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STABILITY AND DUCTILITY OF STEEL STRUCTURES

Edited by
Tsutomu Usami and Yoshito Itoh
Nagoya University
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FOREWORD

The first Japanese session of the International Traveling Colloquium on the Stability of Steel Structures was held in Tokyo in 1976. Now, 21 years later, a session has been hosted in Nagoya. Almost three years have passed since the disastrous earthquake of January 17, 1996, which struck Kobe and the surrounding area, and to the theme of structural stability in the title of past colloquia, we have added that of structural ductility.

The Kobe earthquake caused considerable damage to buildings and civil engineering structures, largely due, it seems, not only to insufficient strength in the steel structures, but also to an insufficiency in ductility. For those of us concerned in this area around the world, this raises numerous questions regarding the seismic development of high performance steels, and the reassessment of structural details and systems.

The contents of this volume are selected from the Nagoya Colloquium proceedings will become an important part of the world literature on structural stability and ductility, and will prove a driving force in the development of future stability and ductility related research and design.

Our special thanks go to the editors, Professors T. Usami and Y. Itoh of Nagoya University for the central role they played in preparing for the Nagoya Session of the 5th International Colloquium (July 29-31,1997), and for their hard work in so successfully carrying out the difficult tasks of the editing work.

Professor Yushi Fukumoto
Chairman of the Organizing Committee
Nagoya Colloquium
January 1998
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PREFACE

The near-field earthquake which struck the Hanshin-Awaji area of Japan before dawn on January 17, 1995, in addition to snatching away the lives of more than 6,000 people, inflicted horrendous damage on the region’s infrastructure, including the transportation, communication and lifeline supply network and, of course, on buildings, too. A year earlier, the San Fernando Valley area of California had been hit by another near-field quake, the Northridge Earthquake, which dealt a similarly destructive blow to local infrastructures. Following these two disasters, structural engineers and researchers around the world have been working vigorously to develop methods of design for the kind of structure that is capable of withstanding not only the far-field tectonic earthquakes planned for hitherto, but also the full impact of a near-field earthquake.

Of the observed types of earthquake damage to steel structures, there are some whose causes are well understood, but many others continue to present us with unresolved problems. To overcome these, it is now urgently necessary for specialists to come together and exchange information. This is the backdrop against which the *Fifth International Colloquium on Stability and Ductility of Steel Structures* was held. The First Session of the Colloquium took place in Budapest, Hungary, in September 1995. This was followed by a Second Session in Chicago, USA, in April 1996, and a Third in Rio de Janeiro, Brazil, in August of the same year. The Nagoya Session in July 1997, Japan, is the Fourth.

In collaboration with the publisher’s Elsevier Science, arrangements have been made to bring out, in book form, a collection of Selected Papers from the Nagoya Session Proceedings. The following points can be made concerning the motives and significance of this publication.

1) The included papers can be presented in a revised form that takes account of the discussion flow at the Colloquium Session.
2) A large number of papers were presented in a subject area of high current interest.
3) The original Session Proceedings were produced solely for the benefit of participants. This does nothing to guarantee a world circulation. This can best be achieved by the publication of a book. Additionally, availability in book form will ensure that these materials are stored in specialist libraries in various countries, where the research results they contain can be of use to students and others.
4) A length limit of eight pages was imposed for the original Proceedings, and many authors found this too restrictive for presenting a detailed account of their work. In the space offered by the book, a more comprehensive description is possible.

Contents were selected for each specialty area after consulting the Japanese and international chair representatives concerned. In order to provide fuller research details, as outlined above, authors were asked to reformulate their contribution so as to bring them up to an average length of twelve pages.

In content, the book will span almost the whole range of steel structure research discussed at the Colloquium Session. During February 1997 the book "Structural Stability Design – Steel and Composite Structures" was published by Elsevier Science (under its Pergamon brand). This was one of the projects undertaken to mark the retirement of Professor Yuhshi Fukumoto. This work, jointly authored by Professor Fukumoto’s associates, focused on the results of steel structure research here in Japan. It is our firm hope that these two volumes, taken together, many greatly contribute to work in this specialty field.

T. Usami and Y. Itoh
Editors
January, 1998
STATE-OF-THE-ART
HISTORY OF RESEARCH AND PRACTICE OF THE STABILITY OF STEEL STRUCTURES IN THE TWENTIETH CENTURY

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ABSTRACT

This paper traces the historical developments on the subject of the stability of steel structures through the present century, touching on the heritage of the previous century and detailing the work left to the researchers in the next one. The historical trace concentrates on the axially loaded column to illustrate the intellectual and technological history as a representative example. The paper ends with an accounting of the state-of-art at the end of our century, and a list of the challenges for researchers for the 21st century.

KEY WORDS

Stability of Structures, History of Stability, Column Strength, Steel Structures, Research Trends

1. INTRODUCTION

This paper is an attempt to put the developments in the area of the stability of steel structures during the Twentieth Century into an historic perspective. This could be a vast undertaking because it encompasses many theoretical and practical concepts. Furthermore, not all developments are known, because of the separation of the world into different nationalities and countries, and papers written in various languages. Many parallel developments on the solution of the same problem occurred in different places at essentially the same time, and often it has been years before this was generally realized. The topic of Structural Stability experienced a huge expansion in the Twentieth Century, branching into many sub-areas which have taken on almost an independent existence of their own. No one can be an expert in all of these fields. Thus the ensuing treatise will be short and incomplete, and it will reflect the bias of the author. He will, however, try to convey the flow of ideas and accomplishments throughout this exciting Twentieth Century.

The Theory of Structural Stability deals with those types of failure of structures which are caused by loss of equilibrium: a small disturbing force causes large displacements, resulting in a catastrophic collapse. The field of Structural Stability has come into its own as a separate sub-discipline of Structural Mechanics mainly as the result of the pioneering text “Theory of Elastic Stability” by Stephen S. Timoshenko which was first published
by McGraw Hill in 1936. This text is still a starting point for all students of the field, and it is a much thumbed
reference to most structural engineers. We must not think, however, that the subject matter stands alone on its
own, because in it are firmly embedded the concepts of the Theory of Elasticity, the Theory of Plasticity,
Materials Science and Mathematics. The history of Structural Stability is thus very much entwined in the
history of these four parent disciplines.

2. BEGINNING OF THE TWENTIETH CENTURY

It is not easy to define a clear boundary in the developments between the end of the Nineteenth and the
beginning of the Twentieth Centuries, since the evolution of the understanding of structural stability was
continuous and the careers of many of the main contributors spanned beyond the year 1900. The new Century
owed much to the previous one. In fact, most of the ideas had been formed, and solutions to many problems
were at hand already. Certainly, most of the Mathematics and Physics underlying Structural Mechanics was
well known, and many applications were available to the practicing engineer. The Theory of Elasticity was
already in a mature state, while the Theory of Plasticity was in its infancy.

The story of the early years of Structural Stability is embedded in the broader context of Mathematics, Physics
and Structural Mechanics, and it is most thoroughly and delightfully related by S. P. Timoshenko in the book
the great mathematicians-physicists-engineers such as the Bernoullis, Euler, Lagrange, Coulomb, Navier,
Young, Cauchy, Poisson, Gaus, Maxwell, Kelvin, Rayleigh and many others, who laid the physical and
mathematical foundations of Structural Mechanics. The industrial advances of the last Century also brought on
many of the problems which needed solution: what is the best way to design a column against buckling; what is
the lateral stability of a thin-walled wide-flange beam; how to proportion the top chord of a pony-truss; what are
the effects of boundary conditions; how to design safe latticed columns; and many others. There are many
names that could be mentioned here, and many places where these solutions were proposed first, but only a few
will be mentioned because of the strong effect of their ideas on the developments in the early part of this
century.

One of the problems discovered early in the last century was the fact that Euler's column formula over-predicted
the capacity of practical columns made from wood, stone, cast and wrought iron, and later, steel. The reasons
for this were that the columns were initially crooked, they had restrained ends, and they failed in the inelastic
range of material behavior. They were thus outside of Euler's assumptions. Two approaches were used to
overcome this disparity:

1. Test a number of columns and propose an empirical column formula (Hodgekinson in the middle of the last
century, and Tetmaier and Bauschinger at the end of it)
2. Test a number of columns and then back-figure an eccentricity which would result in an elastic stress equal
to the yield stress of the material if the column were analyzed as an eccentrically loaded strut using second-
order theory (Rankine).

The resulting column equations were, of course, limited in their applicability. Engesser, Jasinsky and
Considere developed the more general tangent modulus and, later, the reduced modulus theories in the last
decade of the Nineteenth Century. These theories seemed to satisfy the theoreticians for the next fifty or so
years, while the practical designers used the specialized empirical formulas. The detailed history of the
evolution of the axially loaded column has been provided by Bruce G. Johnston (1983).

Of the engineers who straddled the year 1900 and who put their mark on developments in the first half of our
Century, the author would like to single out two: F. Engesser and L. Prandl. Engesser made original and
significant contributions to many of the problems which were to occupy researchers in the next century:
• The inelastic column.
• The built-up column (latticed or laced).
• The pony-truss bridge.
• The method of successive approximation.

Prandl’s doctoral thesis was one of the first solutions to the problem of lateral buckling of beams, and this in itself is of great merit. However, his greatest contribution was the nurturing of two giants who made major contributions to the discipline of Structural Stability: S. P. Timoshenko and T. v. Karman. (Prandl made other major contributions in aerodynamics, and two of his students, Nadai and Prager, founded the Theory Plasticity).

To sum up the Nineteenth Century: most of the problems that were definitively solved in the next century were fairly clearly defined, the needed physics and mathematics was available, and there was an infrastructure of international collaboration which enabled the ideas to sprout and bear fruit. Our discipline inherited much from its forebears, and it was able to build on this fertile foundation.

3. THE MAIN TRENDS OF STRUCTURAL STABILITY IN THE TWENTIETH CENTURY

One of the main trends of research in this Century has been the move from the idealized problems which were soluble by the theories of mechanics using classical mathematical tools to arrive at closed-form formulas, to the solution of more-and-more complex problems which represent the real world. The classic example is the evolution of the axially loaded column from the idea of the perfect column problem of the ideal Euler case, to the initially imperfect column and the beam-column. What has made this extension from idealization to reality possible are the following developments:

• The maturing of structural mechanics, especially the theory of plasticity and the development of the ideas of large deflection mechanics, facilitated clear visualization of the mechanical models.
• The development of classical methods of approximate calculation (energy methods, successive approximation methods, relaxation methods, finite difference methods) which then converged into the modern finite element method in the latter third of our Century permitted the solution of problems which were not able to be solved in a mathematically closed form. In the first half of this Century some of the best minds in applied mathematics created very ingenious tools which performed very complex calculations, without the convenience of digital computers.
• The needs of new industries, the expansion of cities and transportation networks and vehicles, and the devastating wars of the Century provided both the problems in need of solution, as well as the money to obtain the answers.
• The need to arrive at answers, and the intellectual challenge to work on exciting relevant problems, drew hundreds of talented scholars, engineers and mathematicians into the field. Technology evolved sophisticated mechanical and computational devices (testing machines, sensors, computers) which not only helped us in understanding the real world, but also liberated us from much tedious drudgery of measurement and computation.

Conceptually we are able to understand much more than our great-grandfathers at the beginning of the Century. Are we, therefore, doing things in a significantly different way now in the design office when we design structures? Not really, or, being somewhat more optimistic, not yet. Much research on structural stability has concentrated on developing, with ever more sophistication, empirical design equations or design aids. At the end of the Century there is hope that this state of affairs will not prevail too much longer because of a new idea on the horizon called “advanced analysis”. This paper will return at its end to this subject.

The history of structural stability in the Twentieth Century can be roughly divided into the period before and during the Second World War, called here the classical period, and the second half of the Century, named here...
the modern period. The first half of the Century was not affected by the computer, while the second half is
dominated by it. The summing up of the classical knowledge is represented in the English literature of the field
by the books by Timoshenko (1936) “Theory of Elastic Stability”, and Friedrich and Hans Bleich (1952)
“Buckling Strength of Metal Structures”. There are comparable books also in German (Burgermeister and
Steup, 1957) and Russian (Vlasov, 1940). The second part of the Century has considerably expanded the scope
of the classical work to include the concepts of large deformations, imperfections, post-buckling, and interaction
between modes of buckling. In the following parts of this paper the evolution of ideas will be illustrated on hand
of several examples.

4. IN-PLANE BEHAVIOR OF COLUMNS AND BEAM-COLUMNS

The story of columns is outlined in Figs. 1 and 2. During the 19th Century it was found that the Euler theory
needed amending because of the idealizations contained in the assumptions of elastic material and perfect
geometry. By the end of the century three approaches were in existence to deal with this problem:

- Pure empiricism
- The imperfect elastic column
- The tangent modulus and the reduced modulus theory

Intellectually the tangent modulus and the reduced modulus theories posed a very difficult dilemma. The
reduced modulus theory was thought to be correct, but it was difficult to calculate. The tangent modulus theory
was thought to give a lower column strength capacity than the reduced modulus theory, it was easier to
calculate, and thus it was the preferred method of determining column strength. The dilemma was eventually
resolved by F. R. Shanley in 1946. He showed that the tangent modulus was indeed a lower bound, and that the
reduced modulus load was an unattainable upper bound. The true ultimate strength lay somewhere between the
two extremes. Shanley’s model opened up the field to researchers who now measured residual stresses with
modern tools, conducted careful full-scale tests in the laboratory, and performed analyses using the new
understanding of the column behavior and the newly available tool of the computer. By the mid-fifties of our
Century there were column curves for many types of perfectly straight steel columns, and the Column Research
Council (CRC, the original name of the Structural Stability Research Council, SSRC) formulated the
recommendation that column design should be based on the tangent modulus principle in its Technical
memorandum No. 1 in 1952. The CRC also came forth with the “CRC Column Formula”, which is still in use
in some structural standards today. By the end of the 1950-s it was realized that the model of the perfectly
straight column with residual stresses was not a very good model for many practical steel column types,
although it is still the basis design of aluminum column in the USA. The schematic historical time-line of the
straight column is depicted in Fig. 1.

By the end of the 1950-s it was realized that the model of the perfectly straight column with residual stresses
was not a very good model for many practical steel columns. This was already realized at the beginning of the
Century by T. v. Karman (1908), who determined the ultimate strength of an eccentrically loaded column for an
ideal elastic-plastic steel. His ideas were further developed by Chwalla (1928 to 1937), and Jezek (1934) in
Europe, and by Westergaard and Osgood in the US (1928). The European research culminated in the first
modern stability design specification of the Century, the German DIN 4114 (1952). The column design in this
specification is based on a tabulation of maximum permissible stress-versus-slenderness ratio values which were calculated from the Chwalla-Jezek model of eccentrically loaded inelastic columns.
For the decades of the 1950-s through the 1970-s there were essentially two design philosophies in evidence: The CRC model, which included residual stresses and ignored geometric imperfections, and the DIN 4114 model which included geometric imperfections and ignored residual stresses. The computational tools were simply not yet available to account for both phenomena in one model. The proponents of both methods recognized, of course, the shortcomings of their models, and each provided variable factors of safety to account for the neglected items. Thus the design methods, which seemed to be both rational and scientific, were no better than the formulas based on the first yield of an imperfect column. Fortunately, the tools for the determination of the strength of columns by methods which accounted for both the residual stresses and the initial imperfections became available by the 1960-s. The early work of Batterman and Johnston (1967), Bjorhovde and Tall (1971, 1973) and Beer and Schultz (1970) resulted in the epochal collaboration in Europe to test many kinds of steel columns, which served to extend the earlier test performed at Lehigh University in the US, and to arrive at the SSRC Column Curves and the European Column Curves, respectively, which serve as the basis for the design criteria for all modern steel specifications. The schematic historic time-line of the imperfect column development is given in Fig. 2.

The axially loaded column saga has thus a happy ending: we know how to handle any possible case by the conceptual models and the computational tools available to us. The eccentrically loaded column is actually a beam-column without added bending forces. The same tools of analysis and testing can be applied to the in-plane behavior of beam-columns. The vast literature on this subject is summarized in the 4th edition of the SSRC Guide (Galambos, 1988), and the theory and the computational tools are presented in Vol. 1 In-Plane Behavior and Design of the book by Chen and Atsuta (1976) Theory of Beam-Columns.

The research problem of in-plane beam-columns can be also regarded as being solved. The design criteria for beam-columns are, however, not yet as clearly related to the analytical predictions as is the case for columns. There have been attempts of using tabular information obtained by numerical integration in the 1963 specification of the American Institute of Steel Construction, but these did not survive into the next edition. The preferred solution in the world's design standards has been the use of interaction equations which give a simple relation between the axial force and the bending moment which act on the member. Such a method cannot give a uniform safety because it is a very idealized representation of the real strength. The recently emerging
methods of advanced analysis incorporate into the determination of the strength of planar frames an internal finite element scheme which directly determines the strength of the beam-column (Chen and Kim 1997).

**THEORY AND RESEARCH**

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Fig. 2 Geometrically Imperfect columns

5. OUT-OF-PLANE BEHAVIOR

The evolution of the out-of-plane stability of thin-walled open cross-section members follows a similar pattern as for the in-plane behavior discussed above. The solutions are obviously much more complicated because the physics is more complex. The problem of the lateral buckling of narrow rectangular beams was first solved simultaneously by Prandtl in Germany and by Michell in Australia in the year 1899. Timoshenko expanded this work to the elastic buckling of wide-flange beams in 1906. The next major developments were in the 1930-s and 1940-s, spurred by the needs to design metal airplanes. Most of the classical solutions to the elastic lateral-torsional buckling of beams and the torsional and flexural-torsional buckling of columns originated in this period, for example, by Wagner, Kappus, Lundquist, Goodier, Timoshenko, Hoff, Bleich, Stussi, Chwalla, Winter, Hill, DeVries and others. This work is summarized by Bleich (1952) and G. C. Lee (1960). With the wide availability of the computer in the 1960-s there was an explosion of numerical work on the subject of both elastic and inelastic lateral-torsional buckling, until today it can be said that most of the technically possible problems have been solved and the results tabulated or presented in charts for use by design engineers. The solution to the inelastic buckling problem was started a half century after the first elastic problems were solved. The early work on this topic was by B. G. Neal, M. R. Horne, T. V. Galambos, G. C. Lee, Y. Fukumoto, N. S. Trahair and S. Kitipornchai. N. S. Trahair in his book “Flexural-torsional Buckling of Structures” (1993) provides a modern overview of this topic. The research knowledge on out-of-plane behavior of thin-walled open structures is very extensive, including the effects of biaxial bending, and initial imperfections. However, the integration of this information into design criteria is still quite primitive. The most advanced application can be found in the Australian Standard AS 4100-1990.
6. STATUS AT THE END OF THE CENTURY AND PROSPECTS FOR THE FUTURE

Space does not permit the recounting of the history of the advances in all the other problem areas in metal structure stability during the Twentieth Century: Frames, plates, plate-girders, box-girders, arches, shells, reticulated shells, cold-formed structural elements, horizontally curved beams, composite columns, tubes, triangulated structures and so forth. The recounting of these histories is done on the pages of the fifth edition of the SSRC Guide (Galambos 1998). The story for all of these subjects is essentially the same: the initial solution of elastic buckling, and the treatment of the simpler cases which could be achieved in closed form, followed by the clear definition of the appropriate model through testing and computation, and the evaluation by computer methods of the full parameter space of the problem area. In this sense the accomplishments of this Century would seem like the fulfillment of the needs which were defined by the last one. Is there then nothing new which was provided by this Century? Is there anything to do for the Twenty-First century?

There were a number of new and previously not thought of developments in our Century. Some of these are:

- The emergence of the computer and the finite element method. While these are only tools, they do permit an unprecedented freedom in exploring problems.
- The stability of very slender systems, essentially re-uniting the classical stability theory with the mathematics and physics of chaotic systems (e.g. Thompson and Hunt, 1984).
- The stability of structural systems under dynamic and repeated loading due to earthquakes, wind, and industrial excitations (dynamic stability, seismic stability).
- The idea of interaction between various modes of instability, including the post-buckling behavior of very thin plate elements (e.g. combined local and overall stability in cold-formed members, interaction between in-plane and out-of-plane behavior of frames, etc.).
- Stability of composite members (steel-concrete composite columns, local and lateral buckling of high-performance composite structures).
- Stability of wood, masonry and concrete structures.
- Advanced analysis and design wherein the structure is designed as a whole, without having to go back and make a member check apart from the structural analysis.

There are, in fact many areas of research on structural instability which are very active at the end of the Twentieth Century, and which have hardly been touched herein, thus providing ample opportunity for creative research in the next one. The industrial challenges will come from needs for structures in outer space and from the advancements in materials technology.

There will be also some new areas of research emerging in the future. Some of these possibilities are:

- The customary definitions of instability will expand to include a merger with fracture mechanics (a glimpse of this merger can be detected in the book by Bazant and Cedolin 1991).
- We have not yet touched the possibilities of computation of the behavior of very large systems. Certainly, there will be other topics which will be available to challenge the researchers in this exciting field of intellectual endeavor. Clearly, the largest challenge will be to transfer the knowledge gained from research into the hands of the structural engineer.

REFERENCES


2 BEAMS AND BEAM-COLUMNS
MULTIPLE DESIGN CURVES
FOR BEAM LATERAL BUCKLING

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ABSTRACT

This paper develops a consistent set of multiple curves for the design of steel beams against lateral buckling. Multiple curves have already been used for the design of steel compression members against flexural buckling, and are included in a number of design codes. However, it appears that there has been no corresponding systematic development for beam lateral buckling.

KEYWORDS

Beam, buckling, design, steel, strength, structural engineering

1. INTRODUCTION

Multiple design curves for steel compression members were developed because of wide variations in their compression strengths caused by different residual stresses and initial crookednesses resulting from different methods of manufacture. The multiple curves allow compression members with low residual stresses and initial crookednesses to be designed using an appropriately higher capacity curve than for members with high residual stresses and initial crookednesses, so that advantage can be taken of their higher capacities without reducing the safety of the lower capacity members.

There is a similar need for multiple curves for the design of beams against lateral buckling, because there is a very wide range of beam cross-section type, with corresponding ranges in residual stresses and initial crookednesses. Other factors which increase the variability of beam lateral buckling capacities include initial twist, web flexibility, and the material stress-strain curve.

For example, a rectangular hollow section has a comparatively greater minor axis inelastic flexural rigidity because of its two webs than does a single web I-section beam. On the other hand, the flanges of a hollow flange beam are so rigid torsionally that web flexibility reduces the beam’s effective torsional rigidity to a much greater extent than for an I-beam with solid flange plates. Further, cold-formed beams have stress-strain curves, residual stresses, and initial crookednesses and twists which differ from those of hot-rolled I-beams, while the common lipped channel and lipped zed cross-sections of cold-formed beams are markedly different from the I-section of many hot-rolled beams.
This paper proposes a general method for developing a four-part curve for designing beams against lateral buckling, which includes a constant moment capacity region at the local major axis section capacity, an approximately linear inelastic lateral buckling region, an elastic lateral buckling region, and finally another constant moment capacity region at the local minor axis section capacity. This general method may be used to develop a consistent set of multiple design curves for beam lateral buckling.

2. BEAM LATERAL BUCKLING BEHAVIOUR

2.1 Elastic Lateral Buckling

A steel beam which is bent in its stiffer principal plane may buckle elastically in a flexural-torsional mode by deflecting $u$ out of the plane of loading and twisting $\phi$, as shown in Figure 1. The elastic buckling resistance decreases with the beam length $L$, and increases with the minor axis flexural rigidity $E_{ly}$, the torsional rigidity $GJ$, and the warping rigidity $EI_w$. For a simply supported beam in uniform bending, the moment $M_{yz}$ at elastic buckling (Timoshenko and Gere, 1961, Trahair and Bradford, 1991, Trahair, 1993) is given by

$$M_{yz} = \frac{\pi^2 EI_w}{L^2} \left( GJ + \frac{\pi^2 EI_w}{L^2} \right)^{1/2}$$

(1)

![Elevation diagram](image)

![Section at mid-span diagram](image)

![Plan on longitudinal axis diagram](image)

Figure 1: Beam elastic lateral buckling

Extensive research has shown that the bending moment distribution has a very significant effect on the elastic buckling resistance (Trahair and Bradford, 1991, Trahair, 1993), and that uniform bending is the worst case. The effect of the bending moment distribution may be allowed for approximately by using

$$M_m = \alpha_m M_{yz}$$

(2)

for the maximum moment at elastic buckling, in which $\alpha_m$ is a moment modification factor. Values of $\alpha_m$ for many different moment distributions are available (Trahair, 1993), as well as a general approximation (Standards Australia, 1990)

$$\alpha_m = \frac{1.7 M_m}{(M_1^2 + M_2^2 + M_3^2)^{1/2}}$$

(3)
MULTIPLE DESIGN CURVES FOR BEAM LATERAL BUCKLING

in which $M_2, M_1, M_3$ are the moments at the mid- and quarter-points of the beam.

The elastic buckling resistance of a beam is reduced by a transverse load $Q$ which acts at a height $(-y_Q)$ above the axis of the beam, as a result of additional overturning torques generated by the load $Q$, the load height $(-y_Q)$, and the twist rotation $\phi$ of the beam, as shown in Figure 1c. The maximum moment at elastic buckling may often be approximated (Trahair, 1993) by

$$
\frac{M_a}{M_{yz}} = \alpha_m \left\{ \sqrt{1 + \left( \frac{0.4\alpha_m y_Q}{M_{yz} / P_y} \right)^2} + \frac{0.4\alpha_m y_Q}{M_{yz} / P_y} \right\}
$$

(4)

in which

$$
P_y = \pi^2 EI / L^2.
$$

(5)

These formulations for the elastic buckling resistance are for a simply supported beam whose twist rotations $\phi$ are prevented at the supports, but many practical beams may have only elastic end restraints against twist rotation, or may have additional restraints against minor axis rotations $du/dz$ or against warping displacements proportional to $d\phi/dz$. These restraints are often accounted for by substituting an effective length

$$
L_e = kL
$$

(6)

for the actual length $L$ in the above formulations. Information on the effects of elastic restraints is given in Trahair (1993), and incorporated into the design procedures of Standards Australia (1990).

2.2 Strength

The influence of elastic lateral buckling on the strengths of simply supported beams in uniform bending is shown diagrammatically in Figure 2, in which

$$
\lambda = \sqrt{M_{px} / M_{yz}}
$$

(7)

is a modified slenderness and $M_{px}$ is the nominal major axis full plastic moment. At low slendernesses, the strengths of compact beams (which are unaffected by local buckling) rise above the full plastic moment $M_{px}$ due to strain-hardening effects. Beams of non-compact or slender cross-section have their section capacities reduced below $M_{px}$ to $M_{xz}$ by local buckling effects. The strengths of intermediate slenderness beams lie on a transition from the section capacity $M_{xz}$ to the elastic buckling resistance $M_{yz}$. At high slendernesses, the beam strengths are close to the elastic buckling strengths $M_{yz}$, but may rise above them due to strengthening pre- and post-buckling effects (Masur and Milbradt, 1957, Trahair and Woolcock, 1973, Vacharajittiphan et al, 1974, Woolcock and Trahair, 1974, 1975, 1976, Pi and Trahair, 1992a, b) which are unaccounted for in the determination of $M_{yz}$.

Many practical beams are of intermediate slenderness and these fail before the section capacity $M_{xz}$ or the elastic buckling resistance $M_m$ can be reached. This failure is due to premature yielding resulting from residual stresses and early deformations caused by initial curvature, twist, or load eccentricity. Although analytical methods are available (Bild et al, 1992, Pi and Trahair, 1994a, b) for predicting the strengths of intermediate slenderness beams, design formulations are usually based on experimental evidence (Fukumoto and Kubo, 1977a, b, c), such as that shown in Figure 3.
2.3 Other Influences on Beam Strength

Design formulations for beam lateral buckling strengths ignore the effects of pre-buckling deflections which transform the straight beam into a 'negative' arch, and increase its elastic buckling strength by a factor approximated by (Pi and Trahair, 1994a, b)

\[
\frac{M}{M_{xe}} = \frac{1}{\sqrt{1 - \frac{1}{\frac{EI_y}{EI_z} \left(1 - \frac{GJ + \pi^2 EI_w}{L^2} \right)}}}
\]

This factor increases with the ratio of \( I_y / I_x \), and theoretically becomes infinite when \( I_y = I_x \). This is consistent with the common assumption that when \( I_y \) is greater than \( I_x \), then the beam does not buckle.
laterally because it finds it easier to remain in the more flexible plane of bending (however, a beam bent in its more flexible plane by loads high above its axis may buckle torsionally).

While statically determinate beams have slowly rising post-buckling load-deformation paths (Woolcock and Trahair, 1974), redundant beams may have significant increases in strength after elastic buckling (Masur and Milbradt, 1957, Woolcock and Trahair, 1975, 1976). These increases occur as a result of the finite twist rotations that reduce locally the effective bending rigidity in the plane of loading, leading to a favourable redistribution of bending moment. Although significant increases in strength are only realised for very slender beams, this post-buckling behaviour causes the beam strength to remain above the minor axis section capacity \( M_{sy} \), even when this is greater than the elastic buckling resistance, as shown in Figure 2.

Although it is usually assumed in the analysis of lateral buckling that there is no change in the shape of the cross-section, web distortion may reduce the resistances of some beams (Hancock et al, 1980, Bradford, 1992). The reductions depend on the relative stiffness of the web in flexure compared with the torsional stiffness of the flanges, and may be quite pronounced in beams with slender webs and hollow flanges (Pi and Trahair, 1997), as indicated in Figure 4.

Elastic local buckling of a very thin compression flange may reduce a beam’s resistance to overall buckling. Even though local buckling failure is postponed by the post-local buckling resistance, the effective out-of-plane rigidities of the beam are reduced, thus lowering the resistance to lateral buckling. While beams with very thin flanges are rarely used, flanges of intermediate slenderness may buckle locally at loads close to the lateral buckling loads, in which case there may be unexpected strength reductions caused by imperfection sensitivity effects.

![Diagram of cross-section deformations and elastic buckling resistance](image)

**Figure 4:** Lateral-distortional buckling of a hollow flange beam

### 2.4 Design Curves

Design curves for simply supported steel beams in uniform bending are reviewed in Trahair and Bradford (1991) Beedle (1991), and Trahair (1993), and include a number of proposals for multiple design curves, and the use of different curves for hot-rolled and welded beams (BSI, 1990). There are generally considerable variations between the design capacities \( M_{bu} \) as shown for example by the comparison of some hot-rolled design curves (SA, 1990, BSI, 1990, CSA, 1996, AISC, 1993, CEN 1992) in Figure 5. Some of these variations are due to different bending moment distributions, while others are due to the use of different design philosophies (such as using either the means or lower bounds of experimental results such as those shown in Figure 3), or different capacity factors (such as \( \phi = 0.9 \) or 1.0) and load factors.
The design curves of Figure 5 all have a constant moment capacity $M_{bu} = M_{sz}$ at low modified slenderness $\lambda$, and reduced capacities at intermediate slendernesses which reflect the influence of residual stresses on inelastic buckling. Some curves have $M_{bu} < M_{yz}$ at high slenderness, following theoretical predictions of the influences of initial curvature and twist, while others have $M_{bu} = M_{yz}$ at high slendernesses. All curves have $M_{bu} < M_{sy}$ at very high slendernesses.

![Figure 5: Code design curves for hot-rolled beams](image)

### 3. ANATOMY OF A BEAM DESIGN CURVE

A proposal for a generalised design curve for beam lateral buckling is shown in Figure 6, in which the dimensionless capacity $M_{bu}/M_{px}$ of a simply supported beam in uniform bending is plotted against the modified slenderness $\lambda = \sqrt{M_{px}/M_{yz}}$. These parameters demonstrate the principal dependencies of the nominal capacity $M_{bu}$ on the full plastic moment $M_{px}$ and the elastic buckling resistance $M_{yz}$.

