td>25.0</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>16.7</td>
</tr>
<tr>
<td>110</td>
<td>1</td>
<td>8.3</td>
</tr>
</tbody>
</table>

The flow duration curve from the above data shown in Fig. E10.19 (b).

**Fig. E10.19 (b)  Flow Duration Curve**
Power developed,

\[ P = rQgH \eta_0 \times 10^{-3} \text{kW} \]

\[ = \frac{47.08 \times 10^6 \times 9.81 \times 90 \times 0.86}{30 \times 24 \times 3600} \]

\[ = 13.79 \text{ MW} \]

**SHORT-ANSWER QUESTIONS**

10.1 What are the two key parameters of water on which the magnitude of hydro-power depends?

10.2 Enlist the advantages and disadvantages of water power.

10.3 Explain the optimization of hydro-thermal mix in meeting the power demand of a certain region.

10.4 Discuss the factors which should be considered while selecting a site for a hydroelectric plant.

10.5 What do you understand by hydrology? Explain the hydrological cycle.

10.6 What do you mean by (i) hydrograph (ii) Flow duration curve and (iii) power duration curve? What is primary power and secondary power?

10.7 What is a mass curve? What does the slope of the curve at a point indicate?

10.8 Explain what you mean by storage and pondage. Why are they required?

10.9 State the essential elements of a hydroelectric power plant.

10.10 What is a catchment area? Why is a reservoir required?

10.11 State the functions of a dam. How are dams classified? Briefly describe a few important types of dams. How would you select the site and the type of the dam?

10.12 What is a spillway? Why are spillways required? What are the different types of spillways?

10.13 Explain the following terms

(i) Headrace
(ii) Tailrace
(iii) Canal
(iv) Flume
(v) Tunnel
(vi) Pipeline
(vii) Penstock

10.14 What is a surge tank? Why is it important in a hydro-plant?

10.15 What is the function of a draft tube? Briefly explain the different types of draft tubes?

10.16 Enlist the various equipment provided in a powerhouse.

10.17 Explain the different methods of classifying a hydroelectric power plant. What is a run off river plant?

10.18 Explain with a neat sketch a pumped storage plant. What are its advantages?

10.19 What are mini and micro-hydel plants? Why are they important these days?
10.20 How does a hydraulic turbine convert energy? What is a waterwheel?
10.21 Describe the classification of hydraulic turbines in different categories.
10.22 How is the size of turbine ascertained?
10.23 Classify the hydro-turbines according to head, power, size and specific speed.
10.24 What type of turbine would you recommend for the following heads and why?
   (a) 1000 m
   (b) 150 m
   (c) 20 m
10.25 Explain with a neat sketch the principle of operation of a Pelton turbine.
10.26 What are the functions of (a) spear and (b) deflector plate in a Pelton wheel?
10.27 Deduce the ratio between the peripheral velocity of the runner and the velocity of the jet for attaining (a) maximum efficiency and (b) maximum power.
10.28 Explain the following terms.
   (i) Jet ratio
   (ii) Speed ratio
10.29 How are the number of jets in a Pelton wheel ascertained?
10.30 “The number of buckets in a Pelton wheel is a function of the jet ratio”. Explain.
10.31 How is the degree of reaction, $R$, of a hydraulic turbine defined? Explain the cases for $R = 0$, $R = 0.50$ and $R = 1$.
10.32 Explain with a neat schematic diagram the operation of a Francis turbine. What are its advantages?
10.33 Draw the velocity diagrams of an inward-flow Francis turbine and derive the expression of blading efficiency in terms of vane angles.
10.34 What are Kaplan turbines? How is a Kaplan turbine different from a propeller turbine? Explain the characteristic features of a Kaplan turbine.
10.35 What is a Deriaiz turbine? What is its importance?
10.36 What is a bulb turbine? Where is it used?
10.37 Define specific speed of a turbine. Derive its expression in terms of speed, power and head.
10.38 What is scale ratio? What is its importance?
10.39 Define unit speed, unit power and unit discharge and derive their relevant relations.
10.40 What do you understand by cavitation? What are its effects? How can it be minimized?
10.41 Write short notes on the following.
   (i) pitting of turbine blades, and its prevention  (ii) servo-motors.
10.42 What is the synchronous speed of the turbine runner? How is it estimated?
10.43 Explain with a neat sketch the governing principle of an impulse turbine. What are the functions of needle valve and the deflector?
10.44 How is the governing of a reaction turbine carried out? Explain with a neat sketch. What is the function of relief valve?
10.45 When and why are surge tanks and forebays provided? Explain a few types of surge tanks.

10.46 Discuss with neat sketches the characteristic curves related to the performance of hydraulic turbine.

10.47 What is "runaway speed"? How does it affect the turbine design?

10.48 How is the type of turbine selected in a certain hydro-plant? Discuss the effects of head, specific speed, height of installation, the operating characteristics and the capacity on the selection process.

PROBLEMS

10.1 A Pelton wheel is required to develop 4500 kW at 400 rpm operating under an available head of 360 m. There are two equal jets and the bucket angle is 170°. The bucket pitch circle diameter is 1.82 m. Taking $k$ for the buckets as 0.85, determine (a) the efficiency of the runner and (b) the diameter of each jet.

*Ans.* (a) 0.9106 (b) 0.103 m

10.2 In a Pelton wheel the diameter of the bucket circle is 2 m and the deflecting angle of the bucket is 162°. The jet has 165 mm diameter, the pressure behind the nozzle is 700 kPa and the wheel rotates at 320 rpm. Neglecting friction, find the power developed by the wheel and the hydraulic efficiency.

*Ans.* 351.9 kW, 0.616

10.3 A Pelton wheel develops 8 MW under a net head of 130 m at a speed of 200 rpm. Assuming $c_v = 0.98$, hydraulic efficiency = 87 per cent, speed ratio = 0.46 and the ratio of jet-to-wheel diameter = 1/9, determine (a) the flow required (b) the diameter of the wheel (c) the diameter and number of jets needed.

*Ans.* (a) 7.51 m³/s, (b) 2.17 m, (c) 0.242 m, 3.

10.4 A Pelton wheel has to develop 12 MW under a head of 300 m at a speed of 500 rpm. If the diameter of the jet is not to exceed 1/9 of the wheel diameter, estimate the number and diameter of the jets, diameter of the bucket wheel and the quantity of flow. Assume overall efficiency = 88 per cent, $C_v = 0.97$ and $\phi = 0.45$.

*Ans.* $D = 1.32$ m, $d = 0.147$ m, $n = 4$ and $Q = 4.63$ m³/s

10.5 A Pelton wheel to be designed is to run at 300 rpm under an effective head of 150 m. The ratio of the nozzle diameter to the pitch circle diameter is 1/12. Assuming efficiency = 84%, $C_v = 0.98$ and speed ratio = 0.45, determine (a) the diameter of the wheel, (b) diameter of the jet (c) the quantity of water flow (d) the minimum number of buckets required; and (e) the power developed.

*Ans.* (a) 1.55 m (b) 0.129 m (c) 0.694 m³/s (d) 21 (e) 858 kW.

10.6 A jet of 75 mm diameter strikes the bucket of an impulse wheel and gets deflected by an angle of 165°. The speed of the bucket is 45.5 m/s. Find the velocity of the jet for maximum efficiency and the power developed.

*Ans.* 91 m/s, 1545 kW.
10.7 An inward flow reaction turbine having an overall efficiency of 75 per cent delivers 132 kW. The head $H$ is 9 m, velocity of the periphery of the wheel is 54 m/s and the radial velocity is 18 m/s. The wheel makes 120 rpm. The hydraulic losses in the turbine are 20 per cent of the available energy. Determine the discharge at the inlet, the guide blade angle, the wheel blade angle and the diameter and width of the wheel. Assume radial discharge.

*Ans.* $2 \text{ m}^3/\text{s}$, $a_1 = 56^\circ 49'\; \beta_1 = 23^\circ$, $D = 2.86 \text{ m}$ $B = 0.037 \text{ m}$.

10.8 In a Francis turbine of low specific speed, the velocity of flow from inlet to exit of the runner remains constant. If the turbine discharges radially, show that the degree of reaction $R$ can be expressed as

$$R = \frac{1}{2} - \frac{1}{2} \left[ \frac{\cot \beta_1}{\cot \alpha - \cos \beta_1} \right]$$

where $\alpha$ and $\beta_1$ are the guide and runner vane angle respectively and the degree of reaction is equal to the ratio of pressure drop to the hydraulic work done in the runner, assuming that the losses in the runner and negligible.

10.9 A Francis turbine with an overall efficiency of 76 per cent is required to produce 180 kW. It is working under a head of 8 m. The peripheral velocity is 0.25 (2 gH)$^{1/2}$ and the radial velocity of flow is 0.95 (2 gH)$^{1/2}$. The wheel runs at 150 rpm and the hydraulic losses in the turbine are 20 per cent of the available energy. Assuming radial discharge, determine (a) the guide vane angle (b) the wheel vane angle at inlet (c) the diameter of the wheel at inlet; and (d) the width of the wheel at inlet.

*Ans.* (a) $30^\circ 45'$, (b) $35^\circ 12'$, (c) 0.398 m, (d) 0.203 m

10.10 An inward flow reaction turbine works under a head of 22.5 m. The external and internal diameters of the runner are 1.35 m and 1 m respectively. The angle of guide vanes is $15^\circ$ and the moving vanes are radial at inlet. Radial velocity of flow through the runner is constant and there is no velocity of whirl at outlet. Determine the speed of the runner in rpm and the angle of vane at outlet. If the turbine develops 375 kW, find the specific speed. Neglect friction losses.

*Ans.* 206.5 rpm, $19^\circ 53'$, 81.6

10.11 Two inward flow reaction turbines have the same runner diameter of 0.60 m and the same efficiency. They work under the same head and they have the same velocity of flow of 6 m/s. One of the runners $A$ revolves at 520 rpm and has an inlet vane angle of $65^\circ$. If the other runner $B$ has an inlet vane angle of $110^\circ$, at what speed should it run?

*Ans.* 600 rpm

10.12 Water enters an inward flow turbine at an angle of $22^\circ$ to the tangent to the outer rim and leaves the turbine radially. If the speed of the wheel is 300 rpm and the velocity of flow is constant at 3 m/s, find the necessary angles of blades when the inner and outer diameters of the turbine are 0.3 m and 0.6 m respectively. If the width of the wheel at inlet is 0.15 m, calculate the power developed. Neglect the thickness of blades.

*Ans.* $\beta_1 = 59^\circ 54'$, $\beta_2 = 32^\circ 32'$, $P = 61.15$ kW.
size if (a) Francis turbines having specific speed not greater than 200 or 
(b) Kaplan turbines of specific speed not greater than 600, are used.

Ans. (a) 16, (b) 3.

10.21 A run off of 30 m³/s is available at 7.5 m head for generating the desired 
power. The turbine efficiency is 85 per cent. (a) Is it feasible to develop the 
desired power by two turbines with 50 rpm and the specific speed of turbine 
not greater than 450? (b) What type of runner is required to be used? 
(c) What is the diameter of the runner if the speed ratio is 0.85?

Ans. (a) $\text{As/}N_s = 203$, two turbine units can be used. (b) Francis turbine, (c) 3.93 m.

10.22 From the following table of mean monthly discharge, draw the following 
curves.

<table>
<thead>
<tr>
<th>Month</th>
<th>Discharge (m³/s)</th>
<th>Month</th>
<th>Discharge (m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>100</td>
<td>July</td>
<td>1100</td>
</tr>
<tr>
<td>February</td>
<td>325</td>
<td>August</td>
<td>1300</td>
</tr>
<tr>
<td>March</td>
<td>400</td>
<td>September</td>
<td>1000</td>
</tr>
<tr>
<td>April</td>
<td>700</td>
<td>October</td>
<td>800</td>
</tr>
<tr>
<td>May</td>
<td>850</td>
<td>November</td>
<td>600</td>
</tr>
<tr>
<td>June</td>
<td>900</td>
<td>December</td>
<td>300</td>
</tr>
</tbody>
</table>

(a) the hydrograph (b) the flow duration curve.

10.23 From the following table of the mean monthly discharge for 12 months of a 
river at a site, draw (a) the hydrograph and find the average monthly flow, 
and the power available at mean flow of water for head 90 m and overall 
efficiency of generation 90 per cent. Assume 30 days in each month.

<table>
<thead>
<tr>
<th>Month</th>
<th>April</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>August</th>
<th>September</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q - m³ \times 10^6$</td>
<td>500</td>
<td>200</td>
<td>1500</td>
<td>2500</td>
<td>3000</td>
<td>2400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q - m³ \times 10^6$</td>
<td>2000</td>
<td>1500</td>
<td>1500</td>
<td>1000</td>
<td>800</td>
<td>600</td>
</tr>
</tbody>
</table>

(b) Draw the flow duration curve from the data in the hydrograph.

10.24 The following data pertain to a hydroelectric plant. Available head = 140 m, 
catchment area = 2000 sq. km; annual average rainfall = 145 cm, turbine 
efficiency = 85%, generator efficiency = 90%, percolation and evaporation 
losses = 16%. Determine the power developed and suggest the type of turbine 
to be used if the runner speed is to be kept below 240 rpm.

Ans. 8.11 MW, Pelton turbine with 4 jets may be used.

10.25 (a) Discuss the differences between Kaplan, Francis and Pelton turbines and 
state the types of power plants they are suitable for.

(b) At a particular hydroelectric power plant site the discharge of water is 
400 m³/s and the head is 25 m. The turbine efficiency is 88 per cent. The 
generator is directly coupled to the turbine having frequency of generation 
as 50 cycles/s and number of poles as 24. Calculate the least number of 
turbines required if (a) a Francis turbine is used with a specific speed of 300 
(b) a Kaplan turbine with a specific speed of 750 is used.
Diesel electric plants in the range of 2 to 50 MW capacity are used as central stations for small supply authorities and works and they are universally adapted to supplement hydroelectric or thermal power stations where standby generating plants are essential for starting from cold or under emergency conditions.

11.1 APPLICATIONS OF DIESEL ENGINES IN POWER FIELD

The diesel electric power plants are chiefly used in the following field.

(a) **Peak load plant**  Diesel plants can be used in combination with thermal or hydro-plants as peak load units. They can be easily started or stopped at a short notice to meet the peak demand.

(b) **Mobile plant**  Diesel plants mounted on trailers can be used for temporary or emergency purposes such as for supplying power to large civil engineering works.

(c) **Standby unit**  If the main unit fails or cannot cope up with the demand, a diesel plant can supply the necessary power. For example, if water available in a hydro-plant is not adequately available due to less rainfall, the diesel station can operate in parallel to generate the short fall in power.

(d) **Emergency plant**  During power interruption in a vital unit like a key industrial plant or a hospital, a diesel electric plant can be used to generate the needed power.

(e) **Nursery station**  In the absence of main grid, a diesel plant can be installed to supply power in a small town. In course of time, when electricity from the main grid becomes available in the town, the diesel unit can be shifted to some other area which needs power on a small scale. Such a diesel plant is called a “nursery station”.

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(f) Starting stations  Diesel units can be used to run the auxiliaries (like FD and ID fans, BFP, etc.) for starting a large steam power plant.

(g) Central stations  Diesel electric plants can be used as central station where the capacity required is small.

11.2 ADVANTAGES AND DISADVANTAGES OF DIESEL ENGINE POWER PLANT

Following are the advantages of diesel electric stations.

1. It is easy to design and install these electric stations.
2. They are easily available in standard capacities.
3. They can respond to load changes without much difficulty.
4. There are less standby losses.
5. They occupy less space.
6. They can be started and stopped quickly.
7. They require less cooling water.
8. Capital cost is less.
9. Less operating and supervising staff required.
10. High efficiency of energy conversion from fuel to electricity.
11. Efficiency at part loads is also higher.
12. Less of civil engineering work is required.
13. They can be located near the load centre.
14. There is no ash handling problem.
15. Easier lubrication system.

Following are some of the disadvantages in installing diesel units for power generation.

1. High operating cost.
2. High maintenance and lubrication cost.
3. Capacity is restricted. Cannot be of very big size.
5. Cannot supply overload.
6. Unhygienic emissions.

11.3 TYPES OF DIESEL PLANTS

In a diesel engine, air is first compressed to a high pressure and a small volume (volumetric compression ratio varying between 13 and 22) at which the hot air temperature is more than the self ignition temperature of the fuel oil, which is sprayed into the compressed air in fine atomized form. The combustion products expand doing work on the piston till the exhaust valve opens. Exhaust of the products then takes place, at the end of which fresh air is again taken into the cylinder and the cycle repeats itself. There is no spark plug. Fuel oil spray burns in the hot compressed air. Hence, a diesel engine is also called a compression ignition or a C.I. engine.
Diesel engines can be four-stroke and two-stroke, horizontal and vertical, single-cylinder and multi-cylinder, naturally aspirated and supercharged. A textbook on internal combustion engines (like Obert, Lichty, Taylor, Rogowski, Mathur and Sharma, Ganesan, etc. as given in the bibliography at the end of the chapter can be consulted for a good understanding of analysis and operation of diesel engines. Here only some of the salient features of diesel engines are being discussed.

### 11.4 GENERAL LAYOUT

The cross-section of an air cooled IC engine with principal parts is shown in Fig. 11.1. The essential components of a diesel electric plant are shown in Fig. 11.2. It consists of the following elements.

![Diagram of Diesel Engine Components]

**Fig. 11.1 General View of an air-cooled IC engine**
Fig. 11.2 Schematic arrangement of a diesel engine power plant
1. **Engine** It is the main component of the plant and is directly coupled to the generator.

2. **Air intake system** It conveys fresh air through louvres and air filter that removes dirt, etc. causing wear of the engine. Supercharger, if fitted, is generally driven by the engine itself and it augments the power output of the engine.

3. **Exhaust system** It discharges the engine exhaust to the atmosphere. The exhaust manifold connects the engine cylinder exhaust outlets to the exhaust pipe which is provided with a muffler or silencer to reduce pressure on the exhaust line and eliminate most of the noise which may result if gases are discharged directly to the atmosphere. The exhaust pipe should have flexible tubing system to take up the effects of expansion due to high temperature and also isolate the exhaust system from the engine vibration (Fig. 11.3).

![Diesel engine exhaust system](image)

**Fig. 11.3** Diesel engine exhaust system

There is scope of waste heat utilization from the diesel engine exhaust by installing a waste heat boiler to raise low pressure steam which can be used for any process, purpose or for generating electricity. The hot exhaust may also be utilized to heat water in a gas-to-water heat exchanger which can be in the form of a water coil installed in the exhaust muffler. It can also be used for air heating where the exhaust pipe is surrounded by the cold air jacket.

4. **Fuel system** Fuel oil may be delivered at the plant site by trucks, railway wagons or barges and oil tankers. An unloading facility delivers oil to the main storage tanks from where oil is pumped to small service storage tanks known as engine day tanks, which store oil for approximately eight hours of operation (Fig. 11.4). Coils heated by hot water or steam reduce oil viscosity to reduce pumping power.
Fig. 11.4 Fuel storage in a diesel engine power plant

The fuel injection system is the heart of a diesel engine. Engines driving electric generators have lower speeds and simple combustion chambers that promote good mixing of fuel and air. The fuel injection system performs the following functions.

(a) Filter the fuel
(b) Meter the correct quantity of the fuel to be injected
(c) Time the injection process
(d) Regulate the fuel supply
(e) Secure fine atomization of fuel oil
(f) Distribute the atomized fuel properly in the combustion chamber.

Oil is atomized either by air blast or pressure jet. Early diesel engines used air blast fuel atomization where compressed air at about 70 bar was used to atomize as well as to inject the fuel oil. For this an air compressor and a storage tank are needed, which becomes expensive. In pressure jet atomization the fuel oil is forced to flow through spray nozzles at a pressure above 100 bar. It is known as solid injection, which is more common. Solid injection systems may be classified as follows.

(a) Common rail injection system
(b) Individual pump injection system
(c) Distributor system

(a) Common rail injection system A single pump supplies fuel under high pressure to a fuel header or common rail (Fig. 11.5). The high pressure in the header forces the fuel to each of the nozzles located in the cylinders. At the proper time a mechanically operated valve (by means of a push
rod and a rocker arm) allows the fuel to enter the cylinder through the nozzle. The amount of fuel entering the cylinder is regulated by varying the length of the push rod stroke.

![Diagram of a common-rail injection system]

**Fig. 11.5** Common-rail injection system

(b) **Individual pump injection system** Each cylinder is provided with one pump and one injector (Fig. 11.6). The pump directly feeds oil to the cylinder, meters the oil and controls the injection timing. The nozzle contains a delivery valve actuated by fuel oil pressure.

(c) **Distributor system** In this system the fuel is metered at a central point. A pump pressurises, meters the fuel and times the injection (Fig. 11.7). The fuel is then distributed to cylinders in correct firing order by cam-operated poppet valves which open to admit oil to the fuel nozzle.

**Fuel pump** It consists of a plunger \( (L) \) driven by a cam and tappet mechanism, which reciprocates inside a barrel \( B \) (Fig. 11.8). A vertical groove in the plunger leads to a helical groove. The delivery valve \( (V) \) lifts off its seat under oil pressure against the spring force \( (S) \). When the plunger is at the bottom, the supply port \( Y \) and the spill port \( (SP) \) are uncovered and low pressure filtered oil is forced into the barrel. As the plunger moves up, the ports \( Y \) and \( SP \) are closed and oil gets compressed lifting the delivery valve
Fig. 11.6 Individual pump injection system

Fig. 11.7 Distributor fuel injection system

$V$ to enter the injector nozzle through the passage $P$. When the plunger moves up further, the port $SP$ gets connected to the fuel at its top through the vertical groove resulting in a sudden drop in pressure and the delivery valve falls back to its seat against the spring force. The plunger is rotated by the rack $R$ operated by a governor. By rotating the plunger the position of the helical groove relative to the supply port $Y$ can be varied. The length of stroke during
which the oil is delivered is varied and so the quantity of fuel delivered to the engine also varies accordingly.
Fuel injector  It consists of a nozzle valve (NV) fitted in the nozzle body (NB) (Fig. 11.9). The nozzle valve is held on its seat by a spring force (S) acting through

![Diagram of Fuel Injector](image)

**Fig. 11.9 Fuel injector**
the spindle ($E$). A feeling pin ($FP$) at the top indicates whether the valve is working properly or not. The high pressure oil from the fuel pump enters the injector through the passages $B$ and $C$ and lifts the nozzle valve to admit oil into the fuel nozzle that injects oil to the cylinder in fine atomized spray. As the oil pressure falls, the nozzle valve comes back to its seat under spring force and the fuel supply is cut off. Any leakage of fuel (due to wearing out of valve) accumulated above the valve returns to the fuel tank through the passage $A$.

Various types of nozzles are used in CI engines, but the most common types are the single-orifice, multi-orifice and pintle nozzles as shown in Fig. 11.10. A pintle nozzle is clogged less by carbon particles and is thus less expensive to maintain.

![Fuel passage](image1)
Orifice
Single orifice
Multi-orifice
Pintle nozzle

**Fig. 11.10 Various types of nozzles**

5. **Cooling system** The temperature of the gases inside the cylinder may be as high as 2750°C. If there is no external cooling, the cylinder walls and piston will tend to assume the average temperature of the gases which may be of the order of 1000° to 1500°C. The cooling of the engine is necessary for the following reasons.

(a) The lubricating oil used determines the maximum engine temperature that can be used. This temperature varies from 160°C to 200°C. Above these temperatures the lubricating oil deteriorates very rapidly and may evaporate and burn damaging the piston and cylinder surfaces. Piston seizure due to overheating may also occur.

(b) The strength of the materials used for various engine parts decreases with increase in temperature. Local thermal stresses can develop due to uneven expansion of various parts, often resulting in cracking.

(c) High engine temperatures may result in very hot exhaust valve, giving rise to pre-ignition and detonation or knocking.

(d) Due to high cylinder head temperature, the volumetric efficiency and hence power output of the engine are reduced.

Following are the two methods of cooling the engine.

(i) Air cooling

(ii) Water cooling
Air cooling is used in small engines, where fins are provided to increase heat transfer surface area.

Big diesel engines are always water cooled. The cylinder and its head are enclosed in a water jacket which is connected to a radiator. Water flowing in the jacket carries away the heat from the engine and becomes heated. The hot water then flows into the radiator and gets cooled by rejecting heat to air from the radiator walls. Cooled water is again circulated in the water jacket.

Various methods used for circulating the water around the cylinder are the following.

(a) **Thermosiphon cooling** In this method water flow is caused by density difference (Fig. 11.11). The rate of circulation is however slow and insufficient.

![Thermosiphon cooling](image)

**Fig. 11.11** Thermosiphon cooling

(b) **Forced cooling by pump** In this method a pump, taking power from the engine, forces water to circulate, ensuring engine cooling under all operating conditions. There may be overcooling which may cause low temperature corrosion of metal parts due to the presence of acids.

(c) **Thermostat cooling** This is a method in which a thermostat maintains the desired temperature and protects the engine from getting overcooled (Fig. 11.12).

(d) **Pressurized water cooling** In this method a higher water pressure, 1.5 to 2 bar, is maintained to increase heat transfer in the radiator. A pressure relief valve is provided against any pressure drop or vacuum.

(e) **Evaporative cooling** In this method water is allowed to evaporate absorbing the latent heat of evaporation from the cylinder walls. The cooling circuit is such that the coolant is always liquid and the steam flashes in a separate vessel (Fig. 11.13).
Fig. 11.12  *Thermostat cooling*

Fig. 11.13  *Evaporative cooling*
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Fig. 11.15 Full pressure lubrication system

Fig. 11.16 Gear oil pump with relief valve

Fig. 11.17 Full flow dry sump lubrication
Fig. 11.18  By-pass type wet sump lubrication

7. Starting of engine  Following are the three common methods of starting an engine.

(i) By an auxiliary engine, which is mounted close to the main engine and drives the latter through a clutch and gears.

(ii) By using an electric motor, in which a storage battery of 12 to 36 volts is used to supply power to an electric motor that derives the engine.

(iii) By compressed air system, in which compressed air at about 17 bar supplied from an air tank is admitted to a few engine cylinders making them work like reciprocating air motors to run the engine shaft. Fuel is admitted to the remaining cylinders and ignited in the normal way causing the engine to start. The compressed air system is commonly used for starting large diesel engines employed for stationary power plant service.

11.5 COMBUSTION IN A CI ENGINE

In a CI engine combustion of fuel occurs due to the high temperature produced by the compression of air and hence it is an auto-ignition engine. For this, a minimum compression ratio of 12 is required. The efficiency of the cycle increases with higher values of compression ratio, but the maximum pressure reached in the cylinder also increases. This requires heavier construction. The upper limit of compression ratio is a compromise between high efficiency and
low weight and cost. The normal compression ratios are in the range of 14 to 17, but they may be up to 23. The air fuel ratios used in CI engines lie between 18 and 25 as against about 15 in the SI engine. So, for same power CI engines are bigger and heavier than SI engines.

In a CI engine the intake is air alone and the fuel is injected at high pressure in the form of fine droplets near the end of compression. This leads to the delay period, as explained below.

Due to the practical limitations caused by smoke at engine exhaust (smoke limit), CI engines are operated at air fuel ratios higher than the stoichiometric requirement. Due to shortcomings of distribution and limited intermixing of fuel with air within the combustion chamber, CI engines always operate with excess air (unlike SI engines).

In the SI engine there is an ignition delay which occurs between the time the spark is produced and the time when the “actual burning” phase of combustion commences. In a CI engine, the fuel does not ignite immediately upon injection into the combustion chamber (CC). There is a certain period of apparent inactivity between the time when the first droplet of fuel hits the hot air in the CC and the time when it starts through the “actual burning” phase. This period is known as “ignition delay” or “ignition lag”. During this period there is no pressure rise within the cylinder. This delay is indicated on the pressure-time diagram (Fig. 11.19), as the distance between points “a” and “b”. Point “a”

**Fig. 11.19** Pressure-time diagram illustrating ignition delay in a CI engine
represents the time of injection and point "b" represents the time at which the pressure curve, caused by combustion, first separates from the compression pressure (non-firing or motoring) curve. The ignition delay period is divided into two parts—(a) the mixing period, which is the time required for atomization and evaporation of the fuel, and physical mixing with the air and (b) the interaction period, in which molecular interaction prepares the mixture for and initiates, the "actual burning" phase of combustion, which is longer (of the two periods).

Ignition delay has a great influence on combustion rate and on detonation. If the ignition delay is short, there will be smooth operation as shown in Fig. 11.19. However, if the ignition delay is long, fuel droplets accumulate in the CC. When the actual burning commences, there is a very rapid rate of pressure rise, resulting in "jamming" of forces against the piston and rough engine operation. Such a situation produces the extreme pressure differentials and violent gas vibrations known as detonation and evidenced by audible knock. The phenomenon is similar to that in the SI engine. However, in the SI engine detonation occurs near the end of combustion, whereas in the CI engine, detonation occurs near the beginning of combustion (Fig. 11.20).

![Diagrams illustrating knocking combustion in SI and CI engines](image)

**Fig. 11.20**  *Diagrams illustrating knocking combustion in SI and CI engines*

It is necessary to decrease the ignition delay, which varies with the fuel and is measured in terms of cetane number. Ignition delay can be decreased by adding small amounts of certain compounds such as ethyl nitrate, amyl thionitrite and others. The engine variables which affect ignition delay are (a) compression ratio (b) inlet air temperature (c) coolant temperature and (d) engine speed. An increase in compression ratio or inlet air temperature or coolant temperature or engine speed, decreases the ignition delay or knock.
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Fig. 11.21  Diagram illustrating the difference between a high speed and a low speed four-stroke cycle unsupercharged CI engine.

Fig. 11.22  Valve timing diagram for a four-stroke CI engine.
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Brake power \((bp)\) is the output of the engine at the shaft measured by a dynamometer. Absorption dynamometers which are more common can be (a) friction type like prony brake (b) hydraulic and (c) electrical. The brake power is

\[
bp = \frac{2\pi TN}{60} \tag{11.3}
\]

where \(T\) is the torque measured.

Power required to overcome the frictional resistance is the friction power \((fp)\) given by

\[
fp = ip - bp \tag{11.4}
\]

The mechanical efficiency \((\eta_M)\) of the engine is defined as

\[
\eta_M = \frac{bp}{ip} \tag{11.5}
\]

which lies between 80 and 90 per cent.

The \(fp\) is very nearly constant at a given engine speed. If the load is decreased giving lower values of \(bp\), then the variation in \(\eta_M\) with \(bp\) is as shown in Fig. 11.25.

![Fig. 11.25 Variation of mechanical efficiency](image)

The Morse test can be used to measure the \(ip\) of multi-cylinder engines. The engine, say having four cylinders, is run at the required speed and the torque is measured. One cylinder is cut out by disconnecting the injector of a CI engine (or by shorting the spark plug of an SI engine). The speed falls because of the loss of power with one cylinder cut out, but is restored by reducing the load. When the speed has reached the original value, the torque is again measured. It is repeated by cutting out other cylinders one by one. If the values of \(ip\) of the cylinders are denoted by \(I_1, I_2, I_3\) and \(I_4\) and the power losses in each cylinder are denoted by \(L_1, L_2, L_3\) and \(L_4\), then the value of \(bp, B\), at the test speed with all cylinders firing is given by

\[
B = (I_1 - L_1) + (I_2 - L_2) + (I_3 - L_3) + (I_4 - L_4) \tag{i}
\]

If number 1 cylinder is cut out, then the contribution \(I_1\) is lost. If the losses due to that cylinder remain the same as when it was firing, then the \(bp, B_1\), obtained at the same speed is
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Fig. 11.33 Types of positive displacement compressor

(ii) Non-positive displacement type like axial and radial compressors (Fig. 11.34). Turbochargers are superior to superchargers, since the former use the exhaust gas energy during blowdown and the latter consume a part of the engine output.

Fig. 11.34 Types of dynamic or non-positive displacement compressor

Figure 11.35 shows the continuous running performance characteristics of a diesel engine in three modes, i.e. (a) normally aspirated (b) turbocharged and (c) turbocharged with intercooling for a specific engine as given by Eastop and
**Fig. 11.35** Continuous running performance characteristics of a CI engine in three modes: (a) Normally aspirated, (b) Turbocharged and (c) Turbocharged with intercooling

McConkey. It is seen that intercooling of air has a very good influence on the engine performance. Typical performances for a CI engine at full load and part load are shown in Fig. 11.36.