For beams of compact cross-section, the moment capacity is taken as the full plastic moment $M_{px}$ while the slenderness is low ($\lambda \leq \lambda_{mq}$). The limiting slenderness $\lambda_{mq}$ for I-section beams at which the inelastic buckling capacity $M_i$ is equal to $M_{px}$ may be approximated by using a very simplified model of inelastic buckling, according to which

$$ M_i / M_{yz} = E_s / E $$

(9)

in which $E_s$ is the strain-hardening modulus of elasticity. This leads to $\lambda_{pq} = 0.18$ for $E_i/E = 1/30$, but $\lambda_{pq} = 0.26$ is used in SA (1990). For non-compact and slender cross-section beams of low slenderness ($\lambda \leq \lambda_{mq}$), the moment capacity is reduced to the section capacity $M_{sz}$ to allow for local buckling effects.

For beams of intermediate slenderness ($\lambda_{mq} \leq \lambda \leq \lambda_{pt}$), the moment capacity is reduced below the section capacity $M_{sz}$ by an interaction between yielding and elastic lateral buckling which is influenced by the magnitudes of the residual stresses and the geometrical imperfections (initial crookedness and twist). The capacity $M_{it}$ at the upper limit $\lambda_{it}$ of this slenderness range may be approximated by

$$ M_{it} = (f_s - f_r) Z_x $$

(10)

in which $f_s$ is the yield stress, $Z_x$ is the elastic section modulus, and $f_r$ is an equivalent level of residual stress which also allows for the geometrical imperfections. The reduced moment capacity may be approximated by linear interpolation between $M_{px}$ and $M_{it}$. 
For beams of high slenderness ($\lambda_{te} \leq \lambda \leq \lambda_{ey}$), the moment capacity is approximated by the elastic buckling resistance $M_{yz}$. This approximation is based on a compromise between the weakening effects of the geometrical imperfections and the strengthening effects of the pre-buckling deflections. Equation 8 suggests that these latter effects strengthen hot-rolled beam type sections by between 1% and 4% and column type sections between 21% and 30%.

For beams of very high slenderness ($\lambda_{ey} < \lambda$), the moment capacity is approximated by the minor axis section capacity $M_{sy}$. Typical values of $M_{sy}/M_{p}\lambda$ for hot-rolled beam type sections vary between 0.07 and 0.20, and between 0.42 and 0.51 for column type sections.

4. MULTIPLE DESIGN CURVES

Individual design curves may be determined using the model described above and shown in Figure 6 when the values of $M_{p\lambda}, M_{s\lambda}, M_{ie}$ and $M_{sy}, \lambda_{pi}, \lambda_{si}, \lambda_{ie}$, and $\lambda_{ey}$ are known. The values of $M_{p\lambda}, M_{s\lambda}, M_{sy}$ can be determined from the section properties, yield stress, and appropriate design rules for the section capacity. The value of $M_{ie}$ can be approximated using Eqn 10. The value of $\lambda_{pi}$ can be obtained by adapting Eqn 9. The values of $\lambda_{ie}$ and $\lambda_{ey}$ can be obtained by using Eqn 1 with $M_{ie} = M_{ie}$ and $M_{sy} = M_{sy}$, respectively. The values of $\lambda_{si}$ can be obtained by linear interpolation.

Families of design curves can be obtained by using the model described above and shown in Figure 6 by selecting sets of values of $M_{sy}/M_{p\lambda}, f_y/f_y$, and $\lambda_{pi}$. Such a family is shown in Figure 7.
An alternative family of design curves is shown in Figure 8 which is based on the equations

$$\frac{M_{bu}}{M_{ps}} = b + \frac{a - b}{1 + c\lambda^n}$$  \hspace{1cm} (11)

with

$$b = a - 1 = 0.05, 0.10, \ldots 0.25$$  \hspace{1cm} (12)

and

$$n = c = 1.$$  \hspace{1cm} (13)

This family provides a curved inelastic transition as an alternative to the linear transition shown in Figure 7. The positions of these curves can be modified by changing the values of the constants $a$ and $b$, and the shapes by changing the value of the constant $c$ and the index $n$. 

Fig. 8 Alternative family of design curves
5. DESIGN OF STEEL BEAMS

5.1 Experimental Data and Design Curves

Experimental bending strengths $M_e$ may be assessed on a rational basis by adapting the design process of SA (1990), according to which the nominal strength $M_b$ can be determined from the elastic buckling moment $M_m$ and the section capacity $M_{sx}$ by using

$$M_{nw} = M_m / \alpha_m$$

in which $\alpha_m$ is the moment modification factor (Eqn 3 or Trahair, 1993), and

$$\alpha_s = 0.6 \left( \sqrt{\left( \frac{M_{sx}}{M_{nw}} \right)^2 + 3} \right) - \frac{M_{sx}}{M_{nw}} \leq 1.0$$

in

$$M_b = \alpha_m \alpha_s M_{sx} \leq M_{sx}$$

where $\alpha_s$ is the elastic buckling-to-strength conversion of SA (1990).

In the adaptation, this procedure is reversed by using the experimental strength $M_e$ to compute the dimensionless uniform bending strength

$$M_{bu} / M_{sx} = (M_e / \alpha_m) / M_{sx}$$

and by plotting the variation of this with the uniform bending slenderness

$$\lambda = \sqrt{M_{sx} / (M_m / \alpha_m)}$$

or the corresponding lateral-distortional slenderness

$$\lambda_d = \sqrt{M_{sx} / (M_{md} / \alpha_m)}$$

in which $M_{md}$ is the moment at elastic lateral-distortional buckling. This reversed procedure allows $M_{bu} / M_{px}$ to be plotted against $\lambda$ or $\lambda_d$, so that a design curve can be selected that is more relevant than a specific curve such as that of Eqn 15.

Experimental data on the lateral buckling strengths of hot-rolled and welded I-beams under near-uniform bending (Fukumoto and Kubo, 1977a, b, c, Trahair, 1983, Fukumoto et al, 1980, Fukumoto and Itoh, 1981) have been assessed in this way, and are shown in Figure 9 as approximate lower bounds. The lower bounds for hot-rolled I-beams are close to a design curve ($a = 1.07$, $b = 0.12$, $c = 1$, $n = 1$) derived from Eqn 11, while the lower bounds for welded I-beams are close to a second design curve ($a = 0.9$, $b = 0.2$, $c = 2$, $n = 1$).

Also shown in Figure 9 are the corresponding experimental data for cold-formed rectangular hollow sections (Pi and Trahair, 1995, Zhao et al, 1995) and cold-formed lipped channels (Put et al, 1997), and computer-generated data for hollow flange beams (Pi and Trahair, 1997) together with design curves derived from Eqn 11.

It can be seen that individual design curves can be chosen to model closely experimental and computer data on lateral buckling strengths.
5.2 Design Procedure

The first step in the design procedure is to select a design curve relating the nominal dimensionless uniform moment capacity \( M_{bu}/M_{px} \) to the slenderness \( \lambda \) which is appropriate to the type of beam under consideration. The section capacity \( M_{xx} \), the elastic buckling moment \( M_{m} \), and the moment modification factor \( \alpha_{m} \) may then be used in Eqn 18 to determine the particular slenderness \( \lambda \), and the design curve (Eqn 11) to find the corresponding value of \( M_{bu}/M_{px} \). The nominal design capacity can then be determined from

\[
M_{b} = \alpha_{m} M_{bu} \leq M_{xx}
\]  

(20)

6. CONCLUSIONS

There is a need for a set of multiple design curves for the design of steel beams against lateral buckling. Such a set of curves would allow the selection of design curves which are appropriate for different methods of manufacture (hot-rolled, welded, or cold-formed), different cross-sections (I-beams, channels, hollow sections, hollow flange beams, compound sections, etc), and different materials (structural steel, stainless steel, or aluminium).

A general method has been proposed in this paper for developing a four-part curve for the design of beams against lateral buckling. This curve consists of a constant moment region at the major axis section capacity, an inelastic buckling transition, and elastic buckling region, and another constant moment region at the minor axis section capacity.

This method was then used to develop two alternative sets of multiple design curves. Comparisons of one of these with available experimental and computer data allowed appropriate design curves to be selected for hot-rolled and welded I-beams, and for cold-formed rectangular hollow beams, channels, and hollow flange beams.

7. REFERENCES

MULTIPLE DESIGN CURVES FOR BEAM LATERAL BUCKLING


8. NOTATION

\( a, b, c \) Constants defining family of design curves

\( d_c \) Depth of HFB web

\( E \) Young's modulus of elasticity

\( E_s \) Strain-hardening modulus of elasticity

\( f_r \) Equivalent residual stress

\( f_y \) Yield stress

\( G \) Shear modulus of elasticity

\( h \) Distance between flange centroids

\( I_x, I_y \) Second moments of area about \( x, y \) axes

\( I_w \) Warping section constant

\( J \) Torsion section constant

\( k \) Effective length factor

\( L \) Length
**MULTIPLE DESIGN CURVES FOR BEAM LATERAL BUCKLING**

$L_e$  Effective length  
$M$     Moment  
$M_{bp}$ Nominal design moment capacity  
$M_{bu}$ Nominal design moment capacity for uniform bending  
$M_e$ Experimental moment  
$M_i$ Inelastic buckling moment  
$M_{ie}$ Value of $M_i$ at elastic buckling  
$M_m$ Maximum moment  
$M_{md}$ Maximum moment at lateral-distortional buckling  
$M_{m0}$ $= M_m / \alpha_m$  
$M_{pl}$ Full plastic moment about $x$ axis  
$M_{xs}, M_{ys}$ Section moment capacities about $x, y$ axes  
$M_{et}$ Elastic flexural-torsional buckling moment under uniform bending  
$M_{etd}$ Elastic lateral-distortional buckling moment under uniform bending  
$M_1, M_2, M_3$ Moments at quarter-, mid-, and three quarter-points  
$n$ Index defining family of curves  
$P_y$ Column minor axis flexural-buckling load  
$Q$ Transverse load  
$S_x$ Plastic section modulus about $x$ axis  
$u$ Lateral deflection  
$u_B, u_T$ Lateral deflections of bottom and top flanges  
$x, y$ Principal axes of cross-section  
$y_Q$ Distance of load below centroid  
$z$ Longitudinal centroidal axis  
$Z_x$ Elastic section modulus about $x$ axis  
$\alpha_m$ Moment modification factor  
$\alpha_e$ Slenderness reduction factor  
$\phi$ Capacity factor, or  
$\psi$ Angle of twist rotation  
$\psi_B, \psi_T$ Angles of twist rotations of bottom and top flanges  
$\lambda$ Modified slenderness for flexural-torsional buckling  
$\lambda_d$ Modified slenderness for lateral-distortional buckling  
$\lambda_v$ Elastic buckling slenderness limit  
$\lambda_{ie}$ Inelastic buckling slenderness limit  
$\lambda_{pl}$ Plastic slenderness limit  
$\lambda_{si}$ Section capacity slenderness limit
RESTRAINT OF BEAMS BY TRAPEZOIDALLY SHEETING USING DIFFERENT TYPES OF CONNECTION

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ABSTRACT

The stability problem of lateral torsional buckling is substantially reduced by adjacent members like sheeting where stabilization by shear stiffness and by torsional restraint is present. Here the torsional restraint is investigated which is especially influenced by the type of connection. Tests were carried out for the connection by shot fired pins and under high loads. The results are given as values for the torsional restraint coefficient $c_\phi$ of the actual configuration. Account is taken of the width $b_f$ of the beam flange, the thickness $t$ of the sheeting, the type of loading and the magnitude of the load $A$ introduced from sheeting to beam.

Two examples illustrate the application.

KEYWORDS

beams, stability, lateral torsional buckling, stabilization, torsional restraint coefficient, tests, connection, shot fired pins.

1. INTRODUCTION

Slender beams may buckle by a combination of lateral bending and twisting. This stability problem of lateral torsional buckling is especially of interest for beams like purlins. For an easy treatment the beam is usually assumed as an unsupported beam with simple supports. Most specifications define rules for unsupported beams.

In reality unsupported beams as assumed in codes are very rare in practical design. All loads are introduced by neighbouring separate members and their stiffness can be taken into account in much situations. Therefore the risk of lateral torsional buckling may be substantially reduced by adjacent members.

2. INFLUENCE OF RESTRAINT BY ADJACENT MEMBERS

Adjacent members are normally present as individual members like cross beams or as floor elements like sheeting. The restraining effect of cross beams is mainly caused by its bending stiffness and to some extent
by the joints. Contrary to this for sheeting commonly two restraining effects are present:

i) the horizontal deflection of the upper cord of the beam is prevented by the shear stiffness \( R \) of the sheeting, if fasteners between sheeting and beam are present,

ii) the bending resistance of the sheeting in combination with local deformations partly prevents the twisting of the beam.

The effect of horizontal restraint is remarkable, Heil (1994), Lindner (1996) and is in some cases sufficient to reach satisfactory capacity. In other cases additionally the effect of torsional restraint is needed. Therefore here this effect is dealt with only.

It is assumed as a basis for the following design procedure that the torsional restraint acts as an elastic restraint. Therefore it is assumed that the trapezoidally sheeting does not reach its ultimate capacity, which is followed by plastic deformations, at the same time when the beam tries to twist.

A simplified stability check requires a minimum stiffness of the torsional restraint coefficient \( c_\theta \) by Eqn. 1.

\[
c_\theta \geq \left( \frac{M^2}{EI} \right) k_0
\]  

(1)

<table>
<thead>
<tr>
<th>Top flange</th>
<th>Moment distribution</th>
<th>( k_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free ( I )</td>
<td>( + M )</td>
<td>( + M )</td>
</tr>
<tr>
<td>Restraint ( I )</td>
<td>( 0 )</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The factor \( k_0 \) depends on the moment distribution and the fact whether the upper flange of the beam is horizontally restraint or not. Design values for \( k_0 \) can be seen from Table 1, see DIN 18 800 part 2.

If the simplified stability check of Eqn. 1 is not used, the positive influence of the torsional restraint from trapezoidally sheeting can be accounted for by calculating the elastic critical moment \( M_{cr} \) taking into account \( c_\theta \) directly.

The effective torsional restraint coefficient \( c_\theta \) depends on the rigidity of the joint between the beam which should be stabilized and the structural element which acts as an adjacent member. For trapezoidally sheeting three deformation components should be taken into account as shown by Eqn. 2.

\[
\frac{1}{c_\theta} = \frac{1}{c_{\theta M}} + \frac{1}{c_{\theta A}} + \frac{1}{c_{\theta P}} \quad \text{[kNm/m]}
\]  

(2)

where

\( c_{\theta M} \) theoretical value with regard to the stiffness of the adjacent member only
\[ = \frac{4 E I_s}{L_s} \text{ for continuous sheeting,} \]
\( I_s, L_s \) effective moment of inertia, span of the adjacent member,
c_{ef} \quad \text{distorsion of the beam investigated}
= 5770/(d/t^3 + 0.5 b/t^3), \text{dimensions in [cm]},

c_{ea} \quad \text{rigidity of connection.}

The rigidity of connection $c_{ea}$ is most important especially for trapezoidally sheeting. Values for $c_{ea}$ can be obtained by tests only.

Important parameters which influence the results significantly are (see Lindner et al (1986), Lindner et al (1989), Lindner et al (1996)):

- type of the roofing skin (e.g. depth and width of the trapezoidally sheeting, plate thickness $t$),
- positioning of sheeting (positive or negative),
- location and distance of the fasteners (trough, crest, fasteners in every ($b=br$) or alternate ($e=2br$) trough/crest ),
- type of fasteners (selftapping screws diameter 6.3 mm, shot fired pins),
- roofing construction (with or without intervening thermal insulation, type of thermal insulation),
- type of loading (gravity load, uplift load by wind),
- magnitude of loading,
- width $b_f$ of the beam.

Values of $c_{ea}$ given earlier are based on tests with a plate thickness of $t = 0.75 \text{ mm}$, a magnitude of the load acting on the beam of $1.0 \text{ kN/m}$ and selftapping screws. They are included in relevant codes in Europe, like DIN 18 800 part 2 and Eurocode 3 part 1.3.

3. TEST CONFIGURATION

It was shown that a segment of a realistic constructed roof section may be used in tests for different types of loading and constructional details. Test configuration and conducting of the tests were described earlier, (see Lindner et al (1986), Lindner et al (1989), Lindner et al (1996)), see Figure 1.

![Figure 1: Test set up](image)

Three different types of beams were used in the tests: IPE 160, HE160A and IPE 160+U200, see Figure 2. These profiles have a width $b_f$ of the flange of 82, 160, 200 mm. The last combined profile was used because no other profile of $b_f = 200 \text{ mm}$ was available.

As trapezoidally sheeting the types of E40/183 with $t = 0.75$ and $1.00 \text{ mm}$ and E85/280 with $t = 0.75 \text{ mm}$ where used in positive and negative position, see Figure 3.
The connection between purlin and sheeting was executed by self-tapping screws with a diameter of 6.3 mm or shot fired pins of HILTI type ENP2-21L15 which have an approval by Deutsches Institut für Bautechnik. The connectors were placed in every bottom chord \( (e = b_r) \) or in every second bottom chord of the trapezoidally sheeting \( (e = 2b_r) \), see Figure 4.

Different test series were carried out in the years 1994-1996 to investigate especially the influence of the magnitudes of the load for gravity load, the width \( b_r \) of the flange and uplift loading. The results are an extension of the rules in codes already used. They are summarized in the following.

4. TEST CONDUCTION AND TEST RESULTS

The torque \( m \) to the beams is introduced by alternating loads at the cantilevers, see Figure 1, such leading to hysteresis loops. From deflections at different locations of the purlins the twisting \( \Theta \) is calculated.
An example for a torque moment-twist curve measured during the tests is shown in Figure 5. From this figure it can be seen that the stiffness related to a small value $\bar{\vartheta}$ can be much higher than for the value $\bar{\vartheta} = 0.1$ which was taken as an unfavourable reference value. All design values given in DIN 18 800 part 2, Eurocode 3 and Lindner et al (1996) are based on this unfavourable assumption of $\bar{\vartheta} = 0.1$.

![Figure 5: Example for a torque moment-twist curve from test](image)

From the point of practical application it seems to be suitable to use the results in the same way as proposed in the stability code DIN 18 800 part 2 and Eurocode 3 part 1.3. Therefore a basic connection stiffness $c_{bA,k}$ related to a flange width of $b_f = 100 \, \text{mm}$ is defined and given in Table 2. All other parameters are taken into account by additional factors with regard to Eqn. 3.

$$c_{bA,k} = \bar{c}_{bA,k} b_f k_t k_A$$

where

$$k_b = \left(\frac{b_f}{100}\right)^2 \quad \text{if} \quad b_f \leq 125 \, \text{mm}$$

$$k_b = \left(\frac{b_f}{100}\right) 1.25 \quad \text{if} \quad 125 < b_f \leq 200 \, \text{mm}$$

$$k_t = \left(\frac{t}{0.75}\right)^{1.1} \quad \text{positiv position}$$

$$k_t = \left(\frac{t}{0.75}\right)^{1.5} \quad \text{negativ position}$$

for gravity load:

$$k_A = 1.0 + (A - 1.0) 0.08 \quad \text{if} \quad t = 0.75 \, \text{mm} , \text{positive position}$$

$$k_A = 1.0 + (A - 1.0) 0.16 \quad \text{if} \quad t = 0.75 \, \text{mm} , \text{negative position}$$

$$k_A = 1.0 + (A - 1.0) 0.095 \quad \text{if} \quad t = 1.00 \, \text{mm} , \text{positive position}$$

$$k_A = 1.0 + (A - 1.0) 0.095 \quad \text{if} \quad t = 1.00 \, \text{mm} , \text{negative position}$$

for uplift load:

$$k_A = 1.0$$

$A \leq 12 \, \text{kN/m}$ load introduced from sheeting to beam $\frac{t}{b_f} \, \text{[mm]}$, $b_f \, \text{[mm]}$

It can be seen from Table 2 that for selftapping screws and the special type of shot fired pins investigated here the same values may be used with the extension of one case.
TABLE 2
CHARACTERISTIC VALUES FOR THE CONNECTION STIFFNESS $\varepsilon_{\text{BAK}}$, VALID FOR BEAMS WIDTH $b_t = 100$ mm, FASTENING AT BOTTOM CHORD OF TRAPEZOIDALLY SHEETING

<table>
<thead>
<tr>
<th>position of sheeting</th>
<th>distance of fasteners $e / b_t$</th>
<th>selftapping screws 6.3 washer</th>
<th>HILTI shot fired pin ENP2-21L15</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>negative</td>
<td>1.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

gravity load:
1 | x | x | 5.2 | 4.0 |
2 | x | x | 3.1 | 3.1 |
3 | x | x | 3.1 | 3.1 |
4 | x | x | 2.0 | 2.0 |

uplift load:
5 | x | x | 2.6 | 2.6 |
6 | x | x | 1.7 | 1.7 |
7 | x | x | 1.1 | 1.1 |
8 | x | x | 0.6 | 0.6 |

5. STATISTICAL EVALUATION OF THE TESTS

In order to find out whether the proposed characteristic values of Eqn. 3 in combination with Table 2 have a sufficient safety level statistical evaluations were carried out.

For a group of 111 tests for gravity load the relation $\alpha$ between test result and design value calculated by Eqn. 3 is calculated. The results can be seen from Table 3.

TABLE 3
TEST EVALUATION FOR THE RELATION FACTOR $\alpha$ FOR GRAVITY LOAD

<table>
<thead>
<tr>
<th>mean value</th>
<th>standard deviation</th>
<th>standard dev. /mean value</th>
<th>lower limit</th>
<th>simplified lower limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$s$</td>
<td>$s / m$</td>
<td>$m - 0.8s$</td>
<td>$0.8m$</td>
</tr>
<tr>
<td>1.436</td>
<td>0.222</td>
<td>0.159</td>
<td>0.992</td>
<td>1.149</td>
</tr>
</tbody>
</table>

Similar results were obtained for 36 test for uplift load. Because of a higher scattering of the results it was necessary to use a statistical factor of 0.7 instead of 0.8 to get sufficient results.

6. EXAMPLES

6.1 Purlin

A roof is investigated with purlins of IPE 240 section as continuous beams, span $L = 11$ m, loaded by uniformly distributed load. The trapezoidally sheeting $40 \times 183 \times 0.75$ mm is fastened at the bottom chord, $e=b_t$. 

It can be shown due to the rules of DIN 18 800 part 2 that the shear stiffness of the sheeting leads to a horizontal restraint of the upper chord.

![Figure 6: Dimensions for IPE 240](image)

For steel grade St 37 (Fe 360): $M_{pr,k} = 88$ kNm

from textbook $I_z = 284$ cm$^4$

$c_{eq} = 4 \cdot 21000 \cdot 0.00216 / 3.0 = 60.5$ kNm/m

$c_{eq} = 5770 / (24 / 0.62^3 + 0.5 \cdot 12 / 0.98^3) = 53.9$ kNm/m

from Table 2: $c_{eq} = 3.1$ kNm/m

Eqn. 4b: $c_{eq} = 3.1 \times (120 / 100)^2 = 4.46$ kNm/m

Eqn. 3:

$1/c_0 = 1/60.5 + 1/53.9 + 1/4.46$

$c_0 = 3.86$ kNm/m

Eqn. 1 and Table 1:

$c_0 \geq (88.0^2 / (21000 \cdot 0.0284)) \times 0.23 = 2.99$ kNm/m < 3.86 kNm/m

Because the minimum stiffness requirement under the simplified assumption of $k_A = 1.0$ is fulfilled no further check for the purlin with regard to lateral torsional buckling is necessary.

### 6.2 Rafter of frame

The rafter of a frame with the span length of $L = 15$ m, the profil IPE 400 and steel grade St 37 (Fe 360) is investigated as an individual member, see Figure 7. The actions are given by

<table>
<thead>
<tr>
<th>Permanent action</th>
<th>g = 0.95 kN/m$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable action</td>
<td>p = 0.75 kN/m$^2$</td>
</tr>
<tr>
<td>Total load</td>
<td>q = 1.70 kN/m$^2$</td>
</tr>
</tbody>
</table>

![Figure 7: Moment distribution](image)
The rafter is restraint by trapezoidally sheeting E100/t=1.00 mm in positive position, connected every 2.
trough (c = 2b). The sheeting runs from rafter to rafter as a two span member with a = 6.0 m span length.

Using the partial safety factors from DIN 18 800 including $\gamma_m = 1.1$

\[ A = 1.25 \cdot 1.1 \left( 1.35 \cdot 0.95 + 1.50 \cdot 0.75 \right) \cdot 6.0 = 19.9 > 12.0 \text{kN/m} \]

The shear stiffness is calculated due to the approval ("Typenprüfung") of Hoesch (1989).

\[ K_1 = 0.188 \text{ m/kN} \]
\[ K_2 = 16.6 \text{ m}^2/\text{kN} \]
\[ L_s = 2 \cdot 6.0 = 12.0 \text{ m} \]
\[ S = 10^4 / (0.188 + 16.6 / 12) = 6364 \text{ kN} \]
\[ S_{id} = 6364 \cdot 6.0 = 38180 \text{ kN} \]

The shear panel is connected to the rafters and to the edge members as well. But the sheeting is connected in every second rib only and therefore due to DIN 18 800 part 2 20% of $S_{id}$ can bee used only.

\[ R = S_{id} / 5 = 7640 \text{ kN} \]

In order to check whether full lateraly restraint of the top chord may be assumed Eqn. 7 of DIN 18 800 part 2 is used

\[ \lim R = \left( \frac{E \cdot I_w}{L^2} + G \cdot I_T + E \cdot I_x \frac{\pi^2}{L^2} 0.25 \cdot h^2 \right) \frac{70}{h^2} \]

where

$I_w$ warping constant
$I_T$ S.Venant torsion constant
$I_x$ moment of inertia weak axis

\[ \lim R = (21000 \cdot 0.0049 \pi^2 / 15^2 + 8100 \cdot 0.00514 + 21000 \cdot 0.132 \pi^2 0.25 \cdot 0.4^2 / 15^2 ) 70 / 0.4^2 \]

\[ = 22300 \text{ kN} \]

No full restraint can be assumed because of $R < \lim R$.

Simplified check by Eqn. 1:

\[ \text{requ.} c_{\phi,k} = \left( \frac{314^2}{(21000 \cdot 0.132)} \right) 3.5 = 124 \text{ kNm/m} \]

\[ b_{f} / 100 = 180 / 100 = 1.80 \]
\[ k_s = 1.25 \cdot 1.8 = 2.25 \]
\[ k_l = (1.0 / 0.75)^{1.1} = 1.37 \]
\[ k_A = 1.0 + (12.0 - 1.0) 0.095 = 2.05 \]
\[ c_{\phi,k} = 3.1 \text{ kNm/m} \]
Eqn. 3: \[ c_{BA} = 3.1 \cdot 2.25 \cdot 1.37 \cdot 2.05 = 19.6 \text{ kNm/m} \]
\[ c_{ep} = 120 \text{ kNm/m} \]
\[ c_{em} = 137 \text{ kNm/m} \]
\[ c_\theta = 1 / (1/120 + 1/137 + 1/19.6) = 15.0 \text{ kNm/m} \]

The first two values for \( c_{ep} \) and \( c_{em} \) are taken from the book Lindner et al (1994).

The simplified design check leads to:
\[ 15.0 < 124 \text{ kNm/m} \]
and is therefore not sufficient.

Therefore another design check is necessary. In doing this the positive action of the stiffnesses \( c_\theta \) and \( R \) are accounted for by calculating the elastic critical moment \( M_\text{cr} \), see Bamm et al (1995).

The results are given in Table 4.

**TABLE 4**

RESULTS OF LATERAL TORSIONAL BUCKLING ANALYSIS FOR THE RAFTER

<table>
<thead>
<tr>
<th>R [kN]</th>
<th>( c_\theta ) [kNm/m]</th>
<th>( M_\text{cr} ) [kNm]</th>
<th>( x_\text{M} )</th>
<th>( M_\text{s} ) [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>68.5</td>
<td>0.216</td>
<td>65.6</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>233</td>
<td>0.636</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>7640</td>
<td>381</td>
<td>0.826</td>
<td>259</td>
</tr>
<tr>
<td>4</td>
<td>7640</td>
<td>1105</td>
<td>0.983</td>
<td>309 &gt; 303</td>
</tr>
</tbody>
</table>

It can be seen that the influences of the torsional restraint coefficient \( c_\theta \) and the shear stiffness \( R \) are effective and similar in the result. If both stiffnesses are taken into account together 98.3 % of the full plastic moment is reached and therefore the design check leads to 309 > 303 kNm.

7. CONCLUSIONS

Beams which are prone to lateral torsional buckling are restraint in many cases by adjacent members. It is reported on new investigations concerning the torsional restraint by trapezoidally sheeting. For the first time the effects of shot fired pins and the magnitude of the load were accounted for. The results are given as torsional restraint coefficients which can directly be used for practical design. Regulations already given in DIN 18 800 part 2 and Eurocode part 1.3 can be extended by these proposal. The positive effect of the restraint by trapezoidally sheeting are demonstrated by two examples.

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References


ELASTO-PLASTIC BEHAVIOR OF LATERALLY-BRACED COMPRESSION MEMBERS

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ABSTRACT

Elasto-plastic analysis was carried out for the behavior of compression members which had initial deflection expressed by a series of sine waves, and were braced at an arbitrary intermediate point other than the center. A parametric study on the bracing efficiency reveals that the ultimate strength could be increased by bracing, but the efficiency changes depending on the type of initial deflection and the position of the brace, and that if the designer expects the strength of a member braced at the center to be equal to the strength of the member of half length with the same initial deflection, the design bracing force may be taken equal to 2% of the ultimate strength, and the required stiffness ratio of the brace may be about 4, but it is very difficult to achieve the ideal buckling strength of a member whose effective length is equal to half the original length.

KEYWORDS

compression member, buckling, bracing, bracing force, bracing stiffness

1. INTRODUCTION

The design of a compression member which is elastically supported at an arbitrary intermediate point aims to increase the strength by decreasing the effective buckling length. A number of researches on the bracing requirements for the compression member with the intermediate brace have been reported[2-6]. However, in most cases, the initial crookedness was too simplified, and/or only the case of the brace placed at the center of the member was considered. Consequently, the bracing requirements derived were rather too moderate. For example, the conventional design often assumes the force in the brace to be equal to 2% of the force acting on the compression member, but the bracing force well exceeds 2% in the case that the initial crookedness is large, and/or the intermediate brace cannot be arranged at the center. In addition, the research in the past have not fully clarified the phenomenon of the deflection reversal involved in the behavior of the laterally-braced compression member, that is, the deflected configuration of a initially-crooked compression member suddenly shifts from the first mode to the second. There has been no information available on its effect on the bracing requirements. In order to clarify those problems mentioned, elasto-plastic analysis was carried out for the behavior of eccentrically-loaded compression members which had initial deflection expressed by a series of sine waves, and were braced at an arbitrary intermediate point other than the center. The paper first introduces
the results of numerical analysis including the deflection reversal, and then discusses the effects of the initial deflection and the position and the stiffness of the brace on the maximum strength of the compression member and the bracing force.

2. NUMERICAL ANALYSIS

2.1 Stress-Strain Relation

The stress(σ)-strain(ε) curve assumed for the steel compression member is shown in Fig. 1, of which mathematical expression in the inelastic region is given as follows:

\[ \sigma = \frac{1}{1 + e^{\epsilon}} \sigma_y; \quad \alpha = \frac{5}{2} + ln \left( \frac{2}{3} \right) - \frac{E\epsilon}{0.24\sigma_y} \]  

(1-a, b)

where \( \sigma_y \) denotes the yield stress, \( E \) Young’s modulus, and \( e \) the base of natural logarithm. Equation (1) is derived so that the tangent modulus strength \( \sigma_{ct} \) of a compression member calculated from Eq. (1) agrees the buckling strength given by Design Standard for Steel Structures published by Architectural Institute of Japan[H1] shown below:

\[ \sigma_{ct} = \left[1 - 0.4 \left( \frac{\lambda}{\lambda_p} \right)^2 \right] \sigma_y \]  

(2)

where \( \lambda \) denotes the slenderness ratio, and \( \lambda_p = \pi \sqrt{E/0.6\sigma_y} \).

2.2 Model of Analysis

Figure 2 shows the model of a simply-supported compression member of length \( l \), which is intermediately braced at the distance \( l_1 \) from the left end. The brace is shown by a elastic spring with spring constant \( K \). The axial load \( P \) is applied with the eccentricity \( \epsilon_L \) and \( \epsilon_R \) at the left and the right ends, respectively, and the deflection \( y \) is generated by the axial load, in addition to the initial deflection \( y_0 \). The vertical reaction forces at the left and the right supports and at the spring are denoted by \( F_L, F_R \) and \( F \), respectively.

2.3 Analysis of Load-Deflection Relation

The load-deflection relation of the braced member shown in Fig. 2 is analyzed by a conventional numerical integration scheme, dividing each of the left hand and the right hand portions separated at the brace into \( n \) segments. The equilibrium equation of \( j \)th segment is written as follows, referring to Fig. 2:

\[ 0 \leq j < n \quad M_j = P(y_0j + y_j) - F_L j\Delta x_L \]  

(3)
where \( M_j, y_0, \) and \( y_j \) denote the bending moment, the initial deflection, and the additional deflection caused by loading at the subdivision point \( j \), respectively, and \( \Delta x_L \) and \( \Delta x_R \) the lengths of the subdivided elements of the left and the right hand portions, respectively. The central difference expression of the curvature \( \phi_j \) at the point \( j \) is given by

\[
\phi_j = \frac{y_j - 2y_{j+1} + y_{j-1}}{\Delta x^2}
\]

where \( \Delta x \) denotes the length of the subdivided element.

The numerical integration scheme to analyze the load-deflection relation is as follows, referring to the flowchart shown in Fig. 3: First, trial values are assumed for the bending moment at 1st point \( M_j \) and the axial force \( P \) for a given value of the displacement \( y_{\text{given}} \) at the bracing point, and the moment-curvature relation under the axial load \( P \) is independently calculated for the cross section of the compression member using the fiber model. Suppose that the integration proceeds to the point \( j \) and the quantities \( M_j, y_j \) and \( \phi_j \) have been determined at all points from 0 to \( j \). Then, the deflection \( y_{j+1} \) at the point \( j+1 \) is determined from Eq. (5) and then the bending moment \( M_{j+1} \) is determined from Eq. (3) or Eq. (4). The curvature \( \phi_{j+1} \) corresponding to \( M_{j+1} \) is determined from the moment-curvature relation. After repeating this procedure up to the point \( n \), the bracing point, if the value of the deflection \( y_n \) is not sufficiently close to the value of \( y_{\text{given}} \), the procedure should be re-started with another trial value of \( M_j \). Otherwise the same procedure is continued up to the point \( 2n \), the right hand support, and if the value of the deflection \( y_{2n} \) is sufficiently close to zero, the converged solution is obtained. Otherwise the procedure should be re-started with another initial values of \( M_j \) and \( P \) at the beginning. In the numerical calculation, the value of \( n \) was taken equal to 100, and the convergence criteria set for \( y_n \) and \( y_{2n} \) were \(|y_n - y_{\text{given}}| < y_{\text{given}}/500 \) and \(|y_{2n}| < 10^{-6} \), respectively.
3. RESULTS AND DISCUSSION

The variables and their values assigned in the parametric study were as follows:

i) Slenderness ratio ($\lambda = \frac{l}{r}$, $r =$ radius of gyration): $\lambda =$ 60, 120, 180, 240, 300.

ii) Initial imperfection: Three types of combination of load eccentricity and initial deflection expressed by a series of sine waves were considered, that is, the types A, B, and C as shown in Table 1.

iii) Position of the brace ($\eta/l$): The value of $\eta/l$ ranged from 0.05 to 0.95 with an interval of 0.05.

iv) Stiffness ratio of the brace ($k = K/K_{\text{min}}$: $k =$ 0.0, 0.5, 1.0, 2.0, 4.0, 8.0. Here, $K_{\text{min}}$ is the minimum stiffness of the brace required for an ideal compression member braced at the intermediate point to reach the buckling strength calculated with the effective length equal to the length of the longer portion of the compression member separated by the brace, and it is given by Eqs. (6) and (7).

$$K_{\text{min}} = \frac{l_1 + l_2}{l_2} P_{\text{cr}}, \quad P_{\text{cr}} = \frac{\pi^2 E I}{l_a^2}$$

where $l_a$ is the larger of $l_1$ and $l_2$, and the tangent modulus factor $\tau$ is derived from Eq. (1) as follows:

$$\lambda_a \geq \sqrt{\frac{\pi^2 E}{0.6 \sigma_y}} ; \quad \tau = 1.0 \quad \text{(8)}$$

$$\lambda_a < \sqrt{\frac{\pi^2 E}{0.6 \sigma_y}} ; \quad \tau = \left(1 - \frac{0.24 \lambda_a^2 \sigma_y}{\pi^2 E}\right) \frac{\lambda_a^2 \sigma_y}{\pi^2 E} \quad \text{(9)}$$

In addition, the cross section of the compression member is assumed to be square.

3.1 Load-Deflection Relation

It is quite difficult to analyze the elasto-plastic behavior of a laterally-braced compression member, which is a highly non-linear problem since the member changes its deflected shape from the single-curvature type to the double-curvature type as the axial load $P$ increases. Figure 4 and 5 show good examples in which the change of the deflected shape is clearly captured. The relation between the load and the deflection at the bracing point and the deflected shapes shown in these figures are the results of the numerical analysis of compression members with the initial imperfection of the type B, $\lambda =$ 180, $l_1/l = 0.45$ and various values of $k$. Four deflected shapes in Fig. 5 are obtained at four points on the load-deflection curve shown in Fig. 4: 1) maximum strength $P_{\text{max}}$, 2) occurrence of deflection reversal $P_r$, 3) strength reduced to 80% of the maximum strength $P_{0.8}$, and 4) strength reduced to 50% of the maximum strength $P_{0.5}$. It is observed in Fig. 5 that the change of the deflected shape occurs in the case of large value of $k$, while such a change is not observed in the case of small value of $k$.

<table>
<thead>
<tr>
<th>Type</th>
<th>Eccentricity $\epsilon_L = \epsilon_R$</th>
<th>Initial Deflection $y_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>$\frac{1}{10^3} \sin(\frac{\pi x}{l}) + \frac{1}{10^5} \sin(\frac{2\pi x}{l}) + \frac{1}{10^5} \sin(\frac{3\pi x}{l})$</td>
</tr>
<tr>
<td>B</td>
<td>$\frac{l}{20} + \frac{l}{1000}$</td>
<td>$\frac{1}{10^3} \sin(\frac{\pi x}{l}) + \frac{1}{10^5} \sin(\frac{2\pi x}{l}) + \frac{1}{10^5} \sin(\frac{3\pi x}{l})$</td>
</tr>
<tr>
<td>C</td>
<td>$\frac{l}{20} + \frac{l}{500}$</td>
<td>0</td>
</tr>
</tbody>
</table>
3.2 Maximum Strength

Figure 6 shows the relation between the maximum strength ratio of $P_{\text{max}}/P_y$ and the stiffness of the brace $k$, where $P_{\text{max}}$ and $P_y$ denote the maximum strength and yield strength of the compression member, respectively. Solid circle on each curve indicates the value of the standard strength $P_0$. This is the strength calculated for a model with the length $l_y$ shown in Fig. 7, which is separated from the original compression member, and subjected to the load $P$ with the eccentricities $e_L$ and $e_R$. The initial deflection given to this model is the one shown in Table 1 in which $l$ is replaced by $Z$. The value of the stiffness ratio $k$ corresponding to $P_0$ is about 1 in general, but it becomes rather large in the case of the member with small slenderness ratio braced at the center. The stiffness ratio $k$ takes the maximum value 3.7 for the member with $\lambda = 60$ and the initial imperfection of the type C braced at the center, as shown in Fig. 6(e).

Figure 8 shows the relation between the ratio of $P_{\text{max}}/P_y$ and the position of the brace $l/l$ by thin solid lines. It is observed that the value of $P_{\text{max}}/P_y$ increases as the value of the stiffness ratio $k$ increases, and that the point of peak strength shifts to the left from the midpoint with the increase in $k$, in the case of the compression member having the asymmetrical initial imperfection of the types A and B. The peak strength occurs around the point of $l/l = 0.45$ in the case of the type A with $k$ more than 2.

Thick solid and dashed lines in Fig. 8 indicate the values of $P_{\text{cr}}$ and $P_0$, which are the buckling strength of an ideal compression member with the length $l_y$ given by Eq. (7), and the standard strength numerically obtained for a member with the initial imperfection shown in Fig. 7, respectively. In the case of a member with $\lambda = 240$, $P_0$ could be reached, if the member is braced with $k \geq 2$ regardless of the position of the brace. However, if the brace is located around the midpoint, $P_{\text{cr}}$ could not be reached, even if the value of $k$ is increased to 8.

3.3 Bracing Force at Maximum Strength

Figures 9 and 10 shows the values of the bracing force ratio $F/P_{\text{max}}$ plotted against the values of $k$ and $l/l$, where $F$ denotes the bracing force generated when the member strength reaches $P_{\text{max}}$. It is observed in Fig. 9 that the bracing force ratio increases with the increase in $k$ and tends to converge to a certain value in general.
but the shape of the $FIP_{\text{max}}$-$k$ curve quite differs depending on the position of the brace. In the case of the brace located at the center, $FIP_{\text{max}}$ converges at $k$ equal to about 4, and their values are 0.6, 1.7 and 1.9% for the initial imperfection of the types A, B and C, respectively. This result conforms to the conventional design method that the brace is proportioned against 2% of the force working on the compression member. However, the converged value of $FIP_{\text{max}}$ becomes larger as the position of the brace apart from the midpoint, and as the load eccentricity becomes larger in the order of the types A, B and C of the initial imperfection.

In Fig. 10, the values of $FIP_{\text{max}}$ are plotted against the position of the brace $l_f/l$ with varying values of $k$ for the case of $\lambda = 60, 120, 180$ and $240$ with the initial imperfection of the type B. In the case that the brace is located within the range $0.3 \leq l_f/l \leq 0.7$, the value of $FIP_{\text{max}}$ decreases with the increase in $k$ as observed in Fig. 10(d) for $\lambda = 240$, or it does not change much with the change in $k$ as in Fig. 10(a) for $\lambda = 60$. On the other hand, in the case of the position of the brace outside this range, the value of $FIP_{\text{max}}$ increases with the increase in $k$, which means that when the designer tries to increase the stiffness of the brace in order to increase the design strength of a compression member, it may lead to the shortage of the strength of the brace.

4. CONCLUDING REMARKS

1) The change of the deflected shape from the single-curvature type to the double-curvature type occurs in the compression member supported by the brace with large stiffness, but not in the case of the brace with small stiffness.

2) In some cases, the buckling strength $P_{\text{cr}}$ of an ideal compression member with the length $l_a$ could not be reached, even if the stiffness ratio of the brace $k$ is increased to 8, while the standard strength $P_0$ of a member with the length $l_a$ and the initial imperfection could be reached, where $l_a$ denotes the length of the longer portion of the original member separated by the brace.
Behavior of Laterally-Braced Compression Members

Fig. 8: Maximum Strength Ratio-Bracing Point Relations

Fig. 9: Bracing Force Ratio-Bracing Stiffness Ratio Relations
3) The bracing force ratio $F/P_{\text{max}}$ increases with the increase in $k$ and tends to converge to a certain value in general. The converged value of $F/P_{\text{max}}$ becomes larger as the position of the brace apart from the midpoint, and as the load eccentricity becomes larger.

4) The requirements in the brace design are related to both the position of the brace and the stiffness of the brace. In order to achieve the standard strength $P_0$, the brace should be proportioned so that $k \geq 4$ and $F/P_0 \geq 2\%$, in the case that the brace is placed around the center of the compression member. However, in the case of the position of the brace near the end of the compression member, the required value of $k$ may be less than 4, but the required value of $F/P_0$ is much larger than 2\%.

References


INELASTIC BEHAVIOR OF STEEL BEAM-COLUMNS
SUBJECT TO VARYING AXIAL FORCE
AND CYCLIC BENDING MOMENT

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ABSTRACT

Although many experimental studies on the inelastic behavior of steel beam-columns have been conducted, most of the experiments were done under conditions of constant axial force. When multi-story frames are excited by earthquakes, the axial force of beam-columns varies due to the overturning moment and the vertical vibration of the frame. To study the effect of this varying axial force on the inelastic behavior of beam-columns, a series of experiments is conducted. An analytical model for the inelastic behavior of steel beam-columns subject to varying axial force and cyclic bending moment is proposed. The results obtained by the analysis are in agreement with those of the experiments.

KEYWORDS

Steel beam-column, Loading test, Varying axial force, Cyclic bending moment, Vertical vibration, Analytical model, Load deflection curve

1. INTRODUCTION

During earthquakes the columns of multi-story frames are subject to varying axial forces caused by overturning moment and vertical vibration. Many experimental studies of inelastic behavior of steel beam-columns which are subject to cyclic bending moment under the condition of constant axial force have been carried out. However, there have been few experimental studies of the behavior under conditions of varying axial force. Experimental studies on the behavior of steel beam-columns under the condition of varying axial force have been conducted by Ohi, Chin and Takanashi (1992, 1993) and by Yamada, Akiyama and Kuwamura (1994). Ohi et al. carried out static cyclic loading tests and on-line tests on beam-columns subject to a varying axial force due to the overturning moment. Yamada et al. conducted tests using simple beam-column models subject to the monotonic bending moment for the following cases: when the axial force is constant, when it steadily rises or decreases with an increase in the end rotation angle. An experimental study of the beam-column behavior, in cases where cyclic bending moment act on the steel beam-columns under the condition of varying axial force induced by vertical vibration, has yet to be carried out.

In this study, tests of the inelastic behavior of beam-columns which are subject to a varying axial force and a cyclic bending moment simultaneously were conducted.
2. EXPERIMENT

2.1 Test Specimen

In this study, ten 1/4 scale models of a cantilever beam-column were used. Figure 1 shows the shape of the model. It possesses a box type section with the dimension of 120×120×9 mm which is assembled by corner welding of steel plates. A steel block with a center hole through which a pin can pass is welded at the column top and at the lower part rigid beams are welded. The length of the beam-column (L) is 450 mm or 700 mm. The slenderness ratios in the case of 2L for the column length are 19.8 and 30.8 respectively.

![Figure 1: Test Specimen (Dimensions are in mm.)](image)

2.2 Mechanical Properties of Steel

SS400 steel is used for the models. Figure 2 illustrates the stress-strain diagram which was obtained from stub-column tests. The length of member for the stub-column tests is three times the width of the section. The stress-strain relation which is used in the analysis described later is indicated in Figure 3.

![Figure 2: Result of Stub-Column Test](image)

![Figure 3: Stress-Strain Relation Used in Analysis](image)

2.3 Loading Method

Figure 4 shows the experimental setup. Loads are exerted onto the model by using two hydraulic actuators, one acting in a vertical and the other in a horizontal direction. Vertical loading is carried out by load control and horizontal loading is conducted by displacement control.

2.4 Axial Force Loading Patterns

Axial forces are loaded in the following four patterns: C, M, OV and UD.

Loading pattern C corresponds to the condition...
of a constant axial force.

Loading pattern M is for the case in which the axial force rises steadily with an increase in the horizontal displacement.

Loading pattern OV corresponds to the condition of a varying axial force induced by overturning moment. First, the axial force corresponding to the vertical load induced axial force is loaded onto the model under the condition that the horizontal displacement is 0. Next, a varying axial force approximately proportional to the positive and negative cyclic displacement in a horizontal direction is loaded.

Loading pattern UD corresponds to the condition of a varying axial force induced by vertical vibration. With regard to multi-story steel structures, the ratio of the natural vibration period of the primary mode of vertical vibration to that of horizontal vibration is approximately 1/5 ~ 1/10 (Yamazaki and Minami, 1997). Therefore, in these experiments, after loading the axial force equivalent to the axial force induced by a vertical load under the condition of 0 for the horizontal displacement, the varying axial force which is repeated with an increase and a decrease 5 ~ 10 times for every cyclic horizontal loading is imposed. In this paper, the compressive axial force is set to be positive.

2.5 Testing Program

Table 1 shows the testing program. As is explained in the table, the specimen’s name indicates the slenderness ratio and loading pattern of the axial force. When the axial force varies in loading patterns of OV and UD, a mean axial force corresponding to a vertical load induced axial force is set to be 20% or 30% of the yield axial force and the amplitude of the varying axial force is set to be 30% or 40% of the yield axial force. Figure 5 shows controlled horizontal displacement and axial force for each specimen.

3. ANALYSIS

3.1 Assumptions for Analysis

Basic assumptions in analysis are as follows.

(1) Plane sections before deformation remain plane after deformation.
(2) Shearing deformation is ignored.
(3) Effects of shear stress on yield is ignored.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Axial Loading Pattern</th>
<th>L (mm)</th>
<th>Axial Loading Pattern</th>
<th>Lateral Loading</th>
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Note: Specimen name A - 20 - C - 1 (for example) Axial Loading Patterns: C:Constant, M:Monotonic increase, OV:Overturning moment, UD:Vertical vibration
Figure 5: Horizontal Displacement and Axial Force
3.2 **Analytical Method**

The solutions for the axial force increment and the horizontal displacement increment can be obtained using the following method.

In the case of Figure 6(1), (2), in which the relationship between force and displacement in an arbitrary loading step and that in a step which follows it are shown, the following equation regarding the increment of load and displacement can be formed.

\[ \Delta M_f = \Delta Q \cdot L + P \cdot \Delta \delta + \Delta P \cdot \delta \]  \hspace{1cm} (1)

The unknown incremental values of \( \Delta Q \) and \( \Delta M_f \) for the assigned increment \( \Delta P \) and \( \Delta \delta \) can be obtained from the following computation method.

1. The value of \( \Delta M_f \) is assumed.
2. \( \Delta Q \) can be determined by equation (1).
3. The incremental values of both the moment and horizontal displacement at each position can be gained by carrying out numerical integration from the fixed end using the values of \( \Delta M_f \) and \( \Delta Q \).
4. The value of \( \Delta M_f \) is modified and the procedures of (1) ~ (3) are repeated until the value of \( \Delta \delta \) for the displacement increment at the top of the beam-column which was obtained in the numerical integration coincides with the value of \( \Delta \delta \) for the assigned displacement increment.

The following method is used in the numerical integration.

As shown in Figure 7, the member is divided into minute length, \( \Delta z \) for each. The displacement increment \( \Delta \delta \) at location \( z_i \) can be obtained from the following equations using the value of \( \Delta \Phi_i \) for the curvature increment at location \( z_{i-1} \).

\[ \Delta \delta_i = \Delta \delta_{i-1} + \Delta \Phi_{i-1} (\Delta z) + \Delta \Phi_{i-1} \frac{(\Delta z)^2}{2} \]  \hspace{1cm} (2)

\[ \Delta \delta'_i = \Delta \Phi_{i-1} (\Delta z) \]  \hspace{1cm} (3)

Where

\[ \gamma = \frac{d[\ ]}{dz} \]

The moment increment \( \Delta M_i \) at \( z_i \) can be obtained from the following equation.

\[ \Delta M_i = \Delta M_f - \Delta Q \cdot z_i - P \cdot \Delta \delta_i - \Delta P \cdot \delta_i \]  \hspace{1cm} (4)
The curvature increment $\Delta \Phi_i$ for the moment increment $\Delta M_i$ and the axial force increment $\Delta P$ can be obtained by dividing the section into minute elements parallel to the bending axis as shown in Figure 8 and by using the instantaneous stiffness of each element determined in the previous step.

### 3.3 Relationship between Stress and Strain

The data in Figure 3 gained from the stub-column tests is used for a monotonic model. As a hysteresis model, Takanashi & Ohi's model (Ohi et al., 1992) made by connecting an unloading point and a target point with the Ramberg-Osgood function is used. Figure 9 shows the hysteresis characteristics of the material.

$$\left(\sigma - \sigma_u \right) \left[ a + b \left| \sigma - \sigma_u \right|^{r-1} \right] = \varepsilon - \varepsilon_u$$  \hspace{1cm} (5)

The coefficients $\Psi$ (Figure 9) which indicates the degree of Bauschinger's effect and $r$ of the Ramberg-Osgood function in equation (5) are set at 0.80 and 7.0 respectively.

![Figure 9: Stress-Strain Hysteresis Model](image)

### 4. RESULTS

The following symbols are used for the indication of the experimental results.

- $P$ : Axial force,
- $Q$ : Horizontal force,
- $\delta$ : Horizontal displacement,
- $P_y$ : Yield axial force ($A \cdot \sigma_y$),
- $Q_p = M_p / L$,
- $\delta_p = \left( \frac{L^2}{3EI} + \frac{1}{A_S G} \right) M_p$,
- $M_p$ : Plastic moment,
- $I$ : Geometrical moment of inertia,
- $A$ : Sectional area,
- $E$ : Young's modulus,
- $A_S$ : Sectional area for shear deformation,
- $G$ : Shear modulus.
4.1 Axial Force Loading Pattern C, M

Figure 10 illustrates the comparison between the experimental results and the analytical ones in pattern C and pattern M. In pattern C, both monotonic loading (A-20-C-1) and cyclic loading (A-20-C-2, A-20-C-3) are carried out. In pattern M, monotonic loading (A-20-M-1) only is conducted.

4.2 Axial Force Loading Pattern OV

Figure 11 compares the results obtained from the experiment in axial force loading pattern OV with the analytical results (A-20-OV-1, A-30-OV-1). The varying axial force is loaded so that it reach its maximum (P/Py=0.54) at the unloading point of horizontal displacement on the positive side and its minimum (P/Py=0) at the unloading point on the negative side.
4.3 Axial Force Loading Pattern UD

Results of the experiments on monotonic loading (A-20-UD-1) and cyclic loading (A-20-UD-2, A-20-UD-3, A-30-UD-1) in loading pattern UD are shown in Figures 12 and 13. Figures 14, 15, and 16 illustrate the relationship between experimental results and analytical ones in each cyclic loading. In these figures, the axial force, in the case of the increment of horizontal displacement being positive, is shown above the figure indicating the relationship between $Q/Q_p$ and $\delta/\delta_p$. The axial force in the case of the increment of horizontal displacement being negative is shown below the figure. Furthermore, in Figures 12 and 16, the results obtained from an analysis of the relationship between horizontal displacement and horizontal force, which was made under the condition of constant axial forces, being the mean axial force ($P/Py=0.27$), the maximum axial force ($P/Py=0.54$) and the minimum axial force ($P/Py=0$), are shown using broken lines.

4.4 Discussion of Test Results

Load-displacement curves in loading pattern UD show their complicated behavior. However, the analytical results correspond accurately to details of the behavior obtained from the experiments. The moment the axial force value becomes high it can be seen that the horizontal resistance force drops temporarily to a fairly low degree due to the combination of a decrease in plastic moment induced by axial force and the P8 effect (Figures 12 and 13). It is clear from Figures 12 and 16 that the strength of a beam-column subject to varying axial force varies approximately within the range of two strengths, one determined under condition that the axial force is constant with the maximum value of varying force and the other determined under the condition of the axial force being constant with the minimum value. However, if the behavior is examined precisely, it is
Figure 14: Results of A-20-UD-2

Figure 15: Results of A-20-UD-3
found that the horizontal strength under a varying axial force is slightly lower than that under a constant axial force. It is also found that when a beam-column is subject to varying axial force and cyclic horizontal force simultaneously, as can be seen at point A in Figure 14 and point B in Figure 15, there is a case in which the horizontal force lowers at points except for the points of the highest axial force.

5. CONCLUSION

Experiments on the inelastic behavior of beam-columns with a box type section subject to both a varying axial force and a cyclic bending moment were conducted. The moment the axial force reaches a high value it was found that the horizontal strength of the beam-column temporarily dropped to a fairly low degree due to the combination of a decrease in plastic moment caused by axial force and P₆ effects. The analyses resulted in the possibility of estimating the behavior of a beam-column accurately under the loading of a varying axial force.

6. REFERENCES


3 STEEL AND COMPOSITE FRAMES
ABSTRACT

The effective length of a framed column is evaluated on the basis of the elastic buckling analysis of the frame in a conventional design, assuming the vertical load is applied on the centroid of the column, and the effect of the primary bending due to beam load is not considered. This paper presents the results of analysis of inelastic buckling strength of three types of portal frames subjected to beam loads, and the reduction of the buckling strength due to the beam load and the comparison with the strength evaluated by the conventional method are discussed.

KEYWORDS

Frame buckling, buckling analysis, portal frame, beam load, effective length

1. INTRODUCTION

In a conventional design of steel frames, the buckling length of the column is evaluated by the elastic buckling analysis, and it is substituted into an appropriate column curve to obtain the buckling strength. Even though the obtained strength falls into the inelastic range, it is used as the ultimate strength of the column subjected to the compression alone. This process is considered to give the conservative estimate to the inelastic buckling strength of the framed column, as long as the load is applied on the centroid of the column, since the restraint of the beam against the rotation of the column becomes relatively larger after the column enters into the inelastic range than the case of the column remaining elastic. The effect of the primary bending moment generated by the vertical loads on the beam is not considered by any means in the process of determining the effective column length, although it may cause the inelasticity in the column.

There have been a few investigations on the frame buckling in the case that the column and/or the beam entered into the inelastic range due to the primary bending moment caused by the beam load[1, 2, 4, 5], among which Sakamoto, et al. [5] presented the slope-deflection equation for the inelastic member, and applied it to the inelastic buckling analysis of multi-story frames subjected to the distributed beam load. However, more investigation is needed to obtain general information on the effect of the primary bending moment on the buckling strength.
This paper presents first the method of numerical analysis of the inelastic buckling of the frame subjected to beam loads. The results were obtained for three types of portal frames, in which column and/or beam entered into the inelastic range. The effects of column slenderness ratio, beam stiffness ratio, position of the beam load and type of the frame on the buckling strength are discussed, and the accuracy of the conventional strength formula is investigated.

2. MODEL FRAME OF ANALYSIS

Figure 1 shows a model frame, fixed at column bases and permitted to sway, where \( h \) and \( l \) denote the lengths, and \( I_c \) and \( I_b \) the moments of inertia of the column and beam, respectively. The vertical load \( P \) is applied on the centroid of the column, or on the beam at the distance \( l_p \) from the centroid of the column. A fictitious H-shaped cross section, H-100×100×4.49×5.39 mm, is used for the column and beam members, which represents the average sectional properties of H-shaped sections with wide flanges[3].

Figure 2 shows the stress-strain relation of the material assumed in the analysis, which was obtained so that the tangent modulus strength of a compression member calculated from this relation agreed the strength specified by Ref. [6]. It specifies the buckling stress \( \sigma_{cr} \) in the inelastic range as follows:

\[
\sigma_{cr} = \left( 1 - 0.4 \left( \frac{\lambda}{\lambda_p} \right)^2 \right) \sigma_y
\]  

(1)

where \( \lambda, \lambda_p \) and \( \sigma_y \) denote the slenderness ratio, the limiting slenderness ratio and the yield stress, respectively. The proportional limit stress \( \sigma_p \) is set equal to 60% of the yield stress. On the other hand, the inelastic buckling stress is expressed as

\[
\sigma_{cr} = \frac{\pi^2 E_t}{\lambda^2}
\]  

(2)

where \( E_t \) denotes the tangent modulus. Eliminating \( \lambda \) from Eqs. (1) and (2) in view of \( \lambda_p = \pi \sqrt{E / 0.6 \sigma_y} \) leads to the expression of the tangent modulus \( E_t \) as follows:

\[
E_t = \frac{(\sigma_y - \sigma_{cr}) \sigma_{cr} E}{0.24 \sigma_y^2}
\]  

(3)

where \( E \) denotes the Young's modulus. Substituting Eq. (3) into \( E_t = d\sigma / d\varepsilon \) leads to a differential equation, and the solution gives the stress-strain relation in the inelastic range, as follows:

\[
\sigma = \frac{\sigma_y}{1 + e^{\alpha \varepsilon}} ; \quad \alpha = 2.5 + \ln \left( \frac{2}{3} \right) - \frac{E \varepsilon}{0.24 \sigma_y} \quad (\sigma \geq \sigma_p = 0.6 \sigma_y)
\]  

(4)

where \( e \) is the base of natural logarithm. The strain hardening is not considered in Eq. (4).
3. ANALYSIS OF INELASTIC FRAMES WITH VERTICAL LOADS ON COLUMNS

3.1 Method of Analysis

The buckling condition of an elastic portal frame with column ends fixed which is subjected to the vertical load on the centroid of the column and permitted to sway is given as follows:

\[ Z \cos Z + 6k \sin Z = 0, \quad Z = h \sqrt{\frac{P_{cr}}{E I}} \quad (5, 6) \]

where \( k \) denotes the beam stiffness ratio and \( P_{cr} \), the elastic buckling load. Equation (6) gives the expression of \( P_{cr} \) and the buckling stress \( \sigma_{cr} \) in terms of \( Z \) as follows:

\[ P_{cr} = \frac{Z^2 E I}{h^2}, \quad \sigma_{cr} = \frac{Z^2 E}{(h/i)^2} \quad (7, 8) \]

where \( i \) denotes the radius of gyration of the column section. For a given value of \( k \), \( \sigma_{cr} \) can be determined by substituting \( Z \) obtained from Eq. (5) into Eq. (8).

As the column enters into the inelastic range, Eq. (5) still holds by changing the beam stiffness ratio and the elastic modulus to those in the inelastic range. Thus,

\[ Z_i \cos Z_i + 6k_i \sin Z_i = 0, \quad Z_i = h \sqrt{\frac{P_{cr}}{E_i I}}, \quad k_i = \frac{E_i}{E_i} \quad (9, 10, 11) \]

where \( k_i \) denotes the beam stiffness ratio in the inelastic range. Similarly to Eqs. (7) and (8), \( P_{cr} \) and \( \sigma_{cr} \) are expressed as

\[ P_{cr} = \frac{Z_i^2 E_i I}{h^2}, \quad \sigma_{cr} = \frac{Z_i^2 E_i}{(h/i)^2} \quad (12, 13) \]

In the numerical calculation, \( E_i \) is first determined for a given value of \( \sigma_{cr} \) by Eq. (3), and then \( k_i \) and \( Z_i \) by Eq. (11) and Eq. (9), respectively. Finally, the value of \( h/i \) is calculated from Eq. (13).

3.2 Results of Analysis

Figure 3 shows the relation between the slenderness ratio for the column height \( h/i \) and the inelastic buckling load \( P_{cr} \), divided by the yield load \( P_y \), in the case that the vertical load is applied on the centroid of the column. In the figure, the solid line is the result of the inelastic analysis based on Eqs. (9) through (13). The inelastic buckling stress of a framed column in a conventional design is obtained by substituting the buckling length determined on the basis of the elastic bucking analysis into an appropriate column strength curve. The dotted line in Fig. 3 is obtained in such an approximate manner that the buckling stress is determined by substituting the slenderness ratio \( \lambda = \pi h/i Z \) into Eq. (1), in which \( Z \) is determined by Eq. (5). It is clear that the approximation in the conventional design gives a good, conservative estimate to the exact inelastic buckling strength.