### 11.8 LAYOUT OF A DIESEL ENGINE POWER PLANT

The layout of a diesel engine power plant is shown in Fig. 11.37. Diesel engine units are installed side by side with some room left for extension in the future. The repairs and usual maintenance works require some space around the units. The air intakes and filters and exhaust mufflers are located outside. Adequate space for oil storage, repair shop and office are provided as shown. Bulk storage of oil may be outdoor.
Fig. 11.36 Typical performance for a CI engine at (a) Full load and (b) Part load at different speeds

Example 11.1 A four-stroke CI engine of 3.5 litre capacity develops indicated power on average of 13.1 kW/m³ of free air induced per minute, while running at 3600 rpm and having a volumetric efficiency of 82 per cent, referred to free air conditions of 1.013 bar and 25°C. A blower driven mechanically from the engine is proposed to be installed for supercharging. It works through a pressure ratio of 1.75 and has an isentropic efficiency of 70 per cent. Assume that at the end of the intake...
stroke the cylinders contain a volume of charge equal to the swept volume, at the pressure and temperature of the delivered air from the blower. Taking all mechanical efficiencies to be 80 per cent, estimate the net increase in brake power of the engine due to supercharging.

**Solution**

Engine capacity = 3.5 litres = 0.0035 m$^3$
Swept volume = 3600/2 × 0.0035 = 6.3 m$^3$/min
Unsupercharged induced volume = 0.82 × 6.3 = 5.166 m$^3$/min
Blower delivery pressure = 1.75 × 1.013 = 1.7728 bar

\[
T_{2s}/T_1 = \left[ \frac{p_2}{p_1} \right]^{(\gamma - 1)/\gamma} = (1.75)^{(1.4 - 1)/(1.4)} = 1.173
\]

\[
T_{2s} = 1.173 \times 298 = 349.67 \text{ K}
\]

\[
\frac{T_{2s} - T_1}{T_2 - T_1} = \eta_c = 0.7
\]

\[
T_2 - T_1 = \frac{349.67 - 298}{0.7} = 73.8
\]

\[
T_2 = 371.8 \text{ K} = \text{Blower delivery temperature}
\]

The blower delivers 6.3 m$^3$/min at 1.7728 bar and 371.8 K.
Equivalent volume at 1.013 bar and 25°C

\[
= \frac{6.3 \times 1.7728 \times 298}{1.013 \times 371.8} = 8.837 \text{ m}^3/\text{min}
\]

Increase in induced volume = 8.837 – 5.166 = 3.671 m$^3$/min
Increase in indicated power due to extra air induced

\[
= 13.1 \times 3.671 = 48.1 \text{ kW}
\]
Hidden page
\( p_1 = 0.97 \text{ bar}, \quad T_1 = 303K, \quad p_2 = 2.1 \text{ bar}, \quad A/F = 18, \quad T_3 = 580\text{°C}, \quad p_3 = 1.9 \text{ bar}, \quad p_4 = 1.06 \text{ bar}. \)

If the compression were isentropic (Fig. E11.2).

\[
\frac{T_{2s}}{T_1} = \left[\frac{p_2}{p_1}\right]^{(\gamma - 1)/\gamma}
\]

\[T_{2s} = 303 \left[\frac{2.1}{0.97}\right]^{(1.4 - 1)/0.4} = 377.8 \text{ K}\]

\[
\frac{[T_{2s} - T_1]}{[T_2 - T_1]} = 0.75
\]

\[T_2 - T_1 = [377.8 - 303]/0.75 = 402.73 \text{ K}\]

(a) Exit temperature of air from the compressor

\[T_2 = 129.73 \text{°C} \quad \text{Ans.}\]

If the turbine were isentropic,

\[
\frac{T_3}{T_{4s}} = \left[\frac{p_3}{p_4}\right]^{(\gamma_{\text{ex}} - 1)/\gamma_{\text{ex}}}
\]

\[= \frac{580 + 273}{T_{4s}} = \left[\frac{1.9}{1.06}\right]^{(1.33 - 1)/1.33} = 1.1558\]

\[T_{4s} = 853/1.1558 = 738 \text{ K}\]

\[T_3 - T_4 = (T_3 - T_{4s})\eta_T\]

\[853 - T_4 = (853 - 738) \times 0.85 = 97.75\]

\[T_4 = 755.25 \text{ K}\]

(b) Turbine exhaust temperature

\[T_4 = 482.25 \text{°C} \quad \text{Ans.}\]

Compressor power

\[\dot{W}_C = \dot{m}_a \ c_{pa} (T_2 - T_1) = \dot{m}_a \times 1.01 \ (129.73 - 30) = 100.73 \dot{m}_a \text{ kW}\]

Rate of flow of exhaust gases,

\[\dot{m}_{\text{ex}} = \dot{m}_a + \dot{m}_f = \dot{m}_a \ [1 + F/A] = \dot{m}_a [1 + 1/18] = 1.056 \dot{m}_a\]

Turbine power,

\[\dot{W}_T = \dot{m}_{\text{ex}} \times c_{\text{pex}} \times (T_3 - T_4) = \dot{m}_a \times 1.056 \times 1.15 (853 - 755.25) = 118.7 \dot{m}_a \text{ kW}\]

Thus, the mechanical power loss as a percentage of the power generated in the turbine is

\[
\frac{118.7 - 100.73}{118.7} \times 100 = 15.14\% \quad \text{Ans.}\]
Example 11.3 During a test on a diesel engine used for driving a dc generator, the following observations were made.

The output of the generator was 215 A at 210 V, the efficiency of the generator being 85%. The quantity of fuel supplied to the engine was 11.8 kg/h, the calorific value of fuel being 43 MJ/kg. The air-fuel ratio was 18:1. The exhaust gases were passed through an exhaust gas calorimeter for which the observations were as follows:

Water circulated through the calorimeter = 560 litres/h, Temperature rise of water = 38°C, Temperature of exhaust gases at exit from calorimeter = 97°C, Specific heat of exhaust gases = 1.04 kJ/kgK, Ambient temperature = 30°C. If the heat lost to the jacket cooling water was 32% per cent of the total energy released by combustion, draw up an energy balance sheet of the engine.

Solution

Total power generated

\[ = 215 \times 210 = 45150 \text{ W} = 45.15 \text{ kW} \]

Brake power of the engine

\[ = 45.15/0.85 = 53.12 \text{ kW} \]

Energy supplied to the engine

\[ = (11.8/3600) \times 43000 \]

\[ = 140.94 \text{ kW} \]

Rate of flow of exhaust gases

\[ \dot{m}_g = \dot{m}_f + \dot{m}_a \]

\[ = \dot{m}_f (1 + A/F) = (11.8/3600) \times 19 \]

\[ = 0.0623 \text{ kg/s} \]

Heat carried away by exhaust gases

\[ = 0.0623 \times 1.04 \times (97 - 30) + (560/3600) \times 4.187 \times 38 \]

\[ = 4.341 + 24.75 = 29.091 \text{ kW} \]

Heat lost to jacket cooling water = 0.32 \times 140.94 = 45.1 \text{ kW}.

Table E11.3 Energy Balance Sheet

<table>
<thead>
<tr>
<th>Heat supplied to the engine 140.94 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
</tr>
<tr>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
Example 11.4  Following are the observations made for a 20 minute trial of a two-stroke diesel engine.

Net brake load = 680 N, mep = 3.0 bar, N = 360 rpm, Fuel consumption = 1.56 kg, Cooling water = 160 kg, Water inlet temperature = 32°C, Water outlet temperature = 57°C, Air used/kg fuel = 30 kg, Room temperature = 27°C, Exhaust gas temperature = 310°C, Cylinder dimensions = 210 mm bore x 290 mm stroke, Brake diameter = 1m, Calorific value of fuel = 44 MJ/kg, Steam formed per kg fuel in the exhaust = 1.3 kg, specific heat of steam in exhaust = 2.093 kJ/kgK, Specific heat of dry exhaust gases = 1.01 kJ/kgK.

Calculate the indicated power and the brake power and make an energy balance of the engine.

Solution
For a two-stroke engine,

\[
	p_i = \frac{p_i L A N}{60} = \frac{3.0 \times 100 \times 0.29 \times \frac{\pi}{4} \times (0.21)^2 \times 360 \times 1}{60} = 18.08 \text{ kW} \quad \text{Ans.}
\]

\[
\eta_M = \frac{b_p}{p_i} = 0.7089 \text{ or } 70.89\%
\]

Heat supplied during the trial

\[
= 1.56 \times 44,000 = 68,640 \text{ kJ}
\]

Energy equivalent of \( p_i \) in trial period

\[
= 18.08 \frac{\text{kJ}}{s} \times 20 \times 60 = 21,696 \text{ kJ}
\]

Energy carried away by cooling water

\[
= 160 \times 4.187 \times (57 - 32) = 16748 \text{ kJ}
\]

Total mass of exhaust gas

\[
= 1.56 \times 30 = 46.8 \text{ kg}
\]

Mass of steam formed = 1.3 \times 1.56 = 2.028 kg

Mass of dry exhaust gas = 46.8 - 2.028 = 44.772 kg

Energy carried away by dry exhaust gases

\[
= 44.772 \times 1.01 \times (310 - 27) = 12,797 \text{ kJ}
\]

Energy carried away by steam

\[
= 2.028 \left[4.187 (100 - 27) + 2257.9 + 2.093 (310 - 100)\right] = 2.028 (305.65 + 2257.9 + 439.53) = 6090 \text{ kJ}
\]

Total energy carried away by exhaust gases = 12,797 + 6090 = 18887 kJ
Table E11.4 Energy Balance Sheet (for trial period)

<table>
<thead>
<tr>
<th>Energy released by combustion of fuel</th>
<th>68,640 kJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy equivalent of ip</td>
<td>21,696 kJ</td>
</tr>
<tr>
<td>Energy carried away by cooling water</td>
<td>16,748 kJ</td>
</tr>
<tr>
<td>Energy carried away by exhaust gases</td>
<td>18,887 kJ</td>
</tr>
<tr>
<td>Unaccounted for energy loss</td>
<td>11,309 kJ</td>
</tr>
<tr>
<td>(by difference)</td>
<td>68,640 kJ</td>
</tr>
</tbody>
</table>

11.9 GAS TURBINE POWER PLANT

The economics of power generation by gas turbines is now quite attractive due to its low capital cost and its high reliability and flexibility in operation. Another outstanding feature is its capability of quick starting and using a wide variety of fuels from natural gas to residual oil or powdered coal. Due to better materials being made available and with the use of adequate blade cooling, the inlet gas temperature to gas turbine (GT) blades can now exceed 1200°C, as a result of which the overall efficiency of a GT plant can be about 35 per cent, almost the same as that of a conventional steam power plant.

Because of its low weight per unit power, gas turbine is exclusively used to drive aviation systems of all kinds of aircraft. It is also being increasingly used in land vehicles like buses and trucks and also to drive locomotives and marine ships. In oil and gas industries, the gas turbine is widely employed to drive auxiliaries like compressors, blowers and pumps.

11.9.1 Closed Cycle and Open Cycle Plants

The essential components of a gas turbine (GT) power plant are the compressor, combustion chamber (CC) and the turbine. The air standard cycle of a GT plant is the Brayton cycle.

A GT plant can either be open or closed. Figure 11.38 shows the arrangement of an open cycle plant which is more common, where the combustion products after doing work in the turbine are exhausted to atmosphere. In a closed cycle plant (Fig. 11.39), the working fluid (air, helium, argon, carbon dioxide, etc.) is externally heated (by burning fuel or by nuclear reactor) and cooled and it operates in a closed cycle.

The specific advantages and disadvantages of a GT plant for a utility system have been discussed in Section 3.6. Following is given a more detailed account in this regard.
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\[ \eta_{\text{cycle}} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} = 1 - \frac{T_1}{T_{4s}} r_p^{(\gamma-1)/\gamma} \] (11.13)

As \( r_p \) increases, \( \eta_{\text{cycle}} \) increases till Carnot cycle is reached (Fig. 11.41).

![Graph showing \( \eta_{\text{cycle}} \) vs \( r_p \)](image)

**Fig. 11.41 Variation of cycle efficiency with \( r_p \)**

\[ (r_p)_{\text{max}} = \left(\frac{T_3}{T_4}\right)^{1/2} = \left(\frac{T_{\text{max}}}{T_{\text{min}}}\right)^{1/2} \] (11.14)

There is a particular value of \( r_p \) when \( W_{\text{net}} \), i.e. \( W_T - W_C \) becomes maximum (Fig. 11.42).

\[ W_{\text{net}} = Q_1 - Q_2 = W_T - W_C = m_a c_p [T_3 - T_{2s} - T_{4s} + T_1] W_{\text{net}} \] (11.15)

Substituting \( T_{2s} \) and \( T_{4s} \) in terms of \( r_p \) (Eq. 11.12) and since \( T_3 (= T_{\text{max}}) \) and \( T_1 (= T_{\text{min}}) \) are fixed, on differentiation of \( W_{\text{net}} \) with respect to \( r_p \) and making \( dW_{\text{net}}/dr_p \) equal to zero, we get

\[ (r_p)_{\text{opt}} = \left(\frac{T_{\text{max}}}{T_{\text{min}}}\right)^{\gamma/(\gamma-1)} \] (11.16)

![Graph showing \( W_{\text{net}} \) vs \( r_p \)](image)

**Fig. 11.42 Variation of net cycle work with \( r_p \)**

\[ (r_p)_{\text{opt}} = [(r_p)_{\text{max}}]^{1/2} \] (11.17)

On substitution in Eq. (11.14),

\[ W_{\text{net}} = m_a c_p \left[ (T_{\text{max}})^{1/2} - (T_{\text{min}})^{1/2} \right]^2 \] (11.18)
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**Effect of intercooling** By staging the compression process (1-2 and 3-4) with perfect intercooling (2-3), the cycle efficiency decreases, as shown in Fig. 11.44, where the small cycle 1-2-3-4-4'-1 is added to the basic cycle 1-4'-5-6-1, without intercooling. However, it permits more heat recovery from hot gases exiting the turbine at state 6 by heating air leaving the compressor at state 4. For minimum work of compression, the intercooler pressure \( p_i = \left[ p_1 p_2 \right]^{1/2} \), where \( p_1 \) and \( p_2 \) are suction and discharge pressures respectively.

![Figure 11.44](image)

**Effect of reheating** Similarly, by staging the heat supply process with a combustor and a reheater, the cycle efficiency decreases, but it permits more heat recovery from the turbine exhaust gases (Fig. 11.45) (since \( T_6 > T_4 \)), with the result that reheating along with regeneration may bring about an improvement in cycle efficiency. It can be shown that the optimum reheate pressure for maximum work is

\[
p_r = \left[ p_1 p_2 \right]^{1/2}
\]

(11.24)

![Figure 11.45](image)
Figure 11.46 (a) and (b) shows the flow and $T$–$s$ diagrams of a GT plant with intercooling, reheating and regeneration.

\[ W_{net} = W_T - W_C \]
\[ = (\dot{m}_a + \dot{m}_f) c_{pg} (T_3 - T_4) - \dot{m}_a c_{pa} (T_2 - T_1) \]  
\[(11.25)\]

and the heat supply is
\[ Q_1 = \dot{m}_f \times CV \]

Therefore, the overall plant efficiency,
\[ \eta_0 = \frac{W_{net}}{\dot{m}_f \times CV} \]  
\[(11.26)\]
11.9.5 Semi-closed Cycle Gas Turbine Plant

The advantages of the open cycle plant, viz. quick and easy starting and the closed cycle plant, viz. constant efficiency at all loads and higher unit rating permitting the use of higher back pressure, are combined in a semi-closed cycle gas turbine power plant. Here, part of the compressed air is heated by the gases exiting the combustion chamber (CC) and then expanded in an air turbine which drives the compressor, thus operating in a closed cycle. The remaining air is used in the CC to burn fuel, and the combustion products after heating the air expand in a gas turbine to drive the generator before exhausting to the atmosphere (Fig. 11.47a). Figure 11.47 (b) shows a combined combustion chamber and a heat exchanger, where hot gases of combustion leave to expand in the gas turbine in the open cycle and the heated air flows to the air turbine in the closed cycle.

11.9.6 Performance of Gas Turbine Power Plants

The gas turbine plant works under variable load conditions. It is thus necessary to study the effect of load on the cycle efficiency which is directly concerned with the running cost of the plant.
Fig. 11.47 (a) Semi-closed cycle gas turbine plant  
(b) Combined combustion chamber and air heater

It is necessary to study the effect of pressure ratio on the thermal efficiency, air mass flow and specific fuel consumption with regenerative reheat and intercooled cycle, because smaller mass flow rate for the given output reduces the component sizes and the plant capital costs. Lower fuel consumption reduces the running cost of the plant. Some of these characteristics are represented graphically and also discussed.

(a) Part load efficiency  The part load efficiencies for open cycle, closed cycle and semi-closed cycle are shown in Fig. 11.48. The part load performance of the semi-closed cycle is seen to be the best.

Fig. 11.48  Part load efficiencies of different plants
(b) Fuel consumption  The effect of pressure ratio on the specific fuel consumption (sfc) of an open cycle plant with the degree of regeneration as a parameter is shown in Fig. 11.49. It shows that for each degree of regeneration there is an optimum pressure ratio for minimum sfc.

![Graph showing the effect of regenerator effectiveness on specific fuel consumption.](image)

**Fig. 11.49** Effect of regenerator effectiveness on specific fuel consumption

(c) Air flow rate  The effect of pressure ratio on the air mass flow rate for an open cycle plant with the turbine inlet temperature as a parameter is shown in Fig. 11.50. It indicates optimum pressure ratio for different turbine inlet temperatures requiring minimum air flow rates.

![Graph showing the effect of pressure ratio on air mass flow rate per unit output.](image)

**Fig. 11.50** Effect of pressure ratio on air mass flow per unit output
(d) **Thermal efficiency** The effect of pressure ratio of a simple open cycle plant with turbine inlet temperature as a parameter is shown in Fig. 11.51(a) and with compressor inlet temperature as a parameter in Fig. 11.51 (b).

![Graph of turbine inlet temperature as a parameter](image)

![Graph of compressor inlet temperature as a parameter](image)

**Fig. 11.51** *Effect of pressure ratio on thermal efficiency of a simple open cycle plant with (a) Turbine inlet temperature and (b) Compressor inlet temperature as parameters*

(e) **Regeneration** The effect of regeneration on thermal efficiency of a simple cycle taking pressure ratio and the turbine inlet temperature as parameters is shown in Fig. 11.52 (a) and Fig. 11.52 (b) respectively.
Fig. 11.52 Effect of regeneration on thermal efficiency of simple cycle with (a) Pressure ratio as parameter and (b) Turbine inlet temperature as parameters.
11.10 COMPONENTS OF GAS TURBINE PLANT

The construction and operation of the components of a gas turbine plant are necessary for proper understanding and design.

(a) Compressor The high flow rates of air through the turbine and the relatively moderate pressure ratios necessitate the use of rotary compressors. The types of compressor commonly used are the following.

1. Centrifugal compressors
2. Axial flow compressors

A centrifugal compressor consists of an impeller with a series of curved radial vanes as shown in Fig. 11.53. Air is sucked in near the hub, called the impeller eye and is whirled round at high speed by the vanes on the impeller rotating at high rpm. The static pressure of air increases from the eye to the tip of the impeller. Air leaving the impeller tip flows through diffuser passages (scroll) which convert the kinetic energy to pressure energy (Fig. 11.54). The compressors may have single inlet or double inlet. In a double inlet impeller having an eye on either side, air is drawn in on both sides (Fig. 11.55). The impeller is subjected to approximately equal forces in the axial direction. About half the pressure rise occurs in the impeller vanes and half in the diffuser passages.

![Diagram of a centrifugal compressor impeller](image)

Fig. 11.53 Centrifugal compressor impeller

If the air flow into the impeller eye is in the axial direction (Fig. 11.53), the blade velocity diagram at inlet is shown in Fig. 11.56 (a). By using fixed guide blades, the inlet velocity to the impeller eye is inclined at an angle, known as pre-whirl (Fig. 11.56 (b)).

At exit from the impeller the flow is in the radial direction and the blade velocity $V_{b2}$ is larger, since the radius of the impeller is larger at outlet. The blade velocity diagram is shown in Fig. 11.57 (a) being the case of radially inclined blades and (b) being that of blades inclined backwards at an angle $\beta_2$. 

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**Fig. 11.54**  *Centrifugal compressor showing discharge scroll*

The inertia of the air trapped between the impeller blades, however, causes the actual whirl velocity $V_{w_2}'$ to be less than $V_{w_2}$. It is known as *slip*.

Slip factor \( \frac{V_{w_2}'}{V_{w_2}} = \frac{V_{w_2}'}{V_{w_2}}(V_{b_2} - V_{f_2} \cot b_2) \)  \hspace{1cm} (11.27)

Power input \( \dot{m}(V_{b_2} V_{w_2}' - V_{b_1} V_{w_1}) \)  \hspace{1cm} (11.28)

**Fig. 11.55**  *Double-sided impeller of a centrifugal compressor*
For low pressure ratios (less than 4/1) the centrifugal compressor is lighter and is able to operate effectively over a wider range of mass flows at any speed. Using titanium alloys pressure ratios above eight have now been achieved.

For larger units with higher pressure ratios the axial-flow compressor is more efficient and is usually preferred. For industrial and large marine gas turbine plants axial compressors are normally used, although some units may employ two or more centrifugal compressors with intercooling between stages. Centrifugal compressors are cheaper to produce, more robust and have a wider operating range than the axial-flow type.

An axial-flow compressor is similar to an axial-flow turbine with a succession of moving blades on the rotor shaft and fixed blades arranged around the stator (casing). Air flows axially through the moving and fixed blades, with diffuser passages throughout which continuously increase the pressure and decrease the velocity. Stationary guide vanes are provided at entry to the first row of moving blades (Fig. 11.58). The work input to the rotor shaft is transferred by the moving blades to the air, thus accelerating it. The spaces between the blades as well as the stator blades from diffusing passages decreasing velocity and increasing pressure. There can be a large number of stages (5 to 14) with a constant work input per stage.
An equal temperature rise in the moving and fixed blades is usually maintained. The axial velocity of air is also kept constant throughout the compressor. A diffusing flow is less stable than a converging flow as in a turbine and for this reason the blade shape and profile are more important for a compressor than for a reaction turbine.

Typical blade sections of an axial-flow compressor are shown in Fig. 11.59(a) and the corresponding velocity diagrams in Fig. 11.59(b).

\[
\text{Power input } = \dot{m} V_b \Delta V_w \quad (11.29)
\]
From the geometry of the diagram,
\[ \Delta V_w = V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2 = V_f (\tan \beta_1 - \tan \beta_2) \] (11.30)

Degree of reaction,
\[ R = \frac{\text{Enthalpy rise in rotor}}{\text{Enthalpy rise in the stage}} \]
\[ = \frac{h_1 - h_0}{h_2 - h_0} = \frac{V_{r1}^2 - V_{r2}^2}{2V_b \Delta V_w} \]

By re-arrangement,
\[ R = \frac{V_f^2 (\sec^2 \beta_1 - \sec^2 \beta_2)}{2V_b V_f (\tan \beta_1 - \tan \beta_2)} \]
\[ = \frac{V_f}{2V_b} - (\tan \beta_1 + \tan \beta_2) \] (11.31)

Blades are usually of twisted section designed according to free vortex theory (see Cohen et al.).

Due to nonuniformity of the velocity profile in the blade passages the work that can be put into a given blade passage is less than that given by the ideal diagram. It is taken care of by introducing a work done factor, \( y \), defined as

Work done factor,
\[ y = \frac{\text{Actual power input}}{m V_b \Delta V_w} \]

which is about 0.85 for a compressor stage.

(b) Combustion chamber In an open cycle GT plant combustion may be arranged to take place in one or two large cylindrical can-type combustion chambers (CC) with ducting to convey the hot gases to the turbine. Combustion is initiated by an electric spark and once the fuel starts burning, the flame is required to be stabilized. A pilot or recirculated zone is created in the main flow to establish a stable flame which helps to sustain combustion continuously. The common methods of flame stabilization are by swirl flow and by bluff body.

Figure 11.60 shows a can-type combustor with swirl flow flame stabilization. About 20 per cent of the total air from the compressor is directly fed through a swirler to the burner as primary air, to provide a rich fuel-air mixture in the primary zone, which continuously burns, producing high temperature gases. Air flowing through the swirler produces a vortex motion creating a low pressure zone along the axis of the CC to cause reversal of flow. About 30 per cent of total air is supplied through dilution holes in the secondary zone through the annulus round the flame tube to complete the combustion. The secondary air must be admitted at right points in the CC, otherwise the cold injected air may chill the flame locally thereby reducing the rate of reaction. The secondary air not only helps to complete the combustion process but also helps to cool the flame tube. The remaining 50 per cent of air is mixed with burnt
gases in the tertiary zone to cool the gases down to the temperature suited to the turbine blade materials.

**Fig. 11.60** *Can-type combustor with swirl flow flame stabilizer*

Figure 11.61 shows a can-type combustor with a bluff body stabilizing the flame. The fuel is injected upstream into the air flow and a sheet metal cone and perforated baffle plate ensure the necessary mixing of fuel and air. The low pressure zone created downstream side causes the reversal of flow along the axis of the CC to stabilize the flame. Sufficient turbulence is produced in all three zones of the CC for uniform mixing and good combustion.

**Fig. 11.61** *Can-type combustor with bluff-body flame stabilizer*

The air-fuel ratio in a GT plant varies from 60/1 to 120/1 and the air velocity at entry to the CC is usually not more than 75 m/s. There is a rich and a weak limit of flame stability and the limit is usually taken at flame blowout. Instability of the flame results in rough running with consequent effect on the life of the CC.

Because of the high air-fuel ratio used, the gases entering the HP turbine contain a high percentage of oxygen and therefore if reheating is performed, the additional fuel can be burned satisfactorily in HP turbine exhaust, without needing further air for oxygen.
A term "combustion efficiency" is often used in this regard, which is defined as follows.

\[
\text{Combustion efficiency} = \frac{\text{Theoretical fuel–air ratio for actual temperature rise}}{\text{Actual fuel air ratio for actual temperature rise}}
\]

Theoretical temperature rise depends on the calorific value of the fuel used, the fuel-air ratio and the initial temperature of air. To evaluate the combustion efficiency, the inlet and outlet temperatures and the fuel and air mass flow rates are measured. The fuel used in aircraft gas turbine is a light petroleum distillate or kerosene of gross calorific value of 46.4 MJ/kg. For gas turbines used in power production or in cogeneration plants, the fuel used can be natural gas.

In order to give a comparison of combustion chambers operating under different ambient conditions, a combustion intensity is defined as the following.

\[
\text{Combustion intensity} = \frac{\text{Heat release rate}}{\text{Volume of CC} \times \text{inlet pressure}}
\]

The lower the combustion intensity, the better the design. In aircraft a figure of about 2 kW/(m³ atm) is normal, whereas in large industrial plants it is about 0.2 kW/(m³ atm).

(c) Gas turbines  Like steam turbines, gas turbines are also of the axial-flow type (Fig. 11.62). The basic requirements of the turbines are light weight, high efficiency, reliability in operation and long working life. Large work output can be obtained per stage with high blade speeds when the blades are designed to sustain higher stresses. More stages are always preferred in gas turbine power plants, because it helps to reduce the stresses in the blades and increases the overall life of the turbine. The cooling of gas turbine blades is essential for long life as it is continuously subjected to high temperature gases.

Blade angles of gas turbines follow the axial-flow compressor blading (Fig. 11.59 (a)), where the degree of reaction is not 50 per cent. It is usually assumed for any stage that the absolute velocity at inlet to each stage \( (V_1) \) is equal to the absolute velocity at exit from the moving blades (i.e. \( V_2 \)) and that the same flow velocity \( V_f \) is constant throughout the turbine.

**Fig. 11.62** Typical air-cooled gas turbine blade
The degree of reaction, \( R \), as defined for a steam turbine, is valid for gas turbines also. It is the ratio of the enthalpy drop in the moving blades to the enthalpy drop in the stage. As shown in Fig. 11.59 (a), we have

\[
R = \frac{V_f^2 - V_{f_1}^2}{2V_b \Delta V_w} = \frac{V_f^2 (\sec^2 \beta_2 - \sec^2 \beta_1)}{2V_b (V_f \tan \beta_2 + V_f \tan \beta_1)} \]

\[
= \frac{V_f (\tan^2 \beta_2 - \tan^2 \beta_1)}{2V_b (\tan \beta_2 + \tan \beta_1)} = \frac{V_f}{2V_b} (\tan \beta_2 - \tan \beta_1) \quad (11.32)
\]

Putting \( R = 0.5 \) in Eq. (11.32), we get

\[
V_f (\tan \beta_2 - \tan \beta_1) = V_b
\]

or

\[
V_b + V_f \tan \alpha_2 - V_f \tan \beta_1 = V_b
\]

\[
\alpha_2 = \beta_1
\]

It also follows that \( \alpha_1 = \beta_2 \). The fixed and moving blades have the same cross-section and the diagram is symmetrical.

**d** **Vortex blading** is the name given to the twisted blades which are designed by using three dimensional flow equations with a view to decrease fluid flow losses. A radial equilibrium equation can be derived (see the book of Cohen et al.) and it can be shown that one set of conditions which satisfies this equation is as follows.

(a) Constant axial velocity along the blades, i.e.

\[
V_f = \text{constant}.
\]

(b) Constant specific work over the annulus, i.e.

\[
V_b \Delta V_w = \text{constant}.
\]

(c) Free vortex at entry to the moving blades, i.e. \( V_{w1}r = \text{constant} \), where \( r \) is the blade radius at any point.

Since the specific work output is constant over the annulus, it can be calculated at the mean radius, and multiplied by the mass flow rate it becomes the power for the stage. Since the fluid density varies along the blade height, the density at the mean radius can be used, so that \( \dot{m} = \rho_m V_f A \), where \( A \) is the blade annular area.

(d) **Duct work** The duct work consists of ducts between the compressor and the combustion chamber, combustion chamber to the turbine, and the exhaust duct. The ducts must be sized to minimize the pressure losses, as the loss in pressure directly reduces the capacity of the plant.

Ducts should be supported from the floor to reduce vibration. Expansion joints must be provided to allow for dimensional changes due to temperature variation.

### GAS TURBINE FUELS

Gas turbines are basically designed to operate on petroleum-based fuels like natural gas, kerosene, aviation fuel and residual fuel oil. Other fuels like powdered coal, sewage gas, etc. are also being actively considered.
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All these required properties cannot be obtained in one material. Therefore, the selection of material for each component is a difficult job.

1. **Metals for turbine rotor discs** The turbine rotor disc is subjected to centrifugal and thermal stresses. The thermal stresses (due to temperature gradient) can be reduced by using an alloy of high conductivity.

   The disc hub stresses tend to cause tensile deformation. This can be minimised by using a material of low expansion coefficient. Austenitic steels with 12 to 18 per cent chromium, 8 to 12 per cent nickel and small percentages of tungsten, molybdenum and titanium are used for turbine rotor discs.

   These days the turbine discs are cooled by tapping compressed air from the compressor. Therefore, less expensive materials can be used. Ferritic steels having higher creep strength at low temperature (up to 600°C) can be used for the central portion, whereas austenitic steel is used on the outer surface of the ferritic rotor disc.

2. **Material for turbine rotor blade** Blades are subjected to the highest stresses and temperatures. Most satisfactory materials for blades are the stainless steel alloys and 8-20 nickel chromium alloys, known as Nimonic alloys. These alloys have high resistance to oxidation, scaling and deformation and have good creep and fatigue properties. A uniform coating with ceramics (silicon carbide, silicon nitride, aluminium nitride, etc.) on the blades of nimonic alloys provides better mechanical properties. Blades are cooled by compressed air taken by a bleed from the compressor.

3. **Material for combustion chamber** The gas turbine combustion chamber is generally made of Nimonic 75 alloy. This alloy has an excellent creep resistance, capacity to withstand heavy thermal shocks, and high resistance to oxidation.