![Fig. 3: Buckling Strength of Frames with Loads on Columns](image)
4. ANALYSIS OF INELASTIC FRAMES WITH VERTICAL LOADS ON BEAM

4.1 Method of Analysis

In the case that the column and/or the beam enter into the inelastic range due to the primary bending moment generated by the vertical loads on the beam, the effective flexural stiffness of the member in inelastic range changes along the member length. The buckling analysis of such an inelastic frame performed here was separated into two steps. The first step was to obtain the effective flexural stiffness of the members by the elasto-plastic analysis under a given value of the beam load. The model for the analysis in this step is shown in Fig. 4, which is the half of the frame deforming symmetrically. This step only determines the effective flexural stiffness of the inelastic members, and Pδ effect is negligible and was not considered.

The second step was to investigate whether or not the frame buckled under the given load. The model is shown in Fig. 5, where only the anti-symmetrical deflection generated by the buckling is shown, that is, the incremental deflection from the stage shown in Fig. 4. The stiffness matrix of the frame was formed from the effective flexural stiffness obtained in the first step, considering Pδ moment caused by the deflection shown in Fig. 5. The vertical load in this step was treated as if it was applied on the centroid of the column. The strain reversal was assumed not to occur as assumed in the calculation of the tangent modulus load of a centrally-loaded compression member.

In this study, the following three types of frames were treated: A) inelastic columns with elastic beam, B) elastic columns with inelastic beam, C) inelastic columns with inelastic beam. Details of the buckling analysis of Type C is given below referring to the flowchart in Fig. 6. The column and the beam are divided into n and m elements as shown in Figs. 4 and 5, and the lengths of the elements are a and b, respectively.

1) Calculate moment(M)-curvature(ϕ) relation for the beam cross section.
2) Set a value for the beam load P.
3) Calculate M-ϕ relation for the column cross section under the axial load P.
4) Assume the bending moment M_c at the top of the column.
5) Assume the bending moment M_0 at the base of the column.
6) Determine the bending moment M_{ci} at the column subdivision point i using the linearity of the bending moment distribution.
7) Determine the curvature ϕ_{ci} at the point i from the bending moment distribution obtained in 6) and M-ϕ relation obtained in 3).
8) Using the finite difference equation, the relation between ϕ_{ci} and the deflection y_i is given by

$$\phi_{ci} = \left(\frac{d^2 y}{dx^2}\right)_i = -\frac{y_{i+1} - 2y_i + y_{i-1}}{a^2} + \phi_{ci} = a^2 \phi_{ci} + 2y_i - y_{i-1}$$

where x denotes the longitudinal axis of the column. The boundary conditions at the column base are given by

$$y_0 = 0, \quad \left(\frac{dy}{dx}\right)_0 = \frac{y_1 - y_{i-1}}{2a} = 0$$

Determine y_i at each subdivision point (i = 0 to n+1) from Eqs. (14) through (16).
9) Repeat the steps 5) through 8), until the column top deflection y_n becomes equal to zero.
10) Giving the same bending moment as M_c to the left end of the beam, determine the bending moment M_{bj} at the beam subdivision point j from Eqs. (17) and (18).

$$j \leq p : M_{bj} = M_{b_{j-1}} + P \cdot b, \quad j > p : M_{bj} = M_{b_p}$$

where p denotes the subdivision point at which the vertical load P is applied.
11) Determine the curvature $\phi_{q_j}$ of the beam at the point $j$ from the bending moment distribution obtained in 10) and $M-\phi$ relation obtained in 1).

12) Similarly to the case of the column, the deflection $u_{j+1}$ and the boundary conditions at the left end of the beam are given as follows:

$$u_{j+1} = -b^2 \phi_{q_j} + 2u_j - u_{j-1}$$

$$u_0 = 0 \quad \left( \frac{d u}{d s} \right)_0 = \left( \frac{d y}{d x} \right)_0$$

Determine the deflection $u_j$ at each subdivision point ($j = 0 \sim m+1$) from Eqs. (19) through (21).

13) Determine the rotation $\theta_m$ at the center point $m$ of the beam from Eq. (22).

$$\theta_m = \frac{u_{m+1} - u_{m-1}}{2b}$$

Repeat the steps 4) through 12) until the rotation $\theta_m$ becomes equal to zero.
14) Determine the effective flexural stiffness at each subdivision point of the column and the beam from the tangent slope of $M$-$\phi$ relations, based on the values of $M_{ci}$-$\phi_{ci}$ and $M_{bj}$-$\phi_{bj}$ determined above.

15) The differential equations of the equilibrium for the deformed column with the deflection $Y$, and the deformed beam with the deflection $U$ shown in Fig. 5 are given by

$$\frac{d^2}{dx^2}(E I_c) \frac{d^2 Y}{dx^2} + P Y = 0, \quad \frac{d^2}{ds^2}(E I_b) \frac{d^2 U}{ds^2} = 0$$

(23, 24)

where $s$ denotes the coordinate along the longitudinal axis of the beam. The boundary conditions are given as follows:

At the column base, $x = 0$;

$$Y_0 = 0, \quad \left(\frac{dY}{dx}\right)_0 = 0$$

(25, 26)

At the column top, $x = h$ and $s = 0$;

$$-\left(E I_c\right)_n \left(\frac{d^2 Y}{dx^2}\right)_n = -\left(E I_b\right)_0 \left(\frac{d^2 U}{ds^2}\right)_0$$

(27)

$$\left\{\frac{d}{dx} \left[ (E I_c) \frac{d^2 Y}{dx^2} + P Y \right] \right\}_{n-1} = 0$$

(28)

$$U_0 = 0, \quad \left(\frac{d U}{ds}\right)_0 = \left(\frac{d U}{ds}\right)_0$$

(29, 30)

At the center point of the beam, $s = l/2$;

$$U_m = 0, \quad \left(\frac{d^2 U}{ds^2}\right)_m = 0$$

(31, 32)

Compose the frame stiffness matrix $K_M$ by applying the finite difference equations transformed from Eqs. (23) and (24) to the points $i=1\sim n-1$ and to the points $j=1\sim m-1$, respectively, in view of the finite difference expressions of the boundary conditions, Eqs. (25) through (32), where the subscript $t$ denotes the tangent stiffness. Equations (27) and (28) are the equilibrium of the bending moments and the shear force, respectively, the latter being applied to the point $n-1$ instead of $n$, to avoid the inclusion of an additional unknown deflection $Y_{n+2}$.

16) The buckling condition is given by

$$D = \left| K_M \right| = 0$$

(33)

where $D$ is the determinant of the frame stiffness matrix $K_M$. Since the determinant is given by the product of all eigen values of $K_M$, the value of the vertical load $P$ was searched which made the smallest eigenvalue change its sign. Repeat the steps 2) through 15) by increasing the beam load $P$ until Eq. (33) is satisfied.

The values assigned for the parameters in the numerical calculation ranged as follows: $k = 0.5, 1.0, 2.0$, and $h/l = 30 \sim 120$. The ratio of the member length $l/h$ and the ratio of the moment of inertia $I_b/I_c$ were selected so that the designed portal frame fell into one of the three types: A (column-inelastic, beam-elastic), B (column-elastic, beam-inelastic) and C (both column and beam-inelastic). The number of subdivision points $n$ and $m$ were taken equal to 1000 and 200, respectively. Figure 7 shows five cases of the dimensions of model frames analyzed.

4.2 Results of Analysis

Frames of Type A

Figures 8 through 11 show the results of the analysis of Type A frames (column-inelastic, beam-elastic). Figures 8 and 9 show the relations between the position of the beam load $l_P/l$ and the inelastic buckling load $P_{cr}/P_y$ and
the distributions of the curvature $\phi/\phi_y$ and the effective flexural stiffness $(EI)/E_I$ along the column, respectively. The point 1.0 on the ordinate in Fig. 9 indicates the full length $h$ of the column, and $\phi_y$ denotes the yield curvature corresponding to the yield moment of the column section.

It is observed from Fig. 8 that as the beam load parts away from the column, the buckling strength quite reduces...
in comparison with that of the frame subjected to the vertical load on the column. Figure 9 shows that the distribution of the curvature somewhat keeps the linearity and the inelastic zone spreads along the whole length of the column, when the beam load is applied near the column, but the middle part of the column remains elastic as the beam load parts away from the column. The dashed line in Fig. 9 (b) indicates the effective flexural stiffness of the frame which buckles under the vertical load on the column. Note that the value of the buckling load is different in each case shown in Fig. 9, and thus the value of the elastic-limit curvature is different.

Figure 10 shows the relations between the effective length factor $\gamma$ ($= l/L$) and the position of the beam load $l/L$, where $l$ is determined by $l = \pi/\sqrt{EI/P_{cr}}$. The value of $\gamma$ becomes very large, as the beam load parts away from the column, which is more pronounced in the case of $k = 0.5$. Figure 11 shows the non-dimensional relations between the axial force and bending moment generated at the column top at the instance of the buckling, where $M_p$ denotes the full plastic moment of the column section.

**Frames of Type B**

Figures 12 through 15 show the results of the analysis of Type B frames (column-elastic, beam-inelastic). It is observed from Fig. 12 that the buckling strength of a Type B frame is bounded by two values indicated by dotted lines. The upper bound is the elastic buckling strength of the frame subjected to the vertical loads on the column. In the case of the frame with large value of $l/L$, the plastic hinge forms at the end of the beam, and thus the lower bound is given by the elastic buckling strength of a cantilever of the length $h$. Figure 13 shows the reduction of buckling strengths of the frames of Type B with the increase in the value of $l/L$. It is observed from Fig. 14, which shows the distribution of the curvature and the effective flexural stiffness along the beam, that only the end of the beam enters into the inelastic range in the case of small $k$, while the middle portion also becomes inelastic in the case of large $k$.

Figure 15 shows the relations between the effective length factor $\gamma$ and the position of the beam load $l/L$. The value of $\gamma$ increases, as the beam load parts away from the column, but the shape of the curve is quite different from that shown in Fig. 10 for Type A frames. The smallest value of $\gamma$ is obtained for $h/l = 90$ in the region of the small value of $l/L$, while it becomes the largest as the value of $l/L$ increases.

**Frames of Type C**

Figures 16 and 17 compare the results of the analysis of Type C frames (both column and beam-inelastic) with those of Type A frames (column-inelastic, beam-elastic) and Type B frames (column-elastic, beam-inelastic), respectively. In these figures, the dashed and the dotted lines give the limiting values of the beam load determined by the following two formulas, respectively:

$$\frac{M_{max}}{Z_p} = \sigma_y, \quad \frac{\sigma_c}{\sigma_{cr}} + \frac{\sigma_b}{\sigma_y} = 1.0$$

where $Z_p$ denotes the plastic section modulus of the beam cross section, and $\sigma_c$ and $\sigma_b$ the compressive and the flexural stresses in the column, respectively. Equation (34) indicates that the maximum moment in the beam reaches the full plastic value. Equation (35) is the well-known formula to determine the strength of the column subjected to combined axial compression and flexure, in which $\sigma_{cr}$ is calculated from Eq. (1) using the slenderness ratio, $\lambda = \pi h/lZ$, elastically determined from Eq. (5).

It is observed from Fig. 16 that the buckling strength of Type C becomes a little smaller than that of Type A due to the spread of the inelastic zone in the beam, as the value of $l/L$ and $k$ become larger, but the difference is very small. The strength obtained from Eq. (35) very well estimates the buckling strength of the frames of Types A and C. The comparison of the strengths of Types B and C in Fig. 17 indicates that the difference is small in the
region of small value of $l_p/l$, where the strength is nearly equal to the elastic buckling strength of the frame subjected to the vertical load on the column, and in the region of large value of $l_p/l$, where the strength is nearly equal to the value of the beam load at which the plastic hinge forms at the end of the beam. The difference becomes quite large in the mid-region between these two extreme cases, and the estimate by Eq. (35) to the strength of Type B leads to very conservative results.

5. CONCLUSIONS

The buckling analysis was performed for the frame subjected to the vertical load on the column, and for three types of frames subjected to the vertical load on the beam: Types A (column-inelastic, beam-elastic), B (column-
elastic, beam-inelastic) and C (both column and beam-inelastic). The following results were obtained from the analysis.

1) In the case of the frame with the load on the column, the buckling strength obtained by the conventional method used in the design practice gives a good, conservative estimate.

2) The buckling strength of the frame subjected to the beam load reduces as the position of the beam load parts away from the column. The upper bound of the strength is given by the buckling strength of the frame with the load on the column, and the lower bound is given by the load value at which the plastic hinge forms at the end of the beam.

3) The difference between the buckling strengths of the frames of Types A and C is very small, and those strengths are very well estimated by the conventional strength formula for the combined axial and flexural stresses.

4) The strength of the frame of Type B is bounded by the elastic buckling strength of the frame with the load on the column and the elastic buckling strength of the cantilever of the length \( h \). The strength of Type B is nearly equal to the strength of Type C in the region of small or large value of \( l_p/l \), the position of the beam load. Otherwise, the difference is quite large, and the estimate to the strength by the conventional strength formula leads to conservative results.

References


ABSTRACT

For framed structures subjected to quasi-static cyclic loads in the presence of constant loads, a new method is presented for finding the critical steady state, named steady-state limit (SSL), that bounds the following two classes of behavior: one is convergent behavior to steady states and the other is divergent behavior, in which plastic deformations grow exponentially with respect to the number of the cycles. In the proposed method, sequence of steady states, generated under an idealized cyclic loading program with continuously increasing amplitude, is regarded as a continuous path. The SSL is found as the first limit point of the continuous path. Geometrical and material nonlinearities are taken into account using the Total Lagrangian formulation and the bi-linear kinematic hardening rule. The incremental perturbation method is employed to solve the combined and highly non-linear equations.

KEYWORDS

Elastoplastic, Geometrical Nonlinearity, Quasi-Static Cyclic Loads, Frames, Perturbation Method.

1. INTRODUCTION

When elastoplastic beam-columns are subjected to completely reversed cyclic bending with stepwise-ly increasing amplitude under a compressive axial force, its hysteretic behavior is arranged into the following three classes shown in Figure 1: (1) Convergent behavior to a symmetric steady state, in which a pair of the deflected configurations at load reversals is symmetric with respect to the initial member axis; (2) Convergent behavior to an asymmetric steady state, where the deflected shapes involve a certain anti-symmetric mode; (3) Divergent behavior, where deformation grows proportionally
or exponentially with respect to the number of the cycles. The concepts, called the \textit{symmetry limit} (SL) and the \textit{steady-state limit} (SSL), were introduced as the critical steady states that bound these three classes of behavior. The symmetry limit is the critical steady state at which transition from the symmetric steady state to the asymmetric steady state occurs. The SSL is the critical steady state beyond which the beam-columns will no longer exhibit any convergent behavior. To predict the SL and the SSL, the \textit{symmetry limit theory} and the \textit{steady-state limit theory} were developed, respectively (Uetani and Nakamura, 1983).

It might be thought that the SL and the SSL can be found by applying previously established theories. Nevertheless, to the best of authors' knowledge, none of them are directly applicable for the following reasons: (1) \textit{Plastic buckling theory} (Bazant and Cedolin, 1991): The SL and the SSL are phenomenologically and conceptually different from the critical points, such as a branching point and a limit point, of the equilibrium path; (2) \textit{Shakedown theory} and its extensions: In the \textit{classical shakedown theory} (König, 1987), geometrical nonlinearity is completely neglected. Several papers (see e.g. Nguyen, et al., 1983; König, 1987 Gross-Weege, 1990; Polizzotto and Borio, 1996) extended the classical shakedown theory by taking geometrical nonlinearity into account. Nonetheless the extended shakedown theories are not valid when compressive stresses and/or large deformations have strong influence on the structural response. (3) \textit{Numerical methods for response analysis}: It is possible to bound convergence and divergence of plastic deformations (see e.g. Maier et al., 1993). But a number of parametric analyses are required for bounding the structural responses. Moreover, the parametric analyses will never lead to any theoretical condition similar to that for the Euler load.

Recently Uetani (1991) applied the SL theory to multi-story multi-bay planar frames, and showed that the limits exist not only in single members but also in framed structures. In that paper, however, only the idealized and simple frames were treated for which closed-form solutions can be derived. Hence, to investigate the limit states of more complex and practical structures, which generally do not have a SL if they do not have a symmetric shape, it is necessary to establish a method for predicting the SSL using appropriate finite element methods.

The purpose of this paper is to present a new method for finding the SSL of arbitrary shaped frames subjected to dead loads and subsequent cyclic loads. In this paper, Geometrical and material nonlinearities are taken into account using the Total Lagrangian formulation and a bi-linear kinematic hardening rule. Incremental perturbation method (Yokoo et al. 1976) is employed to solve the combined and
highly non-linear equations. In the following sections, governing equations are described first. Then the fundamental concepts of the SSL theory are outlined. Next, using the Taylor-series expansion, a new incremental theory is formulated. Finally, validity of the proposed method is demonstrated through numerical examples.

2. ANALYTICAL MODELS

Let us consider a space truss element shown in Figure 2. In this section, compatibility conditions, equilibrium conditions and constitutive relations are given for the truss element. Large displacements, large rotations and small strain is assumed. The truss is pin-jointed. Buckling of members is ruled out but global buckling is taken into account. We measure stresses and strains using the Total Lagrangian formulation (Crisfield, 1991). Green-Lagrangian strain $\varepsilon$ for the truss element is expressed as

$$\varepsilon = \frac{L^2 - L_0^2}{2L_0^2},$$

where $L$ and $L_0$ are the deformed and initial lengths of the element, respectively. As shown in Figure 2, the relations between the deformed length $L$ and the nodal displacements displacement $u_i$ at the two ends are written as

$$L^2 = (x_4 - x_1)^2 + (x_5 - x_2)^2 + (x_6 - x_3)^2,$$

$$x_i = x_i^0 + u_i, \quad i = 1, \ldots, 6,$$

in which $x_i$ and $x_i^0$ indicate the deformed and initial position of the nodes at the two ends, respectively.
The equilibrium equation for the element is expressed as

\[ f_i = A L_0 \sigma \frac{\partial \varepsilon}{\partial u_i} \]

where \( f_i \) is the nodal force, \( \sigma \) is the second Piola-Kirchhoff stress, and \( A \) is the cross sectional area. Note that summation convention is used only for the subscripts \( i, j \) and \( k \) throughout this paper. Assembling the element equilibrium equations yields the equilibrium equations for the total system.

As a constitutive relation, a bi-linear kinematic hardening rule is employed, which is shown in Figure 3. In terms of Young's modulus \( E \), tangent modulus after yielding \( E_t \), yield stress \( \sigma_y \), plastic strain \( \varepsilon_r \), and \( \alpha = E/E_t \), the kinematic hardening rule is expressed as follows:

\[
\sigma = E(\varepsilon - \varepsilon_r) \quad : \text{for elastic response,} \\
\sigma = \alpha E \varepsilon + (1 - \alpha) \sigma_y \quad : \text{for plastic response in tension,} \\
\sigma = \alpha E \varepsilon - (1 - \alpha) \sigma_y \quad : \text{for plastic response in compression.}
\]

3. OUTLINE

First, let us define the loading conditions. The trusses are subjected to both constant loads and cyclic loads. The constant and cyclic loads are expressed as \( \lambda_0 P_0 \) and \( \lambda_c P_c \), respectively, where \( \lambda \) and \( P \) indicate the load factor and the constant vector, respectively. Displacement and/or force components are included in the constant vectors \( P_0 \) and \( P_c \). The amplitude of the load factor \( \lambda_c \) is denoted by \( \psi \). The equilibrium states at the load reversals \( \lambda_c = \psi \) and \( \lambda_c = -\psi \) are indicated by \( \Gamma^1 \) and \( \Gamma^2 \), respectively.

Uetani and Nakamura (1983) introduced an idealized cyclic loading program, called COIDA. In the COIDA program, as shown in Figure 4, the amplitude of the cyclic forced displacement is continuously increased and, at each amplitude level, loading cycle is repeated for the structure to converge to a steady state. In this paper, based on the idea of the COIDA program, \( \psi \) is continuously increased and the loading cycle is repeated similarly to the COIDA program. For the later formulation, a parameter \( \tau \) is introduced to define the variation of the amplitude \( \psi \).

![Figure 4: The loading cyclic program COIDA](image-url)
Next, the fundamental concepts of the SSL theory are outlined. In a special space illustrated in Figure 5, a steady-state response is regarded as a point, and sequence of the points, generated under the idealized cyclic loading program, is considered to be a continuous path, called steady-state path (SSP). Then the SSL is predicted as the first limit point of the SSP as shown in Figure 6. In the proposed method, the SSP is traced incrementally. And, in tracing the SSP, a steady state is formulated in terms of the state variables at the load reversals in the proposed method. For this purpose, the following hypothesis is introduced:

**Hypothesis:** For all elements, strain reversal occurs only at load reversal $F^+$ or $F^-$.

Note that this hypothesis is applied not for transient responses but for steady-state responses after convergence. In addition, it should be noted that the incremental analysis does not mean tracing all the loading histories incrementally, and, in the present method, the variation of $F^+$ and $F^-$ with respect to $\tau$ is traced.
4. FORMULATION

4.1 Incremental Relations for Variation of Steady State

When all the state variables are known for the current steady states at \( \tau = \tau_h \), our problem is then to determine those for a neighboring steady state at \( \tau = \tau_{h+1} \). Let \( \Delta \tau = \tau_{h+1} - \tau_h \) be an increment of the SSP parameter \( \tau \). Let the superscripts I and II indicate the variables that belong to \( \Gamma^I \) and \( \Gamma^\Pi \), respectively, and let overdot indicate the differentiation with respect to \( \tau \). Then, on the basis of the hypothesis introduced in the last section, the state variables for \( \Gamma^I \) at \( \tau = \tau_{h+1} \) are expressed using the Taylor-series expansion as:

\[
\begin{align*}
U^I(\tau_h + \Delta \tau) &= U^I(\tau_h) + \dot{U}^I \Delta \tau + \frac{1}{2} \ddot{U}^I \Delta \tau^2 + \cdots, \\
F^I(\tau_h + \Delta \tau) &= F^I(\tau_h) + \dot{F}^I \Delta \tau + \frac{1}{2} \ddot{F}^I \Delta \tau^2 + \cdots, \\
e^I(\tau_h + \Delta \tau) &= e^I(\tau_h) + \dot{e}^I \Delta \tau + \frac{1}{2} \ddot{e}^I \Delta \tau^2 + \cdots, \\
\sigma^I(\tau_h + \Delta \tau) &= \sigma^I(\tau_h) + \dot{\sigma}^I \Delta \tau + \frac{1}{2} \ddot{\sigma}^I \Delta \tau^2 + \cdots,
\end{align*}
\]

where \( F \) and \( U \) are the nodal force vector and the nodal displacements vector, respectively, and \( e \) and \( \sigma \) denote the strain vector and the stress vector, respectively. The variables for \( \Gamma^\Pi \) are expressed by replacing the superscript I with II.

4.2 Rate Equations for Variation of Steady State

To trace the SSP, the rate equations are derived by differentiating all the governing equations at \( \Gamma^I \) and \( \Gamma^\Pi \) with respect to the SSP parameter \( \tau \). Differentiation of the compatibility conditions (1) and (2) yields

\[
\begin{align*}
\dot{\epsilon}^I &= \frac{\partial \epsilon^I}{\partial u^I_j} \dot{u}^I_j, \\
\dot{\epsilon}^\Pi &= \frac{\partial \epsilon^\Pi}{\partial u^\Pi_i} \dot{u}^\Pi_i,
\end{align*}
\]

and the rate forms of the equilibrium conditions for an element are given by

\[
\begin{align*}
\dot{f}^I_i &= A L_0 \left( \sigma^I \frac{\partial \epsilon^I}{\partial u^I_i} + \sigma^I \frac{\partial^2 \epsilon^I}{\partial u^I_i \partial u^I_j} \dot{u}^I_j \right), \\
\dot{f}^\Pi_i &= A L_0 \left( \sigma^\Pi \frac{\partial \epsilon^\Pi}{\partial u^\Pi_i} + \sigma^\Pi \frac{\partial^2 \epsilon^\Pi}{\partial u^\Pi_i \partial u^\Pi_j} \dot{u}^\Pi_j \right).
\end{align*}
\]

Before obtaining the stress rate-strain rate relations, let us consider the cyclic responses in the stress-strain plane. These responses are arranged into the four different types as illustrated in Figure 7, where the superscripts c and t indicate the variables at strain reversals in compression and tension,
respectively. For each type of the cyclic responses, the relations between the stresses and the strains at the strain reversals are written as:

\[ \sigma^e = E(\varepsilon^e - \varepsilon_r^e), \]
\[ \sigma^t = E(\varepsilon^t - \varepsilon_r^t), \]

for type E, which is elastic response after some histories of plastic deformations.

\[ \sigma^e = E\varepsilon^e - (1 - \alpha)(E\varepsilon_t^e - \sigma_Y), \]
\[ \sigma^t = \alpha E\varepsilon_t^t + (1 - \alpha)\sigma_Y, \]

for type T, which is the shakedown state whose maximum stress reaches the tensile yield stress.

\[ \sigma^e = \alpha E\varepsilon^e - (1 - \alpha)\sigma_Y, \]
\[ \sigma^t = -(1 - \alpha)(E\varepsilon^e - \sigma_Y) + E\varepsilon_t^t \]

for type C, which represents the shakedown state that starts from and reaches the compressive strain hardening line. And

\[ \sigma^e = \alpha E\varepsilon^e - (1 - \alpha)\sigma_Y, \]
\[ \sigma^t = \alpha E\varepsilon_t^t + (1 - \alpha)\sigma_Y \]

for type P, which indicates so called alternating plasticity. Note that the plastic strain \( \varepsilon_r \) is eliminated in Eqns. (18) and (19). For the type T, \( \sigma^t \) is expressed in two ways using Eqns. (4) and (5), while \( \sigma^e \) is

<table>
<thead>
<tr>
<th>Type</th>
<th>Strain Rate</th>
<th>( C^{cc} )</th>
<th>( C^{ct} )</th>
<th>( C^{tc} )</th>
<th>( C^{tt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>( \dot{\varepsilon}^e \leq 0 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>( \dot{\varepsilon}^e &gt; 0 )</td>
<td>( \alpha )</td>
<td>0</td>
<td>( \alpha - 1 )</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>( \dot{\varepsilon}^t \geq 0 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>T</td>
<td>( \dot{\varepsilon}^t &lt; 0 )</td>
<td>( \alpha )</td>
<td>0</td>
<td>0</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>P</td>
<td>( \dot{\varepsilon}^t \geq 0 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( \alpha )</td>
</tr>
</tbody>
</table>

Table 1: Stress rate-strain rate relations at strain reversals
written using only Eqn. (4). The plastic strains at the strain reversals is eliminated using these three expressions and the following relation

\[ \varepsilon^p_r = \varepsilon^p_t. \]  

For the type C, the plastic strain can be eliminated similarly.

Let us turn now to the main subject, the stress rate-strain rate relations. Differentiating Eqns. (15)-(22), we can express the stress rate-strain rate relations as

\[ \dot{\sigma}^c = C^{oc} \dot{\varepsilon}^c + C^{ct} \dot{\varepsilon}^t, \]  

\[ \dot{\sigma}^l = C^{le} \dot{\varepsilon}^c + C^{lt} \dot{\varepsilon}^t, \]  

in which \( C^{oc}, C^{ct}, C^{le}, \) and \( C^{lt} \) are the coefficients of the strain rates at strain reversals. The coefficients are selected according the type of the current steady-state response and the sign of the strain rates as shown in Table 1. To derive the rate equations for the load reversals \( \Gamma^1 \) and \( \Gamma^8 \), these relations at strain reversals should be transformed into those at load reversals. To this end, the superscripts \( c \) and \( t \) are replaced by \( I \) and \( II \), respectively, or those are replaced with \( II \) and \( I \). After the replacement of the superscripts, we have

\[ \dot{\sigma}^l = C^{II} \dot{\varepsilon}^l + C^{I\!\!II} \dot{\varepsilon}^8, \]  

\[ \dot{\sigma}^8 = C^{I\!\!II} \dot{\varepsilon}^l + C^{II} \dot{\varepsilon}^8. \]  

Substitution of Eqns. (11), (12), (26) and (27) into Eqns. (13) and (14) yields rate equations for the element as

\[ j^1_i = k^{II}_{ij} u^1_j + k^{I\!\!II}_{ij} u^8_j, \]  

\[ j^8_i = k^{I\!\!II}_{ij} u^1_j + k^{II}_{ij} u^8_j, \]  

where

\[ k^{II}_{ij} = AL_0 \left( C^{II} \frac{\partial \varepsilon^1}{\partial u^1_i} \frac{\partial \varepsilon^1}{\partial u^1_j} + \sigma^1 \frac{\partial^2 \varepsilon^1}{\partial u^1_i \partial u^1_j} \right), \]  

\[ k^{I\!\!II}_{ij} = AL_0 C^{I\!\!II} \frac{\partial \varepsilon^8}{\partial u^1_i} \frac{\partial \varepsilon^8}{\partial u^1_j}, \]  

\[ k^{II}_{ij} = AL_0 C^{I\!\!II} \frac{\partial \varepsilon^8}{\partial u^8_i} \frac{\partial \varepsilon^1}{\partial u^1_j}, \]  

\[ k^{I\!\!II}_{ij} = AL_0 \left( C^{II} \frac{\partial \varepsilon^8}{\partial u^1_i} \frac{\partial \varepsilon^8}{\partial u^8_j} + \sigma^8 \frac{\partial^2 \varepsilon^8}{\partial u^1_i \partial u^8_j} \right). \]  

By assembling Eqn. (28), we have the rate equations for the total system in terms of the nodal force vectors \( F^1_g \) and \( F^8_g \) and the nodal displacement vectors \( U^1_g \) and \( U^8_g \)

\[ \dot{F}^1_g = K_{II}^g \dot{U}^1_g + K_{I\!\!II}^g \dot{U}^8_g, \]  

\[ \dot{F}^8_g = K_{I\!\!II}^g \dot{U}^1_g + K_{II}^g \dot{U}^8_g. \]
where $K^{I^1}_g$, $K^{I^2}_g$, $K^{II^1}_g$ and $K^{II^2}_g$ are the coefficient matrices for the nodal displacement rates. The rate equations are solved by using the boundary conditions and the following relations

$$\dot{\lambda}^I = \dot{\psi},$$  \hspace{1cm} (36)

$$\dot{\lambda}^I = -\dot{\psi},$$  \hspace{1cm} (37)

and by specifying the value of $\dot{\psi}$.

### 4.3 Perturbation Equations for Variation of Steady State

A formulation with higher-order derivatives is presented for the SSL analysis. By using the higher-order derivatives, terminal points of incremental steps can be found with the desired accuracy. We derive here only the second-order derivatives for brevity. But more higher-order derivatives can be obtained similarly.

Differentiation of Eqns. (11), (13), and (26) with respect to $\tau$ yields

$$\ddot{\varepsilon}^I = \frac{\partial^2 \varepsilon^I}{\partial u^I_i \partial u^I_j} \dot{u}^I_i \dot{u}^I_j + \frac{\partial \varepsilon^I}{\partial u^I_i} \ddot{u}^I_i;$$  \hspace{1cm} (38)

$$\ddot{f}^I_i = A L_\theta \left\{ \dot{\varepsilon}^I \frac{\partial \varepsilon^I}{\partial u^I_i} + 2\sigma^I \dot{\varepsilon}^I \frac{\partial^2 \varepsilon^I}{\partial u^I_i \partial u^I_j} \dot{u}^I_j + \sigma^I \frac{\partial^2 \varepsilon^I}{\partial u^I_i \partial u^I_j} \ddot{u}^I_j \right\},$$  \hspace{1cm} (39)

$$\ddot{\sigma}^I = C^{II^1} \ddot{\varepsilon}^I + C^{II^2} \ddot{\varepsilon}^I,$$  \hspace{1cm} (40)

where $\dddot{\sigma}^I = \dddot{\varepsilon}^I = 0$ since bi-linear constitutive equation is assumed.

Substituting Eqns. (38) and (40) into Eqn. (39), we obtain the second-order perturbation equations as

$$\ddot{f}^I_i = k^{II^1}_{ij} \dddot{u}^I_j + k^{II^2}_{ij} \dddot{u}^I_j + \dddot{f}^I_i,$$  \hspace{1cm} (41)

$$\ddot{f}^I_i = 2 A L_\theta \dot{\varepsilon}^I \frac{\partial^2 \varepsilon^I}{\partial u^I_i \partial u^I_j} \dot{u}^I_j$$  \hspace{1cm} (42)

$$+ A L_\theta \frac{\partial \varepsilon^I}{\partial u^I_i} \left( C^{II^1} \frac{\partial^2 \varepsilon^I}{\partial u^I_j \partial u^I_k} \dot{u}^I_j \dot{u}^I_k + C^{II^2} \frac{\partial^2 \varepsilon^I}{\partial u^I_i \partial u^I_k} \dot{u}^I_i \dot{u}^I_k \right),$$  \hspace{1cm} (43)

where $k^{II^1}_{ij}$ and $k^{II^2}_{ij}$ are identical to those in Eqn. (28), and hat indicates the terms expressed with the derivatives less than second order. The perturbation equations for the $\Gamma^I$ configuration is obtained by replacing the superscripts I with II and by replacing II with I.

By assembling the perturbation equations for the element, we obtain those for the total system

$$\ddot{F}_g^I = K^I_g \dddot{U}_g^I + K^{II^1}_g \dddot{U}_g^{II^1} + \dddot{F}_g^I,$$  \hspace{1cm} (43)

$$\ddot{F}_g^I = K^{II^1}_g \dddot{U}_g^I + K^{II^2}_g \dddot{U}_g^{II^2} + \dddot{F}_g^I,$$  \hspace{1cm} (44)

in which the coefficient matrices are same as those in the rate equations (34) and (35), and $\dddot{F}_g^I$ and $\dddot{F}_g^{II^2}$ are the terms written with the derivatives less than second order. Before solving the Eqn. (43), the
derivatives less than the second order should be obtained. The perturbation equations are solved by using the boundary conditions and the following relations

\[ \dot{\lambda}^1 = \dot{\psi}, \]  
\[ \dot{\lambda}^2 = -\dot{\psi}, \]  
and by specifying the value of \( \dot{\psi} \). It may be worth noting that the rate equations derived in the last subsection is regarded as the first-order perturbation equations.

4.4 Termination Conditions for Incremental Steps

When the type of the stress-strain cyclic response changes, a different type of stress rate-strain rate relation should be used in Eqns. (26) and (27). Step length \( \Delta \tau \) is therefore determined considering the conditions for the transition of the type of the stress-strain cyclic response. Let \( \sigma_{yt} \) and \( \sigma_{yc} \) denote the subsequent yield stresses in tension and compression, respectively. Then, for every element, \( \Delta \tau \) is calculated using the following conditions:

\[ \sigma'(\tau_{h+1}) = \sigma_{yt}, \]  
\[ \sigma'(\tau_{h+1}) = \sigma_{yc}, \]  
\[ \sigma'(\tau_{h+1}) - \sigma'(\tau_{h+1}) = \sigma_{yt} - \sigma_{yc} = 2\sigma_y \]  
where

\[ \sigma'(\tau_{h+1}) = \sigma'(\tau_h) + \dot{\sigma}'(\tau_h)\Delta \tau + \frac{1}{2} \ddot{\sigma}'(\tau_h)\Delta \tau^2 \]  
\[ \sigma'(\tau_{h+1}) = \sigma'(\tau_h) + \dot{\sigma}'(\tau_h)\Delta \tau + \frac{1}{2} \ddot{\sigma}'(\tau_h)\Delta \tau^2 \]  
in the second-order approximation. Here, the Eqns. (47)-(49) describe the conditions for the transition of the cyclic response for \( E \to T, E \to C, \) and \( E, C \to T \to P, \) respectively. Note that the subsequent yield stresses are expressed in terms of the residual plastic strain at the current steady state as

\[ \sigma_{yt} = \frac{EE_t}{E - E_t} \varepsilon_r(\tau_h) + \sigma_y, \]  
\[ \sigma_{yc} = \frac{EE_t}{E - E_t} \varepsilon_r(\tau_h) - \sigma_y. \]  
Besides the conditions above, the step length \( \Delta \tau \) should be kept small enough to prevent excessive accumulation of truncation errors. Hence the step length \( \Delta \tau \) is selected as the smallest value among the values calculated from the conditions (47)-(49) and the specified maximum allowable value \( \Delta \tau_{max}. \)

4.5 Steady-State Limit Condition

Now, all the derivatives and the step length \( \Delta \tau \) have been obtained. Substituting \( \Delta \tau \) and the derivatives into Eqns. (7)-(10), we have all the state variables at \( \tau = \tau_{h+1}. \) Repeating these procedures, the SSP is traced incrementally.
Since the SSL is characterized as the first limit point of the SSP, the SSL condition is given as

\[ \psi \leq 0 \]  

(Note 54)

Note that, to find the limit point and to trace the SSP after the limit point, a procedure should be employed similar to displacement control schemes (Yokoo et al. 1976; Crisfield, 1991).

5. NUMERICAL EXAMPLES

The SSL analysis is carried out for an arch-type plane truss, whose initial configuration is illustrated in Figure 8. In the proposed method, a hypothesis is introduced and no equilibrium path is traced. Therefore, to investigate the validity of the proposed method, the conventional response analysis, or equilibrium path analysis, is performed and all the loading history is traced. Vertical force \( F_1 \) and completely reversed cyclic forced displacement \( O_3 \) is applied to the two-bar truss shown in Figure 8. The numerical data is given as follows: \( E = 196.1 \) (GPa), \( \alpha = 0.020 \), \( \sigma_y = 294.2 \) (MPa), \( L_0 = 100.0 \) (cm), \( A = 10.00 \) (cm²), \( A_{t_{max}} = 0.001 \), \( P_0 = [0 \text{ (kN)}, 0 \text{ (kN)}, 0 \text{ (cm)}, -1 \text{ (kN)}, 0 \text{ (kN)}, 0 \text{ (kN)}]^T \), \( \lambda_0 = 49.03 \), \( P_c = [0 \text{ (kN)}, 0 \text{ (kN)}, 1 \text{ (cm)}, 0 \text{ (kN)}, 0 \text{ (kN)}, 0 \text{ (kN)}]^T \).

![Figure 8: The steady-state path and the steady-state limit](image1)

![Figure 9: Deformation below and above the steady-state limit](image2)
The result of the SSL analysis is shown in Figure 8. Predicted value of the SSL $\psi_{SSL}/L_0$ is $1.189 \times 10^{-3}$. The results of the conventional analysis are illustrated in Figure 9. The relation between vertical displacement $U_4$ and number of cycles is plotted under the four constant amplitudes $\psi/\psi_{SSL} = 0.90$, 0.98, 1.02 and 1.10. From Figure 9, it is seen that, when $\psi > \psi_{SSL}$, the vertical displacement $U_4$ grows exponentially, whereas $U_4$ converges if $\psi < \psi_{SSL}$. From these facts, it may be concluded that the proposed method is directly verified. In addition, it may be worth noting that SSL is much less than the magnitude of vertical displacement $U_3/L_0 = 2.341 \times 10^{-2}$ at which the reaction force $F_3$ reaches maximum strength under monotonically increasing horizontal displacement.

6. CONCLUSIONS

A new method has been proposed for finding the steady-state limit (SSL) of elastoplastic trusses subjected to quasi-static cyclic loads with continuously increasing amplitude under the action of dead loads. Though the present method is directly applicable only for truss structures, the SSL theory, originally developed for a continuous beam-column model, has been extended for discrete structures. This fact implies that the SSL of arbitrary shaped framed structures can be predicted using the proposed method with appropriate finite element scheme. The SSL has been predicted for two-bar arch type truss in the numerical examples, and verification of the proposed method has been provided by comparing the results obtained by the SSL analysis with those obtained by conventional response analysis.

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REFERENCES

A SIMPLIFIED ANALYSIS OF STEEL FRAMES FAIL BY LOCAL AND GLOBAL INSTABILITY

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ABSTRACT

Moment (M) versus curvature (ϕ) curves of a member under constant normal forces (N) and normal forces (N) versus average axial strain (ε) curves under constant curvature (ϕ) are obtained by FEM analysis of a member of unit length in consideration of the local buckling. These curves are approximated by explicit functions and used to formulate the stiffness matrices in a second order frame analysis. Numerical results of the proposed method for portal frames subjected to vertical and horizontal loads are compared with those of other method and experimental tests, and the validity and the efficiency of the proposed method are shown.

KEYWORDS

Frame analysis, Coupled buckling, Elasto-plastic, Finite displacement, Portal frames, Experimental test

1. INTRODUCTION

There are quite a few researches on coupled local and global buckling of a member like a column or a beam-column, but not many of a framed structure (Usami et al (1988)). It is not impossible to analyze the ultimate strength behavior of a framed structure considering local and global instability if you can use a big size computer. When you want to do this directly by FEM, you have to struggle with a FEM model composed of thousands of plate elements. Considering the computer effort and costs, it seems to be quite difficult to carry out the analysis of complicated actual structures. In this paper, we propose a simplified method to do it by a personal computer.

Suppose a portal frame composed of box-profile cross-section member is the one to be analyzed. One direct method is to use FEM by dividing it into plate elements as shown in Figure 1(a). The proposed method is to
analyze it by dividing it into beam-column elements as shown in Figure 1(c). To do so, the beam-column element should behave like original box cross-section member including local buckling behavior.

Then, as a first step, a member of unit length with a box-profile cross-section are taken out as shown in Figure 1(b) and its inelastic local buckling behavior is analyzed by a second order elasto-plastic FEM. As the results, moment (M) versus curvature (ϕ) curves under various cases of constant axial forces (N) and axial force (N) versus average axial strain (ε) curves under various cases of constant curvatures (ϕ) are obtained and are approximated by explicit polynomial functions. Approximated M-(N)-ϕ curves and N-(ϕ)-ε curves are brought into a usual second order frame analysis to formulate tangent stiffness matrices. By this procedure, the results from the frame analysis provide the ultimate strength behavior of the frame including local and global instability.

Usami et al. (1988) already developed a similar method of analysis for the inelastic behavior of steel frame by using moment-thrust-curvature curves of locally buckled stub-columns. But, their method can not account for the variation of axial force. By comparing the results of the present method with those of Usami et al. and experimental results, the validity of the proposed method is shown.

2. FEM ANALYSIS

A box cross-section member of unit length subjected to normal force N and uniform bending moment M was analyzed by a second order elasto-plastic FEM, where the yield criterion of von Mises and the stress-strain relation of Prandtl-Reuss are assumed (Yamao and Sakimoto (1986)). The numerical model is illustrated in Figure 2, and described as follows:

1) The profile of cross-section is square (d/b = 1.0) and the aspect ratio a/b is 0.7, at which the lowest maximum strength may be expected.

2) The width-to-thickness ratios b/t of the component plates are varied from 20 to 70.

3) The material is assumed to be JIS SM490 and its yield stress σ_y, young's modulus E and poison's ratio ν are σ_y = 3200 kgf/cm² (314N/mm²), E = 2.1x10^5 kgf/cm² (206kN/mm²) and ν = 0.3, respectively.

4) The stress-strain curve is assumed to be tri-linear type, where the strain-hardening initiates at 10 times of the yield strain and the hardening modulus E_s is E/70.

5) Initial deflections of sinusoidal shape were assumed both in longitudinal and transverse directions, where its amplitude is assumed to be b/150 or d/150 according to the tolerance allowed in the Japanese Specifications for Highway Bridges.

6) Residual stresses distributed in trapezoidal pattern are assumed as σ_σ = σ_y, σ_re = 0.3σ_y.
7) A quarter part of the model was analyzed considering the symmetry. The number of elements is 6 in longitudinal direction and along the flange width, but 12 along the web width. Also the plate divided into 6 layers along the thickness for numerical integration.

8) The loading edges of the plates were simply supported.

9) The loads were given by controlling the displacements of nodes along the edges. That is, uniform longitudinal displacements were given in stead of axial force \( N \) and longitudinal displacements which conform the planar rotation of the edge cross-section were given in stead of bending moment \( M \). The values \( N \) and \( M \) are computed from resultants of reactions of edge nodes. In this paper, the positive sign is given to a compressive axial force.

10) The curvature of the member \( \phi \) is computed as \( \phi = \frac{2\theta}{a} \), where \( \theta \) is a rotation of the edge cross section and \( a \) is the length of the member. The average axial strain \( \varepsilon \) is computed from the change of the longitudinal relative displacements between the both end nodes.

3. SUMMARY OF ULTIMATE STRENGTHS OF A UNIT MEMBER

Numerical results of parametric studies for the ultimate strength of a unit member can be approximated by the following equations.

1) Maximum strength of stub-columns under pure compression: \( N_u \)

\[
\frac{N_u}{N_y} = \left( \frac{0.5}{R} \right)^{0.6} \leq 10
\]  

(1)

where \( N_y \) is yield axial force and \( R \) denotes the width-to-thickness ratio parameter given by

\[
R = \frac{b}{t} \sqrt{\frac{\sigma_y}{E}} \sqrt{\frac{12(1-v^2)}{4\pi^2}}
\]  

(2)
2) Maximum strength of short members under pure bending: \( \mu \)

\[
\frac{M_m}{M_p} = 1.147 - 0.3087R \leq 1.0
\]  

(3)

where \( M_p \) is the plastic moment.

3) Maximum bending moment of short members under axial force and bending: \( M_b \)

\[
\frac{M_b}{M_p} = \left( \frac{0.5}{R} \right)^{0.2}
\]  

(4)

4) Interaction strength formula of short members under axial force and bending,

for \( N_y \geq 1.3N_u \),

\[
\left( \frac{N_m}{N_y} \right)^{1.35} + \left( \frac{M_m}{M_y} \right) = 1.0
\]

(5a)

for \( N_y < 1.3N_u \),

\[
\left\{ \frac{N_m + (N_y - 1.3N_u)/2.3}{(N_y + N_u)/2.3} \right\}^{1.1} + \left( \frac{M_m}{M_b} \right) = 1.0
\]

(5b)

Figure 3: Computed \( M-\Phi \) curves and approximated ones

4. \( M-\Phi \) RELATIONS

\( M-\Phi \) curves obtained from the above-mentioned FEM analysis are shown in Figure 3 by solid lines. In this figure, \( M_y \) and \( \Phi_y \) denote the yield bending moment and corresponding curvature.

In order to use this relationship in a second order frame analysis (Komatsu and Sakimoto (1976)), we approximate these curves by explicit equations of second order polynomials (parabola) for the beginning part up to the maximum point \( (M_m, \Phi_m) \) and by two straight lines for the latter descending part as shown in Figure 4.
That is,

for $0 \leq \phi \leq \phi_m$, \[ \frac{M}{M_y} = -\frac{M_m}{M_y} + \left( \frac{\phi_m}{\phi_m - 1} \right)^2 + \frac{M_m}{M_y} \] (6a)

for $\phi_m \leq \phi \leq \phi_c$, \[ \frac{M}{M_y} = \alpha \left( \frac{\phi}{\phi_m} + \frac{\phi_m}{\phi_c} \right) + \frac{M_m}{M_y} \] (6b)

and for $\phi_c \leq \phi$, \[ \frac{M}{M_y} = \beta \left( \frac{\phi}{\phi_c} + \frac{\phi_c}{\phi_c} \right) + \frac{M_m}{M_y} \] (6c)

In these equations, the value $M_m$ can be computed from Eqn.5 and the value $\phi_m$ can be approximated by the following equation,

\[ \frac{\phi_m}{\phi_c} = \exp\{0.989 - 1.422 N_d\} \] (7)

where

\[ N_d = \frac{N/N_y + \left(1 - N_x/N_s\right)/2.0}{\left(1 + N_x/N_s\right)/2.0} \] (8)

The inclinations of the straight lines, $\alpha$ and $\beta$ used in Eqn.6 b,c, were determined from the numerical results by a regression method. The results are,

for $R < 0.5$, $\alpha = \beta = -0.025$ (9a)

for $0.5 \leq R < 10$, $\alpha = -0.147(N/N_y) - 0.133R + 0.0715$ and $\beta = -0.025$ (9b)

and for $10 \leq R$, $\alpha = -0.0323(N/N_y) + 0.0553R - 0.1286$ and $\beta = -0.025$ (9c)

Also, the boundary value $\phi_c$ is given as,

for $R \leq 10$ $\phi_c/\phi_y = 6.0$ and for $R > 10$ $\phi_c/\phi_y = 4.0$ (10)

Finally, M- (N)-$\phi$ relations for square-box members with any width-to-thickness ratio under any combination of forces can be described explicitly by Eqn.6 together with Eqns.1 to 10. The approximated curves are shown by dotted lines in Figure 3 in comparison with the results of FEM analysis (solid lines).
5. $N-(\phi)-\varepsilon$ RELATIONS

$N-(\phi)-\varepsilon$ curves obtained from the FEM analysis are shown in Figure 5 by solid lines. In this figure, $N_Y$ and $\varepsilon_y$ denote the yield axial force and corresponding axial average strain. We approximate these curves by explicit equations of second order polynomials (parabola) for the beginning part up to the maximum point ($N_m, \varepsilon_m$) and by a straight line for the latter descending part. That is,

for $0 \leq \varepsilon \leq \varepsilon_m$,

$$\frac{N}{N_Y} = \frac{N_m}{N_Y} \left( \frac{\varepsilon}{\varepsilon_m} \right)^2$$

and for $\varepsilon_m < \varepsilon$,

$$\frac{N}{N_Y} = \xi \left( \frac{\varepsilon}{\varepsilon_y} - \frac{\varepsilon_m}{\varepsilon_y} \right) + \frac{N_m}{N_Y}$$

where

$$\xi = 0.1126R^2 - 0.1999R + 0.0359$$

The maximum axial force $N_m$ can be determined from various numerical results and given by the following equation;

$$\frac{N_m}{N_Y} = 1.03 \left( 10 - \frac{0.00567(\phi)}{2R} \right)^2$$

The value $\varepsilon_m$ for various numerical results can be approximated by the following equation:

$$\frac{\varepsilon_m}{\varepsilon_y} = 0.14 - 0.168R \left( \frac{\phi}{\phi_y} \right) + 1.955 - 0.0287R$$

The approximated curves are shown by dotted lines in Figure 5 in comparison with the results of FEM analysis (solid lines).
6. METHOD OF FRAME ANALYSIS

The equilibrium equation for the second order frame analysis can be derived as follows (Komatsu and Sakimoto (1976));

\[
(K_p + K_g) \cdot U = P + (\bar{P} - T \cdot \bar{f})
\]

(15)

where

- \( K_p \) = tangent stiffness matrix determined from the tangent of the curves given by Eqns. 6 and 11
- \( K_g \) = initial stress matrix to account for the second order effects
- \( U \) = vector for incremental displacements
- \( P \) = vector for incremental loads
- \( \bar{P} \) = vector for total loads
- \( T \) = transformation matrix from global to local coordinates
- \( \bar{f} \) = vector for total stress resultants

The frame to be analyzed should be divided into suitable number of beam-column elements. Eqn 15 will be solved incrementally by the Newton-Raphson procedure under the given loading conditions. The numerical scheme is as follows:

1) Solve Eqn. 15 for incremental displacements \( U \) under the given load increment \( P \).
2) Compute incremental axial strains, curvatures, stress resultants and total values of those \((e, \phi, \bar{N}, \bar{M}, f)\) as summations of incremental values.
3) Compute \( N_m, \varepsilon_m \) from Eqns 13 and 14 and choose the \( N-(\phi)-\varepsilon \) curves.
4) Compute the tangent axial stiffness \( (EA)_t = dN/d\varepsilon \) and \( N \) from computed \( \varepsilon, \phi \) and using \( N-(\phi)-\varepsilon \) curves given by Eqn. 11. (see Figure 6 (a))
5) Compute \( M_m \) by substituting \( N \) into Eqn. 5. (see Figure 6 (b))
6) Compute \( \phi_m \) from obtained \( N \) and Eqn. 7 and choose the \( M-(N)-\phi \) curves.
7) Compute the tangent flexural stiffness \( (EI)_t = dM/d\phi \) and \( M \) from \( \phi, N \) and \( M-(N)-\phi \) curves given by Eqn. 6 (see Figure 6 (c)).
8) If \( N \) and \( M \) are not close enough to \( \bar{N} \) and \( \bar{M} \), set \( N = \bar{N}, M = \bar{M} \) and return to the stage 2) and repeat the iteration, otherwise
9) Determine the tangent stiffness of a member as the average of those of both edge nodes.
10) Renew the coordinates of all the nodes.
11) Compute the unbalanced force \( (\bar{P} - T \cdot \bar{f}) \)
12) Repeat the Newton-Raphson procedure until the unbalanced force becomes negligibly small.
13) Stop the computation when the displacements diverge caused by structural instability, otherwise proceed to the next loading step.
7. NUMERICAL EXAMPLE

A portal frame composed of box-profile cross-section members shown in Figure 7 is analyzed by the proposed method. The frame is subjected to constant vertical loads $P$ and increasing horizontal load $H$. The vertical load is applied by load control and the horizontal load is controlled by the displacement of the loading point, $\delta$. The span and height are assumed to be 164.8 cm for a frame with $b/t = 30$ and 353.2 cm for a frame with $b/t = 60$ to control the slenderness ratio of the member. The number of beam-column elements is 10 both for the column and beam. The width-to-thickness ratios $b/t$ and the magnitude of the vertical load $P$, that is, its ratio to the axial yielding load $P_y$, are taken as variable parameters.

![Figure 7: Numerical model](image)

![Figure 8: Load versus displacement curves of the portal frames](image)
In the illustration of the results, the horizontal load $H$ and the corresponding displacement $\delta$ are shown in non-dimensional form of $H/H_y$ and $\delta/\delta_y$. $H_y$ is the horizontal load which produces yielding at the cross-section of the clamped end without a vertical load $P$ and $\delta_y$ is the horizontal displacement of the loading point at the corresponding stage. $H_y$ and $\delta_y$ are calculated from the conventional frame formula as follows:

$$H_y = \frac{7I\sigma_y}{2h(b/2)}$$  \hspace{1cm} $$\delta_y = \frac{5H_yh^3}{84EI} \quad (16)$$

in which $I$ is the moment inertia of the member cross-section.

The results of numerical analysis are shown in Figure 8. The results of numerical analysis by Usami et al. are also shown for the comparison purpose. Their method is based on the M- (N)- $\phi$ relation, but the change of axial force is disregarded. From these figures, we can see that the maximum load carrying capacity decreases with the increase in the axial force ratios $P/P_y$ and $b/t$ ratios. It is noteworthy that the frame loses its capacity quite quickly after exceeding the maximum load, that is, after the local failure in one of the members. Also it can be recognized the results from the proposed method coincide well with those by Usami et al.

8. EXPERIMENT OF PORTAL FRAMES

To verify the numerical method an experiment of portal frames under constant vertical forces and increasing horizontal force is conducted. Figure 9 shows dimensions of the specimen. The profile of the cross-section is a welded square box and only the width-to-thickness ratios of the component plates are varied in two specimens. The structural properties and material properties are shown in Table 1 and Table 2. The constant vertical force $P$ is kept at 15% of the yield axial force of the column. The loads are applied by a hydraulic servo-dynamic testing apparatus as shown in Figure 10.

The test specimens were analyzed by the proposed numerical method. The numerical results for the horizontal force and horizontal displacement curves are compared with the test results in Figure 11 and Tables 3 and 4.
TABLE 1
STRUCTURAL PROPERTIES OF THE SPECIMENS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>b (cm)</th>
<th>t (cm)</th>
<th>h=L (cm)</th>
<th>R</th>
<th>( \lambda )</th>
<th>( H_y ) (tonf)</th>
<th>( \delta ) (mm)</th>
<th>( H_p / H_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>11.35</td>
<td>0.55</td>
<td>115</td>
<td>0.468</td>
<td>0.645</td>
<td>9.79</td>
<td>7.74</td>
<td>1.20</td>
</tr>
<tr>
<td>Type B</td>
<td>21.25</td>
<td>0.55</td>
<td>115</td>
<td>0.814</td>
<td>0.345</td>
<td>35.0</td>
<td>4.42</td>
<td>1.18</td>
</tr>
</tbody>
</table>

where \( \lambda = \frac{K h L}{r \pi \sqrt{E}} \)

\( K = \text{effective length}, 1.157 \text{ for this case} \)
\( r = \text{radius of gyration of the cross- section} \)
\( H_p = \text{horizontal force which produces full plastic moment at the clamped end} \)

TABLE 2
MATERIAL PROPERTIES OF THE SPECIMENS

<table>
<thead>
<tr>
<th>( \sigma_y )</th>
<th>E</th>
<th>( \nu )</th>
<th>( \varepsilon_y ) (%)</th>
<th>( \varepsilon_u ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>349 MPa</td>
<td>213 MPa</td>
<td>0.28</td>
<td>0.167</td>
<td>2.177</td>
</tr>
</tbody>
</table>

Figure 10: Apparatus for the model test

The abrupt decrease in the load is caused by the initiation of a crack along a welding line. Because of this crack the maximum load might have been measured somehow smaller. Comparing the maximum loads, numerical results correspond well to the test results except the possible effect of the premature cracking along the welding line of the buckled plate. Comparing the displacements at the maximum load, numerical results do not correspond well to the experimental values especially for Type B. One of the reasons for this is thought to be unexpected rotation of the clamped ends.
Another question how to determine the number of beam-column elements in the numerical analysis. To study the suitable number of beam-column elements, numerical calculations were carried out with variation of the number of elements. As the result, it is found that the maximum load is not influenced, but the inclinations of the degrading curves after the maximum load will be influenced by the number of elements. That is, the inclination of the degrading curve becomes smaller when the number of the elements become fewer.

Considering from the assumption used in this analysis, the length of the beam-column element in the numerical analysis should be equal to or approximately equal to 70% of the width (0.7b) of the cross section. This will be confirmed by the fact that the wave length of the buckled plate at the clamped end was observed to be approximately 0.7b as shown in Figure 12.
9. CONCLUSIONS

A simplified method of analysis for the ultimate strength behavior of steel frames fail by local and global instability was developed. Applicability of the proposed method are shown in the numerical example and experimental test of portal frames. In this paper, the material is restricted to JIS SM490 steel, but from our other studies it is confirmed that this method is also applicable to structures made of other materials.

The proposed method can make a personal computer to be capable of analyzing coupled buckling problems of framed structures, which may be difficult to do without a big size computer. When you analyze the frame studied here by FEM, it may take several days by a personal computer, but it takes only several minutes in the use of the proposed method.

References

ANALYSIS OF NONLINEAR BEHAVIOR OF STEEL FRAMES UNDER LOCAL FIRE CONDITIONS

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ABSTRACT

A finite element approach is presented in this paper for analysis of nonlinear behaviour of steel frames under local fire conditions. The effects of geometric nonlinearity, temperature dependent material nonlinearity and variations in temperature distribution across sections of frame members are considered. Based on the principle of virtual work, the temperature-induced load vector and temperature-dependent geometric stiffness matrix are derived. Following the common procedure of finite element methods, a computer program is developed for calculating either the ultimate load at a specified temperature or critical temperature at a specified load level. The effectiveness of the approach is verified by a good prediction on the behavior of a steel frame experienced fire experiment.

KEYWORDS

Nonlinear analysis, Steel frame, Fire condition, Finite element method, High temperature, Ultimate load, Fire test.

1. INTRODUCTION

Because of the progressive deterioration in mechanical properties of steel with increasing temperatures, when exposed to fire, a bare steel structure will lose its load-bearing capacity in a short period of time. For a long time in the past, fire resistance design of steel structures could only have been based on standard fire test results on protected or unprotected specimens, such a method is time consuming and expensive. To overcome these drawbacks, a considerable amount of work has been done towards developing an alternative method for predicting the behavior of building structures in fire, with the emphasis on the introduction of analytical methods by means of computer simulations. The analytical method offers a cost effective
alternative to the traditional test method, and further more, it permits a more accurate calculation (prediction) of the structural fire response by considering the significance and severity of a real fire, it may therefor lead to a more rational and economical procedure with a more defined and uniform level of safety. This paper is mainly concerned with the analytical treatment of the structural response of steel frames at elevated temperatures.

The behavior of steel structures in fire is very complicated because of the many factors involved, such as the mixed geometrical non-linearity caused by thermal deflections, the complex material non-linearity resulting from the different softening of material due to the non-uniform temperature distribution, and the redistribution of internal forces as a result of the thermal expansion and formulation of "inelastic" zone. Important achievements have been made in the modeling of the behavior of steel structures exposed to fire during recent years, a number of numerical methods based on finite element technique have been proposed for fire resistance analysis of both 2D and 3D steel frames. Some methods can even permit the steel framed floor systems to be analyzed. The paper does not intend to summarize these current developments here, while it is worth mentioning that the Newton-Raphson method dealing with the incremental problems is widely adopted by most of the researchers to calculate the non-linear structural response. In this paper, a direct iteration method capable of predicting the non-linear behavior of steel frames corresponding to any specified load level or temperature distribution is proposed, the procedure can be repeated for ever increasing value of load or temperature and thus be used to calculate the whole structural response at room temperature or under fire conditions, the so-called secant stiffness matrix is used in finite element analysis. In addition to the effects of geometrical as well as material non-linearity, the presented method also permits an accurate consideration of the gradual penetration of inelastic zone. All these considerations have been reflected clearly by an introduction of the corresponding additional stiffness matrices and nodal force vectors in deriving the basic finite element equations.

2. MECHANICAL PROPERTIES OF STEEL AT HIGH TEMPERATURES

Proper models of stress-strain relationships at high temperatures are essential for an accurate prediction of the structural response under fire conditions. In this paper, a simplified trilinear model on the basis of ECCS recommendations, ECCS-T3(1983), is adopted, in which the creep strain is assumed to have been implicitly included. At a specified temperature state, this model of stress-strain curve depends on four material parameters, namely the initial modules of elasticity $E_i$, proportional limit $f_p$, yield point $f_y$ and softening modules of elasticity $E_{pi}$ shown in Figure 1.