4. **Material for compressor** The impeller of centrifugal compressor is subjected to high centrifugal and thermal stresses, the latter being due to the temperature difference between the air inlet and air discharge temperatures. To minimise centrifugal stresses lighter materials like aluminium alloys are used. These alloys suffer from high thermal expansion, for which allowance is provided.

   The axial flow compressor blades are now made of titanium alloys, which are of low density, possess good strength at high temperatures (400-500°C) and are strongly resistant to corrosion. Light weight, good creep strength and fatigue resistance are attractive features of titanium alloys.

11.13 **FREE PISTON ENGINE PLANT**

The free piston engine is usually constructed as an opposed piston, two-stroke diesel cycle with a conventional fuel injection system. It could be used in a gas turbine plant to replace the air compressor and combustion chamber. The operation of such a plant is shown in Fig. 11.63. It comprises five cylinders with two assemblies of pistons that move opposite to each other. A diesel cylinder located at the centre powers the pistons in opposite direction. These pistons are
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11.13.1 Advantages of a Free Piston Engine

1. It is very compact in space and thus has an economical building cost.
2. It has high availability and can run up from cold to full load hardly within 15 minutes.
3. The exhaust gases coming out from the free piston engine are rich in oxygen due to the scavenging air and are at a lower temperature. The plant output can be increased by injecting additional fuel, the combustion of which raises the temperature to a level acceptable to the turbine.
4. The plant can be run at 20–25 per cent overload.
5. It can be built in high capacities.
6. It is heavier than the gas turbine, but lighter than diesel engine.
7. The full load efficiency of the plant is as good as a normal diesel engine.
8. The compressor and the combustor in a gas turbine plant are replaced by the free piston engine which acts as a gasifier converting the fuel and air into a steady supply of hot gas at moderate pressure and temperature (4.5 bar and 500°C). The gas turbine being unburdened with a compressor can develop about three times more of useful power than a conventional gas turbine plant of the same size.

11.13.2 Disadvantages of a Free Piston Engine

1. Lack of design and manufacturing techniques is a deterrent factor for its wide use.
2. There are some problems of starting and controlling the output.
3. There is a problem of synchronization of the free piston engine and the gas turbine.

Example 11.5 The blade velocity at the mean diameter of a gas turbine stage is 360 m/s. The blade angles at inlet and exit are 20° and 52° respectively and the blades at this section are designed to have a degree of reaction of 50 per cent. The mean diameter of the blades is 0.450 m and the mean blade height is 0.08 m. Assuming that the blades are designed according to vortex theory, calculate (a) the flow velocity (b) the blade angles at the tip and the root (c) the degree of reaction at the tip and at the root of the blades.

Solution

Given: \( \beta_1 = 20° = \alpha_2, \beta_2 = 52° = \alpha_1, \ V_{bm} = 360 \text{ m/s}, \ D_m = 0.450 \text{ m} \)
\( h_b = 0.08 \text{ m} \) (Fig. E11.5 (a)).

\[ V_t \tan \beta_2 - V_t \tan \beta_1 = V_{bm} = 360 \]
\[ V_t (\tan 52° - \tan 20°) = 360 \]
\[ V_t = \frac{360}{1.2799 - 0.364} = 393 \text{ m/s} \]  
Ans. (a)
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Fig. 11.5 (b)

\[ V_f \tan \alpha_{1r} = V_{w_0} \]
\[ 393 \tan \alpha_{1r} = 611.76 \]
\[ \alpha_{1r} = 57.28^\circ \]
\[ V_f \tan \alpha_{2r} = \Delta V_{w_r} - V_{w_1r} = 785.79 - 611.76 \]
\[ = 174.03 \]
\[ \alpha_{2r} = 23.88^\circ \]

**Fixed blades (root):** \( \alpha_{1r} = 57.28^\circ, \alpha_{2r} = 23.88^\circ \)  
**Ans. (b)**

For moving blades

\[ V_f \tan \beta_{1r} = V_{w_{1r}} - V_{b_r} = 611.76 - 296 = 315.76 \]
\[ \beta_{1r} = 38.78^\circ \]
\[ V_f \tan \beta_{2r} = V_{b_r} + V_f \tan \alpha_{2r} = 296 + 174.03 \]
\[ = 470.03 \]
\[ \beta_{2r} = 50.1^\circ \]

**Moving blades (root):** \( \beta_{1r} = 38.78^\circ, \beta_{2r} = 50.1^\circ \)  
**Ans. (b)**

\[ V_f \tan \alpha_{1t} = V_{w_{1t}} \]
\[ 393 \tan \alpha_{1t} = 427 \]
\[ \alpha_{1t} = 47.37^\circ \]
\[ V_f \tan \alpha_{2t} = DV_{w_r} - V_{w_{1t}} = 548.57 - 427 \]
\[ = 121.57 \]
\[ \tan \alpha_{2t} = \frac{121.57}{393} \]
\[ \alpha_{2t} = 17.19^\circ \]

**Fixed blades (tip):** \( \alpha_{1t} = 47.37^\circ, \alpha_{2t} = 17.19^\circ \)  
**Ans.**

Similarly for moving blades,

\[ V_f \tan \beta_{1t} = V_{w_{1t}} - V_{b_t} \]
\[ 393 \tan \beta_{1t} = 427 - 424 \]
\[ \beta_{1t} = 0.44^\circ \]
\[ V_f \tan \beta_{2t} = 424 + 121.57 = 545.57 \]
\[ \beta_{2t} = 54.23^\circ \]

**Moving blades (tip)** \( \beta_{1t} = 0.44^\circ, \beta_{2t} = 54.23^\circ \) \( \text{Ans.} \)

**Blade root**

\[ r_r = 0.225 - 0.04 = 0.185 \text{ m} \]
\[ V_{b_t} = 360 \times \frac{0.185}{0.225} = 296 \text{ m/s} \]
\[ V_{w_{im}} r_m = V_{w_{tr}} r_r \]
\[ V_{w_{tr}} = \frac{503 \times 0.225}{0.185} = 611.76 \text{ m/s} \]
\[ \Delta V_{wr} = \frac{\Delta V_{w_{im}} V_{b_m}}{V_{b_t}} = \frac{232593.68}{296} = 7285.79 \text{ m/s} \]

Blade velocity diagram at the blade root is shown in Fig. E11.5 (e).

![Blade velocity diagram](image)

**Fig. E11.5 (e)**

The degree of reaction is given by

\[ R = \frac{V_f (\tan \beta_2 - \tan \beta_1)}{2V_b} \]

At the tip,

\[ R = \frac{393(\tan 54.23^\circ - \tan 0.44^\circ)}{2 \times 424} = 0.64 \text{ or } 64\% \text{ Ans.} \]

At the root,

\[ R = \frac{393(\tan 50.1^\circ - \tan 38.78^\circ)}{2 \times 296} = 0.26 \text{ or } 26\% \text{ Ans.} \]

**Example 11.6** A centrifugal compressor running at 16 000 rpm takes in air at 17°C and compresses it through a pressure ratio of 4 with an isentropic efficiency of 82 per cent. The blades are radially inclined and the slip factor is 0.85. Guide vanes at inlet give the air an angle of pre-whirl of 20° to the axial direction. The
mean diameter of the impeller eye is 200 mm and the absolute air velocity at inlet is 120 m/s. Calculate the impeller tip diameter. Take $c_p = 1.005 \text{ kJ/kgK}$ and $\gamma = 1.4$.

**Solution** With reference to Fig. E11.6 (a).

$$T_{2s}/T_1 = (4)^{(1.4 - 1)/1.4} = 4^{0.286} = 1.487$$

$$T_{2s} = 290 \times 1.487 = 431 \text{ K}$$

$$\Delta T_s = T_{2s} - T_1 = 431 - 290 = 141 \text{ K}$$

$$\Delta T = 141/0.82 = 171.95 \text{ K}$$

$$W_c = mc_p \Delta T = 1 \times 1.005 \times 171.95 = 172.81 \text{ kJ/kg} = \text{power input per kg}$$

Absolute air velocity at inlet (Fig. E11.6 (b))

$$V_1 = 120 \text{ m/s}$$

$$V_{bl} = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.2 \times 16000}{60} = 167.55 \text{ m/s}$$

Pre-whirl angle = $20^\circ$

$$V_{w1} = V_1 \sin 20^\circ = 120 \sin 20^\circ = 41.04 \text{ m/s}$$
At exit of the vanes (Fig. E11.6 (c)),

\[ V_{w_2} = V_{b_2} \]

Slip factor \( V'_{w_2} / V_{w_2} = 0.85 \)

\[ V'_{w_2} = 0.85 \times V_{w_2} = 0.85 \times V_{b_2} \quad (1) \]

Power input per kg \( = V_{b_2} \times V'_{w_2} - V_{b_1} \times V_{w_1} \)

\[ 172.81 \times 10^3 = V_{b_2} \times 0.85 \times V_{b_2} - 167.55 \times 41.05 \]

\[ = 0.85 \times V_{b_2}^2 - 6877.93 \]

\[ V_{b_2} = 459.78 \text{ m/s} = \frac{\pi d_2 \times 16000}{60} \]

Tip diameter, \( d_2 = 0.5488 \text{ m} = 549 \text{ mm} \)

**Ans.**

**Example 11.7** Air enters an axial flow compressor at 25°C and undergoes a pressure increase 6 times that at inlet. The mean velocity of rotor blades is 220 m/s. The inlet and exit angles of both the moving and fixed blades are 45° and 15° respectively. The degree of reaction at the mean diameter is 50 per cent and there are 10 stages in the compressor. If the isentropic efficiency of the compressor is 83 per cent and the axial velocity is taken constant throughout, find the work done factor of the compressor.

**Solution**

**Given:**

\[ V_b = 220 \text{ m/s}, \beta_1 = 45^\circ = \alpha_2, \beta_2 = 15^\circ = \alpha_1 \]

\[ V_{r_2} = V_1, V_{r_1} = V_2. \]

From Fig. E11.7, \( V_1 \cos \alpha_1 = V_2 \cos \alpha_2 \)

\[ V_b = V_{r_2} \sin \beta_2 + V_2 \sin \alpha_2 \]

\[ = V_1 \sin \beta_2 + V_1 \cos \alpha_1 \tan \alpha_2 \]

\[ 220 = V_1 [\sin 15^\circ + \cos 15^\circ \tan 45^\circ] \]

\[ = V_1 (0.2588 + 0.9659) \]

\[ V_1 = 179.64 \text{ m/s} \]

\[ V_2 = \frac{179.64 \times \cos 15^\circ}{\cos 45^\circ} = \frac{173.52}{0.7071} = 245.4 \text{ m/s} \]

\[ \Delta V_w = V_2 \sin \alpha_2 - V_1 \sin \alpha_1 \]
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Solution  The arrangement of the power plant and the corresponding $T$–$s$ diagram are shown in Fig. E11.8 (a) and (b).

For perfect intercooling, the intercooler pressure $p_i$ is given by

\[
p_i = [p_1p_2]^{1/2} = [1 \times 6]^{1/2} = 2.45 \text{ bar}
\]

\[
T_{2s}/T_1 = [p_2/p_1]^{(\gamma-1)/\gamma} = (2.45)^{0.4/1.4} = 1.292
\]

\[
T_{2s} = 1.292 \times 293 = 378.59 \text{ K}
\]

\[
\frac{T_{2s} - T_1}{T_2 - T_1} = \eta_c = 0.82
\]

\[
T_2 - T_1 = \frac{378.59 - 293}{0.82} = 104.38 \text{ K}
\]

\[
T_2 = 397.38 \text{ K}
\]

\[
T_{4s}/T_3 = [p_2/p_1]^{(\gamma-1)/\gamma}
\]

\[
T_{4s} = 293 \times (6/2.45)^{0.286} = 378.54 \text{ K}
\]

\[
T_4 - T_3 = \frac{378.54 - 293}{0.82} = 104.32
\]

\[
T_4 = 397.32 \text{ K}
\]

---

Fig. E11.8
\[ T_6 / T_{7s} = \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\gamma} \]
\[ 1173 / T_{7s} = 6^{0.33/1.33} = 6^{0.248} = 1.5595 \]
\[ T_{7s} = 752.17 \text{ K} \]
\[ T_6 - T_7 = \eta_T (T_6 - T_{7s}) = 0.92 \times (1173 - 752.17) = 1173 - T_7 \]
\[ T_7 = 1173 - 387.16 = 785.84 \text{ K} \]

Regenerator effectiveness

\[ \varepsilon = \frac{T_5 - T_4}{T_7 - T_4} \]
\[ 0.7 = \frac{T_5 - 397.32}{785.84 - 397.32} \]
\[ T_5 = 669.28 \text{ K} \]

Making an energy balance for the combustion chamber,

\[ m_r \times CV \times \eta_{comb} = c_p (1 + m_f) (T_6 - T_5) \]

where \( m_r \) is the mass of fuel consumed per kg of air.

\[ m_r \times 40,800 \times 0.95 = 1.005 (1 + m_f) (1173 - 669.28) \]
\[ m_f = (1 + m_f) 0.0131 \]

\[ \frac{1}{m_f} + 1 = 76.56 \]
\[ \frac{1}{m_f} = 75.56 \]
\[ m_a / m_f = \text{air-fuel ratio} = 75.56 \quad \text{Ans. (a)} \]

\[ W_{GT} = (m_a + m_f) c_p (T_6 - T_7) \]
\[ = \left( 1 + \frac{1}{75.56} \right) \times 1.08 (1173 - 785.84) \]
\[ = 423.67 \text{ kJ/kg air} \]
\[ W_c = m_a c_p [(T_2 - T_1) + (T_4 - T_3)] \]
\[ = 1 \times 1.005 [(397.38 - 293) + (397.32 - 293)] \]
\[ = 209.74 \text{ kJ/kg air} \]
\[ W_{net} = 423.67 - 209.74 = 213.93 \text{ kJ/kg air} \]

Heat supplied = \[ \frac{40,800 \times 0.95}{75.56} = 512.97 \text{ kJ/kg air} \]

\[ \eta_{cycle} = \frac{213.93}{512.97} = 0.417 \text{ or } 41.7\% \quad \text{Ans. (b)} \]

Since air flow rate is 210 kg/s, the power output of the plant at generator terminals

\[ = 213.93 \times 210 \times 0.96 \times 0.95 \]
\[ = 40971.82 \text{ kW or } 40.972 \text{ MW} \quad \text{Ans. (c)} \]

Fuel consumption per hour = \[ 210 \times 3600 \times \frac{1}{75.56} = 10005.29 \text{ kg} \quad \text{Ans. (d)} \]
Specific fuel consumption

\[ \frac{10005.29}{40971.82} = 0.244 \text{ kg/kWh} \]

Ans. (d)

**SHORT-ANSWER QUESTIONS**

11.1 What are the applications of diesel electric power plants?
11.2 Enlist the advantages and disadvantages of diesel engine power plants.
11.3 Which are the different types of diesel engine?
11.4 Name the essential components of a diesel electric plant.
11.5 How is noise of the engine reduced?
11.6 Explain the scope of utilizing the waste heat in the engine exhaust.
11.7 What is an engine day tank? State the functions of a fuel injection system.
11.8 What is solid injection? Briefly explain the different systems of solid injection.
11.9 Explain the operation of a fuel pump. How is the fuel supply to the engine regulated?
11.10 What is a fuel injector? Explain a pintle nozzle.
11.11 Explain the necessity of the cooling system in a diesel engine. What are the methods of cooling the engine?
11.12 Explain the important functions of the lubrication system.
11.13 Discuss the wet sump lubrication system pertaining to a diesel engine.
11.14 How is a diesel engine started?
11.15 What do you mean by auto-ignition? Why is excess air always used in a CI engine?
11.16 Explain the ignition delay in a CI engine. What do you mean by the mixing period and the interaction period?
11.17 Explain the phenomenon of detonation in a CI engine. On what factors does it depend?
11.18 Define cetane number of a fuel. How can fuel knock be controlled?
11.19 Give the typical valve timing diagram of a four-stroke oil engine.
11.20 What is mep? How does an electronic engine indicator help in measuring ip?
11.21 What is a dynamometer? What does it measure? What are the different types of dynamometers?
11.22 Explain the Morse test. What is Willan’s line?
11.23 Define (a) bmep (b) brake thermal efficiency and (c) bsfc.
11.24 What is volumetric efficiency? What are the variables which influence the volumetric efficiency of an engine?
11.25 Give an energy balance of a CI engine.
11.26 Explain the performance characteristics of a CI engine. What do you understand by "Smoke limit"?
11.27 What is the objective of supercharging? Why is it more beneficial in a CI engine compared to an SI engine?
11.28 Explain the main features of supercharging with the help of $p-V$ diagram. What do you mean by mechanical supercharging and turbocharging? What is the effect of intercooling in turbocharging?

11.29 Which are the different types of compressors used for supercharging? Why are turbochargers superior to superchargers?

11.30 Give the layout of a diesel engine power plant.

11.31 Why is power generation by gas turbines attractive these days?

11.32 Give the specific advantages and disadvantages of a gas turbine plant for a utility system.

11.33 Bring out the difference between the closed cycle and open cycle gas turbine power plants.

11.34 Explain the effect of regeneration in a gas turbine plant.

11.35 Discuss the effect of pressure ratio on Brayton cycle output and efficiency.

11.36 Derive the optimum pressure ratio in an ideal gas turbine plant for maximum network. How is the expression modified when compressor and turbine efficiencies are taken into consideration? What is the corresponding maximum net work and the cycle efficiency?

11.37 Discuss the effect of intercooling and reheating in a gas turbine plant.

11.38 What is a semi-closed cycle gas turbine plant? Explain it with the help of a sketch of the plant.

11.39 Discuss the performance characteristics of a gas turbine power plant.

11.40 Explain with a neat sketch the operation of a centrifugal compressor. What is pre-whirl? Why is it provided?

11.41 What is slip? Define slip factor. State the applications of centrifugal compressor.

11.42 What is an axial-flow compressor? What are its applications? What is the basis of its design? How is the degree of reaction defined? What is work done factor? What is its approximate value?

11.43 Explain with a neat sketch the combustion chamber of a gas turbine plant. What are dilution holes? How is flame stabilization secured by (a) a swirler (b) a bluff body?

11.44 Explain the staging of air supply in the forms of primary, secondary and tertiary air in the CC of a gas turbine plant. Why is a large air-fuel ratio used?

11.45 Explain the terms (a) combustion efficiency (b) combustion intensity.

11.46 What are the essential features of gas turbine blades? How are blades cooled?

11.47 Briefly describe the different fuels which can be burnt in gas turbine plants.

11.48 Discuss the materials which are used for gas turbines and compressors. What properties should the blade material possess?

11.49 With the help of neat sketches explain the operation of a free piston engine plant. Explain its applications. What are its advantages and disadvantages?

## PROBLEMS

11.1 During a 60-minute trial of a single cylinder four-stroke oil engine, the following observations were made. Engine dimension $= 0.3 \text{ m} \times 0.45 \text{ m}$ stroke, Fuel consumption $11.4 \text{ kg}$, Calorific value of fuel $= 42 \text{ MJ/kg}$, imep $=$
6 bar, Net load on brakes = 1500 N, N = 300 rpm, Brake drum diameter = 1.8 m, Brake rope diameter = 20 mm, Quantity of jacket cooling water = 600 kg, Temperature rise of cooling water = 55°C, Quantity of air as measured = 250 kg, Exhaust gas temperature = 420°C, Specific heat of exhaust gas = 1.0 kJ/kgK, Ambient temperature = 20°C. Estimate (a) the indicated power (b) the brake power and (c) the indicated thermal efficiency. Draw up an energy balance sheet of the engine.

[Ans. (a) 47.71 kW (b) 42.88 kW (c) 35.86%]

11.2 The following test data refer to a four-stroke four-cylinder diesel engine. Cylinder diameter = 0.35 m, Stroke = 0.40 m, N = 315 rpm, imep = 7 bar; hp of the engine = 250 kW, Fuel consumption = 80 kg/h, Calorific value of fuel = 44 MJ/kg, Hydrogen content in fuel = 13%, Air consumption = 30 kg/min, Cooling water circulated = 90 kg/min, Rise in cooling water temperature = 38°C, Exhaust gas temperature = 324°C, Specific heat of air = 1.005 kJ/kgK, Specific heat of exhaust gas = 1.05 kJ/kgK, Ambient air temperature = 24°C, Specific heat of superheated steam = 2.093 kJ/kgK, Partial pressure of steam in exhaust gases = 0.03 bar. Find (a) mechanical efficiency (b) indicated thermal efficiency, and (c) bsfc. Draw up a heat balance of the engine.

11.3 A four-cylinder, four-stroke diesel engine develops 83.5 kW at 1800 rpm with a bsfc of 0.231 kg/kWh and air-fuel ratio of 23/1. The analysis of the fuel is 87% carbon and 13% hydrogen. The calorific value of fuel is 43.5 MJ/kg. The jacket cooling water flows at 0.246 kg/s and its temperature rise is 50°C. The exhaust temperature is 316°C. Draw up an energy balance for the engine. Take R = 0.302 kJ/kgK and c_p = 1.09 kJ/kgK for the dry exhaust gas and c_p = 1.86 kJ/kgK for superheated steam. The room temperature is 17.8°C and the exhaust gas pressure is 1.013 bar.

[Ans. hp 35.8%, cooling water 22.1%, exhaust 25.3%, radiation and unaccounted 16.9%]

11.4 A six-cylinder, four-stroke CI engine of 75 mm bore and 100 mm stroke has a brake power output of 110 kW at 3750 rpm. The volumetric efficiency at this operating condition referred to ambient conditions of 1.013 bar and 20°C is 80 per cent.

The engine is now fitted with a mechanically driven supercharger which has an isentropic efficiency of 70 per cent and a pressure ratio of 1.6. The supercharged version has a volumetric efficiency of 100 per cent referred to the supercharger delivery pressure and temperature. If it is assumed that the ip developed per unit volume flow rate of induced air at ambient conditions is the same for normal aspiration and supercharging, calculate the net increase in hp to be expected from the supercharged engine. Take the mechanical efficiency of the engine as 80 per cent in both cases and the mechanical efficiency of the drive from the engine to the supercharger is 95 per cent.

[Ans. 64.1 kW]

11.5 A turbocharged six-cylinder four-stroke diesel engine has a swept volume of 39 litres. The inlet manifold conditions are 2.0 bar and 53°C. The volumetric efficiency of the engine is 95 per cent, and it is operating at a load of 16.1 bar
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Determine the power input to the air and the direction of the air at entry to and exit from the rotor and stator blades. Assume air as an ideal gas with $c_p = 1.005 \text{ kJ/kgK}$ and $\gamma = 1.4$.

[Ans. 773 kW]

11.16 An axial flow gas turbine stage is to be designed in accordance with the following data: Blade height = 0.10 m, Rotor speed = 15,000 rpm, Mean blade ring diameter = 0.45 m, Inlet gas pressure = 3.5 bar, Absolute gas velocity at inlet and exit = 280 m/s, Stage isentropic efficiency = 0.90, Inlet gas temperature = 870°C, Exit gas pressure = 1 bar, Degree of reaction at mean blade height = 0.50.

The axial velocity can be assumed constant throughout the rotor and free vortex conditions exist in the space between the stator and rotor. Find (a) the blade angles at mean blade height at stator exit, at rotor inlet and at rotor exit and (b) the degree of reaction at the blade root.

11.17 The polytropic efficiency $\eta_p$ of an expansion or compression process is the isentropic efficiency of an infinitely small stage, so that for an ideal gas $\eta_p = \frac{dh}{dh_s} = \frac{dh}{\gamma dp} = (c_p dT, p)/(RT dp)$. Show that in an expansion process $T_1/T_2 = (p_1/p_2)^{(\gamma-1)/\gamma}$ and for a compression process $T_2/T_1 = (p_2/p_1)^{(\gamma-1)/\gamma \eta_p}$.

11.18 The gas turbine in a cogeneration plant has an output of 150 MW with a thermal efficiency of 35 per cent. The fuel oil used has a calorific value of 43 MJ/kg. The exhaust gas flow rate from the gas turbine is 400 kg/s and its temperature is 550°C. The exhaust gas from the gas turbine passes through a boiler plant and leaves at 90°C. The steam generated is at a pressure of 100 bar and a temperature of 450°C. The feedwater temperature to the boiler is 140°C. The generated steam passes through a steam turbine of 86 per cent isentropic efficiency to exhaust at 5 bar. The boiler has an efficiency of 92 per cent. Determine (a) the mass of fuel used by the gas turbine in t/h (b) the mass flow of steam from the boiler in t/h (c) the output of the steam turbine in MW and (d) the overall efficiency of the total plant. Take $c_p$ of gas turbine exhaust gases as 1.1 kJ/kgK.

[Ans. (a) 35.9 t/h, (b) 252.4 t/h, (c) 40.58 MW (d) 44.5%]

11.19 The air in a gas turbine plant is taken in the LP compressor at 20°C and 1.05 bar. After compressor it is passed through an intercooler where its temperature is reduced to 27°C. The cooled air is further compressed in the HP compressor and then passed to the combustion temperature where its temperature is increased to 750°C by burning the fuel. The combustion products expand in the HP turbine which runs the compressor. Further expansion of the gas continues in the LP turbine which drives the alternator. The gases coming out from the LP turbine at 1.05 bar are used for heating the incoming air from the HP compressor. The pressure ratio of each compressor is 2, isentropic efficiency of each compressor and each turbine is 0.82, the air flow rate is 16 kg/s and the calorific value of fuel is 42 MJ/kg. Neglecting the mechanical, pressure and heat losses in the plant, determine (a) the power output (b) the overall thermal efficiency and (c) the specific fuel
consumption. Take \( c_p = 1.0 \text{ kJ/kgK} \) and \( \gamma = 1.4 \) for air and \( c_p = 1.15 \text{ kJ/kgK} \) and \( \gamma = 1.33 \) for gases.

\[\text{Ans.} \ (a) \ 2.05 \text{ MW}, \ (b) \ 30.9\%, \ (c) \ 0.277 \text{ kg/kWh}\]

11.20 A gas turbine power plant of 10 MW capacity works in a closed cycle using air as the working medium. The plant having a regenerator is designed for maximum specific work output. The inlet air temperature is 300 K and the maximum temperature in the cycle is 960 K. Taking the isentropic efficiency of compressor as 0.8, that of turbine as 0.9, the mechanical efficiency and generator efficiency each as 0.95, the regenerator effectiveness as 0.7, the combustion efficiency as 0.96 and assuming that 90 per cent of the heat released by combustion is transferred to air, determine the fuel burning rate and the air-fuel ratio. Take the calorific value of fuel used as 37 MJ/kg.

\[\text{Ans.} \ 4400 \text{ kg/h, 92.5}\]

11.21 A simple open cycle gas turbine plant works between the pressures of 1 bar and 6 bar and temperatures of 300 K and 1023 K. The calorific value of fuel used is 44 MJ/kg. If the mechanical efficiency and the generator efficiency are 95 per cent and 96 per cent respectively and for an air flow rate of 20 kg/s, calculate (a) the air-fuel ratio (b) the thermal efficiency and (c) the power output.

11.22 An open cycle constant pressure gas turbine plant consists of two compressors with perfect intercooling and a two stage turbine with a reheater. Air enters at 1 bar, 15°C. The maximum pressure ratio and the maximum temperature of the cycle are limited to 5 and 800°C respectively. The reheating takes place at 2.3 bar to 800°C. The isentropic efficiencies of each compressor and each turbine are 0.8 and 0.9 respectively. The calorific value of fuel is 42 MJ/kg. Taking \( c_p = 1.005 \text{ kJ/kgK} \) and \( \gamma = 1.4 \) for air, and \( c_p = 1.15 \text{ kJ/kgK} \) and \( \gamma = 1.33 \) for gases and neglecting pressure and heat losses and if the air flow rate is 25 kg/s, estimate (a) the overall thermal efficiency of the plant (b) the air-fuel ratio (c) the specific fuel consumption, and (d) power output of the plant.

**BIBLIOGRAPHY**

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The demand for electricity in a utility system varies hourly, daily and even from season to season, whereas the supply is fixed according to the installed capacity of the system, which corresponds to the maximum demand plus a reasonable excess to take care of scheduled and unscheduled shutdowns. The result is a large, expensive power plant that operates below its capacity most of the time, thus causing high operating and capital costs.

The objective of energy storage is to offset this adverse effect of fluctuating demand of electricity and to assure a steady output from existing power plants. When the demand is lower than the capacity, energy is stored. When the demand is higher than the capacity, the stored energy is released. It is thus possible to supply electricity reliably, efficiently and economically, meeting the peak electrical demand on short notice.

The need for energy storage was not actually felt till the generating plants were relatively cheap and there was abundant fuel supply. Nature herself stored energy in the form of fuels. The energy density of fossil fuels is about $37 \times 10^6 \text{ kJ/m}^3$ (at 70 bar), while for natural uranium (0.071%, U–235) it is about $10^{14} \text{ kJ/m}^3$. With shrinking fuel availability, the need to conserve natural resources is paramount, for which the following suitable methods of energy management must be adopted.

(a) Meet the peak load demand by interconnecting power networks that might have different power demands on them.

(b) Use modern, more efficient power plants for base-load power generation and old, less efficient power plants for peak-power generation.

(c) Install smaller low capital cost power plants, which may not be highly efficient, as peaking units, e.g. gas turbine unit and small hydroelectric plant.

(d) Incorporate energy storage systems.

Reliability and economy of electricity supply are achieved by having a mix of three types of power plants: a base-load plant, an intermediate plant and a peaking plant. Base-load plants providing the base electrical load to the grid are usually large, i.e. efficient steam power stations powered by fossil or nuclear
fueled and they operate continuously except for scheduled maintenance or forced outages. Intermediate plants are older, less efficient steam plants or repowered combined cycle plants, which operate primarily during hours of high load demand. Peaking plants are specifically designed to provide power during peak demand periods.

The variation of load throughout the day, week and year stimulates a demand for storage especially when the increase in installed capacity of large coal or nuclear plants, (designed to operate at maximum efficiency on their rated power output), exceeds the base load demand and when a future increase in utilization of intermittent and variable energy sources (such as solar, wind or ocean energy) exceeds the utilities' reserve capacities.

Figure 12.1 shows a typical weekly load curve of a utility with and without energy storage. As illustrated by Fig. 12.1 (a) curve, intermediate and peaking power involves extensive generating capacity. The load variation shown here is typical for any European or US utility, but it applies to most other countries, where cheap off-peak electricity rates exist. In countries where this is not the case the daily variation tends to be larger. In any case it appears to be the fact all over the world that installed capacity is about double the yearly average load (Jensen, 1980).

![Diagram of weekly load curve](image)

**Fig. 12.1** Weekly load curve for a typical power utility

(a) Current generation mix

(b) Generation mix with energy storage
If large-scale energy storage were available as illustrated by the curve in Fig. 12.1 (b), then the relatively efficient and economical base-load generation could be increased and the excess beyond off-peak demand (lower shaded areas) could be used to charge the storage system.

Discharge of the stored energy (upper shaded areas) during periods of peak load demand would then reduce or replace fuel-burning peaking plant capacity, thus conserving fuel (mostly oil-based) resources. Use of energy storage to generate peaking power in this manner is termed “peak shaving”. The higher base-load level may replace part of the intermediate generation thus performing load-levelling and enabling the more extensive use of storage to eliminate most or all conventional intermediate cycling equipment, thus reducing installed capacity and saving cost and fuel. Smoothing of the daily load curve leads to reduced stresses of load following operation by the steam plant and consequently reduces maintenance costs.

As shown in Fig. 12.2 the power plant is continuously operated during a day in a base-load mode, as a result of which excess electricity \((ab + cd)\) is produced during the off-peak hours.

![Diagram showing energy storage with constant thermal input as from fossil or nuclear fuel](image)

**Fig. 12.2** Energy storage with constant thermal input as from fossil or nuclear fuel

This excess electricity is stored and then released during the peak demand period \(bc\). Stored energy (area below \(ab + cd\)) is greater than that supplied (area above \(bc\)) since there are conversion losses to and from storage. Various energy storage schemes are the following.