![Figure 1: Trilinear stress-strain model](image)
ANALYSIS OF NONLINEAR BEHAVIOR OF STEEL FRAMES

Calculations of $E_t$, $f_p$, $f_y$, and $E_{pi}$ can be made by following equations:

$$f_y = \left[ 1 + \frac{T_s}{767 \ln \left( \frac{T_s}{1750} \right)} \right] f_{yo} \quad 0 < T_s \leq 600^\circ C$$

$$E_t = (1 - 172 \times 10^{-12} T_s^4 + 118 \times 10^{-9} T_s^5 - 345 \times 10^{-7} T_s^2 + 159 \times 10^{-5} T_s) E_0 \quad 0 < T_s \leq 600^\circ C$$

in which, $T_s$ is the temperature of steel, $f_{yo}$, $E_0$ are yielding point and modules of elasticity of steel at room temperature respectively.

$$f_p = \begin{cases} f_y & T_s \leq 200^\circ C \\ (1 - \frac{T_s - 200}{200}) f_y & 200 < T_s \leq 300^\circ C \\ 0.5 f_y & T_s > 300^\circ C \end{cases}$$

$$E_{pi} = \beta E_t$$

$$\beta = \begin{cases} 0 & T_s < 200^\circ C \\ 1.76 - 1.04 \times 10^{-3} T_s + 2.13 \times 10^{-7} T_s^2 - 1.47 \times 10^{-8} T_s^3 & 200 \leq T_s \leq 600^\circ C \\ 0 & T_s > 600^\circ C \end{cases}$$

The coefficient of thermal expansion of steel at high temperatures is assumed to be a constant, i.e.:

$$\alpha = 1.4 \times 10^{-5} / ^\circ C$$

3. NON-LINEAR FINITE ELEMENT ANALYSIS

3.1 Main Assumptions

The following assumptions are made in order to simplify the analysis:

1) The occurrence of yielding begins at the end of a member and then develops both across the section and along the length of the member;

2) Planes before deformation remain planes after deformation;

3) No out of plane or torsional displacements occur;

4) Shear deformations are neglected;

5) Temperature changes linearly across the section.
3.2 Basic Equations

Usually, fire occurs in one or some compartments and temperature distribution within structural members is not uniform. In addition, external loads applied to the building structure during fire exposure are almost constant. Under the assumption that the residual stress is not considered, the total axial strain at any point of the element section can be expressed in terms of the thermal strain $\varepsilon_t$ and stress related strain $\varepsilon_\sigma$ as follows:

$$\varepsilon = \varepsilon_t + \varepsilon_\sigma$$  \hspace{1cm} (6)

Under the assumption that the temperature changes linearly across the section, $\varepsilon_t$, which is caused by a different temperature change $t_1$ and $t_2$ at the upper and lower surface of the element section respectively, can be given by:

$$\varepsilon_t = \varepsilon_{t1} + \varepsilon_{t2}$$  \hspace{1cm} (7)

in which:

$$\begin{cases} 
\varepsilon_{t1} = \frac{t_1 + t_2}{2} \alpha \\
\varepsilon_{t2} = \frac{t_1 - t_2}{h} \gamma \alpha 
\end{cases}$$  \hspace{1cm} (8)

where $h$ is the height of the cross section; $\gamma$ is the distance at $y$ direction from the point to the axis of symmetry of the cross section.

Taking a steel frame at a specified load and temperature level into consideration, the principle of virtual work results in the following equation:

$$\langle P \rangle \{\delta \Delta \} = \int \sigma \delta \varepsilon \, dV$$  \hspace{1cm} (9)

in which $\langle P \rangle$ is the matrix of external loads, $\{\delta \Delta \}$ is the vector of nodal virtual displacements, $V$ is the volume of the element.

The displacement function of the element at high temperatures are assumed to be the same as those at room temperature:

$$\begin{cases} 
u = \frac{l - x}{l} u_1 + \frac{x}{l} u_2 \\
v = \left( \frac{l^2 - 3x^2}{l^3} + \frac{2x^2}{l^3} \right) v_1 + \left( \frac{xl - 2x^2}{l^2} + \frac{x^2}{l^2} \right) \theta_1 + \left( \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \right) v_2 + \left( \frac{x^2}{l^2} - \frac{x^3}{l^3} \right) \theta_2 
\end{cases}$$  \hspace{1cm} (10)

The only difference here is that it is considered that in Eqn.10, the nodal displacements $\{\Delta \} = \langle u_1, v_1, \theta_1, u_2, v_2, \theta_2 \rangle^T$ have included displacements caused by thermal expansion.

The strain - displacement relation for a beam - column element is written as:

$$\varepsilon = u'' + \frac{1}{2} v'^2 - yv''$$  \hspace{1cm} (11)
Then the virtual strain within the element caused by nodal virtual displacements can be written as follows:

$$\delta e = \delta u' + v'\delta v' - y\delta v''$$

(12)

To consider Eqn.9 by finite element method, the element stiffness equation can be obtained by making use of the stress - strain relation of steel and Eqns.10, 11 and 12. Assembling the stiffness equations of the elements both in fire zone and cold zone by the direct stiffness method the overall equilibrium equations are obtained.

### 3.3 Element Stiffness Equations

**Elastic period: \( \sigma \leq f_p \)**

The stress - strain relation in this period can be written as:

$$\sigma = E,\varepsilon_\sigma$$

(13)

Substituting Eqns.6, 7 into Eqn.13 and then into Eqn.9, we have

$$\{P\}\{\delta \Delta\} = E, \int \varepsilon \delta \varepsilon dV - E,\varepsilon_1 \int \delta \varepsilon dV - E, \int \varepsilon \delta \varepsilon dV$$

(14)

Let:

$$\delta W_1 = E, \int \varepsilon \delta \varepsilon dV$$

$$\delta W_2 = E,\varepsilon_1 \int \delta \varepsilon dV$$

$$\delta W_3 = E, \int \varepsilon \delta \varepsilon dV$$

(15)

\(\delta W_i\) is the same as in the room temperature analysis. Substituting Eqn.12 into Eqn.15, we have

$$\delta W_2 = E,\varepsilon_1 \int (\delta u' + v'\delta v' - y\delta v'')dV = E,\varepsilon_1 \int (\delta u' + v'\delta v')dx$$

(16)

From Eqn.10, there are

$$\nu' = \left(\frac{6x^2}{l^3} - \frac{6x}{l^2}\right) v_1 + \left(1 - \frac{4x}{l} + \frac{3x^2}{l^2}\right) \theta_1 + \left(\frac{6x}{l^2} - \frac{6x^2}{l^3}\right) v_2 + \left(\frac{3x}{l^2} - \frac{2x}{l}\right) \theta_2$$

$$\nu'' = \left(\frac{12x}{l^3} - \frac{6}{l^2}\right) v_1 + \left(\frac{6x}{l^2} - \frac{4}{l}\right) \theta_1 + \left(\frac{6x}{l^2} - \frac{12x}{l^3}\right) v_2 + \left(\frac{6x}{l^2} - \frac{2}{l}\right) \theta_2$$

(17)

Let:
\[
\begin{align*}
B_1 &= \left( \frac{1}{l}, 0, 0, \frac{1}{l}, 0, 0 \right) \\
B_2 &= \left( 0, \left( \frac{6x^2}{l^3} - \frac{6x}{l^2} \right), \left( \frac{4x}{l^2} - \frac{3x^2}{l^3} \right), 0, \left( \frac{6x}{l^2} - \frac{6x^2}{l^3} \right), \left( \frac{3x^2}{l^2} - \frac{2x}{l} \right) \right) \\
B_3 &= \left( 0, \left( \frac{12x}{l^2} - \frac{6}{l} \right), \left( \frac{6x}{l^2} - \frac{4}{l} \right), 0, \left( \frac{6}{l^2} - \frac{12x}{l} \right), \left( \frac{6x}{l^2} - \frac{2}{l} \right) \right)
\end{align*}
\]

Then:
\[
\begin{align*}
\delta u' &= B_1 \{ \delta \Delta \} \\
\delta v' &= B_2 \{ \delta \Delta \} \\
\delta v'' &= B_3 \{ \delta \Delta \}
\end{align*}
\]

Substituting Eqns. 17, 19 into Eqn. 16, we have
\[
\delta W_2 = E_i A \varepsilon_i \{ \delta \Delta \}^\top \left[ \int B_1^\top dx + \int B_2^\top B_2 \{ \Delta \} dx \right]
\]

In the same way:
\[
\delta W_1 = E_i l \left( \frac{l_2 - l_1}{h} \right) \alpha \{ \delta \Delta \}^\top \int B_3^\top dx
\]

Then, making use of Eqns. 15, 20, 21 and 14, one can obtain:
\[
\{ P \} = ([K_1] + [K_2] - [K_3]) \{ \Delta \} - \{ P_1 \}
\]

in which \([K_1], [K_2]\) are element stiffness matrices similar to those for room temperature analysis, and:
\[
[K_3] = E_i A \varepsilon_i \int B_2^\top B_2 dx
\]

\[
\{ P_1 \} = E_i A \varepsilon_i \int B_1^\top dx + E_i l \left( \frac{l_2 - l_1}{h} \right) \alpha \int B_3^\top dx
\]

Softening period \(( \sigma > f_p )\)

In Figure 1, stress - strain relations when \( \sigma > f_p \) are:
\[
\sigma = f_p + \beta \varepsilon_i (\varepsilon_i - \varepsilon_p)
\]
ANALYSIS OF NONLINEAR BEHAVIOR OF STEEL FRAMES

For a partially softening cross section, there is

\[ \int_A \sigma \delta \varepsilon \, dA = \int_{A_1} \sigma \delta \varepsilon \, dA + \int_{A_2} \sigma \delta \varepsilon \, dA \]  

(26)

in which, \(A_1\) is the area of elastic core, \(A_2\) is the area of the softening part of the section.

Substituting Eqn.25 into the second part of Eqn.26, we have

\[ \int_{A_1} \sigma \delta \varepsilon \, dA = \int_{A_1} \delta \varepsilon (1 - \beta) f_p \, dA + \beta \int_{A_2} E_i (\varepsilon - \varepsilon_c) \delta \varepsilon \, dA \]  

(27)

So, Substituting Eqn.27 into Eqn.26 and integering along the length of the element, then substituting into Eqn.14, the following equation is obtained

\[ \{P\} = ([K_1] + [K_2] - [K_3] + [K_4])\{\Delta\} - \{P_i\} + \{P_p\} \]  

(28)

in which:

\[ [K_1] = (1 - \beta) f_p A_2 \int B_1^{-1} B_1 \, dx \]  

(29)

\[ [P_p] = (1 - \beta) f_p A_2 \int B_1^{-1} \, dx \]  

(30)

A computer program NASFAF which means Non-linear Analysis of Steel Frames Against Fire has been developed in this paper based on above analysis. Taking the element temperature distribution and the stress-strain relations of steel at elevated temperatures as the input data, the program permits an calculation of responses of steel frames under any combination of change of loads and temperatures.

4. COMPARISONS WITH TEST RESULTS

4.1 Room Temperature Analysis

![Figure 2: Test frame and loads](image)

\(F_2 = 88995N\)

\(F_1 = 3F_2\)

\(h = 2660mm\)
A one-bay, single story unbraced portal frame with fixed bases as shown in Figure 2 was tested by Arnold et al. (1968). The beam was a 10 I 25.4 stout shape of ASTM-A36 steel and the columns were 5 WF 18.50 stout shape of ASTM-A41 steel. Tension tests were also performed to determine the Stress-strain characteristics of the A36 and A41 steels. Adopting an average set of material properties - yielding point f_y, yielding strain ε_y, and modules of elasticity E as the direct input data of the program NASFAF, the predicted load-deflection relationship as well as the ultimate load has been obtained. Figure 3 shows the comparison between the computer predictions and the reported test results. From Figure 3, we can see that the agreement is satisfactory. Furthermore, the predicted ultimate load is also very close to the test value. All these have confirmed the ability of the method to deal with the structural behavior at room temperature.

It is worth mentioned that in the presented method, the yielding order within structural members can be predicted and the spread of yield both over the cross-section and along the member can also be simulated. The method permits the calculation of inelastic zone parameters such as yielding height over the section and yielding length of a partially inelastic member. For example, in this room temperature analysis, according to the calculation results, first yielding occurred at the top of right column (point A) when P reached a value of about 30 kN., then sections at the bottom of right column (point B) and under the left beam load (point C) began to yield. Once yield occurred at the bottom of left column (point D) when P is about 63 kN., lateral deflection increased quickly until the maximum load is reached. (location of each point is shown in Figure 2. These results are quite in agreement with the analysis and test results provided by Arnold et al. (1968), somewhat difference may exist because of the use of “plastic hinge” theory in their analysis. In addition, calculation results in this paper also show that when P reaches about 65 kN., the upper and lower surfaces of the cross-section at point A are both in yielding state, the yielding height has been more than 40 mm, while yielding length of the right column has become about one-third of the whole length of the member.

Rubert and Schunmann (1985) have performed a series of elevated temperature tests on several plane steel frames and provided results for these tests. One of the tested frame ZSR1 is Shown in Figure 4. The span l was 1200 mm, column height was 1170 mm, room temperature yielding point and modules of elasticity were 355 N/mm² and 210 kN/mm² respectively. One bay of the frame was uniformly heated at a constant rate by electrical elements and the remaining two members were kept at room temperature. Comparison between the predicted deflections and test results illustrated in Figure 3 shows the agreement is satisfactory.
All the tested frames included series EHR, EGR and ZSR were analyzed using the presented method. Figure 6 compares the predicted with measured critical temperatures for all these frames. In all cases, the errors are not more than 10 percent.
In 1995, for the first time in China, the authors, Zhao (1995), conducted a total of four fire tests on steel frames under different load level and heating process. All the test frames are two-dimensional one-story and one-bay unbraced portal frames. During each test, the applied load on the frame was maintained constant while its temperature was increased. The changes in temperature and deflection of the frame were then constantly recorded. The purposes of the test are three-fold, i.e. first, to observe the structural response and the failure pattern of the steel frames in fire; second, to have an understanding of the temperature distribution on the steel profiles and obtain data for use in theoretical analysis; and third, to measure the deflection of the tested frames and provide data for comparisons with analytical results.

One of the test frames is illustrated in Figure 7. Room temperature yield stress and modules of elasticity of A3 steel used for the test frame were 293.5 N/mm² and 2.0 × 10⁵ N/mm² respectively. Different from other tests, this test was conducted in a closed furnace, so the temperature distribution was considered to be uniform. The loading and location of thermal couples are shown in Figure 8.

The measured temperature-time curves are shown in Figure 9. Adopting these measured temperature values directly as input data, lateral deflections are calculated using computer program NASFAF. A comparison between the measured and calculated deflections is shown in Figure 10. As indicated in the figure, a good agreement is obtained.
5. CONCLUSIONS

A direct iteration method for non-linear analysis of two-dimensional steel frame at elevated temperatures has been presented, based on finite element formulation. The method permits effects of geometric nonlinearity, temperature dependent material nonlinearity and variation in temperature distribution across each member to be included. The ability and potential usefulness of the method has been demonstrated by some illustrative examples both for room temperature and high temperature analysis of steel frames.

REFERENCES


EFFECTS OF VISCOUS DAMPING MODELS, HYSTERETIC MODELS AND GROUND MOTION CHARACTERISTICS ON SEISMIC P-DELTA STRENGTH AMPLIFICATION FACTORS

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ABSTRACT

The influence of viscous damping modelling, hysteretic modelling, and frequency content of seismic ground motions on P-delta strength amplification factors for single-degree-of-freedom systems is investigated. Twelve different proportional viscous damping models were examined and the results indicate that P-delta strength amplification factors are not sensitive to the type of damping model. However, both the hysteretic model and the predominant frequency of the ground motion were found to have a significant effect on the increase in strength required to compensate for P-delta effects. The natural period of the system is also a contributing factor. Lower strength amplification is needed for systems with bi-linear slackness or stiffness degrading hysteretic responses when compared to bi-linear models. Low frequency ground motions are more critical for bi-linear models. Under these ground motions, higher strength amplification is required when the structural period of the system is lengthened, regardless of the hysteretic model. Short period structures are more influenced, however, by high frequency ground motions.

KEYWORDS

Bi-linear model, Bi-linear with slackness model, Damping model, Ground motion frequency, Hysteretic model, P-delta effects, Seismic, Stiffness degrading model, Strength amplification factor.

1. INTRODUCTION

Past research (Bernal, 1987, 1990, 1992; Jennings and Husid, 1968; Montgomery, 1981; Neuss and Maison, 1984) has shown that P-delta effects on structures responding in the inelastic range to seismic ground motions generally lead to an increase in lateral deformations and may, eventually, cause dynamic instability and collapse of structures. As current earthquake resistant design provisions in most countries allow for some level of inelastic deformation in structures, attention must be paid to P-delta effects in design. One approach that has been proposed to counteract these effects is to increase the lateral resistance of the structure by applying a strength amplification factor, $\varphi$, to maintain the inelastic deformations at the level experienced without the influence of the gravity loads. For a given structure, the strength amplification factor therefore corresponds to the ratio of the yield strength required to achieve a target ductility including P-delta effects to the yield strength determined for the same ductility when neglecting P-delta effects. The target ductility including P-delta effects can be limited, however, to prevent post-earthquake instability under static gravity loads.
In the past ten years, several researchers have proposed numerical expressions to predict the value of the $\varphi$ factor for a given target ductility and level of gravity loads. Most studies included parametric analyses performed on single-degree-of freedom (SDOF) structural systems which indicated that several parameters could influence the value of $\varphi$. The parameters that can affect the magnitude of the seismically induced inelastic deformations and, potentially, the stability of building structures are shown in Fig. 1. For a given mass and initial stiffness (natural period) and for a given gravity loading ($P$), viscous damping, hysteretic response and ground motion characteristics all play a role in the dynamic equilibrium of a structure subjected to earthquake ground shaking. Viscous damping modelling impacts on the amount of non-yielding energy absorbed by the system whereas hysteretic modelling affects the energy dissipated though inelastic deformation. Ground motion properties of interest include the magnitude, frequency content, duration of strong motion, and the impulsive nature of the signal.

Viscous damping can be modelled using mass and/or stiffness proportional damping based on either the initial elastic or tangent inelastic system properties. Fajfar et al. (1993), Léger and Dussault (1992), Sucuoglu et al. (1994) and Vidic et al. (1994) have shown that the mathematical representation and magnitude of viscous damping can have a significant effect on the ductility demand, energy response, and related code type force reduction factors, for inelastic structures including or excluding geometric stiffness effects. Although the concept of viscous damping in nonlinear analysis is not yet clearly defined, researchers have begun to use different mathematical representation of viscous damping to predict more accurately the inelastic seismic response of structures (Takayanagi and Schnobrich, 1979; Otani, 1980; Sederat and Bertero, 1990; Schiff; 1991). In Japan, it is now current practice to specify Rayleigh damping proportional to the initial elastic stiffness and tangent stiffness for the nonlinear dynamic analysis of steel and reinforced concrete building structures, respectively (Wada et al., 1994). In spite of this, past investigations on P-delta effects on the inelastic dynamic stability of structures only included damping representations that were proportional to the initial elastic structural properties, and no attempt has been made to relate the $\varphi$ factor to the various types of viscous damping models.

Figure 1: P-delta effects on the nonlinear seismic response of building structures

Some attention has been devoted, however, to the possible influence of the hysteretic model, natural period of the system and ground motion characteristics in past studies on seismic stability effects. Bernal (1987, 1990) examined the response of bi-linear hysteretic models with no strain hardening. He noticed that strength amplification factors were weakly correlated with the structural period for periods ranging between 0.1 s and 2.0 s and that ground motions recorded on soft ground could lead to higher strength amplification factors than firm ground accelerograms. With similar hysteretic systems, Mazzolani and Piluso (1993) observed that higher strength amplification was required for short period structures when subjected to ground motions producing higher spectral ordinates in the short period range. Based on limited results, Fenwick et al (1992) also indicated that strength amplification factors could be period dependent and that the frequency content of the seismic excitation could be of significance. MacRae et al. (1993) investigated P-delta effects on bi-linear hysteretic models with strain hardening and stiffness degrading hysteretic models. Stability effects were found to be less important for the latter system.

This paper presents the results of extensive series of analyses that were performed on SDOF systems to better understand the effects of (i) the viscous damping model, (ii) the hysteretic model, (iii) the frequency content of the earthquake ground motions, and (iv) the structural period of vibration on P-delta strength amplification factors.
2. ANALYTICAL AND COMPUTATIONAL PROCEDURES

2.1 P-Delta Effects

Figure 2 shows the response of a bi-linear SDOF system considering P-delta effects upon application of a monotonic lateral load V. The behaviour is modified by adding the ordinates of the curve entitled "P-A" to the basic inelastic load-deflection response of the system. If linear elastic behaviour is assumed, the tangent stiffness, $k_t$, is equal to the initial elastic stiffness, $k_o$, and the effective lateral stiffness considering P-delta effects can be written as $k_e = k_o (1 - \theta)$, where $\theta$ is the stability coefficient. That coefficient is equal to $P/(k_o L)$, where P is the gravity load and L is the height of the structure. It indicates by how much the stiffness of the structure is reduced by the presence of the gravity loading. The stiffness reduction, $\theta k_o$, is also referred to as the geometric stiffness of the system, $k_g$.

Under ground motion excitation, the dynamic equilibrium of the horizontal forces acting on the system is described by:

$$m \ddot{u} + c_t \dot{u} + (k_t - k_g) u = -m \ddot{u}_g$$

(1)

where $m$ is the mass; $c_t$ is the damping coefficient assuming viscous damping; $k_t$ is the tangent stiffness considering only material nonlinearity; $\ddot{u}$, $\dot{u}$, $u$ and $\ddot{u}_g$ are the acceleration, velocity, displacement, and ground acceleration, respectively. The natural frequency corresponding to Eqn. 1 is given by:

$$\omega_{1-g} = \sqrt{[k_t - k_g]/m}$$

(2)

For elastic behaviour considering P-delta effects, $\omega_{1-g}$ is transformed to $\omega_{1-g} = [k_o/m (1 - \theta)]^{0.5} = \omega_o (1 - \theta)^{0.5}$. Thus, gravity loading reduces the natural frequency of the system and, thereby, its dynamic response.

A computer program was developed in this study to solve Eqn. 1 for the various viscous damping and hysteretic models described below. The solution algorithm is based on the Newmark-Beta average acceleration method which has been shown to be convergent and unconditionally stable in the case of negative stiffness with constant damping (Cheng, 1988). A constant time step equal to 0.001 s was considered in the analyses and an equilibrium correction was performed at the end of each time step.

2.2 Viscous Damping Models

The following Rayleigh-type damping representation can be used in nonlinear analyses of MDOF systems:

$$C_t = a_0 M + a_t M + b_t K_t + b_0 K_0$$

(3)

where $C_t$ is the tangent damping matrix, $M$ is the mass matrix, $K_t$ is the tangent stiffness matrix, $K_0$ is the initial stiffness matrix, and $a_0$, $a_t$, $b_0$, $b_t$ are proportionality constants specified by the analyst and usually computed from the natural frequencies of the structures. Note that while the stiffness and inertial properties have clear physical meaning, Rayleigh damping does not (Jeary, 1986; Kareem and Sun, 1990). Nevertheless, it is widely used due to its mathematical simplicity. The coefficients $a_t$ and $b_t$ are generally derived from the initial elastic frequencies. The coefficients $a_t$ are derived from the instantaneous frequencies that change as the response becomes inelastic (Léger and Dussault, 1992). For example, the widely used DRAIN-2D computer program (Kannan and Powell, 1973) allows viscous damping modelling from:
where $K_t$ is the current tangent stiffness matrix including geometric stiffness effects and $K_o$ is the original elastic stiffness matrix of the unstressed structure, excluding geometric stiffness effects. In the context of seismic stability analysis of SDOF systems, mathematical representations of viscous damping can be defined as follows:

\begin{align*}
  c_1 &= (2\xi/\omega_0) (k_o) \\
  c_2 &= (2\xi/\omega_0) (k_o - k_g) \\
  c_3 &= (2\xi/\omega_0) (k_t) \\
  c_4 &= (2\xi/\omega_0) (k_t - k_g) \\
  c_5 &= (2\xi/\omega_0) (k_t) \\
  c_6 &= (2\xi/\omega_0) (k_t - k_g) \\
  c_7 &= [2\xi/(\omega_0 + \omega_t)] (k_o) \\
  c_8 &= [2\xi/(\omega_0 + \omega_t)] (k_o - k_g) \\
  c_9 &= [2\xi/(\omega_0 + \omega_t)] (k_t) \\
  c_{10} &= [2\xi/(\omega_0 + \omega_t)] (k_t - k_g) \\
  c_{11} &= [2\xi/(\omega_0 + \omega_t)] (k_t) \\
  c_{12} &= [2\xi/(\omega_0 + \omega_t)] (k_t - k_g)
\end{align*}

where $\xi$ is the selected damping ratio between the actual damping coefficient and the damping coefficient at critical damping; $\omega_0$ is the initial elastic frequency ($k_o/m^{0.5}$; $\omega_t$ is the tangent frequency for material nonlinearity only ($k_t/m^{0.5}$; $\omega_{t,g}$ is the tangent frequency for geometric nonlinearity only ($[(k_o - k_g)/m]^{0.5}$; $\omega_{t,m}$ is the tangent frequency for material and geometric nonlinearities ($[(k_t - k_g)/m]^{0.5}$. Damping model $c_1$ has been used in previous investigations of the seismic stability of inelastic structural systems. It is proportional to a constant value ($k_o$) and uses initial elastic properties. It is equivalent to mass proportional damping for elastic SDOF systems. In this study, the influence of using the other damping models on the $\phi$ factor was examined. The damping ratio, $\xi$, was kept equal to 5% of critical damping.

2.3 Hysteretic Models

Three hysteretic models were included in this investigation. The restoring force, $F_s$, versus the horizontal displacement response of each model is shown in Fig. 3. For the bi-linear model (Fig. 3a), three strain hardening ratios, $r$, were considered: 0.0, 0.05 and 0.10. The bi-linear model with no strain hardening is referred to herein as the elastic-perfectly plastic (EPP) model and is used as a reference because it has been extensively used in previous research on P-delta effects. Bi-linear models with strain hardening better represent the behaviour of steel frame structures.

To simulate the severely pinched hysteretic behaviour of systems such as tension-only concentrically braced frames or timber structures, a bi-linear model with slackness but no strain hardening was considered in the study (Fig. 3b). Thirdly, a stiffness degrading hysteretic model was developed to reproduce the behaviour of reinforced concrete structures. The model was based on the original formulation by Takeda et al. (1970) but
was modified to include only a bi-linear backbone. The level of strain hardening was set equal to 2.5% based on results by Mitchell and Paultre (1994). In this model, the unloading stiffness, \( k_u \), in each direction progressively softens as a function of the maximum ductility, \( \mu (= u/U_y) \), reached in the corresponding direction:

\[
k_u = k_0 \mu^{-\alpha}
\]  

(17)

The unloading exponent, \( \alpha \), in Eqn. 17 was taken equal to 0.5, which represents a reasonable value for axially loaded columns (Priestley et al, 1996).

2.4 Earthquake Ground Motions

Three sets of ground motions with different frequency content were considered. The normalised spectra of all records is shown in Fig. 4. The ensemble selected by Bernal (1987), which includes firm site records of moderate frequency content, is used as a reference. It is composed of the following earthquake records: 1949 Olympia (S86W), 1940 El Centro (S00E), 1952 Taft (S69E), and 1971 Pacoima (S16E).

The records of the other two groups were selected to achieve a sharp contrast in the shape of the acceleration response spectra. The high frequency group is characterised by a steep drop in spectral ordinates in the 0.2 s to 0.5 s period range. It includes 6 ground motions typical of eastern North America: two 1985 Nahanni records (Battlement Creek N270 and N360), two 1988 Saguenay records (La Malbaie N63 and Les Eboulements N0), and two synthetic ground motions (M7.0 at 20 km and M6.5 at 15 km). Conversely, the low frequency group exhibits peaks of spectral ordinates at periods up to 2 s. That group includes 5 ground motions recorded on soft soil layers: two 1971 San Fernando records (Vanoven St. S00W and S90W), one 1985 Mexico City SCT record (N90W), and two 1989 Santa Cruz records (Hollister South and Pine N00E and S90W).

![Figure 4: Normalised acceleration ground motion spectra (5% damping): (a) high frequency; (b) moderate frequency; (c) low frequency](image)

Other characteristics of the ground motions are presented in Table 1. The peak ground acceleration, PGA, is given as a reference but does not influence the strength amplification factor computed in this study since only the lateral resistance of the structure is varied to achieve, with and without P-delta effects, a specified target ductility level under a given ground motion. The duration, \( D_{5t} \), is taken herein as equal to the time required to build up between 5% and 95% of the total Arias intensity of the ground motion (Léger et al, 1993; Trifunac and Brady, 1975). Ground motion duration increases as the dominant frequency of the shaking decreases. The average value of \( D_{5t} \) is nearly three times longer for low frequency records than for high frequency ones (27.0 s vs 9.7 s).

The number of zero crossing, NZC, and the predominant period of shaking, PPS, defined herein as the ratio of the total duration of the record to half the number of zero crossing, both confirm the differences in the frequency content that exist between each group of records. For instance, NZC for the high frequency records
is, in average, about four times larger than for low frequency ground motions \((814 \text{ vs } 225)\) and this ratio becomes more significant \((394 \text{ vs } 79 = 5.0)\) when considering the number of zero crossing within the duration \(D_T\), \(NZC_T\). The average predominant period of shaking also varies significantly from one group to another: \(0.047\) s, \(0.297\) s and \(0.740\) s for the high, moderate and low frequency records, respectively.

<table>
<thead>
<tr>
<th>Record</th>
<th>Ground Motion Characteristics</th>
<th>Pulse Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PGA ((g))</td>
<td>(D_T) ((s))</td>
</tr>
<tr>
<td>------------------------------------</td>
<td>--------------</td>
<td>-----------------</td>
</tr>
<tr>
<td></td>
<td>High Frequency Group</td>
<td></td>
</tr>
<tr>
<td>Synthetic M7.0 at 20</td>
<td>0.52</td>
<td>6.8</td>
</tr>
<tr>
<td>Synthetic M6.5 at 15</td>
<td>0.13</td>
<td>8.4</td>
</tr>
<tr>
<td>Les Éboulements NO</td>
<td>0.12</td>
<td>11.1</td>
</tr>
<tr>
<td>La Malbaie N63</td>
<td>0.13</td>
<td>8.5</td>
</tr>
<tr>
<td>Nahanni N270</td>
<td>0.19</td>
<td>11.3</td>
</tr>
<tr>
<td>Nahanni N360</td>
<td>0.19</td>
<td>12.0</td>
</tr>
<tr>
<td>average:</td>
<td>0.21</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>Bernal’s High to Moderate Frequency Group</td>
<td></td>
</tr>
<tr>
<td>El Centro</td>
<td>0.35</td>
<td>24.4</td>
</tr>
<tr>
<td>Talf</td>
<td>0.18</td>
<td>28.9</td>
</tr>
<tr>
<td>Olympia</td>
<td>0.28</td>
<td>18.1</td>
</tr>
<tr>
<td>Pacoima</td>
<td>1.17</td>
<td>7.0</td>
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<tr>
<td>average:</td>
<td>0.50</td>
<td>19.6</td>
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<tr>
<td></td>
<td>Low Frequency group</td>
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<td>San Fernando S00W</td>
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<td>25.3</td>
</tr>
<tr>
<td>San Fernando S90W</td>
<td>0.11</td>
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<tr>
<td>Mexico</td>
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<tr>
<td>Santa Cruz N00E</td>
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<td>16.4</td>
</tr>
<tr>
<td>Santa Cruz S90W</td>
<td>0.18</td>
<td>29.7</td>
</tr>
<tr>
<td>average:</td>
<td>0.19</td>
<td>27.0</td>
</tr>
</tbody>
</table>

The number, intensity and duration of acceleration pulses in a ground motion time history may also influence the dynamic response and stability of inelastic structures subjected to earthquake ground shaking. An acceleration pulse is defined as the segment of an accelerogram between any two successive zero crossing points. Severe acceleration pulses produce inertia forces that resemble to “static” horizontal loads which can lead to large lateral deformations and, thereby, increase the potential of instability and collapse. The parameter \(DP\) in Table 1 is the duration of the longest pulse within the time interval \(D_T\). The parameters \(II\), \(I5\) and \(I10\) represent the fraction of the total Arias intensity included in the strongest pulse, the 5 strongest pulses and the 10 strongest pulses, respectively. The entire distribution \((II, I2, \ldots, I10)\) is illustrated in Fig. 5.

In average, the longest pulse in the low frequency records is 14 times longer than in the high frequency time histories \((1.54\ s \text{ vs } 0.11\ s)\). All groups exhibit similar ratios of \(I5\) to \(II\) and \(I10\) to \(II\) but the fraction of energy contained in the largest pulses increases as the frequency content of the records decreases. In average, that fraction for the low frequency records is, in average, about twice as large as that in the high frequency ground motions.
2.5 **Determination of the Strength Amplification Factor, \( \varphi \)**

The single coefficient expression proposed by Bernal (1987) to predict the value of \( \varphi \) for a given target ductility, \( \mu \), and a given coefficient of stability, \( \theta \), was retained to characterise the effects of the various parameters on the strength amplification factor:

\[
\varphi = \frac{1 + \beta \theta}{1 - \theta}, \text{ where } \beta = B (\mu - 1)
\]  

(18)

This equation yields an amplification factor equal to unity when there is no gravity loads on the structure (\( \theta = 0 \)), and approaches infinity as the value of \( \theta \) approaches 1.0. Only the coefficient \( B \) can vary to reflect specific site conditions or a particular structural behaviour.

In this study, a value of \( B \) was computed for each hysteretic model and each ground motion ensemble by regression analyses performed on values of \( \varphi \) obtained for a range of structural periods, stability coefficients, and target ductility levels. The procedure was as follows: 1) for each record and each value of \( \mu \) and \( \theta \), an average strength amplification factor was computed for the range of periods considered; 2) for each pair of \( \mu \) and \( \theta \), the mean value of the average strength amplification factor was computed over the ground motion ensemble; 3) for each value of \( \mu \), a value of \( \beta \) was obtained from a least square fit of the relationships between the mean amplification factors and stability coefficients; 4) finally, a linear regression was performed over the pairs of \( \beta \) and \( \mu \) values to determine the coefficient \( B \). In the study on the effects of using different viscous damping models, the mean value of \( \varphi \) in step 2) was replaced by the amplification factor computed for a 10% probability of exceedance over the ensemble of earthquake records assuming a Gamma distribution. This permitted a direct comparison with the results obtained by Bernal (1987).

When determining the value of \( \varphi \) for a given case, the yield strength associated to a target ductility is obtained through an iterative procedure. The calculations were interrupted when the computed ductility was within 1% of the target value or when the difference between two consecutive values of strength was less than 1%.

For the study on viscous damping models, a period range of 0.2 s to 2.0 s in 0.05 s increments was used. All other analyses were performed for a period range of 0.1 s to 3.0 s in 0.1 s increments. To investigate the possible relationships between the frequency content of the ground motion and the structural period of the models, an additional series of calculations was performed for five sub-groups of three structural periods: 0.1, 0.2 and 0.3 s; 0.5, 0.6 and 0.7 s; 0.9, 1.0 and 1.1 s; 1.4, 1.5 and 1.6 s; and 2.8, 2.9 and 3.0 s. In all cases, the value of the stability coefficient ranged from 0.025 to 0.20 in successive increments of 0.025.
Target ductilities equal to 3, 4, 5 and 6 were considered in the calculations. Lower values were used, however, when determining the lateral resistance of systems with P-delta effects when post-earthquake instability was a possibility. Instability due to P-delta effects may occur during the ground motion as the structure progressively deforms inelastically towards one direction. In addition, instability can also occur after the earthquake if the lateral forces induced by the gravity loads acting on the deformed structure exceed the lateral resistance of the system. This second instability condition must be considered in design and, therefore, was accounted for in the determination of the strength amplification factor. For a given system, post-earthquake instability can be avoided by limiting the residual lateral deformation of the structure. This can be safely achieved by limiting the peak ductility experienced by the structure during the earthquake.

Figure 6 illustrates the procedure for a bi-linear system. Three levels of gravity loading are shown: \( P = 0 \) (no gravity loads); \( P = P_n \), which represents the gravity load condition likely to be present at the time of the earthquake; and \( P = P_u \), where \( P_u \) is the maximum level of gravity load that can be expected in the structure after the earthquake. \( P_u \) can be taken as the ultimate gravity load for which the structure is designed. The stability coefficients \( \theta \) and \( \theta_u \) correspond to the loads \( P_n \) and \( P_u \), respectively.

At the end of the earthquake, the residual deformation is equal to \( u_r \). Upon application of a higher gravity load \( P_u \), the deformation is amplified by an incremental amount, \( u_i \), and instability occurs if the total deformation exceeds the deformation \( \Delta_u \). For known values of \( P_n \) and \( P_u \), the ductility \( \mu_{\text{max}} \), for which the amplified deformation is equal to \( \Delta_u \), can be determined. For an EPP hysteretic model, the expression for \( \mu_{\text{max}} \), as obtained with this approach, is (Bernal 1987):

\[
\mu_{\text{max}} = \frac{1}{\theta} \frac{P_n}{P_u}
\]  

(19)

A ratio of \( P_n \) to \( P_u \) equal to 0.4 is representative of loading conditions prescribed in modern building codes for most types of construction (timber, steel and concrete). This value was therefore adopted in this study. Expressions similar to Eqn. 19 were developed for the other hysteretic models and the resulting values of \( \mu_{\text{max}} \) are presented in Fig. 7 for the range of interest of stability coefficients and target ductilities.

In the calculation of the \( \varphi \) factor, the target ductility is limited to \( \mu_{\text{max}} \) when assessing the resistance required with P-delta effects. This limit is not applied when determining the resistance without P-delta effects. The \( \varphi \) factor so obtained then eliminates the possibility of post-earthquake instability under gravity loads.
3. EARTHQUAKE RESPONSE ANALYSIS

3.1 Effects of Viscous Damping Models on the Strength Amplification Factor

Numerical properties of viscous damping models

Before considering the influence of the various damping models on the $\phi$ factor, an examination of the numerical properties of the models is essential to understand their effects and, more importantly, make a proper selection for analysis purposes.

In models described by Eqns. 5 to 10, P-delta effects reduce the stiffness of the system ($k_0 - k_g$ or $k - k_e$ vs $k_0$ or $k$), and damping coefficients are then also reduced ($c_2$ vs $c_1$, $c_4$ vs $c_3$, etc.). Conversely, higher damping is introduced when the geometric stiffness is included in the evaluation of the natural frequency in the proportionality constant ($c_7$ vs $c_1$, $c_8$ vs $c_2$, etc.). When both the stiffness and the frequency effects due to geometric nonlinearity are considered, the combined effect is a reduction in damping since the frequency is proportional to the square root of the reduced stiffness ($c_6$ vs $c_1$, $c_{10}$ vs $c_3$ and $c_{12}$ vs $c_5$). Including P-delta effects may also lead to a negative stiffness in the post-elastic range. In such a case, there is no real instantaneous natural frequencies for the system and damping coefficients $c_{11}$ and $c_{12}$ are not defined.

To get some insight into the actual significance of damping models $c_1$ to $c_{12}$, a bilinear SDOF structure with a 0.1 s period and parameters of $\xi_0=0.05$, $\theta=0.08$, $r=0.1$ and $\eta=0.5$ has been analysed under the SOOE El Centro record. The parameter $\eta$ is the ratio of the design base shear at yield to the maximum effective force applied during the earthquake: $\eta = V_y / (m \dot{u}_{\text{gmax}})$. It is used to characterise the strength of the structure relative to the peak ground acceleration of the record, $\dot{u}_{\text{gmax}}$.

**Table 2**

<table>
<thead>
<tr>
<th>$i$</th>
<th>elastic</th>
<th>inelastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>0.92</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>0.32</td>
</tr>
<tr>
<td>6</td>
<td>0.92</td>
<td>0.06</td>
</tr>
<tr>
<td>7</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>8</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>9</td>
<td>1.04</td>
<td>0.10</td>
</tr>
<tr>
<td>10</td>
<td>0.96</td>
<td>0.02</td>
</tr>
<tr>
<td>11</td>
<td>1.04</td>
<td>0.71</td>
</tr>
<tr>
<td>12</td>
<td>0.96</td>
<td>0.14</td>
</tr>
</tbody>
</table>

![Figure 8: Time history (partial) of $c_1/c_i$.](image-url)

Table 2 shows the relative magnitude of the damping coefficients when this system responds in the elastic and inelastic ranges. It is expressed as the ratio of $c_i$ to $c_1$, the latter representing the most common damping model. Strain hardening has been included to avoid a negative stiffness such that instantaneous real frequencies can be computed at each time step as required for models $c_{11}$ and $c_{12}$. Figure 8 shows a 2.2 s window of the time history of the ratio of $c_i$ to $c_1$. In Fig. 8, models with similar damping values upon inelastic response have been gathered into 5 groups and only the value for one model per group is shown. For the system analysed, all damping models yield similar values of damping when the response is elastic. All damping coefficients are then smaller than $c_1$, except for damping models $c_7$, $c_9$ and $c_{11}$ for which the geometric stiffness term is considered only in the computation of the proportionality constant. Upon yielding of the system, however, all models for which damping is proportional to the tangent stiffness matrix, $k_t$, exhibit a significant reduction in $c$ values. For models $c_1$, $c_2$, $c_7$ and $c_8$, the damping value remains unchanged.
Upon yielding, the difference between the models is strongly influenced by the amount of strain hardening in the system. Figure 9 shows the variation of the ratio \( c_i / c_i \) when \( r \) is varied from 0 to 0.25 for the studied model. Of course, damping coefficients \( c_1, c_2, c_7 \) and \( c_9 \) remain constant but all other coefficients generally increase with strain hardening. The rate of increase is different from one model to another, especially when \( r \) is close to or smaller than \( \theta \). When the strain hardening ratio is smaller than the stability coefficient \( (r < \theta) \), the coefficients \( c_{11} \) and \( c_{12} \) are not defined and models \( c_4 \), \( c_6 \) and \( c_{10} \) exhibit negative damping, which means that energy is fed into the yielding structure.

**Effects of viscous damping models on the strength amplification factor**

Figure 10 shows a typical amplification spectrum obtained for the SOOE El Centro record, a target ductility of 4 and a stability coefficient equal to 0.10. As shown, there are only minor variations in the amplification among the different damping models except for model \( c_6 \) which requires higher strength amplification to maintain the ductility demand to the desired level. For small values of the strain hardening ratio such as the value of 0.0001 used in this example, it is found that the damping coefficient \( c_6 \) is very sensitive to \( r \) and can take very large negative value when P-delta effects are included. If the strain hardening parameter \( r \) is slightly increased (e.g. \( r = 0.01 \) in this example), the difference between model \( c_6 \) and the other models is eliminated.

A complete statistical analysis with Bernal's group of moderate frequency ground motion was carried out with damping models \( c_1, c_4 \) and \( c_6 \). Damping model \( c_1 \) was considered in the study for comparison purposes since it corresponds to the model used by Bernal (1987). The damping coefficient in models \( c_4 \) and \( c_6 \) is proportional to the tangent stiffness and instantaneous frequency of the system, respectively. Model \( c_4 \) was selected because it is currently used in practice for the analysis of concrete structures. Model \( c_6 \) was considered because it generally yields responses that are significantly different of other models. A bi-linear hysteretic model with \( r = 0.0001 \) was used to simulate elastic-perfectly plastic behaviour.

The values of \( B \) (Eqn. 18) obtained from the analysis are 1.78, 1.76 and 3.78 for models \( c_1, c_4 \) and \( c_6 \), respectively. For model \( c_1 \), the value of \( B \) obtained in this study is 5% lower than the value proposed by Bernal. The difference can be attributed to the stringent tolerance limits imposed in this study in the iterative process for determining the lateral resistance. Using a damping coefficient proportional to the tangent
stiffness produced results very similar to those obtained with the basic damping model \( c_1 \). As expected, model \( c_6 \) led higher strength amplification demand, which is due to the small amount of strain hardening exhibited by the system considered. The sensitivity of model \( c_6 \) to strain hardening was investigated for this analysis and \( B \) was found to reduce from 3.78 to 2.09 when \( r \) was set equal to 0.005 instead of 0.0001. This new \( B \) value is only 17% higher than the value computed with model \( c_1 \), which indicates that the influence of this model reduces very rapidly when strain hardening is included. It is worth noting that model \( c_6 \) would very unlikely be selected in practice for elastic-perfectly plastic systems as it feeds energy in the system instead of dissipating it.

From this study, it can be concluded that the influence of viscous damping models on the strength amplification factor, \( \varphi \), is very limited. As reported by Tremblay et al. (1996), however, changing the damping model may affect other nonlinear seismic response parameters.

### 3.2 Effects of Hysteretic Models on the Strength Amplification Factor

In this second parametric study, the hysteretic model was varied and the three different ground motion ensembles were considered. The damping model \( c_1 \) was used in all cases. The values obtained for \( B \) for each hysteretic model and ground motion ensemble, as computed for the 0.1-3.0 s period range, are presented in Table 3 and Fig. 11. For a given set of ground motions, all bi-linear models, with and without strain hardening, generally led to comparable values of \( B \). The bi-linear model with slackness and the stiffness degrading hysteretic models both require lower strength amplification factors. For instance, for the moderate frequency earthquake group, the \( B \) value for these two hysteretic models is equal to 67% and 39%, respectively, of the \( B \) value computed for the EPP model. This result confirms the finding by MacRae et al. (1993) that P-delta effects are less significant on stiffness degrading systems than on bi-linear models.

<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.40</td>
<td>1.53</td>
<td>1.82</td>
</tr>
<tr>
<td>b</td>
<td>1.42</td>
<td>1.71</td>
<td>1.91</td>
</tr>
<tr>
<td>c</td>
<td>1.39</td>
<td>1.53</td>
<td>1.57</td>
</tr>
<tr>
<td>d</td>
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<tr>
<td>e</td>
<td>0.62</td>
<td>0.60</td>
<td>0.60</td>
</tr>
</tbody>
</table>

(1) See Figure 11

This remarkable difference can be attributed to the fact that bi-linear models do not experience any degradation of their stiffness during an earthquake, as opposed to the bi-linear with slackness and stiffness degrading models. Past research (Bernal, 1987; Montgomery, 1981) has shown that P-delta effects are more important during yielding excursions, when the effective lateral stiffness of the system is close to zero or even negative. Assuming that the target ductility is reached after several inelastic cycles, bi-linear systems would be yielding over a longer period of time than the two other models when reaching the target ductility and, therefore, would be more influenced by gravity loading. This is also in agreement with MacRae et al. (1993) who suggested that the severity of P-delta effects for bi-linear models with low strain hardening was related to the number of inelastic cycles experienced by the structure.

Another result of significance in Table 3 is the variation of \( B \) when the strain hardening ratio is increased for the bi-linear models. For these systems, increasing the strain hardening ratio is known to generally reduce P-
delta effects (Jennings and Husid, 1968; MacRae et al., 1993) but such a trend is not observed in Table 3: for each set of ground motions, the B coefficient increases when r is increased from 0 to 0.05, and then decreases when r is increased further from 0.05 to 0.10.

In this case, increasing the strain hardening ratio reduces the strength required to achieve a given ductility, both with and without P-delta effects. Thus, the influence on the $\phi$ factor varies depending whether the reduction in strength is more important with or without the P-delta effects. The calculations reveal that the answer depends on the value of the stability coefficient.

For example, when P-delta effects are included, the reduction in strength is relatively more important for low values of $\theta$. In this case, an increase in the strain hardening quickly produces beneficial changes in the effective lateral stiffness of the system upon yielding. The stiffness rapidly approaches zero and eventually becomes positive, which represents a much more stable condition. This results in a significant reduction in the strength required to achieve a given target ductility, which translates in a reduction of the strength amplification factor. For high values of $\theta$, the yield stiffness of the system with P-delta remains negative even when strain hardening is increased and the favourable influence of strain hardening is relatively more pronounced for the systems without P-delta effects. This behaviour is illustrated in Figure 12 for the moderate frequency ground motion group.

B values in Table 3 have been obtained for the whole range of stability coefficients and, consequently, do not reflect the relative influence of $r$ and $\theta$ on the strength amplification factor. This suggests that an expression which could account for the relative value of $r$ and $\theta$ would be more appropriate than Eqn. 18 for structures exhibiting a bi-linear response with strain hardening.

### 3.3 Effects of Ground Motion Frequency Content and Structural Period on the Strength Amplification Factor

Table 3 and Figure 11 indicate that a higher strength amplification is generally required for standard bi-linear models when the frequency content of the ground motions is lowered. The opposite is observed for the bi-linear model with slackness whereas the stiffness degrading systems do not seem to be influenced by the characteristics of the ground motions.

For the EPP model, similar B values are obtained for the high and moderate frequency content accelerograms. The increase of $\phi$ under low frequency earthquakes is in agreement with the observation by Bernal (1990) who compared strength amplification factors obtained from 5 ground motions on soft soils to those computed with 10 firm site records. Bernal attributed this increase to the longer duration of the soft soil accelerograms rather than to the frequency content of the earthquakes. The former has been shown to be of significance on P-delta effects (Fenwick et al., 1992) and all low frequency ground motions used in this study are much longer in duration than the high frequency time histories (Table 1). The more severe acceleration pulses contained in the low frequency ground motions may also have contributed to this result.

In Table 4 and Fig. 13, the influence of the ground motion is examined further in combination with that of the structural period for the EPP, bi-linear with slackness (BLS), and stiffness degrading (SDM) hysteretic models. The response of bi-linear systems with strain hardening was similar to that of the EPP model.
**Table 4**

VALUES OF B FOR THE EPP, BI-LINEAR WITH SLACKNESS (BLS), AND STIFFNESS DEGRADING (SDM) HYSTERETIC MODELS FOR THE SUB-GROUPS OF STRUCTURAL PERIODS

| Period Groups (s) | High Frequency | | Moderate Frequency | | Low Frequency | |
|-------------------|----------------|----------------|-------------------|----------------|----------------|
|                   | EPP | BLS | SDM | EPP | BLS | SDM | EPP | BLS | SDM |
| 0.1 to 3.0        | 1.40 | 1.14 | 0.62 | 1.53 | 1.03 | 0.60 | 1.82 | 0.97 | 0.60 |
| 0.1 to 0.3        | 1.71 | 0.78 | 0.67 | 1.32 | 0.57 | 0.48 | 0.60 | 0.15 | 0.36 |
| 0.5 to 0.7        | 1.76 | 0.90 | 0.60 | 1.42 | 0.88 | 0.63 | 1.44 | 0.55 | 0.47 |
| 0.9 to 1.1        | 1.35 | 1.00 | 0.53 | 1.50 | 1.29 | 0.69 | 1.13 | 0.67 | 0.54 |
| 1.4 to 1.6        | 1.23 | 1.02 | 0.69 | 1.64 | 0.94 | 0.50 | 2.22 | 1.23 | 0.62 |
| 2.8 to 3.0        | 0.97 | 1.45 | 0.60 | 1.53 | 1.22 | 0.69 | 1.74 | 1.49 | 0.66 |

Similitudes exist between the shape of the acceleration spectra shown in Figure 4 and the variation of the B values with the structural period shown in Fig. 13. For instance, it is the high frequency earthquake group that produces the highest values of B for the 0.1-0.3 s period range (1.71, 0.78 and 0.67 for the EPP, BLS and SDM models, respectively). For the same structural periods, the value of B progressively decreases when ground motions of lower frequency are considered. A similar but opposite trend can be observed for the group of structural periods ranging from 2.8 to 3.0 s for the EPP model. However, the factor for long period BLS and SDM systems is not as influenced by the ground motion frequency content.

For all models subjected to the low frequency accelerograms, the values of B gradually increase when longer structural periods are considered. In that case, the assumption that longer seismic duration of low frequency
earthquakes is responsible for higher P-delta strength amplification factors for longer structural periods appears to be correct since these structures have more time to respond in their own frequency. It does not explain, however, the smaller amplifications observed for short period structures under the same ground motions when compared to those obtained under the high frequency earthquakes.

These observations partly confirms the suggestion by others (Mazzolani and Piluso, 1993; Fenwick et al., 1992) that P-delta strength amplification factors are related to the shape of the elastic acceleration spectra. There is no direct relationship, however, between the spectral ordinates and the values of B and, in some cases, very dissimilar trends also exist. For example, the variation of the B value with the period for the BLS model under the high frequency group of earthquake records or for all models under the moderate frequency group of records do not match the shape of the corresponding spectrum. The effects of the ground motion frequency content and of the structural periods appear to be complex and other parameters are also likely to contribute such as the ground motion duration, the inelastic behaviour, etc.

In this study, the B values were obtained for all hysteretic models and groups of ground motions assuming that Eqn. 18 was applicable to all these cases. The φ factors predicted by Eqn. 18 using the computed B values were therefore compared to the actual amplification factors to assess the adequacy of this equation for the various hysteretic models and ground motions. The best correlation was obtained for the EPP model with Bernal's group of earthquake records (Fig. 14a). These represent the conditions for which Eqn. 18 was derived. For other hysteretic models, the values predicted by Eqn. 18 generally overestimate the amplification factor, except for high values of 0 and μ, in which cases the predicted values are not conservative. The most noticeable differences were observed for the bi-linear model with r = 0.10 (Fig. 14b). For a given hysteretic model, the accuracy of the prediction does not vary significantly, however, from one group of ground motions to another. These results suggest that further work is needed to formulate expressions that better predict the φ factor for hysteretic models other than the elastic-perfectly plastic one.

![Figure 14: Comparisons between actual and predicted (Eqn. 18) strength amplification factors for moderate frequency ground motions.](image)

4. CONCLUSIONS

Parametric studies have been performed to examine the influence of the viscous damping modelling, the hysteretic behaviour and the characteristics of ground motions on the seismic P-delta strength amplification factor for single-degree-of-freedom systems with various structural periods.

For elastic-perfectly plastic systems, similar amount of strength amplification is required to compensate for P-delta effects regardless whether the damping coefficient is proportional to the initial stiffness or the tangent stiffness of the system. However, viscous damping models proportional to the tangent stiffness of the system are influenced by the amount of strain hardening present. If the stability coefficient exceeds the strain...
hardening ratio, energy is fed into the structure by these damping mechanisms when yielding occurs. Judgement must then be exercised when using these models.

The hysteretic model, the frequency content of ground motions, and the structural period were found to significantly influence the strength amplification factor. More importantly, the effects of these parameters appear to be interrelated:

(i) The bi-linear with slackness and the stiffness degrading hysteretic models, which experience some degree of cumulative damage during an earthquake, generally require a lower strength amplification factor to counteract P-delta effects than the standard bi-linear hysteretic models with or without strain hardening. For bi-linear models with strain hardening, the amplification factor was found to depend on the relative value of the strain hardening ratio and the stability coefficient.

(ii) When all structural periods were considered (0.1 to 3.0 s), the P-delta strength amplification factors for the bi-linear models progressively increases when the frequency content of the ground motion is lowered. For the same period range, the factor $\phi$ for the other two models was found to be less sensitive to the frequency content of the seismic excitations. A slight decrease was however observed for the bi-linear with slackness and stiffness degrading hysteretic models when lower frequency ground motions were considered.

(iii) When the combined effects of the ground motion frequency content and the structural period are examined, all models exhibit a significant increase of the factor $\phi$ with the structural period when subjected to the low frequency ground motions. For all hysteretic models, the strength amplification factor for short period structures decreased when the frequency content of the ground motion was lowered. This suggests that there exists a relationship between the elastic acceleration response spectrum and the strength amplification factor. No other definite trends could be observed, however, between the $\phi$ factor and the structural period for high and moderate frequency ground motions.

Further research is needed to develop a more general expression that could predict accurately the strength amplification factor for various types of hysteretic models, while accounting for ground motion characteristics and structural periods. Gravity load effects on the time history of the inelastic seismic response of structures may need to be examined more closely, with attention paid to the length and number of yielding events and the possible influence of the pulse characteristics of the ground motions.

5. ACKNOWLEDGEMENTS

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6. REFERENCES


QUASI-STATIC CYCLIC AND PSEUDO-DYNAMIC TESTS ON COMPOSITE SUBSTRUCTURES WITH SOFTENING BEHAVIOUR

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ABSTRACT

Six full-scale steel-concrete composite substructures, part of a moment-resisting rigid frame were built and tested both in a quasi-static cyclic and a pseudo-dynamic fashion as part of an investigation on aseismic design of composite systems. The specimens embodied two composite beams with full shear connection and four companion beams with two different degrees of partial shear connection, all with exterior rigid joints. The tests were conducted in a low-cycle high amplitude regime. Thereby, stiffness, strength, ductility and energy consumption properties of substructures under reversed cyclic displacements as well as the effects of local buckling on the properties of beam-to-column joints were analysed. Main test data are commented upon and substructure seismic performances are assessed. Finally, a comparison between experimental and design strength capacity predicted by the relevant European code is provided.

KEYWORDS
Steel-concrete substructure, partial shear connection, beam-to-column joint, cyclic test, pseudo-dynamic test, strain-softening behaviour, local buckling.

1. INTRODUCTION

The current level of knowledge in composite construction embodying beams exposed to high-amplitude cyclic oscillations in which the floor slab acts as diaphragm in transmitting inertial forces is quite limited. Indeed, the part of Eurocode 8 (1994) corresponding to steel-concrete composite systems appears as an informative Annex, owing to the lack of confidence and information. Thereby, researches and code-related activities in the area of composite building structures located in high earthquake-prone zones are in progress.

On the European side, eight pseudo-dynamic tests were conducted by Meinsma (1996) on reduced-scale two-hinged portal frames, characterised by different steel-concrete connecting devices. Tests emphasised the good performances of composite beams with full shear connections and no local buckling in steel beams occurred. However, objections were posed to the adoption of the partial shear connection concept in seismic design, based upon the strength deterioration exhibited by stud shear connectors in pull-push and push-out tests (Kindmann and Meinsma, 1996). Analytical studies were performed by Broderick (1996), among others, who compared the seismic performance of both steel and composite moment frames designed according to Eurocode 8 (1994). The benefits provided by composite beams as part of the lateral force transmitting system...
on frame performances were evident, also with regard to the hogging-moment critical regions. However, only composite beams with full shear connections were analysed. Along the same line, an experimental study is underway at the ELSA laboratory on a full-scale composite frame (Pinto and Calvi, 1996). The study focuses mainly on understanding the participation of the slab in the moment transfer between beams and columns. Even in these tests, a full degree of interaction is adopted for the stud connector design.

On the North-American side, a major step taken in recent years was the development of the first U.S. code on seismic design of composite structures, i.e. the NEHRP provisions (1994). As far as composite beams are concerned, equations are provided to guarantee ductility in dissipative zones for beams which are intended as part of the primary lateral-force-resisting system. Moreover, it is suggested to reduce the nominal stud shear strength by 10 to 25 per cent, when stud shear connectors experience large reversed oscillations. Investigations on the seismic performance of composite frames are conducted currently in Japan too. For the sake of brevity, it is only mentioned the experimental study conducted by Yamamoto and Ohtaki (1997) on a full-scale three-storey and two bay frame. It is constituted by composite beams and reinforced concrete columns. Results showed that the aforementioned structural system is endowed with excellent seismic properties. Nonetheless, only full interaction was adopted for the composite beam design which was achieved by means of stud shear bolts.

In order to address the problem of composite beams with different degrees of shear interaction in a context of low-cycle large-amplitude reversed oscillations, Bursi and Ballerini (1996) and Bursi and Gramola (1997), tested six composite substructures with full and partial shear connection both in a cyclic and pseudo-dynamic regime. Moreover, eleven companion elemental pull-push specimens were exposed to a multi-specimen test program under variable- and constant-amplitude displacement, in order to analyse the low-cycle fatigue behaviour of headed stud shear connectors (Bursi and Ballerini, 1997). Indeed, the stiffness and strength deterioration after significant slip can have a significant impact on the structural system's serviceability.

In this paper, the overall performances of the aforementioned substructures are analysed and commented upon together with an assessment of their seismic performances. Moreover, other relevant phenomena as web and lower flange buckling which were induced in the hogging moment critical region of the composite beams are commented upon. Finally, a comparison between the maximum strength of composite substructures and the section capacity prediction according to the relevant Eurocode 4 (1992) is performed.

2. SUBSTRUCTURE SPECIMENS AND QUASI-STATIC TEST PROCEDURES

2.1 Steel-concrete composite substructure specimens

Six full-scale composite substructures were tested by Bursi and Ballerini (1996) and Bursi and Gramola (1997), respectively. They represent a full scale model of part of a moment-resisting frame, in which the floor slab acts as a diaphragm in transmitting inertial forces. Since the intent of the research was to study the composite action in a cyclic regime, a conventional degree $F_c/F_{cf}$ of shear connection based upon monotonic push-test results, (Eurocode 4, 1992) and equal to 1.36, 0.68 and 0.41, was estimated for the composite beams with full shear connection (F.C.), intermediate partial connection (I.P.C.) and low partial connection (L.P.C.), respectively. For each degree of shear connection two specimens were tested. The F.C. and I.P.C. specimens exhibited similar seismic performances, the L.P.C. substructures were hence designed and tested in a second phase in order to cover the range of shear connection of practical interest. As low-cycle fatigue criteria which usually govern this design are not available, only the aforementioned strength criterion for shear connection design was adopted.

The geometrical characteristics of composite substructures are shown schematically in Fig. 1. Columns were designed so as to meet the so-called strong column-weak beam concept. Thereby, all inelastic phenomena were confined in composite beams. Beam-to-column joints were designed to be rigid and full strength. (Eurocode 8, 1992). To this end, the beam section was welded to the column whilst additional rebars were located in the deck around the HE 360B section. In the beam web adjacent to the reinforcing plates, see Fig.
1, the amount of reinforcements, 8 12, was kept minimum in order to lower the neutral axis in the steel section and, as a result, delay local buckling. Thereby, the composite beam cross-section and all adjacent cross sections were classified as Class 1 (Eurocode 4, 1992), except L.P.C. specimens which were classified as Class 3, owing to the improved strength properties of rebars. As a matter of fact, to achieve behaviour factors greater than 4, Eurocode 8 (1994) would prescribe Class 1 cross-sections only. Welded headed shear studs were adopted in this test program, because they represent the most popular mean of shear connection today. They were designed to fail by shearing, thus avoiding brittle concrete-pullout failure. However, vertical separation and bending forces effects were not checked in the connector design. Moreover, both strength and ductility of connectors were enhanced by using large stud spacing and ribs parallel to the direction of the applied shear forces, as suggested by Hawkins and Mitchell (1984).

The measurement apparatus is shown schematically in Fig. 1. It comprises both a LVDT transducer in order to acquire the roller displacement and the load cell internal to the actuator to detect the reaction force. The reaction frame movement is detected by means of one LVDT at the base support. Moreover, both the composite beam axial displacement and the storey drift were detected by means of an external digital transducer (DT) at the steel beam centroid and a LVDT located at the exterior column flange, respectively, both depicted in Fig. 1. The interface slip between the steel beam and the composite slab is detected by means of coupled LVDTs located at Secs. 1-4 and shown in the same figure. The vertical separation was measured in the same aforementioned sections for the L.P.C. specimen only. In order to appraise internal forces in steel beams, flange strains were detected by means of linear strain gauges located at Secs. 2 and 4. Within these sections also rebar axial deformations were monitored whilst each stud connector was equipped with four strain gauges, labelled from 1 to 4, as highlighted in Fig. 1. As a result, both the effective width of rebars and the deformation state related to bending as well as vertical separation in stud connectors could be monitored at each loading stage.