- (a) Pumped hydro
- (b) Compressed air energy storage
- (c) Energy storage by flywheels
- (d) Electrochemical energy storage
- (e) Magnetic energy storage
- (f) Thermal energy storage
Pumped hydro storage is the only large scale energy storage method which is highly developed and used in power systems. For decades, utilities have used pumped hydro storage as an economical way to utilize off-peak energy, by pumping water to a reservoir at a higher level. During peak load periods the stored energy is discharged through the pumps, then acting as turbines, to generate electricity to meet the peak demand. Energy is thus stored as hydraulic potential energy by pumping water from a low-level into a higher level reservoir. When the discharge of the energy is required, the water is returned to the lower reservoir through turbines which drive electricity generators.

Pumped hydro storage usually comprises the following. An upper reservoir, waterways, a pump, a turbine, a motor, a generator and a lower reservoir as shown schematically in Fig. 12.3.

![Diagram of pumped hydroelectric energy storage](image)

**Fig. 12.3** Pumped hydroelectric energy storage

Potential energy stored by raising mass of water $m$ to an elevation $H$ is

$$PE = mgH$$

(12.1)

The operating heads on the pump-turbine in the pumping mode $H_p$ and in the turbine mode $H_T$ are

$$H_p = H + H_1$$

(12.2)

and

$$H_T = H - H_1$$

(12.3)

where $H$ is the static head and $H_1$ represents the losses during flow.

The pumping power $P_p$ and the generating power $P_T$ are

$$P_p = \frac{Q_p \rho g H_p}{\eta_p}$$

(12.4)
and

\[ P_T = [\rho g H_T Q_T] \eta_T \]

(12.5)

where \( Q \) represents the volumetric flow rate (\( m^3/s \)), \( \rho \) is the density of water (1000 kg/m\(^3\)) and \( \eta \) is the efficiency.

1000 kg of water raised by 100 m will store \((1000) \ kg \times (9.81) \ m/s^2 \times (100) \ m\) or \(9.81 \times 10^5 \) J or 0.2725 kWh of energy. Thus large masses of water must be elevated to sufficiently large heights to store large quantities of energy.

One or both of these reservoirs may be artificially excavated or may be a natural river or lake. Pumped hydro systems may be above ground which can be of high head and medium head, or underground. When topography does not permit the former, underground reservoir may be used. A typical above ground pumped hydro system of a conventional design is shown in Fig. 12.4. A surge tank is built near the mouth of the pressure tunnel to relieve the pipes of undue inertia pressure set up in the tunnel when the flow is checked following a reduction of load. Should the pressure exceed a certain predetermined value, water merely spills over the lip of the surge tank. The surge tank also provides a reservoir of water that can be drawn upon when the load on the turbine suddenly increases.

![Fig. 12.4 Schematic of a conventional above ground pumped hydro storage system](image)

The losses in pumped hydro systems include the following:

(a) motor and pump losses
(b) flow losses during upflow
(c) seepage into ground
(d) leakage of water from pipes and equipment
(e) evaporation during storage
(f) turbine and generator losses
(g) flow losses during downflow

The combined efficiency of a pumped hydro system, called the *turnaround efficiency*, \( \eta_{TA} \), is defined as

\[ \eta_{TA} = \frac{\text{Total energy output}}{\text{Total energy input during a charge-discharge cycle}} \]

(12.6)

In most plants, it is around 65 per cent.
Figure 12.5 gives the outline of the 400 MWe. Crauchan Pumped Hydrostation in Scotland in conjunction with a thermal power plant. Pumping is carried out mostly at night and at weekends to meet day time peak load on next day.

**Fig. 12.5** Crauchan station of the north of Scotland Hydroelectric Board

### 12.2 COMPRESSED AIR ENERGY STORAGE (CAES)

During off-peak hours, excess energy is used to compress air and store it in reservoirs, aquifers or caverns. The stored energy is then released during periods of peak demand by expansion of the air through an air turbine. The turnaround efficiency is the same as that of a pumped hydro system, i.e. 65 per cent.

Three types of reservoirs can be used to store compressed air. They are salt caverns, aquifers and hard-rock caverns. Aquifers are naturally occurring porous rock formations. These underground reservoirs are, however, subjected to fluctuations in pressure, temperature and humidity.

When air is compressed for storage, its temperature will rise according to Eq. (12.7).

\[
T_2 = T_1 \left[\frac{P_2}{P_1}\right]^{(n-1)/n}
\]

Eq. (12.7)

The heat of compression may be retained in the compressed air. This is called adiabatic storage and results in high storage efficiency, since more energy is recovered by expansion, with pressure lines diverging. If the heat of compression is allowed to dissipate, additional heat could be added by burning fuel to retain the high storage efficiency. This is called a hybrid system.

#### 12.2.1 Adiabatic Storage System

A simple adiabatic compressed air energy storage (CAES) system is shown in Fig. 12.6. During off-peak hours, electrical energy from the main power plant is used by the motor-generator (MG) set operating in the motor mode to drive the compressor C. The compressed air passes through a packed bed P for sensible thermal energy storage and then to a constant pressure underground reservoir (R). The constant pressure is obtained by displacing water to a pressure-
compensation pond that has a constant head above the reservoir. During peak hours, air from the reservoir flows through the packed bed \((P)\) picking back sensible heat, then through the air turbine that now drives the \(MG\) set in the generator mode. Clutches separate the compressor during the peak periods and the turbine during off-peak periods.

Air reservoir volume of the cavern \((V)\) greatly depends on the storage pressure. For a peak load capacity of 1500 MWh, the required volume is

\[
V = 2 \times 10^6 \text{ m}^3 \text{ for 10 bar storage pressure}
\]

and

\[
V = 64,000 \text{ m}^3 \text{ for 100 bar storage pressure}
\]

In most cases, the volume of packed bed \((P)\) is about one-tenth of the reservoir volume \((R)\).

**Fig. 12.6** Simple single-stage adiabatic compressed air energy storage system with pressure compensation pond

\[
V = 2 \times 10^6 \text{ m}^3 \text{ for 10 bar storage pressure}
\]

and

\[
V = 64,000 \text{ m}^3 \text{ for 100 bar storage pressure}
\]

In most cases, the volume of packed bed \((P)\) is about one-tenth of the reservoir volume \((R)\).
12.2.2 Hybrid System

In a simple gas turbine power plant comprising a compressor, a combustion chamber and a turbine, about two-thirds of the power generated is consumed by the compressor and only one-thirds of the total power is available in the form of electricity ($\eta \sim 33\%$). In a hybrid system, fuel is burnt in the compressed air. As shown in Fig. 12.7, during off-peak hours at night, the MG set operating as a motor drives the compressor. The compressed air is stored in a pressure vessel or cavern. During peak hours on the next day, air is taken from the store, heated in a combustor by burning fuel and then expanded in a turbine to the ambient pressure. The MG set now operating in the generator mode supplies electricity to the power system.

![Diagram of hybrid system](image)

**Fig. 12.7** Development of the CAES concept (a) Simple gas turbine cycle (b) Gas turbine cycle modified to CAES configuration

In a hybrid CAES scheme all the energy produced by the turbine is available for electricity generation during peak hours, since the compressor has already done its work during off-peak hours and thus does not consume any power from the turbine output now.

The first CAES system to be built was a 290 MW plant designed by Brown Boveri and built at Huntorf, Germany for a utility of Humburg which is in operation since 1978. It uses two salt caverns with a total volume of 300000 m$^3$.
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Fig. 12.9  *Flywheel with mass concentrated in the rim*

and

\[ W = \frac{1}{2} mr^2 \omega^2 \]  \hspace{1cm} (12.10)

Thus the energy content depends on the total mass to the first power and on the angular velocity (or rpm) to the second power. In order to obtain high energy content, high angular velocity is more important than the total mass of the rotary system.

The energy density \( W_m \), i.e. the amount of energy per kg, is thus

\[ W_m = \frac{1}{2} \rho \omega^2 r^2 \]  \hspace{1cm} (12.11)

The volume energy density \( W_v \) is then

\[ W_v = \frac{1}{2} \rho \omega^2 r^2 \]  \hspace{1cm} (12.12)

where \( \rho \) is the mass density.

High angular velocity depends on the strength of the material. The tensile stress \( \sigma \) in the rim is given by

\[ \sigma = \rho \omega^2 r^2 \]  \hspace{1cm} (12.13)

Now the maximum kinetic energy per unit volume will be

\[ (W_v)_{\text{max}} = \frac{1}{2} \sigma_{\text{max}} \]  \hspace{1cm} (12.14)

If the dimensions of the flywheel are fixed, the main requirement is high tensile strength. Also,

\[ (W_m)_{\text{max}} = \frac{1}{2} \frac{\sigma_{\text{max}}}{\rho} \]  \hspace{1cm} (12.15)

A light material (low density) with a high tensile stress is suitable. Fibre composites with strengths higher than steel and much lower mass densities are an obvious choice. The tensile strength and mass densities of some fibres are given in Table 12.1.
Table 12.1 *Strength and density of some fibres*

<table>
<thead>
<tr>
<th>Fibre</th>
<th>Strength (GN/m²)</th>
<th>Mass density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>3.5</td>
<td>2500</td>
</tr>
<tr>
<td>Silica</td>
<td>6.0</td>
<td>2200</td>
</tr>
<tr>
<td>Carbon</td>
<td>2.6</td>
<td>1900</td>
</tr>
<tr>
<td>Chrysolite asbestos</td>
<td>4.5</td>
<td>2500</td>
</tr>
</tbody>
</table>

For comparison the value of strength for steel wire (0.9% carbon) is 4.2, but the mass density is rather high (7.9).

The factor 0.5 in Eq. (12.15) relates to a simple rim flywheel. But the expression is valid for any flywheel, made from material of uniform mass density ρ so that

\[
(W_m)_\text{max} = k_m \frac{\sigma_{\text{max}}}{\rho} \tag{12.16}
\]

The value of \( k_m \) depends on the geometry of the flywheel and \( k_m \) is called the shape factor or the mass efficiency factor. The value of \( k_m \) comes from the expression of moment of inertia \( I \), the values of which for some shapes of rotating bodies are shown in Fig. 12.10. Values of \( k_m \) for a number of flywheel shapes are given in Table 12.2.

![Fig. 12.10](image)

**Fig. 12.10** *Moment of inertia for disc, cylinder and bar shaped flywheel*

Table 12.2 *Flywheel shape factors*

<table>
<thead>
<tr>
<th>Flywheel shape</th>
<th>( k_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant stress disk</td>
<td>0.931</td>
</tr>
<tr>
<td>Flat unpierced disk</td>
<td>0.606</td>
</tr>
<tr>
<td>Thin rim</td>
<td>0.500</td>
</tr>
<tr>
<td>Rod or circular brush</td>
<td>0.333</td>
</tr>
<tr>
<td>Flat pierced disk (o.d./i.d. = 1.1)</td>
<td>0.305</td>
</tr>
</tbody>
</table>
The shape factor $k_m$ is a measure of the efficiency with which the flywheel’s geometry uses the material’s strength. The ideal shape would be a constant stress disc where all the material is uniformly stressed biaxially and $k_m = 1$, which is not the case for a solid flywheel.

In order to obtain maximum energy storage density, a special design has been proposed, where maximum stress is obtained throughout the flywheel. Such flywheels are thickest near the axis and thinnest near the rim. The shape factor of these truncated conical discs is about 0.8.

Other designs have to be applied to flywheels made of fibre materials, where the tensile strength is high only in one direction. Composite materials have poor transverse strength and a radially thin rim has the material in hoop stress utilizing its tangential stress capability.

The energy stored in a flywheel is

$$KE = \frac{1}{2} l\omega^2 = \frac{1}{2} mR^2 (2\pi N)^2$$

$$= 2\pi^2 mR^2 N^2$$  \hspace{1cm} (12.17)

where $m =$ mass of flywheel, $R =$ radius of gyration and $N =$ rps.

The energy absorbed (or released) by a flywheel between speeds $N_1$ and $N_2$, 

$$\Delta E = 2\pi^2 mR^2 (N_2^2 - N_1^2)$$

$$= 2\pi^2 mR^2 (N_2 + N_1) (N_2 - N_1)$$  \hspace{1cm} (12.18)

The ratio of the variation in rotational speed to the mean speed ($N$) is called the coefficient of speed fluctuation, $k_s$.

$$k_s = \frac{N_2 - N_1}{N} = \frac{(N_2 - N_1)^2}{N_1 + N_2}$$

where

$$N = (N_1 + N_2)/2$$

$$\Delta E = 4\pi^2 mR^2 k_s N^2$$  \hspace{1cm} (12.19)

The value of $k_s$ depends upon the desired closeness of speed regulation. For engines, $k_s$ may vary between 0.005 and 0.2.

An important consideration in flywheel design is the stress level a flywheel rotating at a very high speed is subjected to. The theoretical maximum specific energy $W_m$ (energy stored per unit mass, $E/m$) is given by Eq. (12.16). The maximum volumetric specific energy, $(W_v)_{\text{max}}$ or $(E/V)_{\text{max}}$ as given by Eq. (12.14) may be written for any flywheel as

$$(E/V)_{\text{max}} = (W_v)_{\text{max}} = k_v \sigma_{\text{max}}$$  \hspace{1cm} (12.20)

where $k_v$ is the volume efficiency ratio, the value of which indicates how well a particular flywheel design utilizes the material strength and fills the cylindrical volume around the flywheel.

The principal parameters that determine the suitability of flywheels for energy storage are the two efficiency ratios $k_m$ and $k_v$ as well as the stress and density. The values of $k_m$ and $k_v$ depend upon the type of material (isotropic, uniaxial composite, variable density) as well as flywheel shape (disc, drum, rod). The strength-density ratio, $\sigma/\rho$, is high for such materials as glass or silica fibres as given in Table 12.1.
The high specific design stresses of carbon fibre/epoxy and Kevlar-fibre/epoxy allow storage of a large amount of energy in a relatively light flywheel. The minimum energy capacity of $3.6 \times 10^9$ J required by a power system could be achieved by multi-ring flywheel energy storage made of hoop-wound Kevlar-fibre/epoxy material. The angular speed of this flywheel would be about 3000 rpm. The diameter of the largest ring would be about 5 m, the length about 5 m and the total mass of the flywheel would be $130 \times 10^3$ kg.

The turnaround efficiency $\eta_{TA}$ of a flywheel energy storage system during the charge-store-discharge period will depend on the duration of the store period. There are two main sources of losses in the flywheel, windage and bearing. Windage losses can be reduced to a low level by running the flywheel in a vacuum chamber. Bearing losses for a typical 200t rotor have been estimated at $2 \times 10^5$ J/s. Based on this figure, the $\eta_{TA}$ is about 85 per cent for discharge immediately after charge, but falls to 78 per cent after five hours and 45 per cent after 24 hours of the keeping regime.

**12.4 ELECTROCHEMICAL ENERGY STORAGE**

The most traditional of all energy storage devices for power systems is electrochemical energy storage, which can be classified into three categories, primary batteries, secondary batteries and fuel cells. The common feature of these devices is that stored chemical energy is converted to electrical energy. The main attraction of the process is that its efficiency is not second law or Carnot cycle limited, unlike thermal processes. Primary and secondary batteries utilize the chemicals built into them, whereas fuel cells have chemically bound energy supplied from the outside in the form of synthetic fuel (hydrogen, methanol or hydrazine). Unlike secondary batteries, primary batteries cannot be recharged when the built-in active chemicals have been used and therefore strictly they cannot be considered as genuine energy storage. The term "batteries" will refer to secondary batteries in the following text.

Batteries and fuel cells consist of two electrode systems and an electrolyte, placed together in a special container and connected to an external source or load (Fig. 12.11).

![Diagram of a battery and a fuel cell](image)

(a) Primary or secondary cell
(b) Fuel cell

**Fig. 12.11 Comparison of a battery and a fuel cell**

These two electrodes, fitted on both sides of an electrolyte and exchanging ions with the electrolyte and electrons with the external circuit, are called the anode (−) and cathode (+) respectively (Fig. 12.12).
Hidden page
used for generation in small, local DC power systems were usually shut down at night and the demand was met by lead-acid batteries which were charged during the day. These batteries were also used in several US towns to feed DC electricity to electric street cars during rush hour. The growth of large centralized AC power systems and cheap coal and oil generated electricity relegated batteries to emergency standby duty for DC auxiliaries.

The battery market is still dominated by the lead-acid battery invented by Plante in 1859. It is the oldest chemical storage device. The battery (Fig. 12.13) consists of alternate pairs of plates, one pure lead in spongy form and the other lead coated with lead dioxide, immersed in a dilute solution of sulphuric acid which serves as an electrolyte. During discharge both electrodes are converted into lead sulphate (PbSO₄). Charging restores the positive electrode to lead dioxide and the negative electrode to metallic lead. The battery deteriorates gradually in performance due to irreversible physical changes in the electrodes and ultimately fails after 1000–2000 cycles.

![Diagram of a lead-acid storage battery]

**Fig. 12.13** Lead-acid storage battery

The overall cell reaction is as follows.

\[
\begin{align*}
\text{Anode} & \quad \text{Cathode} \\
Pb & \quad \text{PbSO}_4 + 2\text{H}_2\text{O} \quad + \quad \text{PbSO}_4 \\
+ 2\text{H}_2\text{SO}_4 & \quad + \quad \text{PbO}_2
\end{align*}
\]

During discharge the cathode is positive and the anode negative. The reasons for its wide use are the following:

(a) Relatively high nominal voltage, 2.0 V/Cell
(b) Versatility in providing high or low current
(c) High degree of reversibility, capable of hundreds of charge/discharge cycles
(d) Relatively cheap and easy to fabricate.

The main drawbacks of these batteries are, however, low energy density, long charge time and the need for careful maintenance. About half the weight of a lead-acid battery is occupied by inert materials, e.g. grid metal, water, separators, connectors, terminals and cell containers. Attempts to reduce the weight and hence to increase the energy density have made the use of low-density grid materials. The use of carbon fibres in the positive electrode has resulted in reduction in weight and also in increased power capability.

Industrial development on alkaline electrolyte batteries such as Ni-Zn, Fe-Ni, etc. aims to produce improved energy storage systems for traction applications. The nickel-zinc battery is analogous to the much more expensive nickel-cadmium battery. The cell reaction is the following:

\[
\text{Cathode} \quad \text{Anode} \\
2\text{NiOOH} + 2\text{H}_2\text{O} + \text{Zn} \xrightleftharpoons{\text{Discharge}}^{\text{Charge}} 2\text{Ni(OH)}_2 + \text{Zn(OH)}_2
\]

The Ni-Zn cell’s main drawbacks are its short cycle life, separator stability, temperature control, high cost and mass production problems. Poor cycle life is caused by the high solubility of reaction products at the zinc electrodes. Redeposition of zinc during charging results in the growth of dendrites which penetrate the separators and cause internal short circuit. Attempts have been made to suppress the growth of zinc dendrites during charging by vibrating the zinc electrode.

The \textit{nickel-iron battery} is an alkaline storage battery using KOH as the electrolyte. The cell reaction is:

\[
\text{Cathode} \quad \text{Anode} \\
2\text{NiOOH} + 2\text{H}_2\text{O} + \text{Fe} \xrightleftharpoons{\text{Discharge}}^{\text{Charge}} \text{Ni(OH)}_2 + \text{Fe(OH)}_2
\]

The main drawback of Ni—Fe batteries for electric vehicle applications has been their low energy density. It also suffers from a poor peaking capability and low cell voltage. Recently, however, some improvements have been brought about.

The iron-air battery consists of an anode using iron as active material and a cathode taking oxygen from the air. The cell reaction is:

\[
\text{Anode} \quad \text{Cathode} \\
\text{Fe} + \text{H}_2\text{O} + \frac{1}{2}\text{O}_2 \xrightleftharpoons{\text{Discharge}}^{\text{Charge}} \text{Fe(OH)}_2
\]

The battery suffers from high self-discharge of the iron electrode at low temperatures, poor charge efficiency and limited power capacity (max 30–40 W/kg).
The zinc-air battery has a highly concentrated KOH electrolyte and the electrochemical reaction is between oxygen from the air and zinc metal. The cell reaction is:

\[
\text{Anode} \quad \text{Cathode} \quad \begin{array}{c}
\text{Discharge} \\
\text{Charge} \\
\end{array} \\
\text{Zn} + \frac{1}{2} \text{O}_2 \quad \text{ZnO}
\]

As with the iron-air system, the zinc-air couple has a poor overall charge-discharge efficiency due to the polarization losses associated with the air electrode.

The zinc-chlorine battery attempts to overcome the replating problem of zinc by using an acid electrolyte. The cell reaction is:

\[
\text{Anode} \quad \text{Cathode} \quad \begin{array}{c}
\text{Discharge} \\
\text{Charge} \\
\end{array} \\
\text{Zn} + \text{Cl}_2 \cdot 6\text{H}_2\text{O} \quad \text{ZnCl}_2 + 6\text{H}_2\text{O}
\]

The main problem of this battery is its heavy weight, a $10^{12}$ J storage device requiring 60 tonnes of Cl₂, which itself needs utmost safety.

Advanced batteries like sodium-sulphur couple using a solid state electrolyte and lithium-sulphur couple using a fused salt electrolyte are still under development.

The *sodium-sulphur* battery has a ceramic electrolyte ($\beta$-alumina) which can conduct sodium ions. The cell reaction is:

\[
\text{Anode} \quad \text{Cathode} \quad \begin{array}{c}
\text{Discharge} \\
\text{Charge} \\
\end{array} \\
\times \text{Na} + y \text{S} \quad \text{Na}_x\text{S}_y
\]

Advanced batteries have provided the stimulus for more interesting work in solid state chemistry and electrochemistry in recent years. The seminal discovery was the observation of very high sodium ion mobility above room temperature in sodium beta-alumina. The beta-alumina was thought to be an isomorph of aluminium oxide, and its crystal structure was determined before the Second World War. However, its unusual electrical properties were discovered only recently. When the electrical conductivity of beta alumina was measured at 300°C, it became clear that the charge carriers were exclusively sodium ions and not electrons. The application of such materials to a modern generation of power batteries was quickly realised. In the sodium-sulphur battery, patented by Ford, instead of solid electrodes separated by a liquid electrolyte, as in the conventional lead-acid car battery, sodium beta-alumina is used as a solid electrolyte, specifically conducting sodium ions, between liquid electrodes of sodium metal and sulphur (Fig. 12.14). The cell voltage, 2.08V, is derived from the chemical reaction between sodium and sulphur to produce sodium polysulphide and the theoretical energy density, about 750 Wh/kg (or $2.7 \times 10^6$ J/kg), is much higher than 170 Wh/kg (or $0.61 \times 10^6$ J/kg) of the lead-acid battery.

The lithium-sulphur battery consists of liquid lithium and sulphur electrodes and an electrolyte of molten LiCl-KCl eutectic at an operating temperature in the 380–450°C range. The cell reaction is
**Fig. 12.14** Schematic diagram of the sodium-sulphur battery which uses a sodium beta alumina solid electrolyte as the separator between liquid electrodes (sodium anode and sulphur cathode). The operation temperature is 300–400°C

Highly corrosive liquid lithium attacks the ceramic insulators and separators and shortens the cell’s life. Efficiency is not very high because of self-discharge caused by lithium dissolving in the molten LiCl-KCl electrolyte.

The use of lithium-aluminium alloys and iron sulphide as electrodes has led to the development of more efficiency Li-S cells with good energy densities. The reactions are the following.

<table>
<thead>
<tr>
<th>Anode</th>
<th>Cathode</th>
</tr>
</thead>
<tbody>
<tr>
<td>4LiA1</td>
<td>FeS₂</td>
</tr>
<tr>
<td>Discharge</td>
<td>Charge</td>
</tr>
<tr>
<td>4LiA1</td>
<td>Fe + 2Li₂S + 4A1</td>
</tr>
</tbody>
</table>

The lithium-titanium disulphide cell has a lithium metal anode and a cathode of TiS₂. The electrochemical discharge reaction proceeds via the insertion of lithium ions between adjacent sulphur layers.
Hidden page
where \( n \) is the number of electrons needed to get one atom or molecule of \( X \) into its ionic form in the electrolyte and \( a_1 \) and \( a_2 \) are the activities at electrodes 1 and 2. \( F \) is the Faraday constant (96 000 coulomb/mol).

We can make use of such a cell in the following ways.

(i) If \( a_1 > a_2 \) and \( X \) is continuously added on the left and removed on the right, we have a source of energy—a concentration fuel cell.

(ii) If \( T \) and \( a_1 \) are known, we can measure \( a_2 \)—we have an electrochemical sensor.

(iii) If we apply a greater voltage than \( E \) in the opposite sense, we can drive \( X \) from one side to the other—hence we have an ion pump or an electrolyser.

As a fuel cell is an electrochemical cell which can continuously change the chemical energy of a fuel and oxidant to electrical energy with high efficiency, a variety of synthetic fuels have been tried, such as hydrogen, methanol, ammonia and methane. A diagram of hydrogen-oxygen fuel cell is shown in Fig. 12.16.

![Fig. 12.15 Schematic diagram of an electrochemical fuel cell](image)

**Fig. 12.15** Schematic diagram of an electrochemical fuel cell

![Fig. 12.16 A hydrogen-oxygen fuel cell](image)

**Fig. 12.16** A hydrogen-oxygen fuel cell

Hydrogen-oxygen fuel cells with the overall reaction

\[
H_2 + \frac{1}{2} \text{O}_2 \rightarrow \text{H}_2\text{O}
\]

are attractive because of their high energy density, no pollution and high cell efficiency. A good hydrogen or oxygen conductor should be used as the electrolyte.
The maximum energy available from a chemical reaction is $-\Delta G$, the change in the Gibbs free energy for the reaction. The corresponding ideal voltage for a reversible reaction is

$$V_0 = \frac{-\Delta G}{nF} \tag{12.22}$$

where $n$ is the number of electrons transferred in the reaction and $F$ is the Faraday constant (96 000 coulomb/mol). Now,

$$G - G^0 = n \bar{R} \ T \ln a \tag{12.23}$$

where $a$ is the activity given by $f/f^0$, $f$ being the fugacity (see Nag, 1995).

Per unit mol, $-\Delta G = \bar{R} \ T \ln \frac{a_2}{a_1} \tag{12.24}$

On substituting result of Eq. (12.24) in Eq. (12.22), it becomes $V_0 = \bar{R} \ T/nF \ln \frac{a_2}{a_1}$, which is the same as Eq. (12.21).

### 12.5 MAGNETIC ENERGY STORAGE

Electrical resistance of metals depends on temperature. In 1911, Kammerlingh Onnes found that electrical resistance of mercury drops to zero in the neighbourhood of absolute zero temperature. Onnes called this phenomenon "superconductivity". All metals exhibit this property. The temperature below which a metal becomes a superconductor is called the transition or critical temperature. All superconducting metals have transition temperatures in the cryogenic range (0-123 K).

Superconducting electromagnets were constructed in 1970 for magnetohydrodynamic power generation. These were also used in bubble chambers to cool electric generators, motors and transformers. Electric power transmission and distribution can best be accomplished by the use of high-purity aluminium cables operating at liquid hydrogen temperatures ($\sim 20$ K). Commercial success depends upon whether the metal, the gas and the refrigeration systems can be acquired and operated economically.

The concept of superconducting magnetic energy storage is based on the principle that energy can be stored in the magnetic field associated with a coil. If a coil is made of a material in a superconducting state (i.e. maintained below the critical temperature), then once it is charged, the current will not decay and the magnetic energy can be stored indefinitely. The stored energy can be released back to the network by discharging the coil.

Energy stored in a coil in which the current $I$ flows is

$$E = \frac{1}{2} LI^2 \tag{12.25}$$

where $E$ is in $J$, $L$ is the inductance in henry ($V$ - $s/A$) and $I$ is the current in $A$. The inductance $L$ of a coil depends on its dimensions, which for a coil with conductors of a rectangular cross-section (Fig. 12.17) may be characterized by
\[ \xi = 2R/(ab)^{1/2} \]  
\[ \delta = a/b \]  
and  
\[ V = 2 \pi R ab = \frac{8 \pi R^3}{\delta^2} \]  
where \( R = \) mean radius of coil, \( m \); \( a \) and \( b = \) width and depth of conductor, \( m \); \( V = \) volume of conductor in one coil turn, \( m^3 \). 
The inductance \( L \) is given by  
\[ L = f(\xi, \delta) RN^2 \]  

Fig. 12.17 Dimensions of a cylindrical coil with rectangular conductors

where \( f(\xi, \delta) = \) form function, \( V.s/A.m \); \( N = \) number of turns of coil. 
Energy stored in a coil, from Eq. (12.25).  
\[ E = \frac{1}{2} f(\xi, \delta) RN^2 j^2 \]  
The current density \( j \) is  
\[ j = Nl/ab \]  
Substituting Eq. (12.30) in Eq. (12.29) and simplifying  
\[ E = \frac{1}{4} \pi^{5/3} f(\xi, \delta) \xi^{-2/3} V^{5/3} j^2 \]  
The coil that gives the maximum value of inductance per unit volume, i.e. maximum \( L/V \) ratio is called the Brooks coil. It has  
\[ a = b, R = \frac{3}{2} b, d = 1, x = 3 \]  
Energy stored in a Brooks coil is given by  
\[ E_B = 3.028 \times 10^{-8} V^{5/3} j^2 \]  
For a cylindrical coil, other than Brooks, the energy stored is  
\[ E = \frac{F}{F} E_B \]  
where \( F \) is less than 1.0 and is a function of \( \xi \) and \( \delta \). An important parameter is the volume of material needed per unit of energy stored, i.e.
\[
\frac{V}{E} = \frac{V}{F E_B} = \frac{0.33 \times 10^8}{F V^{2/3} j^2}
\]  

(12.34)

For a Brooks coil, \( F = 1 \). The cost of the coil is proportional to its volume. Therefore, from Eq. (12.34), it is seen that the cost per unit energy stored is inversely proportional to \( V^{2/3} \) and to \( j^2 \). For stability consideration, the current density \( j \) is limited to values between \( 50 \times 10^6 \) and \( 100 \times 10^6 \) A/m\(^2\). Thus, huge structural mass is required to contain the magnetic field energy. For stainless steel, it would be about 160 kg/kWh, which is cost prohibitive. Much of research on magnetic energy storage for utility use is in progress in the USA and Japan.

### 12.6 THERMAL ENERGY STORAGE

Direct storage of heat in insulated solids or fluids is possible even at low temperatures, but energy can only be recovered effectively as heat. Thermal energy storage (TES) is ideally suited for applications such as space heating, where low quality (temperature) energy is required. It has found wide use in many industrial applications such as the manufacture of cement, iron and steel, glass, aluminium, paper, plastics and rubber and in food processing.

Following are the two distinct thermal energy storage mechanisms.

(a) Sensible heat storage, based on the heat capacity of the storage medium.
(b) Latent heat storage, based on the energy associated with a change of phase for the storage medium (melting evaporation or structural change).

#### 12.6.1 Sensible Heat Energy Storage

Energy can be stored as sensible heat by virtue of a rise in temperature of the storage medium, such as water, liquid or a solid.

Storage density (kJ/m\(^3\)) is = Temperature difference (°C) × Specific heat (kJ/kg °C) × Density of material (kg/m\(^3\))

\[
= \rho \cdot c_p \cdot \theta
\]

This system is simple in concept, but has the disadvantage of variable temperature operation and low storage density. Sensible energy storage could employ one of the following devices.

(a) Pressurized water storage
(b) Organic liquid storage
(c) Packed solid beds
(d) Fluidized solid beds

An example of pressurized water sensible heat storage system in a power plant is shown in Fig. 12.18, where the primary heat source is either a fossil fuelled furnace or a nuclear reactor. The base load plant is capable of supplying more steam than needed during periods of low demand. The excess steam is fed from the turbine at high pressure (as in feedwater heating) during these periods of
Fig. 12.18  Schematic flow diagram of a power plant with a pressurized water sensible energy storage system

low demand. This extracted steam is fed to steam accumulators and mixed with water, thus producing saturated pressurized water. The accumulators are discharged during periods of high demand through a small peaking turbine. Discharge continues until a low specified pressure is reached in the accumulators. Steam at low and varying temperature enters the peaking turbine.

Typical values of accumulator high and low pressures are 20 bar (t_{sat} = 212 °C) and 2 bar (t_{sat} = 120 °C).

Steam condenses in water during accumulator charge and re-evaporates during discharge. The storage medium is the pressurized water in accumulators and operates over a relatively wide temperature range.

Storage density of thermal energy utilized in the peaking turbine per unit volume of the high pressure saturated water is

$$\frac{1}{v_{f,1}} (h_{f,1} - h_{f,2})$$  \hspace{1cm} (12.2)

where $v_f$ and $h_f$ are the specific volume and enthalpy of saturated water respectively, and the subscripts 1 and 2 referred to the high (stored) and 1 (emptied) pressures. Therefore, for 20 bar and 2 bar pressures,

Storage density $= \frac{908.5 - 504.8}{0.0011766} = 343$ 107 kJ/m$^3$ = 95.3 kWh/m$^3$

$\Delta T = 212 - 120 = 92$°C
Storage density is approximately 1 kWh/(m³ °C) over the temperature range.

The electric-energy density obtained by the peaking turbine-generator depends upon

1. Thermal turnaround efficiency, $\eta_{TA}$
2. Peaking turbine-generator efficiency, $\eta_{PT}$

The value of $\eta_{TA}$ is a function of the following:

(a) Losses associated with sensible heat transfer to and from the steel walls, structural members of the accumulators and the interconnecting pipework.
(b) The transient heat losses to the environment.

Now, $\eta_{TA}$ is the ratio of

\[
\frac{\text{Energy stored in the accumulator structure}}{\text{Energy stored in the contained water at a given pressure}}
\]

which is

\[
\frac{(\pi DLt) \rho_s c_s}{\left(\frac{\pi}{4} D^2 L\right) \rho_f c_f} = \frac{4t \rho_s c_s}{D \rho_f c_f} = 2(P/\sigma) \frac{\rho_s c_s}{\rho_f c_f}
\]

(12.36)

since

\[\sigma = PD/2t, \text{ or } 2t/D = P/\sigma\]

where $D$ is the diameter of cylindrical accumulator, $L$ its length, $t$ its thickness, $P$ the pressure, $\sigma$ the wall stress, $\rho$ the density, $c$ the specific heat and the subscripts $s$ and $f$ denote solid and liquid respectively. The volumetric heat capacities are roughly equal, i.e. $\rho_f c_f = \rho_s c_s$. The ratio of $P/\sigma$ is of the order of 0.03 for steel. Thus the contribution of losses by sensible heat transfer to the walls of the vessel is very small and may be ignored.

The convective heat losses from the water to the environment are therefore the major contributor to the thermal turnaround efficiency. They vary with time and depend upon the water temperature and the overall heat transfer coefficient $U$ between the water and the outside environment.

Let us consider saturated water of volume $V$, at instantaneous temperature $T$, while the surroundings are at temperature $T_\infty$ (Fig. 12.19). Let $\theta = T - T_\infty$, so that $d\theta = dT$. Assuming a lumped capacity system with low Biot number,

**Fig. 12.19**

Rate of decrease of internal energy of water

\[= \text{Rate of heat loss to the surroundings}\]

\[\rho_f V c_f \frac{d\theta}{dt} = -UA \theta\]
where $A$ is the heat transfer surface area and $t$ the time.

$$\frac{d\theta}{\theta} = \frac{UA}{\rho_f c_f V} dt$$

which on integration gives

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-(UA/d)/(\rho_f c_f V)}$$  \hspace{1cm} (12.37)

Let the time constant $\tau$ be defined by

$$\tau = \frac{\rho_f c_f V}{UA} = \rho_f c_f \left[ \frac{\pi D^2 L}{4} \right] \frac{1}{\pi D L U}$$

$$= \frac{D \rho_f c_f}{4U}$$  \hspace{1cm} (12.38)

Equation (12.37) can be re-arranged as

$$1 - \frac{T - T_\infty}{T_i - T_\infty} = 1 - e^{-v/\tau}$$

$$\frac{T - T_i}{T_\infty - T_i} = 1 - e^{-v/\tau}$$  \hspace{1cm} (12.39)

If the liquid at fully charged condition is at temperature $T_1 (= T_i)$, its temperature $T$ decreases with time $t$ due to heat losses (Fig. 12.20), so that

$$\frac{T(t) - T_i}{T_\infty - T_i} = 1 - e^{-v/\tau}$$  \hspace{1cm} (12.40)

![Diagram]

**Fig. 12.20** Stored liquid temperature-time change in sensible energy storage. $T_1 =$ fully charged, $T_2 =$ end of energy withdrawal, $T_\infty =$ ambient

Assuming a storage period of $t_s$ at which the water temperature decreases to $T_s$.

$$\frac{T_s - T_1}{T_\infty - T_1} = 1 - e^{-ts/\tau}$$  \hspace{1cm} (12.41)
The thermal turnaround efficiency is given as

\[ \eta_{TA} = \frac{\text{energy left in storage at } T_s \text{ (after heat losses)}}{\text{original energy stored}} \]

\[ = \frac{h_s - h_2}{h_1 - h_2} = \frac{T_s - T_2}{T_1 - T_2} \]

\[ \therefore 1 - \eta_{TA} = 1 - \frac{T_s - T_2}{T_1 - T_2} = \frac{T_1 - T_2}{T_1 - T_2} \]

\[ = \frac{T_1 - T_2}{T_1 - T_2} \left[ 1 - e^{-t_s/\tau} \right] \left[ \frac{T_{\infty} - T_1}{T_s - T_1} \right] \]

\[ = \frac{T_1 - T_{\infty}}{T_1 - T_2} \left[ 1 - e^{-t_s/\tau} \right] \]

\[ \therefore \eta_{TA} = 1 - \frac{T_1 - T_{\infty}}{T_1 - T_2} \left[ 1 - e^{-t_s/\tau} \right] 
\]

(12.42)

Thus \( \eta_{TA} \) strongly depends on the ratio \( t_s/\tau \), where \( t_s \) is several hours in a daily storage system. The only variable in \( \tau \) is \( U \), which heavily depends on accumulator insulations, design and location. High values of \( \tau \) will result for low values of \( U \).

Accumulators may be constructed above ground or underground. Underground accumulators are more costly, but have heavier insulation and higher \( T_{\infty} \) and hence higher \( \eta_{TA} \). The choice of above ground on underground accumulators is decided by cost, efficiency, operational problems and safety.

The efficiency of the peaking turbogenerator \( \eta_{PT} \) is low because of variable inlet conditions, the use of low temperature saturated steam, small size and absence of feedwater heating. An efficiency of 20 to 25 per cent is reasonable, while the base load plant has an efficiency of 33 to 40 per cent.

There are three main ways of operating the accumulator, viz. variable pressure, expansion or displacement.

(a) **Variable Pressure Accumulator** It is also called Ruths accumulator (Fig. 12.21). When fully charged, almost all its volume is filled with saturated hot water, with a small cushion of saturated steam above it. In the discharge mode, steam is drawn off from the top and as the pressure in the steam cushion decreases, some water flashes into steam.

(b) **Expansion accumulator** It is shown in Fig. 12.22. When fully charged, the accumulator is almost full of hot water with a small steam cushion, as in the variable pressure mode. As hot water is drawn from the bottom during discharge, some water flashes to steam, which reduces the pressure and temperature of saturated water and steam. All the water can be removed to the flash evaporator with a reduction of pressure of about 30 per cent. The water from the last flash evaporator is collected and stored.


**Fig. 12.21** Variable pressure accumulator

**Fig. 12.22** Expansion accumulator with flash evaporator

**c** Displacement accumulator  It is always completely full of water. When fully charged, it contains hot water at the desired temperature. When fully discharged, all the water is cold. As shown in Fig. 12.23, hot water is injected at the top during charge and removed from the top during discharge. Cold water leaves and enters at the bottom. Since hot water has lower density than cold water, it will float at the top. A sharp temperature gradient or thermocline separates the hot and cold water. During discharge, one or more flash evaporators are used to
Fig. 12.23  Displacement accumulator with flash evaporators

generate steam for peaking turbines. The water drained from the evaporators and the condensate from the turbine are returned to the accumulator as cold water. During charge, steam is mixed with cold water taken from the bottom to raise the temperature to the desired level. Cold water equal in mass to the steam is returned to the boiler inlet feedwater to generate more steam.

12.6.2 Latent Heat Energy Storage

Energy is stored in the form of the latent heat caused by phase change either by melting a solid or vaporizing a liquid. Energy release is accomplished by reversing the process, i.e. by solidifying the liquid or condensing the vapour.

Storage density = latent heat × density

Since the latent heats are much larger than the specific heats, the energy storage density is larger than that in sensible heat storage. The system has the additional advantage of operating at constant temperature. It also has the advantage of a wide choice of materials with different fusion and evaporation temperatures to suit the peaking turbine.

Storage materials must possess, in addition to proper phase transition temperature and high latent heat, good thermal conductivity, containability, chemical stability, nontoxicity and low cost. No material meets all these requirements, but some fluoride salts meet some of them. One of the most suitable salts for latent heat storage is the 70% NaF-30% FeF₂ eutectic salt, which has a fusion temperature of 680 °C and the highest storage density (of all salts) of 1500 MJ/m³. ZnCl₂ is another salt with a fusion temperature of about 370 °C and an energy density of about 400 MJ/m³. The following fluoride mixtures in Table 12.4 are chemically stable and can be contained in chromium nickel steel, suitable as storage for heat engines.
Table 12.4  Melting points of some storage media

<table>
<thead>
<tr>
<th>Storage media</th>
<th>Melting point (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sodium-magnesium fluoride, NaF/MgF₂</td>
<td>832</td>
</tr>
<tr>
<td>Lithium-magnesium fluoride, LiF/MgF₂</td>
<td>746</td>
</tr>
<tr>
<td>Sodium-calcium-magnesium fluoride, NaF/CaF₂/MgF₂</td>
<td>745</td>
</tr>
<tr>
<td>Lithium-sodium-magnesium fluoride, LiF/NaF/MgF₂</td>
<td>632</td>
</tr>
</tbody>
</table>

12.7  CHEMICAL ENERGY STORAGE

The heat of reaction of reversible chemical reactions is used to store thermal energy during endothermic reactions and to release it during exothermic reactions. Like latent heat energy storage, this form also offers large energy storage densities. The following reversible reaction along with some others as given in Table 12.5 have been suggested for energy storage.

\[
\text{CO} + 3\text{H}_2 \xrightleftharpoons[\text{Endo}]{\text{Exo}} \text{CH}_4 + \text{H}_2\text{O}
\]

For reaction from right to left at 298K, 1 atm

Energy balance is

\[
H_R + Q = H_P
\]

\[
(h_f^0)_{\text{CH}_4} + (h_f^0)_{\text{H}_2\text{O}} + Q = (h_f^0)_{\text{CO}} + 3 \times 0
\]

\[-74.9 + (-286) + Q = -110.6 + 0
\]

\[
Q = 250.3 \text{ kJ/gmol}
\]

The reaction is endothermic and energy is absorbed. The reverse reaction from left to right results in \(Q = -250.3 \text{ kJ/gmol}\), i.e. energy is released and the reaction is exothermic. The endothermic reaction is called reformation and the exothermic reaction is called methanation.

Table 12.5  Reversible chemical reactions under consideration for energy storage

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Temperature range, K</th>
<th>Heat of reaction at 298K, kJ/gmol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CO + H₂</td>
<td>&lt;=&gt; CH₄ + H₂O</td>
<td>700 -1200</td>
</tr>
<tr>
<td>2. 2CO + 2H₂</td>
<td>&lt;=&gt; CH₄ + CO₂</td>
<td>700 -1200</td>
</tr>
<tr>
<td>3. C₆H₆ + 3H₂</td>
<td>&lt;=&gt; C₆H₁₂</td>
<td>500 -750</td>
</tr>
<tr>
<td>4. C₇H₈ + 3H₂</td>
<td>&lt;=&gt; C₇H₁₄</td>
<td>450 -700</td>
</tr>
<tr>
<td>5. C₁⁰H₈ + 5H₂</td>
<td>&lt;=&gt; C₁₀H₁₈</td>
<td>450 -700</td>
</tr>
<tr>
<td>6. C₂H₄ + HCl</td>
<td>&lt;=&gt; C₂H₅Cl</td>
<td>420 -770</td>
</tr>
<tr>
<td>7. CO + Cl₂</td>
<td>&lt;=&gt; COCl₂</td>
<td></td>
</tr>
</tbody>
</table>
A schematic of a power plant with a chemical storage system is shown in Fig. 12.24. During periods of low demand, some heat from the primary heat source is diverted to the reformer (endothermic reactor) to convert the products CH₄ + H₂O to the reactants CO + 3H₂, which are stored in a vessel at high pressure, about 70 bar, but at ambient temperature. During periods of high demands, these reactants are fed to the methanator (exothermic reactor) where heat is generated to run a peak turbine. In the methanator the reactants are converted to the products CH₄ + H₂O, which are stored in a separate vessel for later use in the reformer during periods of low demand. A thermal turnaround efficiency of this system is estimated as 85 to 90 per cent.

The two storage vessels and the two reactors operate at different pressures. Storage pressures need to be high to minimize vessel size and cost and the reformer has to operate at low pressure to maximize the rate of endothermic reaction.

Fig. 12.24  *Schematic flow diagram of a power plant with a chemical storage system using the reaction CO + 3H₂ → CH₄ + H₂O*
12.8 HYDROGEN ENERGY

Hydrogen is widely regarded as the ultimate fuel and energy storage medium for future centuries. It can be derived from water (by electrolysis) using any source of high quality energy and it can be combusted back to water in a closed chemical cycle without any pollution. The potential of hydrogen for the storage and cheap transmission of energy over long distances has led to the concept of hydrogen economy (Fig. 12.25).

![Diagram of hydrogen economy fuel system]

**Fig. 12.25 The hydrogen economy fuel system**

Hydrogen can be produced by the following methods.

1. Catalytic steam reforming of natural gas
2. Chemical reduction of coal (water gas reaction)
   \[\text{C} + \text{H}_2\text{O} \rightarrow \text{CO} + \text{H}_2\]
   or
   \[\text{CO} + \text{H}_2\text{O} \rightarrow \text{CO}_2 + \text{H}_2\]
3. Industrial photosynthesis
4. Ultraviolet radiation
5. Partial oxidation of heavy oils
6. Electrolytic decomposition of water

The electrolytic decomposition of water (electrolysis) comprises two processes.

- Cathode: \[2\text{H}_2\text{O} + 2e^- \rightarrow \text{H}_2 + 2\text{OH}^-\]
- Anode: \[2\text{OH}^- \rightarrow \frac{1}{2} \text{O}_2 + \text{H}_2\text{O} + 2e^-\]

According to Faraday's law of electrolysis the mass of hydrogen m discharged may be obtained from
\[ m = \frac{A I t}{FZ} \]  

where \( A \) = atomic weight, \( I \) = electric current through the electrolyte, \( t \) = duration of electrolysis, \( Z \) = valency and \( F \) = Faraday constant = 96500 coulomb/kgm.

The overall reaction is

\[ \text{H}_2\text{O} \rightarrow \text{H}_2 + \frac{1}{2} \text{O}_2 - 237 \text{ kJ/kmol} \]

Hydrogen may be used as a primary fuel for peak power generation producing water in the reaction. During off-peak hours hydrogen may be produced by electrolysis of water with excess power available. The hydrogen based power utility concept is shown schematically in Fig. 12.26.

The main drawback of hydrogen is its extreme flammability and the problem of storing the gas under pressure. Liquefaction could simplify storage but it consumes a lot of energy.

**Fig. 12.26** Hydrogen-based power utility concept

Methane may be produced by reactions with hydrogen.

\[ \text{CO} + 3\text{H}_2 \rightleftharpoons \text{CH}_4 + \text{H}_2\text{O} \quad Q = -205 \text{ kJ} \]

or

\[ \text{C} + 2\text{H}_2 \rightleftharpoons \text{CH}_4 \quad Q = -73 \text{ kJ} \]

Methane may be used by consumers as an ordinary fuel, without being highly inflammable as hydrogen.

Ammonia may be more attractive as a fuel, which can be produced from hydrogen by combining it with nitrogen, under 200–500 bar pressure and at 720 K in presence of a catalyst (Haber process)

\[ \text{N}_2 + 3\text{H}_2 = 2\text{NH}_3 \quad Q = -90 \text{ kJ} \]
Dissociating ammonia back into hydrogen and nitrogen can be done by passing the gas through a hot tube. The advantages of storing ammonia, relative to storing hydrogen, are the following.

(a) safer storage  
(b) energy density is higher  
(c) easier to liquefy

### 12.8.1 Storage of Hydrogen

Hydrogen may be stored as (i) compressed gas (ii) chemical compound (iii) liquid (iv) metallic hydrides.

(i) Since the cost of liquefaction is high, the bulk storage of hydrogen can be made in the form of compressed gas in underground caverns, where it can be stored like natural gas.  
(ii) Hydrogen in chemical combination with other elements like methane and ammonia is more amenable to storage.  
(iii) Liquid hydrogen has a mass energy density three times greater than oil. Its use is attractive for heavy surface transport and aircraft. For energy storage, liquid hydrogen is not so attractive because of its low density. Ammonia and methane, as liquids, are more efficient in this aspect.  
(iv) The principal disadvantages of gaseous hydrogen as a storage medium is that it takes up large space, it is explosive and it is not leak-proof. Liquid hydrogen, highly cryogenic and inflammable, is costly too. These drawbacks can be fought back if hydrogen is stored in hydride form (Table 12.6). The aim is to select a hydride which can be thermally decomposed in a reversible manner so that hydrogen may be withdrawn or replenished from or to the vessel when necessary. Some of the desired features of a suitable hydride store are the following:

1. High hydrogen content per unit mass of metal.  
2. Low dissociation pressure at moderate temperatures.  
3. Constancy of dissociation pressure during the decomposition time.  
4. Safe on exposure to air.  
5. Low cost.

<table>
<thead>
<tr>
<th>Table 12.6 Mass and volume energy densities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gas at 150 atm, 20°C</strong></td>
</tr>
<tr>
<td>140000</td>
</tr>
<tr>
<td><strong>Liquid -252°C</strong></td>
</tr>
<tr>
<td><strong>Metal hydride (including metal)</strong></td>
</tr>
<tr>
<td><strong>Oil (for comparison)</strong></td>
</tr>
</tbody>
</table>

The exothermic chemical reaction of hydride formation from metals (Me) and hydrogen (H₂) is as follows:

\[
\text{H}_2 + \text{Me} \quad \text{Charging (heat released) hydride + heat}
\]

\[
\text{Discharging (heat added)}
\]
Low temperature FeTi hydride, with a low energy requirement for hydrogen release, has been developed by Brookhaven National Laboratory, USA. Magnesium-based high temperature hydride has attracted interest because magnesium is a readily available and cheap metal. Both materials release hydrogen endothermically, thus creating no safety problem. The mass energy densities \( W_m \) of hydrides based on Ti and Mg are the following:

\[
\begin{align*}
\text{FeTiH}_{1.7} & \rightarrow \text{FeTiH}_{0.1} \quad 516 \text{ Wh/kg} \quad (1856 \text{ kJ/kg}) \\
\text{Mg}_2\text{NiH}_4 & \rightarrow \text{Mg}_2\text{NiH}_{0.3} \quad 1121 \text{ Wh/kg} \quad (4036 \text{ kJ/kg}) \\
\text{MgH}_2 & \rightarrow \text{MgH}_{0.005} \quad 2555 \text{ Wh/kg} \quad (9198 \text{ kJ/kg})
\end{align*}
\]

When hydrides are used as hydrogen stores for heat engines or for domestic heaters, the waste heat from them can be returned to the hydride. If the amount of waste heat is less than the heat needed for hydrogen release, then the waste heat energy can be stored in the metal hydride, acting as a thermal storage device. A combination of hydrides with different release temperatures may be used for different applications such as heat pumps, central air conditioning and even for renewable energy storage systems.

**Example 12.1** In a simple CAES system, the average air flow into a cavern of 64,000 m\(^3\) is at the rate of 8300 m\(^3\)/h. Air enters the compressor at 1 bar, 20°C and leaves at 100 bar, the polytropic compressor efficiency being 70 per cent. If the peaking turbine efficiency is 60 per cent and air is stored in the cavern at 100 bar, 20°C, estimate (a) the compressed air temperature (b) the storage time (c) the total energy storage in MWh (d) the total energy delivered by the peaking turbine. Take \( \gamma = 1.4 \), \( c_p = 1.005 \text{ kJ/kg K} \) and \( R = 0.287 \text{ kJ/kgK} \) for air.

**Solution** With reference to Fig. E12.1

\[
\frac{T_{2s}}{T_1} = (p_2/p_1)^{(\gamma - 1)/\gamma} = (100)^{0.4/1.4} = 3.7325
\]

\[
\therefore \quad T_{2s} = 1093.62 \text{K}
\]

\[\text{Fig. E12.1}\]
\[
\frac{T_{2s} - T_1}{T_2 - T_1} = 0.7
\]
\[
\therefore \quad T_2 - T_1 = \frac{800.62}{0.7} = 1143.74
\]
\[
\therefore \quad T_2 = 1436.74 \text{K} = 1163.7 \text{°C} \quad \text{Ans. (a)}
\]

Specific volume of air at 100 bar, 20°C
\[
\frac{RT}{p} = \frac{0.287 \times 293}{100 \times 100} = 84.09 \times 10^{-4} \text{m}^3/\text{kg}
\]

Mass flow rate of air into the cavern
\[
\frac{8300}{84.09 \times 10^{-4} \times 3600} = 274.18 \text{ kg/s}
\]

Rate of energy storage
\[
\dot{m}_a \ c_p (T_2 - T_1) = 274.18 \times 1.005 \times 1143.74 \times 10^{-3} = 315.16 \text{ MW}
\]

Storage time
\[
\frac{64000}{8300} = 7.71 \text{h} \quad \text{Ans. (b)}
\]
\[
\therefore \text{ Total energy storage} = 315.16 \times 7.71
\]
\[
= 2429.88 \cong 2430 \text{ MWh} \quad \text{Ans. (c)}
\]

Total energy delivered by the peaking turbine
\[
= 2430 \times 0.6 = 1458 \text{ MWh} \quad \text{Ans. (d)}
\]

**Example 12.2** In a pressurized water sensible heat storage system, steam is extracted from the turbine during off-peak hours and fed into steel accumulators of volume 175000 m³. The accumulators are 4 m in diameter each and are well insulated so that \( U = 1.5 \text{ W/m}^2\text{K} \). The maximum and minimum storage pressures are 20 bar and 2 bar. The ambient temperature is 20°C. The specific heat of water may be taken as 4.35 kJ/kgK. Assuming the thermal turnaround efficiency of 96 per cent and a peaking plant efficiency of 25 per cent, calculate (a) the storage time (b) the total energy stored in the accumulators and (c) the total energy that can be delivered by the peaking turbine.

**Solution** At 20 bar
\[
T_1 = 212.37 \text{°C}, \quad h_{f,1} = 908.5 \text{ kJ/kg},
\]
\[
\nu_{f,1} = 0.0011766 \text{ m}^3/\text{kg}
\]

At 2 bar,
\[ T_2 = 120.23^\circ C, h_{f,2} = 504.8 \text{ kJ/kg}, \]
\[ v_{b2} = 0.0010605 \text{ m}^3/\text{kg} \]

At \[ 20^\circ C, h_f = 293 \text{ kJ/kg} \]

Taking an average density of water as
\[
\frac{1}{2} \left[ \frac{1}{v_{f,1}} + \frac{1}{v_{f,2}} \right] = 896.43 \text{ m}^3/\text{kg}
\]

The time constant \( \tau = \frac{D \rho_c c_f}{4U} \)

\[
4 \times 869.43 \frac{\text{kg}}{\text{m}^3} \times 4.35 \times 1000 \frac{\text{J}}{\text{kg K}} \times \frac{4 \times 1.5 \frac{\text{W}}{\text{m}^2 \text{K}} \times \frac{3600 \text{s}}{1 \text{h}}}{4 \times 1.5 \frac{\text{W}}{\text{m}^2 \text{K}} \times \frac{3600 \text{s}}{1 \text{h}}}
\]
\[ = 700.374 \text{ h} \]

The thermal turnaround efficiency
\[
\eta_{TA} = 1 - \frac{T_1 - T_{in}}{T_1 - T_2} = [1 - e^{-ts/\tau}] \]
\[ 0.96 = 1 - \frac{212.37 - 20}{212.37 - 120.23} \left[ 1 - e^{-ts/\tau} \right] \]
\[ 1 - 2.0878 \left[ 1 - e^{-ts/\tau} \right] = 0.96 \]
\[ 2.0878 \left[ 1 - e^{-ts/\tau} \right] = 0.04 \]
\[ 1 - e^{-ts/\tau} = 0.019159 \]
\[ e^{-ts/\tau} = 0.980841 \]
\[ e^{ts/\tau} = 1.0193 \]

\[ t_s/\tau = 0.0193 \]

\[ \therefore \text{ Storage time} = 0.0193 \times 700.374 = 13.517 \text{ h} \]

Ans. (a)

Total volume of accumulators = 175000 m\(^3\).

Mass of water needed to be flashed
\[ m = \frac{175000}{v_{f,1}} = \frac{175000}{0.0011766} = 1.4873 \times 10^8 \text{ kg} \]

Total energy stored in accumulators
\[ = m(h_{f,1} - h_{f,2}) \]
\[ = 1.4873 \times 10^8 (908.5 - 504.8) \]
\[ = 600.423 \times 10^8 \text{ kJ} = 0.166784 \times 10^8 \text{ kWh} \]
\[ = 0.166784 \times 10^5 \text{ MWh} = 16678.4 \text{ MWh} \]

Ans. (b)

Total energy that can be delivered by the peaking turbine
\[ = 16,678.4 \times 0.96 \times 0.25 = 4002.82 \text{ MWh} \]

Ans. (c)
Hidden page
12.25 What is a fuel cell? How is it different from a battery? Give the essential functions of a fuel cell. What is the emf of a fuel cell? On what factors does it depend?
12.26 Explain a hydrogen-oxygen fuel cell? Why is it attractive?
12.27 What do you mean by superconductivity? What is the concept of superconductive magnetic energy storage?
12.28 What is a Brooks coil? What is the amount of energy stored in a Brooks coil?
12.29 Why is huge structural mass required to contain magnetic energy?
12.30 What are the two distinct thermal energy storage systems? What is the typical value of storage density of sensible heat energy storage?
12.31 What is the thermal turnaround efficiency? On what factors does it depend?
12.32 How is convective heat loss from the stored saturated water estimated? Why is the energy stored in the accumulator structure very small compared to energy stored in water?
12.33 Show that the thermal turnaround efficiency of sensible heat energy storage system is given by

\[ \eta_{TA} = 1 - \frac{T_1 - T_\infty}{T_1 - T_2} \left[ 1 - e^{-t_s/\tau} \right] \]

where \( t_s \) is the duration of energy storage and \( \tau \) is the time constant.
12.34 Explain with a neat sketch a pressurized sensible heat storage system where the primary heat source is either a fossil-fueled furnace or a nuclear reactor.
12.35 What are the three main ways of operating an accumulator? Explain a Ruths accumulator. Why is steam cushion provided?
12.36 Explain the operation of an expansion accumulator.
12.37 Explain a latent heat energy storage system. What are its advantages and disadvantages compared to sensible heat energy storage?
12.38 What should be the properties of storage materials for storing latent heat? What are the suitable salts?
12.39 Explain the principle of chemical energy storage.
12.40 For the reversible chemical reaction

\[ \text{CO} + 3\text{H}_2 \rightleftharpoons \text{CH}_4 + \text{H}_2\text{O} \]

what do you mean by reformation and methanation?
12.41 Explain with a neat diagram of a power plant having a chemical energy storage system utilizing the reaction given in the Question 12.40.
12.42 What is the advantage of hydrogen as an energy storage medium? What is hydrogen economy?
12.43 What are the different methods of producing hydrogen?
12.44 What are Faraday’s laws of electrolysis?
12.45 How can hydrogen be used for peak power generation? What are the drawbacks of hydrogen as a fuel?
12.46 What are the advantages of storing ammonia. How can ammonia be produced from hydrogen?
12.47 Describe the different methods of storing hydrogen. Explain how hydrogen is stored in hydride form.

12.48 What are the desirable features of a hydride to store hydrogen?

12.49 How is hydrogen released from a hydride?

12.50 What are the advantages of magnesium-based hydrides? Give the names of some hydrides based on Ti and Mg.

## PROBLEMS

12.1 A pumped-hydro energy storage system with an elevation of 40 m is considered for a power grid which has a load pattern of 600 MW during 18h and 1200 MW during 6h for one day. Calculate (a) the power output of a power plant in MW that would meet the load demand with and without energy storage and (b) the volume of water in m³ that must be pumped to meet storage demand. The electric generator efficiency is 0.8 and density of water is 1000 kg/m³.

12.2 Calculate the air flow, compressed air temperature and storage volume for a 1500 MWh peaking unit charging for 7.5 h. Assume compressor inlet at 1 bar, 20°C, compressor exit at 100 bar, a compressor polytropic efficiency of 70 per cent, a peaking turbine efficiency of 60 per cent and a constant specific heat of air \( c_p = 1.05 \text{ kJ/kgK} \). Take \( R = 284.75 \text{ J/kgK} \) for air.

\[ \text{Ans. } 1162°C, 7.5 \times 10^6 \text{ kg}, 62,575 \text{ m}^3 \]

12.3 An adiabatic CAES system is required to generate 250 MW during 6 peak hours from storage during 12 h of operation of the power plant. There is no intercooling or reheat used. The compressor and turbine have polytropic exponents of 1.5 and 1.3 and pressure ratios of 100 and 80, respectively and mechanical efficiencies of 0.92 each. The MG set has a combined mechanical electrical efficiency of 0.97. Atmospheric air is at 1 bar, 20°C. Air enters the heat storage packed bed at 90 bar, 100°C. The packed bed has heat losses of 10 per cent and air leakage losses of 1.5 per cent. Calculate (a) the turbine air mass flow rate during peak operation in kg/s (b) the minimum volume of air storage cavern in m³ (c) the energy storage capacity in MWh (d) the compressor power input, in MW and (e) the turnaround efficiency of the system. Take \( c_p = 1.05 \text{ kJ/kgK} \) for air.

12.4 A magnetic energy storage coil is constructed of a conductor of square cross-section 400 mm \( \times \) 400 mm and a mean diameter of 32 m. Calculate the number of turns necessary for a stored energy of 5000 MWh if the current is 160000 A. Take the energy stored \( E = FE_B \), where \( E_B \) is the energy stored in a Brooks coil and \( F = 0.15 \).

\[ \text{Ans. } 11 \text{ turns} \]

12.5 A flywheel in the form of a disc 8 m in diameter and 3 m thick runs at 3000 rpm. It is made of an anisotropic filament composite material of a uniform density 2160 kg/m³. Calculate (a) the energy in the flywheel, in kWh (b) the change in rotational speed and corresponding energy that can be extracted from the flywheel if the coefficient of speed fluctuation may not exceed 0.01.
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13

Non-conventional Power Generation: Direct Energy Conversion

Fossil fuels (coal, fuel oil and natural gas) still meet the major part of our energy demand and are getting fast depleted. Moreover, there are serious pollution hazards like greenhouse effect and global warming which occur due to fossil fuel burning. The use of nuclear power too has its own problems and nuclear fusion is yet to be realized in practice. We have thus been forced to look for nonconventional power generation systems so as to reduce fossil fuel consumption by increasing the conversion efficiency from fuel to electricity. A substantial fuel economy can be achieved by converting “heat” (internal energy) directly to electricity by eliminating the link process of producing mechanical energy via steam (Rankine cycle). Major direct energy-conversion devices are magnetohydrodynamic, thermionic and thermoelectric generators and fuel cells. In this chapter we will describe the following unconventional energy-conversion systems which have considerable influence on the energy scenario of the future:

1. Magnetohydrodynamic (MHD) power generation
2. Thermionic power generation
3. Thermoelectric power generation
4. Fuel cells
5. Geothermal energy
6. Hydrogen energy system

13.1 MAGNETOHYDRODYNAMIC (MHD) POWER GENERATION

Of all the direct energy conversion methods exploitable, the MHD power generation seems to be the most promising for a utility system. The maximum limiting temperature for turbine blades being 750–800°C, the MHD generator is capable of tapping the vast potential offered by modern furnaces, which can reach temperatures of more than 2500 K, and up to 3000 K with preheating of air.
Faraday’s law of electromagnetic induction states that when a conductor and a magnetic field move relative to each other, an electric voltage is induced in the conductor. The conductor may be a solid, liquid or gas. In an MHD generator, the hot ionized gas replaces the copper windings of an alternator. When a gas is heated to high temperatures, the valence electrons of the excited atoms move on to higher quantized orbits and ultimately, at certain energy levels they fly off and become free electrons. For a gas to be conducting, a certain number of free electrons must be present along with an equal number of ions and the main body of neutral atoms. Since a very high temperature is required to ionize a gas (thermal ionization) which cannot be endured by the materials available, the hot gas is seeded with an alkali metal, such as cesium or potassium (K₂CO₃ or KOH) having a low ionization potential (energy needed to ionize one g mol of atoms) before the gas enters the MHD duct. An adequate electrical conductivity of the order of 10 mho/m can thus be realized at somewhat lower temperatures in the range 2200–2700 °C.

A simple view of the MHD generator is shown in Fig. 13.1. The duct through which the electrically conducting ionized gas flows has two sides supporting a strong transverse magnetic field of 4 – 5 tesla (1 tesla = 10⁴ gauss) at right angles to the flow and the other sides forming the faces of electrodes which are joined through an electrical circuit. As the hot ionized gas or plasma enters the MHD duct, due to the effect of the strong magnetic field and the consequent Lorentz force, there is a decrease in the kinetic energy of the plasma, and the electrons and ions get deposited on the opposite electrodes. The power generated per unit length is approximately proportional to \( \sigma u B^2/\rho \), where \( \sigma \) is the electrical conductivity, \( u \) is the velocity of the gas, \( B \) is the magnetic field strength and \( \rho \) is the density. The power produced being dc, the conversion to ac is done by an inverter. Figure 13.2 shows the principal components of a typical MHD plant and its cycle of operations on \( T-s \) diagram. It is a Brayton cycle with MHD generator replacing the combustor of the conventional GT plant.
13.1.1 Combined MHD-Steam Power Plant

If the gas entering the MHD duct at about 3000 °C could be expanded to the ambient temperature of 30 °C, the Carnot efficiency would have reached 90%. Unfortunately, the MHD power output is restricted because by the time the gas temperature falls to 2000 °C the electrical conductivity becomes very low with the electrons combining with ions to form neutral atoms, and the generator then ceases to operate satisfactorily. Therefore, the MHD generator is used as a topping unit and the MHD exhaust at about 2000 °C is utilized in raising steam to drive turbine and generate electricity in a conventional steam power plant used as a bottoming unit (Fig. 13.3). If the fraction $z$ of the fuel energy is directly converted to electricity in the MHD generator, the remainder $(1 - z)$ is converted with an efficiency $\eta'$ in the bottoming steam plant so that the overall efficiency is

$$\eta = z + \eta' (1 - z)$$  \hspace{1cm} (13.1)

If $z = 0.3$ and $\eta' = 0.4$, then $\eta = 0.58$, which is a good power plant efficiency.
MHD-topped steam plants can operate either in an open cycle or in a closed cycle. An open cycle scheme is shown in Fig. 13.3. The products of combustion

**Fig. 13.3** An open cycle MHD-steam power plant
with highly preheated air are seeded with 1% potassium before they enter the MHD duct at about 2500 – 3000 K, where some part of the internal energy of the gas (plasma) is directly converted to dc electricity, and then by dc – ac inverter to ac power. The high temperature exhaust from the MHD duct is then used in the preheating of air and in raising steam. In the steam cycle, the feedwater heaters and reheaters have not been shown. The combustion air can also be preheated indirectly using an auxiliary combustion system. Oxygen-enriched air is used when the preheated air temperature is not high enough. Since the products of combustion are exhausted to atmosphere, the oxides and hydroxides of the seeding element cause severe air pollution. The use of an electrostatic precipitator helps in the recovery of the seed, which can be used again, and also in the abatement of atmospheric pollution. An MHD–steam power plant with coal as the fuel is shown in Fig. 13.4.

In the closed cycle scheme, helium (or argon) gas seeded with cesium is heated in a nuclear reactor, passed into the MHD duct and then into the steam generating system (Fig. 13.5). A gas turbine plant can also be used as a bottoming unit (Fig. 13.6). Since the combined plant operates over a larger temperature difference, the efficiency will obviously be higher.

For MHD duct walls, the material has to stand up to temperatures above 2200 °C and the corrosive atmospheres of alkali-seeded gases. The duct wall will also need to be an electrical insulator at these temperatures. Materials used are magnesium oxide, strontium zirconate and hafnia. Electrodes in the dc MHD generator perform the same function as brushes in a conventional dc generator. Tungsten or carbon electrodes have been used. Electrodes are often segmented to reduce energy losses due to Hall effect [20]. To produce a strong magnetic field, electromagnets used consume a lot of electricity. To reduce the power consumption of these electromagnets, cryogenic or superconducting coils at liquid helium temperatures have been suggested.

Although most of the developmental efforts on MHD were based on fuels like natural gas, kerosene, benzene, toluene, fuel oil, etc., coal is inherently a better fuel than others, because it contains less hydrogen, and thus the sink of electrons in the flow created by the OH-ions is reduced. The only fuel which has better characteristics than coal is char, which contains almost no hydrogen and, in general, results in a 25% increase in the performance of the generator (Womack, 1969). It was found that there was no deterioration of the channel through chemical or thermal action of coal-slag deposits, or of the electrical performance of the duct, and there was no loss of the insulator property through penetration of the potassium seed by the coal combustion products. In fact, the slag deposits protect the electrodes and insulators and improve the breakdown properties of the channel by reducing the electrical field gradient in the flow direction.
Fig. 13.4 Component arrangement and system temperatures for a coal-fired MHD/steam power plant. (A. W. Postlethwaite and M. M. Sluyter. Mechanical Engineering, March 1978.)
**Fig. 13.5** *Closed cycle MHD-steam power plant*

The oldest MHD-steam power plant is the 75 MW unit (U–25 facility) in the erstwhile USSR, of which 25 MW is generated by MHD means. In 1981 another 1000 MW plant was commissioned near Moscow. Under the joint Indo-USSR program, a prototype of an MHD generator having 5 MW thermal input with coal gas as the fuel using 40% enriched air, 5 bar combustor pressure, a magnetic field of 5 tesla, and a seed of 50% $\text{K}_2\text{CO}_3$ in water has been built up at BHEL, Tiruchirapalli, in collaboration with BARC, Bombay [1]. The technology of MHD power generation is poised for a big leap, and is now a major contender for future power plant schemes.
**Fig. 13.6** Closed cycle MHD–gas turbine power plant

### 13.2 THERMICONIC POWER GENERATION

A thermionic generator transforms “heat” directly into electrical energy by utilizing thermionic emission. Any metal has free electrons. A metal electrode, which is called the *emitter*, is heated until it is hot enough to release electrons from its surface. The electrons cross a small gap and accumulate on a cooled metal electrode, called the *collector*. To minimize energy losses as electrons cross the gap, the space between the electrodes is either maintained at a high vacuum or filled with a highly conducting plasma like ionized cesium vapour. The electrons enter the collector and return through an external load to the emitter, thereby producing electrical power (Fig. 13.7). The emitter is positively charged, called the cathode, and the collector is negatively charged, called the

**Fig. 13.7** A basic thermionic generator
anode. A thermionic generator is thus a cyclic heat engine and its maximum efficiency is limited by Carnot’s law. It is essentially a low-voltage high-current device, where current densities of 20 – 50 amp/cm² have been achieved at voltages from 1 to 2 V. Thermal efficiencies of 10–20% have been realized, but higher values are certainly possible in the near future.

The emission of an electron from a metal surface is opposed by a potential barrier equal to the difference between the energies of an electron outside and inside the metal. Thus, a certain amount of energy must be spent to release the electron from the surface, which is referred to as surface work function (\( \phi \)). Based on statistical mechanics, the maximum electron current per unit area that an emitting surface can provide is given by the Richardson-Dushman equation:

\[
J = A_1 T^2 \exp \left( -\frac{\phi}{kT} \right)
\]

(13.2)

where \( J \) is the current density (amp/m²), \( T \) is the absolute temperature (K), \( \phi \) is the work function (eV), \( k \) is the Boltzmann constant \( (1.38 \times 10^{-23} \text{ J/molecule-K}) \) and \( A_1 \) is the emission constant which is equal to 120 amp/(cm²·K²). For different materials, the work function varies from 1–5 eV \((1 \text{ eV} = 1.602 \times 10^{-19} \text{ J})\).

The kinetic energy of the free electrons at absolute zero would occupy quantum states, or discrete energy levels from zero up to some maximum value defined by the Fermi energy level, \( \epsilon_f \) [24]. Each energy level contains a limited number of free electrons just like an electron orbit containing a limited number of electrons. Above absolute zero temperature, some electrons may have energies higher than the Fermi level. The electrons may be assumed to be vibrating about \( \epsilon_f \) with an amplitude of vibration depending on the temperature. The energy that must be supplied to overcome the weak attractive force on the outermost orbital electrons is the work function \( \phi \), so that the electron leaving the emitter has an energy level \( \phi + \epsilon_f \).

When heat is supplied to the emitter, some of the high-energy free electrons at the Fermi level get the necessary energy—energy equal to emitter work function \( \phi_e \) to escape the emitter surface—move through the gap, strike the collector and give up their K.E. \( \epsilon_f(a) \) plus the energy equal to collector (anode) work function \( \phi_a \). This energy is rejected as heat from the low temperature collector.

The electron energy is reduced to the Fermi energy level of the anode, \( \epsilon_f(a) \), but this energy state is higher than that of the electron at the Fermi energy level of the cathode, \( \epsilon_f(c) \), so that the electron can pass through the external load from the anode to the cathode. Cathode materials should thus have low Fermi levels while anode materials must have high Fermi levels. In an electron beam, the average kinetic energy of an electron is given by \( 2kT \), \( k \) being the Boltzmann constant.

The positively charged cathode tends to pull the electrons back and the electrons already in the gas exert a retarding force on the electrons trying to cross. This produces a space charge barrier.
Figure 13.8 shows a thermionic generator with an interspace retarding potential equivalent to $\delta$ volts above the anode work function $\phi_a$. Due to this potential barrier, $V_c > \phi_c$ and $V_a > \phi_a$, and the current density from cathode to anode and that from anode to cathode (back emission) are given by

$$J_c = A_1 T_C^2 \exp \left( -\frac{V_c}{KT_C} \right) \text{amp/cm}^2$$

$$J_a = A_1 T_a^2 \exp \left( -\frac{V_a}{KT_a} \right) \text{amp/cm}^2$$

(13.3)

The output voltage across the electrical resistance ($R_o$) is

$$V_o = V_c - V_a = \phi_c - \phi_a = \frac{1}{e} [\epsilon_f(a) - \epsilon_f(c)]$$

(13.4)

Each electron must overcome the interspace potential ($V_c - \phi_c$) and the work function $\phi_c$ as it leaves the cathode. In doing this work, it carries away the net energy

$$Q_{1c} = J_c(V_c - \phi_c + \phi_c) = J_c V_c \text{ W/cm}^2$$
Each electron also carries away its own K.E. which is $2KTC_e$. This component of energy transfer is

$$Q_{2c} = J_c \frac{2KTC_e}{e} \text{ W/cm}^2$$

The back emission from the anode must similarly carry energy to the cathode. The net rate of energy supply to the cathode would thus be

$$Q_1 = J_c \left( V_c + \frac{2KTC_e}{e} \right) - J_a \left( V_a + \frac{2KTC_a}{e} \right)$$  

(13.5)

where $e = 1.602 \times 10^{-19}$ coulomb.

The power output from the generator is

$$W = V_o (J_c - J_a)$$  

(13.6)

The thermal efficiency of the thermionic generator would be given by

$$\eta = \frac{V_o (J_c - J_a)}{J_c \left( V_c + \frac{2KTC_e}{e} \right) - J_a \left( V_a + \frac{2KTC_a}{e} \right)}$$  

(13.7)

where $V_o = V_c - V_a$. Substituting

$$\frac{V_c}{KT_c} = \beta_c, \quad \frac{V_a}{KT_a} = \beta_a \quad \text{and} \quad \frac{T_a}{T_c} = \theta,$$

The efficiency can be written as

$$\eta = \frac{(\beta_c KT_c - \beta_a KT_a)(J_c - J_a)}{J_c \left( \beta_c KT_c + \frac{2KTC_e}{e} \right) - J_a \left( \beta_a KT_a + \frac{2KTC_a}{e} \right)}$$

$$= \frac{(\beta_c - \theta \beta_a)[1 - \theta^2 \exp(\beta_c - \beta_a)]}{(\beta_c + 2) - \theta^2 (\beta_c + 2\theta) \exp(\beta_c - \beta_a)}$$  

(13.8)

It is found that for all values of $\theta$, the efficiency curve peaks are very near to the value of $\beta_a$ equal to $\beta_c$ [5].

Putting $\beta_c = \beta_a$, Eq. (13.8) becomes

$$\eta_{\text{max}} = [1 - \theta] \frac{\beta}{\beta + 2} \left[ \frac{1 - \theta^2}{1 - \theta^2 (\beta + 2\theta)/(\beta + 2)} \right]$$  

(13.9)

It is interesting to note that the final bracketed term is very nearly equal to unity so that

$$\eta_{\text{max}} = [1 - \theta] \frac{\beta}{\beta + 2}$$  

(13.10)

Here, $(1 - \theta)$, i.e. $(1 - T_a/T_c)$ is the Carnot cycle efficiency. If $\beta = 18$,

$$\eta_{\text{max}} = 0.9(1 - \theta)$$
Again, $\eta_{\text{max}}$ occurs when $\beta_a = \beta_c$, i.e.,

\[
\frac{V_c}{T_c} = \frac{V_a}{T_a} \quad (13.11)
\]

To reduce the space charge barrier and to promote electron emission from the cathode, ionized cesium vapour is made to fill the gap. To achieve a higher degree of ionization of the cesium, the emitter temperature must be of the range $1500 - 1600 \, ^{\circ}\text{C}$.

### 13.2.1 Thermionic Generator as Topping Unit

Attempts are being made to use thermionic generators for utility applications. The fuel elements of nuclear reactors may be the most suitable high temperature heat sources for thermionic generators (Fig. 13.9). The fuel element 1 containing the fissile material carries the cathode 2 which is surrounded by the anode 3 separated from the cathode by a space 4 filled with ionized cesium vapour. The anode is cooled on the outside by the coolant flowing through the annulus 5. Some of the energy released by nuclear fission is thus directly converted to electricity by thermionic means, and the remainder is converted in a bottoming steam plant, yielding a higher overall plant efficiency.

![Schematic of a thermionic generator in a nuclear reactor](image)

**Fig. 13.9** Schematic of a thermionic generator in a nuclear reactor

In the steam generator of a fossil-fueled steam power plant, the riser tubes are located in the radiant zone of the furnace. The energy of high temperature combustion gases can be partly converted to electricity if the riser tubes are provided with cathode and anode of a thermionic generator with the interspace filled, as usual, with ionized cesium vapour (Fig. 13.10). The use of the hot combustion gases to produce extra energy before the steam cycle in a topping unit improves the overall plant efficiency.

Another interesting application is in a topping cycle combining an MHD generator with a thermionic generator (Fig. 13.11 and Fig. 13.11a). The waste
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High temperature emitters may be made of materials like tungsten or rhenium. Ceramic shields must be provided to protect them from the corrosive combustion gases. Collectors may be made of molybdenum coated with cesium.

**Fig. 13.11a** Block diagram for a thermionic topped steam power plant

### 13.3 THERMOELECTRIC POWER GENERATION

When the junctions of two dissimilar wires $A$ and $B$ are maintained at two different temperatures a potential difference is developed (Fig. 13.12). It is called the Seebeck effect and it is the basis of temperature measurement by a thermocouple. The Seebeck coefficient or thermoelectric power is defined as

$$\alpha_{A,B} = \lim_{\Delta T \to 0} \frac{\Delta V}{\Delta T}$$

**Fig. 13.12** A thermocouple
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Let $m$ be the resistance ratio defined by

$$m = \frac{R_L}{R_p + R_n}$$

so that

$$1 + m = \frac{R_p + R_n + R_L}{R_p + R_n}$$

and from Eq. (13.22),

$$I = \frac{\alpha_{p,n} (T_1 - T_o)}{(R_p + R_n)(1 + m)}$$

The useful power is then (from Eq. 13.20),

$$W_L = \frac{\alpha_{p,n}^2 (T_1 - T_o)^2}{(R_p + R_n)(1 + m)^2} (R_p + R_n)m = \frac{m \alpha_{p,n}^2 (T_1 - T_o)^2}{(1+m)^2 R_p + R_n}$$

and the heat input (from Eq. 13.19) is

$$\dot{Q}_1 = \alpha_{p,n}^2 \frac{T_1 (T_1 - T_o)}{(R_p + R_n)(1 + m)} - \frac{1}{2} \frac{\alpha_{p,n}^2 (T_1 - T_o)^2}{(1 + m)^2 (R_p + R_n)} + (K_p + K_n)(T_1 - T_o)$$

The efficiency of the thermoelectric generator is given as

$$\eta = \frac{\dot{W}_L}{\dot{Q}_1} = \frac{\frac{m}{(1+m)^2 R_p + R_n} \frac{\alpha_{p,n}^2 (T_1 - T_o)^2}{(R_p + R_n)(1 + m)}}{\alpha_{p,n}^2 \frac{T_1 (T_1 - T_o)}{(R_p + R_n)(1 + m)} - \frac{1}{2} \frac{\alpha_{p,n}^2 (T_1 - T_o)^2}{(1 + m)^2 (R_p + R_n)} + (K_p + K_n)(T_1 - T_o)}$$

$$= \frac{T_1 - T_o}{T_1 \left(1 + m\right) - \frac{1}{2} \frac{T_1 - T_o}{T_1} + \frac{(K_p + K_n)(R_p + R_n)(1 + m)^2}{\alpha_{p,n}^2 \cdot T_1}}$$

Let $z$, called the figure of merit, be defined by

$$z = \frac{\alpha_{p,n}^2}{(K_p + K_n)(R_p + R_n)}$$

It consists of only material properties of the two semiconductors. It can be seen from Eq. (13.27) that as $z$ increases, $\eta$ will increase. If $R = R_p + R_n$ and $K = K_n + K_p$, for a pair of materials when the product $(R \cdot K)$ is minimum, $z$ (and hence $\eta$) is maximum.

Now,

$$R \cdot K = \left[ \frac{\rho_p L_p + \rho_n L_n}{A_p} \right] \left[ \frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n} \right]$$
Let $A_p/L_p = \gamma_p$ and $A_n/L_n = \gamma_n$, so that

$$R \cdot K = \left[ \frac{\rho_p}{\gamma_p} + \frac{\rho_n}{\gamma_n} \right] (k_p \gamma_p + k_n \gamma_n)$$

$$= k_p \rho_p + k_p \rho_n \frac{\gamma_p}{\gamma_n} + k_n \rho_p \frac{\gamma_n}{\gamma_p} + k_n \rho_n$$

(13.30)

Once the materials have been selected, the variables are $\gamma_n$ and $\gamma_p$, the area-to-length ratios of the legs. To minimise $R \cdot K$,

$$\frac{d(R \cdot K)}{d(\gamma_n/\gamma_p)} = 0 = k_n \rho_p - k_p \rho_n \left( \frac{\gamma_n}{\gamma_p} \right)^{-2}$$

$$\therefore \quad \frac{\gamma_n}{\gamma_p} = \left[ \frac{k_n \rho_p}{k_p \rho_n} \right]^{1/2}$$

(13.31)

On substitution in Eq. (13.30),

$$R \cdot K_{\text{min}} = k_p \rho_p + k_p \rho_n \left[ \frac{k_n \rho_p}{k_p \rho_n} \right]^{1/2} + k_n \rho_p \left[ \frac{k_n \rho_p}{k_p \rho_n} \right]^{1/2} + k_n \rho_n$$

$$= \left[ \sqrt{(\rho_p k_p)} + \sqrt{(\rho_n k_n)} \right]^2$$

(13.32)

and the maximum figure of merit is given by

$$z_{\text{max}} = \left[ \frac{\alpha_{p,n}}{\sqrt{(\rho_p k_p)} + \sqrt{(\rho_n k_n)}} \right]^2$$

(13.33)

From Eq. (13.27),

$$\eta = \frac{T_i - T_o}{T_i} \frac{m}{(1 + m) - \frac{1}{2} \frac{T_i - T_o}{T_i} + \frac{(1 + m)^2}{z T_i}}$$

(13.34)

For given values of $T_i$ and $T_o$, and with the maximum $z$ for a given pair of materials, $\eta$ depends on $m$, and to find the optimum value of $m$,

$$\frac{d\eta}{dm} = 0$$

which gives

$$m_{\text{opt}} = \left[ 1 + z \frac{T_i + T_o}{2} \right]^{1/2} = m_o, \text{ say}$$

(13.35)

By rearrangement,

$$m_o + 1 = \frac{z}{m_o - 1} \frac{T_i + T_o}{2}$$
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From this, it would seem that the greater the number of stages, the greater the efficiency. However, the maximum stage efficiency is limited by the temperature range over which it works, as given by Eq. (13.36). If each stage is optimized for geometry and resistance ratio, each stage efficiency can be improved. The greater the number of optimized stages the great the efficiency, and hypothetical infinite staging should produce a theoretical maximum value of efficiency over a given temperature range.

Figure 13.15 presents a 3-stage thermoelectric generator with the stages operating at different temperatures and the sizes successively reduced.

![Diagram of a 3-stage cascade thermoelectric generator](image)

**Fig. 13.15** A simple 3-stage cascade thermoelectric generator

The output voltage can be increased by putting a number of couples in series (Fig. 13.16). Such an arrangement is called a *thermopile* which can measure an emf signal even for a small temperature difference.

### 13.3.2 Materials of Thermoelectric Elements

Bismuth telluride, lead telluride, germanium and other semiconductor materials have properties suitable for thermoelectric generation. Alloying and "doping" make it possible to produce *p*-type and *n*-type materials. In the temperature
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Fig. 13.18  A schematic of combined thermoelectric-steam power plant

The waste heat of gas turbines and diesel engines can be utilized for thermoelectric power generation. Even stack gases can be used to produce electricity by thermoelectric means (Fig. 13.19). The metal stack consists of a series of rings of two alternate metals connected alternately at the inner and outer annular edges, and are thermally and electrically insulated. Pairs of a series of thermoelectrodes produce cumulative voltage.

Fig. 13.19  Thermoelectric waste heat stack

The decay heat of radioisotopes has been used for the operation of small (0.1 kW) thermoelectric generators.

Notwithstanding their comparatively low thermal efficiency, thermoelectric generators are already recognised as very convenient direct energy conversion systems, due to their simple and compact construction and the absence of moving parts, and have a promising future in utility systems as well as for waste heat recovery.
13.4 FUEL CELLS

The fuel cell converts chemical energy directly into electrical energy in a reaction that eliminates combustion of the fuel. Unlike a heat engine that operates on a thermodynamic power cycle, the performance of the fuel cell is not restricted by the second law of thermodynamics.

Some discussion on fuel cells was undertaken in the previous chapter. The sign convention for the cathode (+) and the anode (−) is the same for batteries and fuel cells, and for thermoelectric, thermionic and MHD generators; negative ions or electrons flow from the cathode to the anode within the device, so that the conventional current flow is from the cathode to the anode in the external circuit. The elemental particles are referred to as charge carriers. The negative charge carriers may consist of electrons or of atoms or molecules with negative charges or electrons. The positive charge carries may consist of atoms or molecules that have lost some of their electrons, or may be an electron hole (space left by the departure of an electron).

The charge of an electron (e) is 
$$-1.60 \times 10^{-19} \text{ coulomb}.$$  

To move one electronic charge over a distance with a potential difference of −1 volt, an energy of 1 eV (electron volt) is required.

$$1 \text{ eV} = -1.602 \times 10^{-19} \times -1 \text{ coulomb-volt} = 1.602 \times 10^{-19} \text{ joule}$$

A dry cell battery is shown in Fig. 13.20. The chemical equations show the separate reactions at the anode and at the cathode, and also the overall reaction of the whole cell.

Anode reaction: Electrons lost to external circuit (oxidation)

$$\text{Zn} + 2\text{OH}^- \rightarrow \text{Zn(OH)}_2 + 2e^-$$
$$\rightarrow \text{ZnO} + \text{H}_2\text{O} + 2e^-$$

Cathode reaction: Electrons gained from the external circuit (reduction)

$$\text{HgO} + \text{H}_2\text{O} + 2e^- \rightarrow \text{Hg} + 2\text{OH}^-$$

Cell reaction: Anode and cathode materials eventually depleted.

$$\text{Zn} + \text{HgO} \rightarrow \text{ZnO} + \text{Hg}$$

The anode reaction is essentially oxidation of zinc. This can be imagined as taking place in two steps as shown. The cathode reaction is essentially reduction of the mercuric oxide to mercury. It is typical of batteries that the electrodes and
sometimes the electrolyte are chemically changed and exhausted so that eventually the reaction must come to a stop.

A fuel cell could be considered as an electric battery in which both the fuel and the oxidizer are continuously replaced (Fig. 13.21). The anode and the cathode material do not normally enter into the chemical reactions although they may act as catalysts. The two electrodes must also serve the function of preventing the non-ionized fuel and oxidizer into the electrolyte between the two.

**Fig. 13.21** Fuel cell operation

Fuel cells might be divided into basic categories according to whether the product of the overall reaction must be disposed of in the cathode plenum space or in the anode plenum space, and whether the current flow through the electrolyte is a transfer of negative ions from the cathode to the anode or a transfer of positive ions in the opposite direction. Three types are illustrated in Fig. 13.22. More complicated fuel cells may actually operate as combinations of these basic types. Fuel cells are also classified according to the temperatures at which they operate.
13.4.1 Typical Fuel-Cell Reactions

In a fuel cell, the type of reactions taking place is determined by the fuel and oxidizer combination, by the composition of the electrolyte, and by the materials and the catalytic effect of cathode and the anode surfaces.

In Figs. 13.23, 13.24, 13.25 and 13.26, four different fuel cell reactions are presented. In each figure, the anode, cathode and overall cell reactions are given and the sides from which the products are removed are noted. The first two figures of this group are both hydrogen-oxygen cells. Although the product is water in both examples, the reactions are quite different. In Fig. 13.23, the charge carrier through the electrolyte is a positive hydrogen ion, while in Fig. 13.24, it is a negative hydroxyl ion.

Anode: $2H_2 \rightarrow 4H^+ + 4e^-$
Cathode: $4e^- + O_2 + 4H^+ \rightarrow 2H_2O(l)$
Cell: $2H_2 + O_2 \rightarrow 2H_2O(l)$
**Fig. 13.23** Hydrogen-oxygen fuel cell, Type A

**Fig. 13.24** Hydrogen-oxygen fuel cell, Type C

Anode: \[ 2H_2 + 4OH^- \rightarrow 4H_2O + 4e^- \]

Cathode: \[ 2O_2 + 4H_2O + 4e^- \rightarrow 2H_2O + 2H_2O + 4OH^- \rightarrow O_2 + 2H_2O(l) \]

\[ \therefore \quad O_2 + 4H_2O + 4e^- \rightarrow 4OH^- + 2H_2O(l) \]

Cell: \[ 2H_2 + O_2 \rightarrow 2H_2O(l) \]
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Fig. 13.26  Schematic representation of the methane-oxygen fuel cell

13.4.2 Thermodynamics of Fuel Cell Reactions

The fuel cell operates as a steady state, steady flow system (Fig. 13.27) in which fluid enters at state 1 and leaves at state 2. Ideally the system should operate isothermally.

Writing the S.F.E.E.,

\[ H_1 + Q = H_2 + W \]

\[ \therefore \quad W = H_1 - H_2 + Q \quad \text{(13.39)} \]

For ideal isothermal heat transfer,

\[ Q = T (S_2 - S_1) \quad \text{(13.40)} \]

In terms of Gibbs function,

\[ G_2 - G_1 = H_2 - H_1 - T (S_2 - S_1) \quad \text{(13.41)} \]

The work for an isothermal reversible system

\[ W_{\text{(max)}} = G_1 - G_2 = -\Delta G \quad \text{(13.42)} \]

Thermal efficiency of an ideal fuel cell
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**Hydrogen-Oxygen Fuel Cell**

\[
H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)
\]

\[
\Delta G = \sum_{P} \Delta G - \sum_{R} \Delta G, \text{ where } P = \text{products}, \quad R = \text{reactants}
\]

\[
= -237.3 - 0 = -237.3 \text{ kJ/kg mol}
\]

Similarly,

\[
\Delta H = -285.99 \text{ kJ/kg mol}
\]

\[
\therefore \quad \eta_i = \frac{237.3}{285.99} = 0.83 \text{ or } 83\%
\]

\[
V_i = \frac{\Delta G}{nF} = \frac{-237.3}{1 \times 2 \times 96.529} = -1.23 \text{ V}
\]

**Carbon-Oxygen Cell**

\[
\Delta G = -394.57 \text{ kJ/kg mol}, \quad \Delta H = -393.7 \text{ kJ/kg mol}
\]

\[
\eta_i \equiv 100\%, \quad V_i = \frac{394.57}{1 \times 4 \times 96.529} = -1.02 \text{ V}
\]

**Methane-Oxygen Cell**

\[
CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O (l)
\]

\[
\Delta G = -818.48 \text{ kJ/kg mol}, \quad \Delta H = -890.91 \text{ kJ/kg mol}
\]

\[
\eta_i = \frac{818.48}{890.91} = 0.92 \text{ or } 92\%
\]

\[
V_i = \frac{-818.48}{8 \times 96.529} = -1.06 \text{ V}
\]

If 1 gmol of CH$_4$ is consumed in 1 min, estimate the ideal power output.

\[
P_i = \dot{n}F V_i = \frac{1 \times 8 \times 96.529 \times 1.06 \text{ gmol}}{60 \text{ s}} \times \frac{\text{J}}{V \times \text{gmol}} \times V
\]

\[
= 13.643 \text{ W}
\]

**13.4.3 Different Fuel Cells**

Fuel cells were originally used for manned space missions where the hydrogen and oxygen were stored in their pure form as liquids and the resulting combustion product, namely, water, was then used by astronauts for drinking. However, for terrestrial uses, in addition to utilizing the fuel and oxidant in liquefied form, cheaper methods are also utilized. These involve using hydrogen-containing gases or liquids for the fuel at the anode and air-containing oxygen at the cathode. Such cells are not as efficient as using H$_2$ and O$_2$ in pure form, but they are much cheaper.

Examples of hydrogen-containing gases and liquids which have been used and can still be used are as follows:
Hydrocarbons: methane \( \text{CH}_4 \), ethane, \( \text{C}_2\text{H}_6 \), acetylene \( \text{C}_2\text{H}_2 \), ethylene \( \text{C}_2\text{H}_4 \), propylene \( \text{C}_3\text{H}_6 \), propane \( \text{C}_3\text{H}_8 \), methanol \( \text{CH}_3\text{OH} \), hexane \( \text{C}_6\text{H}_{14} \), butene \( \text{C}_4\text{H}_8 \), butane \( \text{C}_4\text{H}_{10} \), pentane \( \text{C}_5\text{H}_{12} \), benzene \( \text{C}_6\text{H}_6 \), toluene \( \text{C}_7\text{H}_8 \), heptane \( \text{C}_7\text{H}_{16} \), octane \( \text{C}_8\text{H}_{18} \), nonane \( \text{C}_9\text{H}_{20} \), decane \( \text{C}_{10}\text{H}_{22} \).

Nitrogenous: ammonia \( \text{NH}_3 \), hydrazine \( \text{N}_2\text{H}_4 \).

The hydrocarbons are ‘cracked’ with steam giving rise to \( \text{CO} \), \( \text{CO}_2 \), and \( \text{H}_2 \). When fuel \( \text{H}_2 \) is blown through a porous metallic electrode consisting of catalysts such as platinum and noble metals, the hydrogen molecule loses two electrons (\( 2e^- \)) and becomes a doubly charged ion (\( 2\text{H}^+ \)). This is an oxidation process. Because of electrons accumulating on the surface of the metallic anode and the electrolyte acquiring positively charged ions adjacent to the electrode, a charge separation occurs resulting in a potential difference, positive on the electrolyte side and negative on the anode side, much like the plates of a capacitor. The \( \text{H}^+ \) ions pass through the electrolyte such as KOH, in which the bond is ionic with \( \text{K}^+ \) and \( \text{OH}^- \) ions being present.

At the cathode, electrons returning from the external circuit combine with oxygen and react with water in the KOH solution of the electrolyte to form hydroxyl ions:

\[
\text{O}_2 + 4e^- + 2\text{H}_2\text{O} \rightarrow 4\text{OH}^-
\]

The oxygen suffers a reduction process through combining with the electrons. These enter the electrolyte and maintain the strength of the KOH, transporting the electrons from the cathode to the anode. The \( \text{H}^+ \) and \( \text{OH}^- \) ions combine to form \( \text{H}_2\text{O} \) and go into solution.

The electrodes must be good electrical conductors and highly resistant to corrosive environment. They must also be catalytic to perform charge separation, but not take part in any chemical reaction themselves. Because fuel cells work best with platinum and other precious metals, nearly 25% of the cost of the cell is in these electrodes. The electrolyte is the carrier of charges and can be either acidic or alkaline, and be in liquid or solid state. Regeneration in which the product materials can be re-converted to fuel and oxidant reduces costs.

Five types of fuel cells developed so far are as follows:

(a) Phosphoric Acid Fuel Cell (PAFC)

The cell operates at about 200°C, \( \text{H}_2-\text{O}_2 \) cell, high pressure, efficient, 1 MW and above, 13.8 kV, platinum electro-catalyst.

(b) Alkaline Fuel Cell (AFC)

\( \text{H}_2 \) and \( \text{O}_2 \) in pure form, KOH electrolyte: electrodes porous Ni substrate with Pt support.

(c) Solid Polymer Electrolyte Fuel Cell (SPEFC)

Operates at temperatures below 100°C, high polymer electrolyte and Pt electro-catalyst.

(d) Molten Carbon Fuel Cell (MCFC)

Operates at high pressure and temperature. Electrolyte consists of molten carbonate of sodium or potassium (\( \text{NaCO}_3 \) or \( \text{KCO}_3 \)). Electrodes are made of Ni for the anode and Ag for the cathode.

(e) High Temperatures Solid Oxide Fuel Cell (HTSOFC)
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Power Plant Engineering

13.9 What do you understand by (a) surface work function, (b) Fermi energy?
13.10 What is interspace retarding potential?
13.11 What is Richardson-Dushman equation? What is its relevance in thermo-
nionic power generation?
13.12 What is the reason of filling the gap with ionized cesium vapour?
13.13 How does a thermonic generator fit as a topping unit in a (a) nuclear reactor, 
(b) riser tube of a boiler?
13.14 Explain a combined MHD-thermonic-steam power plant.
13.15 What are the suitable materials for the emitters and collectors of a thermionic 
generator?
13.16 What do you understand by (a) Seebeck effect, (b) thermoelectric power?
13.17 What do you mean by (a) Peltier heat, (b) Peltier coefficient?
13.18 Explain the principle of thermoelectric power generation.
13.19 What do you understand by “figure of merit”? When does its value become 
the maximum?
13.20 What is the optimum resistance ratio for (a) maximum power, (b) maximum 
efficiency?
13.21 Explain the cascade multi-stage operation of thermoelectric generators.
13.22 What are the suitable materials for thermoelectric elements?
13.23 How can a thermoelectric generator be incorporated in the fuel elements of 
nuclear reactors?
13.24 Explain how a thermoelectric generator can be used as the topping unit of a 
steam power plant.
13.25 Explain how the waste heat of gas turbines and diesel engines can be utilized 
for thermoelectric power generation.
13.26 Briefly discuss the merits of thermoelectric power generators.
13.27 How does a fuel cell operate? How is it different from a battery?
13.28 Explain the reactions in a dry cell battery.
13.29 What are the three types of full cell reactions? Give the hydrogen-oxygen, 
carbon-oxygen and methane-oxygen fuel cell reactions.
13.30 Explain the thermodynamics of fuel-cell reactions.
13.31 What is a Faraday? What do you mean by the ideal voltage of a fuel cell?
13.32 Explain the PAFC, AFC, MCFC and HTSOFC.
13.33 Give the applications of fuel cells.
13.34 Explain geothermal heat as an energy source. What are the sites where 
geothermal power plants have been established?
13.35 Give the five types of geothermal systems in commercial use.
13.36 Enumerate the advantages of hydrogen as a fuel.
13.37 Explain the different methods of producing hydrogen.

PROBLEMS

13.1 The duct of an MHD generator has a constant spacing between electrodes of 
0.4 m. Each electrode has an area of 0.5 m². Ionized gas with an electrical 
conductivity of 30 (ohm-m)⁻¹ flows through the duct at an average velocity of
800 m/s. A cross magnetic flux of 2.5 Wb/m² is applied, and an external resistance (load) of 0.04 ohm is connected across the electrodes. What is the theoretical power output?

13.2 A thermionic generator with a cathode work function of 2.2 V and an anode work function of 1.6 V operates at a cathode temperature of 1400 K. For ideal values, assume zero retarding voltage in the interspace. (a) What should be the approximate anode temperature? (b) What would be the ideal current density? (c) Calculate the ideal power output per cm² and the thermal efficiency.

13.3 Design a thermoelectric generator to operate from a heat source of 1000 K and to reject heat at 600 K. The required output is 50 W at 6 V. The properties of the materials to be used are: \( \alpha_{p,n} = 0.001 \) V/K, \( k_p = 0.03 \) W/cm-K, \( k_n = 0.02 \) W/cm-K, \( \rho_p = 0.005 \) ohm-cm, \( \rho_n = 0.006 \) ohm-cm. Assume the thermoelectric elements to be 1 cm in length.

[Ans. 0.51 cm², 0.685 cm²]

13.4 Certain elements A and B have the following properties in the temperature range of interest:

\[
\begin{align*}
\alpha_{p,n} & = 0.003 \text{ V/K}, \\
K_p, K_n & = 0.04 \text{ W/K} \\
R_p, R_n & = 0.025 \text{ ohm}
\end{align*}
\]

The elements operate between junction temperatures of 1250 K and 750 K. Determine (a) the maximum output and the efficiency at maximum output, (b) input power and terminal voltage at no load, and (c) the input power and current under short-circuit condition.

[Ans. (a) 11.25, 12.4%, (b) 40 W, 1.5 V, (c) 130 W, 30 A]

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**BIBLIOGRAPHY**

Fossil fuels are primarily used to generate electricity, to produce heat in industrial processes as well as to drive engines for transport purposes in land, water and air. The products of combustion of such fuels are CO₂, CO, H₂O, SO₂, NO₂, NO, etc. For many years, CO₂ was regarded as a benign gas as long as it was not inhaled in great amounts, for it is not poisonous. Since CO₂ is released into the atmosphere, photosynthesis in which green plants use CO₂ to produce sugars like glucose, kept levels of CO₂ in balance to sustain the biosphere. Indeed, plants and therefore life, could not survive without CO₂. But gradually, as the decades went by, scientists began to realise that the massive increase of CO₂ in the atmosphere due to industrial activities worldwide and deforestation has been causing a great harm to the world. This CO₂ gas acts like the glass in a greenhouse, trapping the sun’s heat and causing the earth’s atmosphere to warm up.

Unlike CO₂, carbon monoxide is poisonous. It asphyxiates, or suffocates, people who inhale it. It causes headache, dizziness and confusion. It tricks red blood cell which feed CO to the brain, instead of oxygen, leading to its malfunction.

Nitrogen and oxygen combine during combustion (at high temperature) to form NO, NO₂, N₂O, and so on, termed together as NOX. These pollute the air which when inhaled, turns into acid in the lungs which eats away at the lungs spongy structure causing it to lose the capacity to absorb oxygen from the air. Less oxygen is thus passed into the blood stream as a result of which a person suffers shortness of breath, and lung and bronchial illnesses, leading often to death.

If there is sulphur in fuel, it is emitted in the form of sulphur oxide like SO₂ or SO₃, termed together as SOX. It causes similar health problems as NOX especially those of the respiratory organs. In the atmosphere it may combine with rainwater to form sulphuric acid turning the rainwater acidic. When this rainwater falls on the earth, it scorches the earth making the soil absolutely infertile and destroys aquatic life in rivers and lakes.
Pollutants in the air (acids, smog, dust, etc.) clog the tiny pores in leaves, stopping trees from photosynthetic activity and respiration which helps plants breakdown food to give them energy.

A large source of pollution comes from burning wastes in incinerators. If burning occurs without any effort to collect the dust and debris, these rise high in the air and stay there, becoming part of the dirt contained in smog, eventually falling as part of the dust and grit, along with chemicals released from plastics, packing and other disposable articles. Thus, incinerators, manufacturing chemineys belching out smoke and debris, exhaust pipes of cars and trucks, and stacks of electric power plants are sources of air pollution.

14.1 GREENHOUSE EFFECT

A greenhouse is designed with a transparent roof and side panels, normally made of glass, that allow sunlight to reach the plants kept inside, keeping them warm, so that in the middle of a frigid winter outside, a temperatre or tropical climate keeps the plants lush and green inside. The greenhouse effect for the earth works in the same way. Certain gases in the atmosphere act as a transparent roof, the most abundant of these "greenhouse gases" being carbon dioxide. The others are methane, carbon monoxide, hydrocarbons and chlorofluorocarbons (CFCs). These gases are transparent, so that the high-temperature radiation from the sun passes through the atmosphere and reaches the earth, but the low-temperature radiation from the earth is prevented from escaping into the outer space by the greenhouse gases (Fig. 14.1). The energy absorbed and trapped by the CO₂ gas heats up the atmosphere causing global warming.

With the increasing energy needs of a growing population and economy, the increase of fossil fuel consumption has caused the atmospheric CO₂ level to rise, which in turn causes the earths' temperature to rise. This has started to melt the ice caps at the North and South Poles, retreating the glaciers and snowlines and causing the ocean levels to rise. It is projected that at this rate of CO₂ release, the oceans will rise 2.0 – 2.5 metres by the end of this century, as a result of which the coastal plains, which are the most fertile, will be severely flooded with the consequent shortage of habitable and agricultural land.

A great deal of CO₂ is dissolved in the sea. If the temperature rises due to increasing CO₂ levels, then the top 70 m of seas will also heat up, releasing some of the dissolved CO₂ which will further add to the greenhouse effect. This will increase the temperature of the seas even more, releasing even more CO₂, and so on. This is called the catastrophic greenhouse effect. Also, an increase in the sea's surface temperature would substantially increase the number and severity of hurricanes leading to hundreds of deaths, millions becoming homeless and also great property damage.

The greenhouse effect causes heavy rainfall and consequent flooding in one part of the world, while droughts occur in other parts of the world to balance the water cycle. Both cause hardships to society and enormous economic losses.
14.2 ACID RAIN

When water vapour in clouds condenses into water droplets, they fall as rain through the atmosphere mixing with polluting gases such as oxides of sulphur, nitrogen and carbon, forming dilute acids—sulphuric acid, nitric acid and carbonic acid. The first two are very strong corrosive acids and are cited as the main culprits causing acid rains. However, the third one, carbonic acid, although much weaker than the first two, could actually be more damaging because it is produced in much greater quantities.

The rain containing these acids falls everywhere—into lakes, rivers and oceans, over forests, fields and farms and on to homes, buildings and structures. Everything which comes into contact with rainwater is subjected to the corrosive
effects of the acids, which are harmful to everything, both to living beings and to material objects, and as the acid concentrations increase, the rainwater becomes more destructive and the acid concentrations increase as fossil fuel consumption increases.

As a result of acid rains, the waters in lakes, ponds or estuaries become more and more acidic, affecting greatly the flora and fauna, decimating fish and all aquatic organisms. It causes irreparable damage to forests and farms, affecting the quality and quantity of farm produce. Acid rain damages the protective coatings of paint and exterior metallic trims on cars, buses and other vehicles. Acidified drinking water can cause various ailments, especially in the kidneys and urinary tracts. Hundreds of species of plants and animals become extinct every year due to the effects of fossil fuel generated pollutants and acid rains.

### 14.3 SMOG

The basic difference between fog (water vapour condensing on solid particles) and smog in which a complex organic compound, per oxyacetyl nitrate (PAN) condenses or gets adsorbed on pieces of particles like flyash or on dust floating in the air. It causes irritation to the eyes, attacks bronchial tubes and even causes death of a person having a respiratory disease, if the smog is thick enough.

### 14.4 NUCLEAR RADIATION

A properly constructed well-maintained nuclear reactor is harmless. However, nuclear plants are not always well managed as it happened in Three-Mile Island (USA) and Chernobyl (erstwhile USSR). Emitting radiations of high as well as low intensity may be extremely hazardous to human life and animals. So, nuclear power plants may also not be so benign as they are thought out to be. Any kind of failure, either human or otherwise, may cause a great catastrophe.

### 14.5 SOLAR ENERGY

The radiation continuously showered on earth by the sun represents the most basic and inexhaustible source of energy which is the mother of all forms of energy—conventional or non-conventional, renewable or nonrenewable, the only exception being nuclear energy. Plants use solar energy to effect photosynthesis, converting the carbon derived from atmospheric CO₂ to plant tissues. This gives rise to plant biomass. Animals including humans use plant biomass to get food energy. The animal biomass results from the animal tissues generated by food consumption. This plant or animal biomass becomes food energy for carnivores and the source of work. Apart from food, the plant biomass is also a major energy source—it is either directly converted into heat by burning or is converted to chemicals like methane, methanol, coke, etc. which in turn become sources of energy.
The heat from the sun causes continuous evaporation of water from the oceans, lakes, rivers, plants and soil. The sun also heats up the air. Due to the differences in the nature of terrains, altitudes and distances from the sun, this heating is not uniform. The air acquires different temperatures at different points horizontally as well as vertically. This leads to winds—slow and fast, providing wind energy. The evaporation of water coupled with action of winds drives what we call hydrological cycle leading to rains and changes in weather. Water stored at an elevation when allowed to flow by gravity, is a major source of energy—hydropower or hydel energy. The waves generated in oceans by winds, and the gravitational pull of the sun and moon contain in them wave energy. The sun heats up the top layer of oceans (up to the depths to which its light penetrates). The bottom darker layers in oceans remain cooler than the top layers. This difference in temperatures is used to generate OTEC-Ocean Thermal Energy Conversion.

Thus all renewable energy forms, with the exception of breeder reactors, are solar in origin.

The nonrenewable sources such as petroleum, coal and lignite, called fossil fuels, are also solar in origin—generated by the action of heat, pressure and time on forests and animals that existed some millions of years ago.

Solar energy is thus very large and inexhaustible: The power from the sun intercepted by the earth is about $1.8 \times 10^{11}$ MW, which is many thousands of times larger than the present consumption rate of all commercial energy sources. Thus solar energy, in principle, could supply all the present and future energy needs of the world.

In addition to its size, solar energy has two other big advantages. Firstly, unlike fossil fuels and nuclear power, it is an environmentally clean source of energy. Secondly, it is free and available in adequate quantities in almost all parts of the world.

However, there are certain problems associated with its use, such as (1) it is a dilute source of energy, hardly exceeding 1 kW/m². Thus large collecting areas are required in many applications resulting in excessive costs. (2) Availability of solar energy varies widely with time in the day-night cycle as well as from season to season. Consequently, the energy collected when the sun is shining must be stored for use during periods when it is not available.

Various methods of solar energy utilization are given in Table 14.1. It is seen that the energy from the sun can be used directly and indirectly. The direct means include thermal and photovoltaic conversion, while the indirect means include the use of water power, the winds, biomass and OTEC.

| Table 14.1  Methods of Solar Energy Utilization |
| Solar Energy Utilization |
| Direct Methods          | Indirect Methods            |
| Thermal                | Photovoltaic               | Water power | Wind | Biomass | OTEC | Wave energy |

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energy can be stored as sensible heat or latent heat. Sensible heat storage is usually done in an insulated container containing a liquid like water or a porous solid in the form of pebbles or rocks (Fig. 14.6). The first type is preferred with liquid collectors, while the second type is compatible with air heaters. In the case of latent heat storage, heat is stored in a substance when it melts and extracted when the substance freezes.

![Diagram of two forms of sensible heat storage](image)

**Fig. 14.6 Schematic of two forms of sensible heat storage:** (a) liquid, (b) porous solid

A novel device which combines the functions of both collection and storage is the *solar pond*. It consists of an expanse of water about a metre or two in depth in which salts like sodium or magnesium chloride are dissolved. The concentration of the salt is more at the bottom and less at the top. Because of this, the bottom layers of water are denser than the surface layers even if they are hotter and thus natural convection does not occur. The heat from the sun’s rays absorbed at the bottom of the pond is retained in the lower depths and the upper layers of water act like a thermal insulation.

### 14.5.2 Solar Thermal Power Generation

Solar thermal power cycles can be broadly classified as low, medium and high temperature cycles. Low temperature cycles generally use flat-plate collectors so that maximum temperatures are limited to about 100°C. Medium temperature cycles work in the range of 150°C to 300°C, while high temperature cycles work at maximum temperatures above 300°C.

**Low Temperature Rankine Cycle** A typical system working on a low temperature Rankine cycle is shown in Fig. 14.7. The energy of the sun is collected by water flowing through an array of flat-plate collectors. To get still
higher temperature, booster mirrors which reflect radiation on to the flat-plate collectors are sometimes used. The hot water at about 100°C is stored in a well-insulated thermal storage tank. From here, it flows through a vapour generator through which the working fluid of the Rankine cycle is also passed. The working fluid has a low boiling point. Consequently, vapour at about 90°C and a pressure of a few atmospheres leaves the vapour generator. This vapour then executes a regular Rankine cycle by flowing through a prime mover, a condenser and a pump. The working fluids normally used are organic fluids like methyl chloride and toluene and refrigerants like R-11, R-113 and R-114. Since the cycle operates through a small temperature difference \( t_{\text{sat,vap}} - t_{\text{cond}} \approx 55^\circ\text{C} \), the Rankine efficiency hardly exceeds 7-8%. The efficiency of the collector system is of the order of 25%. Hence an overall efficiency of only about 2% is obtained.

**Fig. 14.7 Low temperature Rankine cycle**

Plants of this type of French design having capacities up to about 50 kW have been installed in many parts of the world. However, such plants have been found to be costly, because of the large collector area required. If it is coupled to solar ponds, it may be cost effective, as done in Israel [1].

**b) Medium Temperature Rankine Cycle** Temperatures around 150 to 300°C can be attained by using arrays of cylindrical parabolic focusing collectors. The axes of these collectors could be oriented north-south or east-west and tracking about these axes would be required. The fluid flowing through the absorber tubes is a high-boiling-point liquid. It is stored in tanks and drawn through a heat-exchanger in which it transfers heat to high pressure water which is converted to steam. The high pressure steam executes a Rankine cycle. Fluids other than steam have been considered. Fluorine-based fluids are quite suitable, yielding a cycle efficiency of about 20% and an overall efficiency of 10%.

**c) High Temperature Rankine Cycle** For generating temperatures higher than 400°C and for operating Rankine cycles with efficiencies comparable to those obtained in conventional power plants, a concept which is being seriously considered both in the United States and Europe is the Central Tower Concept (Fig. 14.8). Solar radiation reflected from heliostats, i.e. arrays of large mirrors, is here concentrated on a boiler situated at the top of a supporting tower. This thermal input to the boiler is used to operate a high-temperature Rankine cycle. At the moment, a few prototypes are either in operation or under construction.
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Using solar energy, hydrogen can be produced in four different ways—direct heat, thermochemical, electrolytic and photolytic.

In the direct-heat method, water is heated to form steam and then steam is superheated to about 1400°C or more, at which stage the molecules of steam start splitting apart to form hydrogen and oxygen. As the temperatures are further increased, the rate at which the steam molecules split apart increases. The same effect can also be achieved by reducing the steam pressure. Thus higher temperatures and lower pressures are the best ways to produce hydrogen using the direct-heat method. Large concentrating parabolic mirrors would then be used to focus the solar energy on to containers of water. Such a system is called a solar furnace, since it produces very high temperature—steam without pollution.

In the thermochemical method, temperatures as high as 2500-3000°C are not required to split steam. If much cooler steam at 300-1000°C is passed over powdered iron, the iron soaks up the oxygen, forming iron oxide—rust—and leaving hydrogen. The rust can be then heated to make it release the oxygen, leaving us with unrusted, powdered iron again. By doing this over and over again, with large amounts of powdered iron, we could obtain a supply of hydrogen gas.

In the electrolytic method, cells similar to the cells in a car battery are used to produce hydrogen and oxygen from water. Each cell consists of two electrodes immersed in an electrolyte of water plus some chemicals that conduct electricity well, and is connected to a direct current (d.c.) electricity supply. When enough electricity is supplied between the electrodes to cause a current to flow, oxygen is produced at one end (the anode) and hydrogen at the other (the cathode). Instead of splitting steam with heat, we are splitting water with electricity.

In the photolytic method, the sun’s energy is directly used to split water into hydrogen and oxygen. Water ‘soaks up’ minute light particles in the sun’s rays, called photons. When water has absorbed enough photons, it splits into hydrogen and oxygen. This phenomenon is called photolysis. The photons in the ultraviolet portion of sunlight have the higher energy needed for the direct photolysis of water. However, most of the ultraviolet radiation is absorbed in the upper atmosphere by the ozone layer. Consequently, not much UV radiation reaches the earth.

For large-scale storage of solar hydrogen, unused caverns resulting from mining, etc., can be used (see Chapter 12). On a small-scale, hydrogen can be transported and distributed as a gas in a tanker truck, but for industries requiring lots of energy, pipelines are the most economical way to transport and distribute large quantities of hydrogen.

Hydrogen can be converted to electricity using three different systems—gas turbines, steam turbines and fuel cells (Fig. 14.11). Gas turbines can be run on hydrogen gas instead of natural gas, generating mechanical or electrical energy clearly and efficiently. Steam could also be produced cleanly by burning hydrogen in pure oxygen, which can be operated in a conventional Rankine cycle. There is also a third method of converting hydrogen into electricity that is
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Hydrogen is the best fuel for fuel cells. If this hydrogen is obtained from a fossil fuel or an alcohol fuel like methanol, then we will still have the pollutants that cause the greenhouse effect, the acid rains and the smog. But when hydrogen is obtained from water, we don’t get those pollutants. One day, every house, factory, shop, office block, etc., could be powered by its own fuel cell, but we must ensure that the hydrogen used to fuel the cell is obtained from the safest source—water.

14.5.4 Photovoltaic Conversion

In photovoltaic conversion, solar radiation falls on semi-conductor devices called solar cells which convert the sunlight directly into electricity. A schematic diagram of a photovoltaic cell (PV-cell) or solar cell is given in Fig. 14.12. It relies on the effect that light has on the junction between two types of semiconductors called $p$-type and $n$-type. $N$-type has an excess of electrons and $p$-type has a shortage of electrons. When a bright light shines on a cell, energy from the light (photons) enables electrons to break free from the junction between them. This is called the photoelectric effect. For single-crystal silicon (4 valence electrons), ‘$p$’ is obtained by doping silicon with boron (3 valence electrons) and is typically 1 $\mu$m thick; ‘$n$’ is obtained by doping with arsenic or phosphorous (5 valence electrons) and is typically 800 $\mu$m thick. The sun’s photons strike the cell on the microthin $p$-side and penetrate to the junction to generate electron-hole pairs. When the cell is connected to a load, as shown, the electrons will diffuse from $n$ to $p$. The direction of the current $I$ is in the opposite
Diagrams of (a) a silicon cell, (b) another view of a PV cell (c) a solar cell

**Fig. 14.12**

Fig. 14.13 Typical performance characteristics of a silicon solar cell at two solar radiations

direction of the electrons. Typical voltage current characteristics are shown in Fig. 14.13 at two different solar radiation levels, for each of which $V_{oc} =$ open-
circuit voltage, $I_{SC} = \text{short-circuit current}$. The ideal power of the cell is $V_{OC} \cdot I_{SC}$. The maximum useful power is the area of the largest rectangle that can be formed under the $I-V$ curve. If the voltage and current corresponding to this situation are denoted by $V_m$ and $I_m$ then the maximum useful power is $V_m \cdot I_m$. The ratio of the maximum useful power to the ideal power is called the fill factor ($k$). Typical values of these factors for a silicon cell are:

$$V_{OC} = 450 \text{ to } 400 \text{ mV}, \quad I_{OC} = 30 \text{ to } 50 \text{ mA/cm}^2, \quad k = 0.65 \text{ to } 0.80.$$  

Solar cells in the form of thin films or wafers convert from 3% to less than 30% of incident solar energy into d.c. electricity. Connection of such cells into series-parallel configurations permits the design of solar ‘panels’ with high voltages as high as several kilovolts. Combined with energy-storage and power-conditioning equipment, these cells can be used as an integral part of a complete solar-electric conversion system.

The remarkable simplicity of a solar-voltaic system would make it appear a highly desirable energy system for terrestrial purposes, apart from its use in space applications. The principal advantages are that they have no moving parts, require little maintenance, work quite satisfactorily with beam or diffuse radiation and can work as modular systems ranging from a few watts to megawatts. However, the extremely high costs of development and fabrication of solar arrays have discouraged widespread use of such cells.

Monocrystalline silicon is costly to produce since it cannot be made in a continuous automated fabrication process. Considerable work is in progress to develop continuous production processes for cell manufacture. One of the most interesting applications for the large scale use of photovoltaic cells has been suggested by Glaser [4]. He had proposed a concept of satellite power station in which a satellite would be placed in a geosynchronous orbit far above the earth (Fig. 14.14). The satellite would consists of a large array of solar cells, many square kilometres in area. The d.c. power generated by these cells, when solar radiation falls on them, would be fed to microwave oscillators and large antennas would beam the output of the oscillators towards the earth. Antennas on the earth would receive the microwave beam energy, which would then be converted back to d.c. power. The array is space would not be subject to any day-night cycles and it would receive a much higher intensity of solar radiation than an earth, which is claimed to be a big advantage.

**Fig. 14.14** Satellite solar power station concept
14.6 WIND ENERGY

Wind is induced chiefly by the uneven heating of the earth’s crust by the sun. Thus wind energy is rightly an indirect form of solar energy. Winds can be classified as planetary and local. Planetary winds are caused by greater solar heating of the earth’s surface near the equator than near the north or south poles. This causes warm tropical air to rise and flow through the upper atmosphere towards the poles and cold air from the poles to flow back to the equator nearer to the earth’s surface. The direction of motion of the planetary winds is affected by the rotation of the earth. Local winds are caused by differential heating of land and water, and also by hills and mountain sides. Windmills played an important role in water pumping throughout the world. Recent development of wind energy has concentrated on the generation of electricity.

The function of a windmill is to extract energy from the wind and to produce mechanical energy which may then be converted to electricity. Many types of windmills have been designed and developed. However, only a few have been found to be practically suitable and useful. Some of these are: (i) Multiblade type, (ii) Sail type, (iii) Propeller type, (iv) Savonius type, and (v) Darrieus type. The first three have a horizontal axis, while the last two have a vertical axis (Fig. 14.15). A sketch of a historic four-blade Dutch windmill is also shown in the same figure. Both the multiblade and sail-type mills run at low speeds of 60 to 80 rpm. The propeller type has two or three aerofoil blades and run at speeds

![Diagram of windmill types](image-url)

**Fig. 14.15** Types of windmill rotors
and Savonius types are much lower than the values for the propeller and the Darrius types. It is also seen that the highest values of $C_p$ are obtained with the propeller type. The variation of $C_p$ for an ideal rotor without having any losses is also shown. It is assumed that only the wind blowing in the area $A_i$ is blown over the turbine and it is streamlined (Fig. 14.16a). Some thermal augmenters are often used to increase the pressure difference between the inlet and outlet sides of turbine rotor, e.g., air on the downstream side is heated by burning waste.

**Fig. 14.16 (a) Theory of wind energy conversion**

The following properties hold:

1. Mass of air flowing past an area $A_i$ per second
   \[ \dot{m}_i = \rho A_i V_i = \rho A_t V_t \text{ kg/s} \]

2. Kinetic energy per second = power available in wind
   \[ P_i = \frac{1}{2} \dot{m}_i V_i^2 = \frac{1}{2} \rho A_i V_i^3, W \]

3. Power available in the wind at exit
   \[ P_e = \frac{1}{2} \rho A_e V_e^3 = \frac{1}{2} \rho A_t V_t V_e^2 \]

4. Power developed by an ideal rotor or turbine
   \[ P_t = P_i - P_e = \frac{1}{2} \rho A_t V_t (V_i^2 - V_e^2) \]

5. We can eliminate $V_e$ by assuming
   \[ a = \frac{V_e}{V_i} \]
   and
   \[ V_t = \frac{1}{2} (V_e + V_i) \text{ giving } V_e = 2V_t - V_i \]
   \[ V_i^2 - V_e^2 = (V_i + V_e)(V_i - V_e) = 2V_t(V_t - 2V_t + V_i) \]
   \[ = 2V_t \cdot 2(V_t - V_i) = 4V_t V_i \left(1 - \frac{V_i}{V_t} \right) \]
   \[ = 4 \frac{V_t V_i}{V_i^2} \left(1 - \frac{V_i}{V_t} \right) V_i^2 = 4a (1 - a) V_i^2 \]

6. \[ \therefore P_t = \frac{1}{2} \rho A_t V_t \cdot 4a (1 - a) V_i^2 \]
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A square float moves up and down with the water, guided by four vertical manifolds that are part of a platform (Fig. 14.18). The platform is stabilized within water by four large underwater floatation tanks so that it is supported by buoyancy forces and no significant vertical or horizontal displacement of the platform due to wave motion occurs. A piston attached to the float moves up and down inside a cylinder. The piston-cylinder arrangement is used as a reciprocating compressor. The downward motion of the piston draws air into the cylinder via an inlet check valve. This air is compressed by upward motion of the piston and is supplied to the four underwater floatation tanks through an outlet check valve via the four manifolds. The four floatation tanks thus serve the dual purpose of buoyancy and air storage, and the four vertical manifolds serve the dual purpose of manifolds and float guides. An air turbine is run by the compressed air which is stored in the buoyancy-storage tanks, which in turn drives an electrical generator, producing electricity which is then transmitted to the shore through an underwater cable.

![Diagram of a float wave-power conversion machine]

**Fig. 14.18** Schematic of a float wave-power conversion machine

In a *high-level reservoir wave machine* (Fig. 14.19), a magnification piston is used and the pressurized water is elevated to a natural reservoir above the wave generator which has to be near a shoreline or to an artificial water reservoir. The water in the reservoir is made to flow through a turbine coupled to an electric generator and then back to the sea level. Calculations show that a 20 m diameter generator can produce 1 MW power.
Fig. 14.19 Schematic of a high-level reservoir wave machine (hydraulic accumulator wave machine)

A Dolphin-Type Wave-Power machine (Fig. 14.20), designed by Tsu Research Laboratories in Japan, consists of a dolphin, a float, a connecting rod and two electric generators. The float has two motions. The first is a rolling motion about its own fulcrum with the connecting rod. The other is a nearly vertical or heaving motion about the connecting rod fulcrum. It causes relative revolving movements between the connecting rod and the stationary dolphin. In both cases, the movements are amplified and converted by gears into continuous rotary motions that drive the two electrical generators.

Fig. 14.20 Schematic of a Dolphin type wave generator

Hydraulic accumulator wave machines are also used, where instead of compressing air, the water itself is pressurized and stored in a high-pressure accumulator or pumped to a high-level reservoir, from which it flows through a water turbine electric generator. A different idea of extracting energy from ocean
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earth every 24 h 50 min. During this time the tide rises and falls twice, resulting in a tidal cycle that lasts 12 h 25 min. The tidal range \( R \) is defined as

\[
R = \text{water elevation at high tide} - \text{water elevation at low tide}
\]

The rise and fall of the water level follows a sinusoidal curve, shown with point \( A \) indicating the high tide point and point \( B \) indicating the low tide point (Fig. 14.24). The average period of time for the water level to fall from \( A \) to \( B \) and then rise from \( B \) to \( C \) is each approximately equal to 6 h 12.5 min.

At times during full or new moon, when the sun, moon and earth are approximately in one line, the gravitational forces of the sun and moon are enhanced. These high tides are called *spring tides*.

![Fig. 14.24 Tides of sea](image)

On the other hand, near the first and third quarters of the moon, when the sun and moon are at right angles with respect to the earth, *neap tides* occur. The tidal range is then very small, the high tides are lower and the low tides are higher than the average. Thus the range varies during the 29.5 day lunar month (Fig. 14.25). It is the maximum at the time of the new and full moons (spring tides) and the minimum at the time of the first and third quarter moons (neap tides). The spring-neap tidal cycle lasts one-half of a lunar month. A typical mean range is roughly one-third of the spring range.

The variations in the periodicity and monthly and seasonal ranges must be taken into account in the design and operation of tidal power plants. The tides are, however, predictable, and fairly accurate tide tables are usually available. Tidal ranges vary from one location to another. They have to be very large to justify the huge costs of building dams and associated hydro-electric power plants.

The tides along most coastlines are about a metre high, but in constructed areas they may rise by 10 metres or more. In these constricted areas the most effective tidal power plants are located. A dam or sluice gate is placed across an ocean bay or estuary. An incoming tide fills up the enclosed basin while passing through a row of hydraulic turbines.
After the basin is filled with water, the gates are closed and the turbines are shut down. Then the turbine blades are reversed and the gates are opened again to let the water surge out. Thus turbines would be rotated either way to generate electric power.

A tidal power development scheme essentially involves the construction of a long barrier across a bay or estuary to create a large basin on the landward side. The barrier includes dykes, gate-controlled sluices, and the power house. Tidal power schemes may have the following different configurations:

1. Single basin, single-effect tidal power scheme.
2. Single basin, double effect tidal power scheme.
3. Linked basin scheme

In the *Single Basin, Single-Effect Tidal Power Scheme*, the basin is filled by keeping the sluices open and letting the water flow from the sea to the basin during the high tide (Fig. 14.26). Power is generated by letting the water flow from the basin to the sea through the turbines during the low tide.
Fig. 14.26 (b)  Single basin, single effect tidal power scheme: schematic top view

In a Single Basin Double-Effect Tidal power Schemes, power is generated during flood (high) tide, with water flowing from the sea to the basin through the turbines and also during ebb (low) tide, with water flowing from the basin to the sea through the turbines (Fig. 14.27). In this case, turbine blades should be reversible with proper blade angles depending upon the direction of flow.

Fig. 14.27 (a)  Single basin, double-effect tidal power scheme
In a Linked Basin (Double Basin Single Effect Tidal) Power Scheme (Fig. 14.28), there are two basins on the landward side with the powerhouse located in the barrier between the two basins. Power is generated by water flowing from the high basin to the low basin through the turbines and water flowing from the low basin to the sea during ebb tide. Turbo-generators should be capable of efficient generation at low heads, and consequently, of handling large discharges. The layout of a typical tidal power plant is shown in Fig. 14.29.
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having 24 bulb turbines of 10 MW each. In India, the prospective sites for tidal power exist in Gujarat and West Bengal (Sunderbans).

14.8 GEOTHERMAL ENERGY

Geothermal energy is primarily energy from the earth’s own interior. The natural heat in the earth has manifested itself for thousands of years in the form of volcanoes, lava flows, hot springs and geysers. The interior of the earth is thought to consist of a central molten core surrounded by a region of semifluid material called the mantle (Fig. 14.30). This is covered by the crust, which has a depth of about 30 to 90 km. The temperature in the crust increases with the depth at the rate of about 30°C/km. Below the crust, the molten mass, called

![Different layers in the cross-section of the earth](image_url)

**Fig. 14.30** Different layers in the cross-section of the earth

![A typical geothermal field](image_url)

**Fig. 14.31** A typical geothermal field
magma, is in the process of cooling at the rate of 0.063 \( \text{W/m}^2 \). Figure 14.31 shows a typical geothermal field. The hot magma near the surface (A) solidifies into igneous rock (B) or volcanic rock. Groundwater that finds its way down to this rock through fissures in it will be heated by the heat of the rock or by mixing with hot gases and steam emanating from the magma. The heated water will then rise connectively upward and into a porous and permeable reservoir (C) above the igneous rock. This reservoir is capped by a layer of impermeable solid rock (D) that traps the hot water in the reservoir. The solid rock has fissures (E) that act as vents of the giant underground boiler. The vents show up at the surface as hot springs, fumaroles, and geysers, fumaroles, or hot springs. A well (H) taps steam from the fissure for use in a geothermal power plant. Geothermal steam is of two kinds: magmatic steam that originates from the magma itself and meteoritic steam with groundwater heated by the magma. The latter is the largest source of geothermal steam.

Not all geothermal sources, however, produce steam. Some are lower in temperature so that there is only hot water. Some receive no groundwater at all and contain only hot rock. Geothermal sources are therefore of three kinds: (1) hydrothermal, (2) geopressurized, and (3) petrothermal.

**Hydrothermal systems** are those in which water is heated by contact with the hot rock which can be either vapour-dominated or liquid-dominated.

In **vapour-dominated systems** the water is vaporized into steam that reaches the surface in a relatively dry condition at about 250°C and rarely above 8 bar. This steam is suitable for use in power plants with the least cost. However, corrosive gases and erosive material are discouraging.

In **liquid-dominated systems** the hot water trapped underground is at a temperature range of 174°C to 315°C. When tapped by wells drilled, the water flows either naturally to the surface or pumped up to it. The drop in pressure to about 8 bar or less causes it to flash to a two-phase mixture of low quality i.e. liquid-dominated. It contains large concentrations of dissolved solids ranging from 3000 to 25,000 ppm. Power production is adversely affected because these solids precipitate and cause scaling in pipes and heat transfer surfaces. Liquid-dominated systems, are however, much more plentiful, and the US Geological Survey estimated 900 to 1400 quads \( Q (1 Q = 10^{15} \text{ Btu}, \text{about } 10^{18} \text{ J}) \) of energy available in these systems.

**Geopressurized systems** are sources of water or brine that has been heated in a manner similar to hydrothermal water, except that this water is trapped in much deeper underground aquifers (2400 to 9100 m deep) at relatively low temperature (\(~ 160°C\)) and very high pressure (\(\geq 1000\) bar) with high salinity (\(H-10\%\)) and is often referred to as *brine*. Also, it is saturated with natural gas, mostly methane, thought to be the result of decomposition of organic matter. There is economic feasibility of generating electricity by a combined cycle, one that involves the combustion of methane as well as heat from the thermal energy of hot water.
In *petrothermal systems*, magma lying close to the earth’s surface heats overlying rock and when no underground water exists, there is simply hot dry rock (HDR). The temperatures of HDR vary between 150°C to 290°C. This energy, called petrothermal energy, represents by far the largest resource of geothermal energy. Since the HDR is largely impermeable, to make it permeable, fracturing methods are considered which involve drilling wells into the rock and then fracturing by high-pressure water or nuclear explosives.

Figures 14.32 and 14.33 show a schematic and T-s diagrams of vapour-dominated power system. The steam at the well (1) at about 200°C is nearly saturated and may have a shut-off pressure up to 35 bar. Pressure drops through the well cause it to slightly superheat at the well head (2) where the pressure rarely exceeds 7 bar. It then goes through a centrifugal separation to remove particulate matter and then enters the turbine after an additional pressure drop (3). The steam expands through the turbine and enters the condenser at 4. Direct-contact condensers are more effective and less expensive than surface-type condensers. The turbine exhaust steam at 4 mixes with cooling water (7) that comes from a cooling tower. The mixture of 7 and 4 is saturated water (5) that is pumped to the cooling tower (6). A steam-jet ejector (SJE) is used to rid the condenser of the noncondensable gases and to minimize their corrosive effect. Examples of vapour-dominated system are the plants at the Geysers, USA, Larderello, Italy and Matsukawa, Japan.

Liquid-dominated systems can be of two types:

---

**Fig. 14.32  Schematic of a vapour-dominated powerplant**
Fig. 14.33  *T-s diagram of the cycle shown in Fig. 14.32*

1. Flash steam system, 2. Binary cycle system. Flash steam system is illustrated by the flow and *T-s* diagrams in Figs. 14.34 and 14.35. Water from the underground reservoir at 1 reaches the well head at 2 at a low pressure. Process 1-2 is a throttling process that results in a two-phase mixture of low quality at 2. This is throttled further in a flash separator to state 3. The mixture is now separated into dry saturated steam at 4 and saturated brine at 5. The latter is reinjected into the ground. The dry steam at about 8 bar is expanded in a turbine to 6 and mixed with cooling water in a direct-contact condenser with the mixture at 7 going to a cooling tower. Flashed-steam systems have been used widely in Japan, New Zealand, Italy, Mexico and elsewhere.

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**Fig. 14.34**  *Schematic of a liquid-dominated single-flash steam system*
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Hidden page
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14.29 Explain the scope of generating electricity from biomass. With the help of a typical biogas plant, explain the functions of the digester and the dome and how are animal wastes utilized to produce biogas. How can India benefit from installation of biogas plants?

BIBLIOGRAPHY

Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Hidden page
Accumulator 841
Acid rain 898
Activation energy 236
Activity 602, 835
Adiabatic flame temperature 204
Adiabatic storage 818
After condenser 574
Air-cooled blades 791
Air cooler section 573
Air preheater 374
- L Jungstom 378
- Tubular 375
Air removal 571
Air-in-cloth ratio 400
Alpha decay 606
Anaerobic digestion 173
Anion exchanger 406
Approach 580
Aquatic farm 174
Archimedes number 280
Ash handling system 402
Ash softening temperature 163
Atomic mass unit 604
Atomic number 602
Axial flow compressor 788
Back pressure turbine 28
Baghouse 400
Balanced draught 195
Barn 617
Base load 7
Batteries 827, 879
Becquerel 610
Beta decay 606
Binary vapour cycle 105
Binding energy 604
Biochemical conversion 173
Bioconversion processes 174
Biogas plants 935
Blade fastenings 501
Blade friction factor 454
Blade height 478
Blade stresses 500
Blade thickness factor 479
- Blowdown 410
- Boilers 317
- Bent tube 323
- Cross drum 323
- Fire tube 318
- Fluidized bed 379
- Forced circulation 331
- La Mont 353
- Longitudinal drum 322
- Natural circulation 329
- Nomenclature 326
- Once through 344
- Package 320
- Stirling 325
- Straight tube 321
- Supercritical 354
- Utility 317
- Water tube 321
- Water wall 336, 351
Boiling water reactor 637
- External circulation 638
- Internal circulation 638
Brayton cycle 775, 857
Brayton-Rankine combined cycle 111
Breeder reactor 642
Breeding ratio 642
Bubbling fluidized bed 279
- Atmospheric 379
- Pressurized 391
Bull turbine 695
Burners
- Combined 269
- Distributed mixing 266
- Fuel oil 306
- Gas 307
- Low NOX 267
- Multifuel 268
- Pulverized coal 263
- Straight flow 263
- Vertical 272
<table>
<thead>
<tr>
<th>Term</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vortex</td>
<td>265</td>
</tr>
<tr>
<td>Burner arrangement</td>
<td>269</td>
</tr>
<tr>
<td>Bypass governing</td>
<td>497</td>
</tr>
<tr>
<td>CANDU reactor</td>
<td>643</td>
</tr>
<tr>
<td>Capacity factor</td>
<td>4</td>
</tr>
<tr>
<td>Carnot cycle</td>
<td>48</td>
</tr>
<tr>
<td>Carryover efficiency</td>
<td>471</td>
</tr>
<tr>
<td>Cascade multistage operations</td>
<td>874</td>
</tr>
<tr>
<td>Casing arrangement</td>
<td>484</td>
</tr>
<tr>
<td>Catchment area</td>
<td>662</td>
</tr>
<tr>
<td>Cavitation</td>
<td>698</td>
</tr>
<tr>
<td>Central tower concept</td>
<td>904</td>
</tr>
<tr>
<td>Centrifugal compressor</td>
<td>785</td>
</tr>
<tr>
<td>Centrifugal fans</td>
<td>198</td>
</tr>
<tr>
<td>- Forced draught</td>
<td>195</td>
</tr>
<tr>
<td>- Induced draught</td>
<td>195</td>
</tr>
<tr>
<td>- Output control</td>
<td>198</td>
</tr>
<tr>
<td>Centrifugal governor</td>
<td>495</td>
</tr>
<tr>
<td>Cetane number</td>
<td>754</td>
</tr>
<tr>
<td>Chain grate stoker</td>
<td>245</td>
</tr>
<tr>
<td>Chain reaction</td>
<td>612</td>
</tr>
<tr>
<td>Chemical energy storage</td>
<td>844</td>
</tr>
<tr>
<td>Chimney, maximum discharge</td>
<td>193</td>
</tr>
<tr>
<td>Choked flow</td>
<td>439</td>
</tr>
<tr>
<td>Circulating fluidized bed</td>
<td>289</td>
</tr>
<tr>
<td>- Atmospheric</td>
<td>290</td>
</tr>
<tr>
<td>- Combustion</td>
<td>287</td>
</tr>
<tr>
<td>- Pressurized</td>
<td>291</td>
</tr>
<tr>
<td>Circulating water system</td>
<td>579</td>
</tr>
<tr>
<td>Circulation</td>
<td>329</td>
</tr>
<tr>
<td>- Forced</td>
<td>331</td>
</tr>
<tr>
<td>- Natural</td>
<td>329</td>
</tr>
<tr>
<td>- Types</td>
<td>343, 396</td>
</tr>
<tr>
<td>Cladding</td>
<td>626</td>
</tr>
<tr>
<td>Coal analysis</td>
<td></td>
</tr>
<tr>
<td>- Proximate</td>
<td>160</td>
</tr>
<tr>
<td>- Ultimate</td>
<td>166</td>
</tr>
<tr>
<td>Coal gasifiers</td>
<td>160, 296</td>
</tr>
<tr>
<td>Coal liquefaction</td>
<td>172</td>
</tr>
<tr>
<td>Coal properties</td>
<td>161</td>
</tr>
<tr>
<td>- Ash softening temperature</td>
<td>162</td>
</tr>
<tr>
<td>- grindability</td>
<td>161</td>
</tr>
<tr>
<td>- Heating value</td>
<td>161</td>
</tr>
<tr>
<td>- Sulphurcontent</td>
<td>161</td>
</tr>
<tr>
<td>- Swelling index</td>
<td>161</td>
</tr>
<tr>
<td>- Weatherability</td>
<td>161</td>
</tr>
<tr>
<td>Coal types</td>
<td>159</td>
</tr>
<tr>
<td>Coal-based combined cycle</td>
<td>133</td>
</tr>
<tr>
<td>Coal-water mixture</td>
<td>166</td>
</tr>
<tr>
<td>Coefficient of discharge</td>
<td>444</td>
</tr>
<tr>
<td>Coefficient of velocity</td>
<td>444</td>
</tr>
<tr>
<td>Cogeneration</td>
<td>78</td>
</tr>
<tr>
<td>Collection efficiency</td>
<td>385, 399</td>
</tr>
<tr>
<td>Combined cycle power generation</td>
<td>119</td>
</tr>
<tr>
<td>Combined MHD-steam plant</td>
<td>858</td>
</tr>
<tr>
<td>Combustion</td>
<td>177</td>
</tr>
<tr>
<td>- Actual air-fuel ratio</td>
<td>179</td>
</tr>
<tr>
<td>- Basic conditions</td>
<td>177</td>
</tr>
<tr>
<td>- Char</td>
<td>287</td>
</tr>
<tr>
<td>- Equation</td>
<td>182</td>
</tr>
<tr>
<td>- Excess air</td>
<td>178</td>
</tr>
<tr>
<td>- Fuel oil</td>
<td>304</td>
</tr>
<tr>
<td>- Gas</td>
<td>307</td>
</tr>
<tr>
<td>- Stoichiometric air</td>
<td>188</td>
</tr>
<tr>
<td>Compounding of steam turbines</td>
<td>457</td>
</tr>
<tr>
<td>Compressed air energy storage</td>
<td>818</td>
</tr>
<tr>
<td>Condensate polishing</td>
<td>407</td>
</tr>
<tr>
<td>Condensers</td>
<td>562</td>
</tr>
<tr>
<td>- Barometric</td>
<td>565</td>
</tr>
<tr>
<td>- Contact type</td>
<td>563</td>
</tr>
<tr>
<td>- Jet</td>
<td>565</td>
</tr>
<tr>
<td>- Surface type</td>
<td>565</td>
</tr>
<tr>
<td>Condenser efficiency</td>
<td>573</td>
</tr>
<tr>
<td>Condition line</td>
<td>489</td>
</tr>
<tr>
<td>Conduits</td>
<td>673</td>
</tr>
<tr>
<td>Cooling towers</td>
<td>580</td>
</tr>
<tr>
<td>- Approach</td>
<td>580</td>
</tr>
<tr>
<td>- Calculations</td>
<td>586</td>
</tr>
<tr>
<td>- Cooling efficiency</td>
<td>581</td>
</tr>
<tr>
<td>- Dry type</td>
<td>583</td>
</tr>
<tr>
<td>- Forced draught</td>
<td>581</td>
</tr>
<tr>
<td>- Induced draught</td>
<td>581</td>
</tr>
<tr>
<td>- Natural draught</td>
<td>582</td>
</tr>
<tr>
<td>- Packing or fill</td>
<td>581</td>
</tr>
<tr>
<td>- Range</td>
<td>581</td>
</tr>
<tr>
<td>- Wet type</td>
<td>580</td>
</tr>
<tr>
<td>Corner firing</td>
<td>279</td>
</tr>
<tr>
<td>Corona</td>
<td>397</td>
</tr>
<tr>
<td>Coupled cycles</td>
<td>107</td>
</tr>
<tr>
<td>Courses</td>
<td>342</td>
</tr>
<tr>
<td>Critical discharge</td>
<td>439</td>
</tr>
<tr>
<td>Critical mass</td>
<td>613</td>
</tr>
<tr>
<td>Critical pressure ratio</td>
<td>437</td>
</tr>
<tr>
<td>Critical size</td>
<td>613</td>
</tr>
<tr>
<td>Critical speeds</td>
<td>504</td>
</tr>
<tr>
<td>Critical velocity</td>
<td>504</td>
</tr>
<tr>
<td>Cross-compounded turbine</td>
<td>435</td>
</tr>
<tr>
<td>Crushers</td>
<td>281</td>
</tr>
</tbody>
</table>
Curie 610
Curtis stage 461
Cyclone furnace 272
Cyclone separator 339, 384

Dams 668
  - Earth 671
  - Gravity 669
  - Rock fill 671
Dolphin wave machine 921
Dampers 197
Deaerator 72
Decay constant 608
Deflector 683
Degree of reaction 468, 688
Degree of saturation 587
Degree of supercooling 450
Degree of supersaturation 450
Demand factor 5
Demineralization plant 406
Depreciation 12
  - Sinking fund 14
  - Straight fund 13
Deriaz turbine 694
Desuperheater 368
Deutch equation 399
Dew point temperature 183
Diagram efficiency 455
Diagram work 455
Diesel engine power plant 737
  - Cetane number 754
  - Cooling system 745
  - Combustion 751
  - Energy balance 760
  - Fuel system 739
  - Layout 765
  - Lubrication system 748
  - Morse test 757
  - Performance characteristics 754
  - Starting 751
  - Supercharging 761
  - Volumetric efficiency 759
Diffusion control 239
Diffuser 430
Disc friction loss 487
Dispersion 192
Dissociation 209
Distributor 274
Diversity factor 5
Dolphin type wave machine 920

Doubling time 2, 642
Downcomers 328
Draft tubes 675
Drain cooler 62
Draught system 189
  - Balanced 195
  - Forced 194
  - Induced 195
  - Mechanical 194
Drift eliminator 583
  - Drum 337
  - Drum internals 338
  - Drum level control 365
  - Dry bottom furnace 260
  - Dry bulb temperature 588
  - Dry cell battery 827
  - Dual pressure steam cycle 120
  - Dummy piston 499

Economic scheduling 17
Economiser 43, 356
Efficiency
  - Auxiliaries 72
  - Boiler 76
  - Carnot 48
  - Cogeneration plant 79
  - Collection 385, 399
  - Combined cycle 108
  - Condenser 573
  - Cooling 581
  - Generator 77
  - Hydraulic 686
  - Nozzle 443
  - Overall 76
  - Rankine cycle 42
  - Stage 491
  - Stirling cycle 56
  - Turbine 76
  - Turnaround 817
  - Vacuum 573
Electric generator 508
Electrochemical energy storage 826
Electrostatic precipitator 397
Elutriation 280
Emergency governor 497
Emulsion firing 165
Emulsion phase 280
Energy balance 187
Energy farm 174
Energy storage 813

Copyrighted material
Generator cooling 510
Global warming 897
Gland leakage 448
Greenhouse effect 897
Grinability 161
GT-ST combined plant 110

Half life 608
Hardness 404
Heat of combustion 199
Heat rate 43
Heater drip 61
Heater train 66
Heaters, feedwater 575
  - Closed 575
  - Open 578
Heating value 183
Heavy water reactor 643
Hybrid storage system 820
Hydraulic turbines 679
  - Classification 679
  - Efficiency 686
  - Governing 699
  - Performance 704
  - Selection 708
  - Size 682
Hydroelectric power plant
  - classification 676
Hydrogen energy 846, 889
  - Production 847, 889
  - Storage 848
Hydrogen zeolite 406
Hydrograph 662
Hydrological cycle 661
Hydro-thermal mix 659

Ideal regenerative cycle 56
Ideal working fluid 104
Ignition delay 752
Impulse turbines 451
Incremental heat rate 76
India’s nuclear power programme 645
Indian energy scenario 20
Industrial wastes and byproducts 166
Integrated gasification combined cycle 135, 303
Intermediate load 2
Internal treatment 409
Isotope 609
Jet ratio 684
K-capture 607
Kaplan turbine 693
Kinetic control 240
Kinetics of combustion 235
Knocking combustion 753
La Mont boiler 503
Labyrinth seal 488
Lacing wire 503
Latent heat storage 843
Law of diminishing return 68
Law of mass action 236
Layout of diesel engines 765
Layout of steam power plant 74
Lime-soda process 405
Liquid metal fast breeder reactor 641
Load curve 4
Load duration curve 2, 9
Load factor 3
Location of power plant 9
Logarithmic energy decrement 614
Loop seal 384
Losses in steam turbines 485
L-valve 383

Mach number 434
Magnetic energy storage 835
MHD power generation 856
Mass curve 664
Mass defect 604
Mass number 602
Mass transfer coefficient 239
Mean temperature of heat addition 48
Mechanical draught 194
Metastable equilibrium 449
MHD-steam power plants 859
Minimum fluidization velocity 274
Moderating power 619
Moderating ratio 619
Moderator 619
Morse test 757
Multiplication factor 623

Natural circulation 329
Natural draught 490, 582
Natural gas 165
Net cycle heat rate 78
Neutrino 603
Neutron cross-section 620
Neutron emission 607
Neutron energy 613
Neutron flux 618
Neutron life cycle 622
Non-mechanical valves 383
Nozzle governing 496
Nozzle
- Efficiency 443
- Flow area 445
- Height 477
- Thickness factor 447
- Types 445
Nuclear cross-section 616
- Absorption 618
- Capture 618
- Fission 618
- Scattering 618
Nuclear fission 610
Nuclear fusion
Nuclear reactions 604
Nuclear stability 604
Nucleate boiling
Nucleus 602
Nusselt equation 332

Ocean energy system 927
- OTEC 917
- Tidal energy 922
- Wave energy 918
Oil burners 306
Once-through boilers 352
Optimum degree of regeneration 65
Optimum reheat pressure 55
Orsat analyzer 180
OTEC 917
Overall plant efficiency 76
Overfeeding 241

Package boiler 320
Packed bed 277
Packing or fill 881
Parallel exhausts 481
Parsons turbine 468
Partial admission 480
Passout turbine 80
Peak load 7
Peak shaving 815
Peaking turbine 838, 845
Peltier effect 870
Pelton wheel 682
Pendent superheater 362
PFBC based combined cycle 134
PFBC loop 299
pH value 409
Phosphate softening 405
Photovoltaic conversion 909
Pinch point 47, 121
Plant use factor 5
Platens 363
Pondage 666
Positron 603
Positron decay 607
Pour point 164
Power coefficient 242
Powerhouse 675
Power plant economics 10
Present worth concept 15
Pressure compounding 459
Pressure pulse velocity 433
Pressurized fluidized bed 391
Pressurized water reactor 633
Pressurizer 636
Primary superheater 359
Priming 411
Propeller turbine 691
Psychrometric calculations 586
Pulverized coal firing 248
Pulverizers 282
Pumped hydro 816
Pumped storage plant 678
Pumping power 632

Radiant superheater 361
Radioactive decay 606
Radioisotope 609
Range 581, 924
Rankine cycle 42
- External irreversibility 47
- Internal irreversibility 46
Reaction turbines 466
Reactivity 624
Reactor types 632
Regeneration 55
Regenerative feedwater heating 58
Regimes of fluidization 276
Reheat cycle 53
Reheat factor 489
Reheater 364
Repowering 139
<table>
<thead>
<tr>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserve factor</td>
</tr>
<tr>
<td>Resonance region</td>
</tr>
<tr>
<td>Richardson-Dushman equation</td>
</tr>
<tr>
<td>Rigid shaft</td>
</tr>
<tr>
<td>Riser</td>
</tr>
<tr>
<td>Riser tubes</td>
</tr>
<tr>
<td>Scale ratio</td>
</tr>
<tr>
<td>Scattering</td>
</tr>
<tr>
<td>Schmidt number</td>
</tr>
<tr>
<td>Sensible heat storage</td>
</tr>
<tr>
<td>Sherwood number</td>
</tr>
<tr>
<td>Shocks</td>
</tr>
<tr>
<td>Shrouding</td>
</tr>
<tr>
<td>Sinking fund</td>
</tr>
<tr>
<td>Slip factor</td>
</tr>
<tr>
<td>Slip ratio</td>
</tr>
<tr>
<td>Slip velocity</td>
</tr>
<tr>
<td>Slugging</td>
</tr>
<tr>
<td>Small hydro</td>
</tr>
<tr>
<td>Smog</td>
</tr>
<tr>
<td>Sodium zeolite softening</td>
</tr>
<tr>
<td>Solar cell</td>
</tr>
<tr>
<td>Solar energy</td>
</tr>
<tr>
<td>- air heater</td>
</tr>
<tr>
<td>- thermal power</td>
</tr>
<tr>
<td>Solar furnace</td>
</tr>
<tr>
<td>Solar hydrogen</td>
</tr>
<tr>
<td>Sonic velocity</td>
</tr>
<tr>
<td>Sorbent</td>
</tr>
<tr>
<td>Spear valve rod</td>
</tr>
<tr>
<td>Specific speed</td>
</tr>
<tr>
<td>Specific humidity</td>
</tr>
<tr>
<td>Speed regulation</td>
</tr>
<tr>
<td>Sphericity</td>
</tr>
<tr>
<td>Spillways</td>
</tr>
<tr>
<td>Spinning reserve</td>
</tr>
<tr>
<td>Spontaneous combustion</td>
</tr>
<tr>
<td>Spreader stoker</td>
</tr>
<tr>
<td>Stack</td>
</tr>
<tr>
<td>Stagnation properties</td>
</tr>
<tr>
<td>Steam purity</td>
</tr>
<tr>
<td>Steam drum</td>
</tr>
<tr>
<td>Steam generators</td>
</tr>
<tr>
<td>Steam jet air ejector</td>
</tr>
<tr>
<td>Steam pressure control</td>
</tr>
<tr>
<td>Steam rate</td>
</tr>
<tr>
<td>Steam temperature control</td>
</tr>
<tr>
<td>Steam washing</td>
</tr>
<tr>
<td>STIG</td>
</tr>
<tr>
<td>Stirling boiler</td>
</tr>
<tr>
<td>Stirling cycle</td>
</tr>
<tr>
<td>Stokers</td>
</tr>
<tr>
<td>Stoichiometric air</td>
</tr>
<tr>
<td>Storage</td>
</tr>
<tr>
<td>Storage density</td>
</tr>
<tr>
<td>Straight line method</td>
</tr>
<tr>
<td>Structure of atom</td>
</tr>
<tr>
<td>Structure of CFB</td>
</tr>
<tr>
<td>Subsonic nozzle</td>
</tr>
<tr>
<td>Supercharging</td>
</tr>
<tr>
<td>Superconductivity</td>
</tr>
<tr>
<td>Supercritical boiler</td>
</tr>
<tr>
<td>Supercritical pressure cycle</td>
</tr>
<tr>
<td>Supercritical velocity</td>
</tr>
<tr>
<td>Superheater</td>
</tr>
<tr>
<td>- Convective</td>
</tr>
<tr>
<td>- Pendant</td>
</tr>
<tr>
<td>- Platens</td>
</tr>
<tr>
<td>- Radiant</td>
</tr>
<tr>
<td>Supersaturated flow</td>
</tr>
<tr>
<td>Supersonic nozzle</td>
</tr>
<tr>
<td>Supervisory instruments</td>
</tr>
<tr>
<td>Supplementary firing</td>
</tr>
<tr>
<td>Surface condensers</td>
</tr>
<tr>
<td>Surge tanks</td>
</tr>
<tr>
<td>Synthetic fuels</td>
</tr>
<tr>
<td>Tandem compounded turbine</td>
</tr>
<tr>
<td>Tangential burners</td>
</tr>
<tr>
<td>Tapered blades</td>
</tr>
<tr>
<td>Terminal temperature difference</td>
</tr>
<tr>
<td>Terminal velocity</td>
</tr>
<tr>
<td>Thermal energy storage</td>
</tr>
<tr>
<td>Thermal flywheel</td>
</tr>
<tr>
<td>Thermal neutrons</td>
</tr>
<tr>
<td>Thermal reactor</td>
</tr>
<tr>
<td>Thermionic power generation</td>
</tr>
<tr>
<td>Thermochemical conversion</td>
</tr>
<tr>
<td>Thermoelectric power generation</td>
</tr>
<tr>
<td>Thermoelements</td>
</tr>
<tr>
<td>Thermopile</td>
</tr>
<tr>
<td>Three-zone FW heater</td>
</tr>
<tr>
<td>Throttle governing</td>
</tr>
<tr>
<td>Tidal energy</td>
</tr>
<tr>
<td>- Linkage basin</td>
</tr>
<tr>
<td>- Range</td>
</tr>
<tr>
<td>- Schedule</td>
</tr>
<tr>
<td>- Single basin</td>
</tr>
<tr>
<td>- Double basin</td>
</tr>
</tbody>
</table>
Tides-spring and neap 924
Tilting burner 372
Total dissolved solids 337
Travelling grate stoker 244
Transport disengaging height 280
Tube mills 282
Turbines
- Bulb 695
- Deriaz 694
- Francis 689
- Gas 772
- Hydraulic 679
- Impulse 451
- Kaplan 693
- Pelton 682
- Propeller 691
- Reaction 466
- Steam 451
- Wind 912
Turbine blading 451
Turbine governing 494
Turbine heat rate 176
Turbogenerator 509
Turbulent bed 280
Turbulent burner 265
Turnaround efficiency 817
Turning gear 508
Twisted blades 482

Underfeed stoker 247
Underground coal gasification 170
Unit discharge, speed and power 697

Vacuum efficiency 573
Valve timing diagram 755
Velocity coefficient 444
Velocity compounding 461
Velocity ratio 455
Vibrating grate stoker 247
Void fraction 278, 333
Volumetric efficiency 759
Volumetric heat source 626
Vortex blading 792
Vortex burner 265

Water tube boiler 321, 342
Water wall 351
Wave energy 918
Wet bulb temperature 580
Williams line 758
Wilson line 450
Wilson plot 569
Wind energy 912
Wind turbines 912
- Coefficient of performance 913
- Power availability 914
- Speed ratio 914
- Types 912

Zeuner's relation 441
Hidden page
Hidden page
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