2.2 Quasi-static cyclic tests

To begin with, quasi-static cyclic lateral displacements were applied to three substructures according to the ECCS short testing procedure (1986). Thereby, the ensuing stiffness, strength, ductility and energy absorption properties of each F.C., P.S.C. and L.P.C. specimen, respectively, were appraised. Moreover, as described in the sequel, the ECCS procedure (1986) allowed a comparative assessment among the specimen performances to be made. In the aforementioned procedure, the horizontal displacement \( e \), which was detected by means of the LVDT located at the exterior column flange, see Fig. 1, was assumed to be the prime parameter of the test control. Thereby, the main response of each substructure was expressed in terms of a hysteretic load-displacement \( (F - e) \) relationship.

In accordance with the ECCS short testing procedure (1986), the amplitude of initial cycles was selected small enough to detect the onset of yielding. Thereby, a conventional elastic limit state, characterised by a displacement \( e \), and the corresponding force \( F_y \) were defined on the skeleton curves. After testing, bi- and trilinear approximations of the skeleton curves were defined more rigorously based on best-fitting and energy-equivalence criteria, as illustrated schematically in Fig. 2. The aforementioned values for positive as well as negative hemicycles are collected in Column 8 and 5 of Table 1, respectively. As a result, it was readily feasible to characterise and compare the substructure responses in terms of stiffness, strength and ductility.

2.3 Pseudo-dynamic tests

From the perspective of reliable seismic testing procedures, the problem of testing structural systems with the conventional quasi-static cyclic approach consists in the uncertainty to relate the cyclic response to the seismic performance. Moreover, the ECCS procedure (1986) seems to be severe for the assessment of ductility levels and may not allow the potential strength of the tested component to be achieved. Thereby, pseudo-dynamic tests were performed on three additional companion specimens. These tests entail strength, ductility and energy absorption capabilities of specimens for a given random excitation to be assessed.
Figure 1: Test substructure characteristics and measurement apparatus

Figure 2: Bi- and trilinear approximation of the test load-displacement responses

Figure 3: Schematic of the pseudo-dynamic test set-up

Figure 4: Quasi-static hysteresis loops of reaction force vs. horizontal displacement of substructures: a) full shear connection; b) intermediate partial shear connection; c) low partial shear connection
The set-up adopted to carry on pseudo-dynamic tests is reported in Fig. 3, schematically. It reproduces the scheme developed by Shing et al. (1991), in which the time integrator is implemented with a dual displacement control by using both an external digital displacement transducer (DT) and an internal analogue transducer (LVDT), as highlighted in Fig. 1. This scheme prevents the deformation of the reaction frame supporting the actuator from affecting the actual structural displacement, whilst avoiding any external disturbance to the transducer used in the servo-control loop. The time integrator adopted to perform the test is implicit and unconditionally stable and, it is characterised by optimal characteristics of accuracy and numerical dissipation. In the solution process, both the initial stiffness operator and a modified Newton-Raphson iteration method are employed. The initial stiffness operator was estimated by means of the specimen responses provided by quasi-static cyclic tests. From a numerical stability standpoint, substructures may exhibit a severe softening behaviour when subjected to strong earthquake excitations owing to local and/or global buckling or strain-softening. Thereby, the time integrator incorporates an adaptive time-stepping strategy in order to avoid the divergence of the Newton-type iterative technique (Bursi et al., 1994).

3. MAIN RESULTS AND PERFORMANCE EVALUATION OF SUBSTRUCTURES

3.1 Quasi-static cyclic tests

For brevity, only the main results are illustrated in the sequel. The hysteresis loops of the reaction force developed by the F.C. substructure versus the controlled displacement are plotted in Fig. 4a. Hysteresis cycles are stable. Moreover, the inelastic hysteretic behaviour exhibited by the specimen is governed by steel beam yielding for positive (pull) loads and rebar yielding as well as concrete fracturing for negative (push) loads. In line with the predictions of Eurocode 4 (1992), based on the limiting breadth over thickness ratios for Class 1 section, web local buckling followed by flange buckling occurred at the first negative hemicycle characterised by a partial ductility ratio $e/\epsilon_y$ equal to 4. Both web and flange buckling happened at 250 mm from the column flange beyond the reinforcing plates, see Fig. 1. However, during alternate cycles, local buckling did not reduce substantially the load-carrying capacity of the substructure. In detail, owing to the reversed cyclic displacement regime, the right flange buckled and the left flange was straightened. Whereas in next reversal, the left flange buckled and the right one was straightened. All buckles occurred outwards in the bottom flanges whilst the buckle size was confined between two stiffeners. In spite of local buckling, positive as well as negative ductility levels $e/\epsilon_y$ equal to 6 were withstood by the substructure. Collapse was governed by crushing and uplifting of the composite slab at Sec. 4 (i.e. at the end of the additional rebars located around the HE 360B section). However, noticeable rotation capacity was provided by the specimen.

In order to verify the capabilities of section capacity calculations of Eurocode 4 (1992), which is calibrated indeed for monotonic loading, composite sections of the beam were analysed by using a plastic (stress-block) resistance moment idealisation. In this respect, mean reaction forces corresponding to the maximum bending resistances of the composite beam without partial safety factors were computed by adopting actual material properties. They were evaluated by using an average effective width $w_{\text{eff}}$ equal to 820 mm (Bursi and Ballerini, 1996). Predictions are indicated in Fig. 4a by horizontal hatched lines. The discrepancy between the code prediction and the plastic failure strength collected in Column 5 of Table 1, emphasises the limited adequacy of a non-seismic code when cyclic loading governs the structural response. As far as serviceability limit states are concerned, interstorey drift limits corresponding both to 2.0 and to 2.5 per cent (Exposure Group I and II, respectively, NEHRP, 1994), are depicted in Fig. 4a by vertical hatched lines, based on the assumption of a storey height of 3.5 m. With respect to stiffness, strength and displacement ductility, the substructure behaviour is quite satisfactory. It is worth noting that besides serviceability, the aforementioned limits control indirectly the stability of a structure and provide added strength and stiffness in moment frames.

The reaction force-controlled displacement loops relevant to the I.P.C. substructure are reported in Fig. 4b. To a certain extent, even these hysteresis cycles are stable. Web and flange buckling revealed at the first hemicycle with $e/\epsilon_y$ equal to 4, with akin characteristics of the companion F.C. specimen. At the third positive hemicycle of the same set, weld beads between the beam bottom flange and the column fractured. In spite of this collapse mode, a negative hemicycle at a ductility ratio with $e/\epsilon_y$ equal to 6 was achieved, followed by a
Figure 4: continued

**LOW PARTIAL CONNECTION**
- Cyclic test
- Code prediction

Figure 5: Reaction force vs. slip of 16 mm headed stud shear connectors

**TABLE 1**

PARAMETERS OF THE Hysteretic RESPONSE APPROXIMATION FOR CYCLIC TESTS

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<tbody>
<tr>
<td></td>
<td>$K^*$</td>
<td>$K_0^*$</td>
<td>$F^*$</td>
<td>$F_0^*$</td>
<td>$F_{max}^*$</td>
<td>$F_{max}^<em>$/F_0^</em></td>
</tr>
<tr>
<td>Pull F. C.</td>
<td>16.9</td>
<td>2.1</td>
<td>205.0</td>
<td>261.4</td>
<td>358.7</td>
<td>1.4</td>
</tr>
<tr>
<td>L.P.C.</td>
<td>15.2</td>
<td>1.9</td>
<td>200.0</td>
<td>264.7</td>
<td>347.3</td>
<td>1.3</td>
</tr>
<tr>
<td>L.P.C.</td>
<td>13.5</td>
<td>1.0</td>
<td>208.9</td>
<td>326.3</td>
<td>389.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Push F. C.</td>
<td>12.0</td>
<td>2.1</td>
<td>106.4</td>
<td>211.6</td>
<td>276.4</td>
<td>1.3</td>
</tr>
<tr>
<td>L.P.C.</td>
<td>12.2</td>
<td>2.2</td>
<td>101.6</td>
<td>209.8</td>
<td>271.3</td>
<td>1.3</td>
</tr>
<tr>
<td>L.P.C.</td>
<td>11.5</td>
<td>1.4</td>
<td>125.0</td>
<td>276.2</td>
<td>336.7</td>
<td>1.2</td>
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Figure 6: Mean absorbed energy ratio vs. partial ductility for quasi-static cyclic tests:
- a) positive hemicycles
- b) negative hemicycles

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positive hemicycle at the same ductility level. Nonetheless, the arguments posed in favour of the F.C. substructure for what strength and ductility are concerned, render the I.P.C. substructure behaviour quite satisfactory. Also for the aforementioned specimen, the reaction force values predicted by Eurocode 4 (1992) and corresponding to the maximum bending capacity at Sec. 4, can be compared to the measured values by means of Fig. 4b. Even for this test, Eurocode 4 (1992) appears of limited applicability. Anew, if the substructure is assumed to be part of a framing system with an interstorey height $h$ equal to 3.5 m, its hysteretic behaviour appears to be satisfactory with respect to the interstorey drift limits (NEHRP, 1994).

The hysteresis loops relevant to the L.P.C. substructure are reported in Fig. 4c. The substructure exhibits stable hysteresis cycles. However, it must be pointed out that the L.P.C. substructures own better material properties compared to the previous companion substructures. Also in this specimen, web and flange buckling revealed at the first hemicycle with $e/e_y$ equal to 4 and with akin characteristics of those of the companion F.C. specimen, even though the section was classified as Class 3 (Eurocode 4, 1992). Moreover, the concrete deck did not fracture whilst the failure was governed by lateral torsional buckling. In spite of Eurocode 4 cross-section classification, the L.P.C. substructure is able to develop its inherent plastic strength, whilst ductility properties allow the interstorey drift limits to be fulfilled without strength loss. Even in these conditions, the reaction force value predicted by Eurocode 4 (1992) and corresponding to the maximum bending resistance of Sec. 4 can be compared to the experimental values shown in Fig. 4c. For this particular test, Eurocode 4 strength prediction appears to be adequate for the reasons explained in the following.

Insofar as section capacity calculations are concerned, the limited applicability of Eurocode 4 (1992) can be attributed to different reasons. On the one hand, the actual strength of substructures deteriorates, among many other reasons, through cyclic energy release processes which result from concrete crushing caused by longitudinal compressive stresses as well as tensile cracking. As a result, the bounding between the concrete and the connectors is gradually reduced and loss of stiffness and strength is accumulated. Thereby, an unsafe overestimation of strength has to expected by the application of a code which was calibrated for a static regime. On the other hand, it is evident that the stud shear connectors may not reach the ultimate strength at the plastic failure load, as directly implied by code calculations (Eurocode 4, 1992). As a result, a satisfactory comparison between test and code prediction is expected only in those cases, like the L.P.C. substructure one, in which all connectors tend to reach their own ultimate strength capacity.

For brevity, only the reaction force versus the averaged concrete deck-top flange interface slip of the L.P.C. substructure detected at Sec. 3 (see Fig. 1) is illustrated in Fig. 5. The low degree $F_s/F_{ct}$ equal to 0.41 of shear connection, determines a significant amount of interface slip developed (~5.37 mm ± 9.89 mm). It has to be noticed that the aforementioned slip range is greater than the one observed in the corresponding pull-push tests (Bursi and Ballerini, 1997). Moreover, as expected, lower range of slips (~0.51 mm ± 0.47 mm) and (~0.84 mm ± 1.50 mm) were detected at the same section for the F.C. substructure and I.P.C substructure, respectively. As mentioned in Sec. 2.1, the vertical separation was monitored for the L.P.C. substructure only. At Sec. 3, a maximum vertical separation of 2.3 mm, about 23 per cent of the corresponding horizontal slip was observed.

Table 1 collects the overall stiffness, strength and ductility parameters for the F.C., I.P.C. and L.P.C. substructures, respectively. The aforementioned parameters were reckoned by means of the bi- and trilinear approximations of the skeleton curves, as schematically highlighted in Fig. 2. Within the F.C. and I.P.C. substructures one can readily observe the good performance of the I.P.C. substructure in term of strength and displacement ductility if compared to the F.C. substructure. As a result, if 70 per cent of the required shear connection degree is provided to the substructure, fully composite capacity can be achieved. Inevitably, local buckling in the hogging moment regime mitigated the performances of both substructures in term of strength. Nonetheless, ductility properties were more than satisfactory. As far as the properties of the L.P.C. substructure are concerned, the good strength characteristics mainly depend on the increased material properties both of rebars and of the steel beam. However, the limited reduction of stiffness properties combined with the maximum $e_{max}/e_y$ and ultimate $e_{max}/e_y$ displacement ductility values confirm the good performances of the L.P.C. substructure in a cyclic regime.
Figure 7: N69W component of 1952 Taft earthquake:  
a) ground acceleration; b) Fourier amplitude

Figure 8: N69W component of 1952 Taft earthquake:  
a) acceleration spectrum; b) displacement spectrum

Figure 9: Pseudo-dynamic hysteresis loops of reaction force vs. horizontal displacement of substructures:  
a) full shear connection; b) intermediate partial shear connection; c) low partial shear connection
A meaningful comparison among the specimens is obtained by plotting the mean relative absorbed energy per cycle versus the partial ductility $e/e_y$, as suggested by the ECCS Recommendations (1986). Hemicycles are not symmetrical and thereby, both the positive, see Fig. 6a, and the negative force regime, see Fig. 6b, are kept separated. From the plots one can observe the best performance of the I.P.C. substructure exhibited for positive hemicycles, owing to the dissipation mechanisms developed in the concrete deck. However, at displacement ductilities greater than 4, well beyond the interstorey drift limit of 2.5 per cent, the F.C. substructure exhibits a better behaviour. As far as the L.P.C. substructure is concerned, the absorbed energy ratio versus the positive partial ductility is lower compared to the other substructures owing to the limited dissipation in the concrete deck. Indeed, in the sagging moment regime the dissipation was caused by the steel beam and the stud shear connectors. For negative hemicycles, the absorbed energy ratios are similar for all substructures, mainly because, after concrete cracking, the dissipation behaviour is governed by the steel section yielding as well as stud shearing.

3.2 Pseudo-dynamic tests

Main results provided by the three companion substructures subjected to pseudo-dynamic tests are analysed and commented upon in this section. The N69W component of the 1952 Taft earthquake as well as the relevant acceleration Fourier amplitude are illustrated in Fig. 7a and 7b, respectively. Moreover, in Fig. 7b, the elastic and dominant inelastic fundamental period of the F.C. substructures are reported as reference. Since substructures increase their fundamental period owing to their inelastic behaviour, several frequency component of the accelerogram excite the specimens. Both the acceleration and the displacement spectrum of the aforementioned accelerogram corresponding to a damping ratio of about 4 per cent, are reported in Fig. 8a and 8b, respectively. Such a value represents the mean equivalent viscous damping inherent in the F.C. and I.P.C. substructure including the friction in the set-up. It was measured in free vibration pseudo-dynamic tests in a linear elastic range. The choice of the N69W accelerogram enabled displacements to failure to be imposed to the substructures. The peak ground acceleration was stepped-up from 0.1g corresponding to a presumably linear elastic substructure behaviour to 2.0g which determined the ultimate limit state in all substructures. Thereby, both cyclic and pseudo-dynamic test results were compared in a straightforward manner and the effectiveness of the aforementioned test procedures was checked. Moreover, during the pseudo-dynamic tests a concentrated mass equal to 0.013235 kNsec$^2$/mm was adopted at the actuator end.

The reaction force-displacement responses corresponding to a peak ground acceleration of 2.0g are plotted in Fig. 9a, 9b and 9c for the F.C., I.P.C. and L.P.C. substructure, respectively. One can observe that the energy absorption capabilities of specimens under pull loading, i.e. positive displacements, were not exploited fully. In contrast, in the pushing regime web and flange buckling at 250 mm from the column flange beyond the reinforcing plates determined substructure failure, and reduced the energy consumption characteristics of specimens. Nonetheless, whilst the F.C. substructure was able to limit the maximum displacement in the negative displacement range, see Fig. 9a, both the I.P.C. and the L.P.C. substructure experienced displacements beyond the maximum displacement capacity of the actuator (-250 mm). The characteristics of local buckling were very akin to those observed in the relevant quasi-static cyclic tests. As far as serviceability limit states are concerned, with reference to interstorey drift limits corresponding to 2.0 as well as to 2.5 per cent (NEHRP, 1994), a satisfactory behaviour of specimens can be observed.

The mean reaction forces predicted by Eurocode 4 (1992) without partial safety factors are indicated by means of horizontal hatched lines in Fig. 9. They correspond to the maximum resistance of composite substructures both for the sagging and the hogging moment regime. From the perspective of strength design, one can notice that these predictions are not applicable because random-amplitude displacements govern the structural response. In pseudo-dynamic tests, indeed, the maximum strength exhibited by the specimens was dictated by random displacements and, as a result, substructures experienced early large plastic excursions in the pulling regime. Thereby, a reduction of the yield stress of the steel section upon reloading was caused by the Bauschinger effect. The aforementioned phenomenon clarifies the limited accuracy of Eurocode 4 (1992). The reaction force of the L.P.C. substructure versus the averaged slip measurements detected at Sec. 3 (see Fig. 1) is illustrated in Fig. 10. It is apparent that L.P.C. substructure has experienced less concrete deck-top
TABLE 2
PARAMETERS OF THE HYSTERETIC RESPONSE APPROXIMATION FOR PSEUDO-DYNAMIC TESTS

<table>
<thead>
<tr>
<th>PULL</th>
<th>$K_*^+$ (kN/mm)</th>
<th>$K_*^-$ (kN/mm)</th>
<th>$F_*^+$ (kN)</th>
<th>$F_*^-$ (kN)</th>
<th>$F_{max}^+/F_*^+$</th>
<th>$e^+_y$ (mm)</th>
<th>$e_{max}^+$ (mm)</th>
<th>$e_y^+$ (mm)</th>
<th>$e_{max}^+ / e_*^+$</th>
<th>$e_*^+/e_y^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F. C.</td>
<td>14.2</td>
<td>2.4</td>
<td>195.9</td>
<td>253.2</td>
<td>1.2</td>
<td>17.8</td>
<td>41.6</td>
<td>42.0</td>
<td>2.3</td>
<td>2.4</td>
</tr>
<tr>
<td>L.P.C.</td>
<td>13.5</td>
<td>2.8</td>
<td>192.4</td>
<td>262.8</td>
<td>1.2</td>
<td>19.4</td>
<td>39.2</td>
<td>39.2</td>
<td>2.0</td>
<td>2.0</td>
</tr>
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<td>12.8</td>
<td>1.1</td>
<td>191.0</td>
<td>255.6</td>
<td>1.2</td>
<td>20.0</td>
<td>63.6</td>
<td>63.6</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>PUSH</td>
<td>$K_*^+$ (kN/mm)</td>
<td>$K_*^-$ (kN/mm)</td>
<td>$F_*^+$ (kN)</td>
<td>$F_*^-$ (kN)</td>
<td>$F_{max}^+/F_*^+$</td>
<td>$e^+_y$ (mm)</td>
<td>$e_{max}^+$ (mm)</td>
<td>$e_y^+$ (mm)</td>
<td>$e_{max}^+ / e_*^+$</td>
<td>$e_*^+/e_y^+$</td>
</tr>
<tr>
<td>F. C.</td>
<td>11.3</td>
<td>2.2</td>
<td>97.9</td>
<td>202.9</td>
<td>1.3</td>
<td>17.9</td>
<td>51.1</td>
<td>154.5</td>
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<td>8.6</td>
</tr>
<tr>
<td>L.P.C.</td>
<td>11.5</td>
<td>1.5</td>
<td>98.2</td>
<td>203.4</td>
<td>1.2</td>
<td>17.6</td>
<td>59.6</td>
<td>137.5</td>
<td>3.4</td>
<td>7.8</td>
</tr>
<tr>
<td>L.P.C.</td>
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<td>2.5</td>
<td>117.0</td>
<td>191.4</td>
<td>1.2</td>
<td>17.4</td>
<td>33.0</td>
<td>150.2</td>
<td>1.9</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Figure 9: continued

Figure 10: Reaction force vs. slip of 16 mm headed stud shear connectors

Figure 11: Absorbed energy vs. full ductility for pseudo-dynamic tests (2.0 g p.g.a.):
a) positive hemicycles; b) negative hemicycles
flange interface slip (-0.62 mm ÷ 3.37 mm) when compared to the interface slip developed in the companion quasi-static cyclic test, see Fig. 5. In addition, the aforementioned interface slip range is higher than the corresponding slip range (-0.25 - 0.19 mm) of the I.P.C. specimen. But, surprisingly, the I.P.C. specimen has a slip range lower than the one (-0.82 mm ÷ 0.78 mm) experienced by the F.C. substructure. Clearly, the large flexibility exhibited by the reduced number of stud shear connectors both in I.P.C. and L.P.C. substructures mitigates the shear flow at the concrete deck-top flange interface. As a result, the interaction and internal force distribution is modified. Also for this test series, the vertical separation was monitored for the L.P.C. substructure only. The maximum separation at Sec. 3 was 0.83 mm, i.e. about 25 per cent of the corresponding horizontal slip. In detail, the vertical separation required by the random excitation was lower than that observed in the relevant cyclic test, though in percentage, it was higher than the corresponding horizontal slip.

Even for pseudo-dynamic test data, the overall response properties were assessed by means of bi- and trilinear approximations of the skeleton curves as illustrated in Fig. 2. From Table 2 one can observe that both the I.P.C. and the L.P.C. substructure can offer, in terms of strength, performances similar to the one provided from the F.C. substructure. As far as ductility properties are concerned, substructures with partial shear connection offer good performances in terms of maximum $\epsilon_{max}/\epsilon_y$ and ultimate $\epsilon_{max}/\epsilon_y$ displacement ductilities. Moreover to a large extent, the resources of substructures were not exploited under sagging moment whilst ductility demands were higher in the hogging moment regime, in which composite beams were susceptible to local buckling.

Due to the random nature of displacement cycles in a pseudo-dynamic test, it is unfeasible to define the absorbed energy ratio as a function of the partial ductility. Thereby, owing to the fact that all substructures embody similar geometrical characteristics they were compared in terms of absorbed energy versus full ductility (ECCS, 1986). The corresponding relationships are plotted in Fig. 11a for the positive force (pull) regime and in Fig. 11b for the negative (push) one, respectively. First, one can observe the different displacement ductility levels required to the substructures by the random excitation. In addition, one can notice in Fig. 11a the good performance exhibited by the L.P.C. substructure at significant full ductility levels. Indeed, in the negative force regime, see Fig. 11b, all substructures show similar performances. As a result, the energy consumption properties among the substructures are rather similar though the degree of shear connection is different.

4. CONCLUSIONS

The general shortcomings associated with the use of composite beams in earthquake prone zones, i.e. the severe strength deterioration of the partial shear connection as well as the large susceptibility to buckling in hogging moment regions are not confirmed by these tests. As a matter of fact, quasi-static cyclic test results on composite substructures with full and partial shear connection verify that composite beams with partial connection can perform satisfactorily in terms of strength, ductility and energy consumption capability similar to the companion full shear connection beams. A similar behaviour can be expected under severe earthquakes as proven by pseudo-dynamic test results. However, the aforementioned performances require a strict control of the amount and quality of reinforcements as well as of the class of steel sections located in severe hogging moment regions. This control entails a ductile plastic behaviour of critical composite sections. Moreover, stud shear connectors have to be designed to exhibit a ductile behaviour whilst splitting and concrete pull-out have to be prevented.

Finally, predictions based on codes calibrated upon monotonic loading overestimate the actual strength capacity of composite beams and, thereby, appear to be inadequate when large alternate displacements govern the structural response. Moreover, no indications about the actual ductilities of member sections are available in seismic codes. The aforementioned inaccuracies combined with lack of information may hamper the use of the capacity design in composite beams and may represent a major obstacle in the widespread use of composite systems in earthquake prone zones.
5. ACKNOWLEDGEMENTS

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6. REFERENCES


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PSEUDO-DYNAMIC TESTS AND ANALYSIS ON SEMI-RIGIDLY JOINTED STEEL FRAMES

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ABSTRACT

This paper presents the results from monotonic loading tests, cyclic loading tests and pseudo-dynamic tests on semi-rigidly jointed steel frames. There are two types of test specimen used as semi-rigid connections: split-tee type and angle type. In the pseudo-dynamic tests, the applicability of sub-structuring techniques to the earthquake response simulation on semi-rigidly jointed 2-story steel frames is demonstrated, and the influence of pinching effects in the restoring force characteristics on the global response of semi-rigidly jointed steel frames is discussed.

KEYWORDS

semi-rigid connection, monotonic loading test, cyclic loading test, earthquake response, sub-structuring technique, pinching effect, steel frame, brace

1. INTRODUCTION

Welded connections are widely used in beam-to-column connection of steel frame as rigid connections. But some diaphragms should be welded to the joints to obtain sufficient rigidity and strength, and it is observed in the recent earthquake damage, that the strain concentration in the vicinity of the weld may cause the fracture when loaded by severe earthquakes. Instead of such a welded rigid connection, another details semi-rigidly connected by cleats and mechanical fasteners are sometimes used in European and American countries. In Japan, however, these types of semi-rigid connection are not so popular except systematically prefabricated low-rise residential buildings, usually braced frames.

The reason is that the structural design of middle-rise unbraced frame is mainly controlled by drift limitation, and the usage of semi-rigid joints will make it more stringent. Even with this demerit, the fabrication error of members can be easily absorbed with such a detail, and the construction and quality controls become easier. Furthermore, there are various combinations of connection stiffness and strength available corresponding to various types of semi-rigid details, and then it is possible to control the collapse mode and the energy absorp-
tion capacity of frames to a severe earthquake by an appropriate use of semi-rigid connections. In addition, stiffness and strength of semi-rigidly jointed frames can be enhanced by adding earthquake resisting elements like braces to dissolve the demerit mentioned above.

In this study, two types of semi-rigidly jointed beam specimens are fabricated and tested: one is connected by split-tees and the other is connected by top, seat, and double web angles. Quasi-static loading tests are performed to identify the restoring characteristics including pinching effect, and pseudo-dynamic tests are carried out to demonstrate the applicability of sub-structuring technique to the earthquake response simulation on semi-rigid jointed 2-story steel frames. In this paper, the results of monotonic and cyclic loading tests as well as the earthquake response simulation on semi-rigidly jointed 2-story frame are presented.

2. BRIEF DESCRIPTION OF QUASI-STATIC LOADING TESTS

The setup for testing is shown in Fig.1, where a test specimen composed of a beam and a connection is loaded as a cantilever beam. The lower end of beam is jointed to base block through a semi-rigid joint, and the other end is the pinned end loaded by an actuator. Two types of joint details are used as follows:

(A) Split-tee type: Fig.2 shows the details of split-tee type connection. These split-tees are made of JIS steel grade SS400 and cut from rolled H-shaped section, H-150 × 150 × 7 × 10. Four high-strength bolts are used in each of web and flange of tee. Pretension in high-strength bolts is about 11.4 ton, and bolt-hole clearance is 2.0 mm.

(B) Angle type: Fig.3 shows the details of angle type connection. In this case, top and seat flange angles and double web angles are used, which are made of JIS steel grade SS400 rolled angle, L-75 × 75 × 9 × 8.5. As for the mechanical fasteners, the same high-strength bolts are used in the same conditions with the split-tee type.
Each type of joint detail is used both in monotonic and cyclic loading tests. The beams to be connected are commonly made of JIS steel grade SS400 rolled H-shaped section, H-250 × 125 × 6 × 9.

3. RESULTS OF LOADING TESTS

The material properties obtained from material tests are shown in Tab.1. The collapse mechanical models of split-tee connection and angle connection are shown in Fig.4, and the calculation results of ultimate strength are also shown in Tab.1. The inelastic behaviors of split-tee type and angle type observed in the monotonic loading tests are shown in Fig.5. The vertical axis represents for the ratio of moment of beam end to fully-plastic moment of beam, while the horizontal axis represents the rotation angle of beam including rotation of joint. Initial slippage of bolted joints occurs in the early stage of plastic range. In the case of angle type, the
restoring characteristics are similar to a bilinear curve, and in the case of split-tee type, tangent stiffness after yielding decreases slightly when the specimen comes close to the ultimate state. The slip coefficient measured from the tests is around 0.39.

Fig. 6 shows hysteresis behaviors observed in the cyclic loading tests. As for the initial slip loads, they almost agree with the slip loads measured from the monotonic loading tests. The end moment, \( M/M_p \), is kept less than 0.8 during all the tests, and the beams stay in elastic range. The inelastic energy absorption is done by split-tees and angles completely, and deformation concentrates at the semi-rigid joint. The split-tee type has larger yield strength and less stringent pinching effect than those of the angle type. The relationship between slip resistance and loading cycle is shown in Fig. 7, where the vertical axis is the value of \( M_s/M_{so} \). \( M_s \) denotes the slip resistance observed at each loading cycle, and \( M_{so} \) denotes that of monotonic loading test. In the case of split-tee type, the level of slip resistance gradually decreases, according to the loading cycles and rotation amplitudes, to 60% of initial slip resistance finally.

Fig. 8 shows the rotation range when pinching is observed. The ratio of the pinching range to the whole rotation amplitude, \( L_s/A \), are plotted to the ratio, \( A/\bar{A} \), where \( A \) denotes rotation amplitude at each loading cycle and \( \bar{A} \) denotes the pinching range corresponding to bolt-hole clearance. The relationship shown in the figure looks like approximately linear, and then the pinching range \( L_s \) can be expressed by a quadratic function of the rotation amplitude \( A \).

4. BRIEF DESCRIPTION OF EARTHQUAKE RESPONSE TESTS

The models for testing are shown in Fig. 9, model B is different from model A by adding fictitious braces as earth-
quake resisting elements, the 2-story moment frames are composed by using split-tee or angle type semi-rigid joints. Four earthquake response tests are performed, each model with split-tee type connections and angle type connections. In these tests, sub-structuring pseudo-dynamic test techniques are applied, and the columns are assumed as elastic elements and simulated in computer as fictitious structures. The beams and their connections are extracted for loading tests performed in parallel with analysis. In the case of model B, the fictitious braces are simulated by using slip model. The contribution of these braces in strength is taken as 0.5, which is the ratio of strength of braces to that of the whole system.

The testing system is shown in Fig. 10. Being as loading apparatus, the actuators and the controller are connected to the computer through two kinds of interface boards (Analog to Digital and Digital to Analog). In the test, the value of load is read from the load cell attached to the actuator and feed back to the computer system as beam restoring force so that the hybrid response analysis can be performed on a whole structural system including fictitious elements. The beam specimens and the semi-rigid connection details (split-tee or angle type) are the same as in the quasi-static loading tests. The moment of inertia of fictitious column is assumed to 2.8 times of beam specimen \( I_e = 10125 \text{cm}^4 \) and mass of each story is assumed to be 15.0/980 ton·cm\(^{-1}\)·sec\(^2\) and concentrated at each node. The average value of initial elastic stiffness of split-tee type and angle type, both measured from the quasi-static loading tests, is around 850 ton·m/rad. The fundamental natural period of the unbraced frame becomes 1.0 sec if based on this average stiffness value. The natural periods based on actual stiffness \( K \) of each details are:

(a) Split-tee type, \( K = 1000 \text{ton·m/rad} \)

Model A: \( T_l = 0.94 \text{sec} \), Model B: \( T_l = 0.54 \text{sec} \)

(b) Angle type, \( K = 720 \text{ton·m/rad} \)

Model A: \( T_l = 1.08 \text{sec} \), Model B: \( T_l = 0.66 \text{sec} \)

5. EQUATION OF MOTION

The equation of motion and the attendant equilibrium equation of 2-story moment frame can be formed as

\[
[M]\ddot{X} + [K]X + [R] = -[M]y
\]

\[
[K_1]\ddot{X} + [K_1][\Delta \theta] + [\Delta M_i] = 0
\]

in which \( X_i \) = displacement of \( i \) story; \( \theta_i \) = beam end rotation angle including deformation of a beam; \( R_i \) = restoring force of braces of \( i \) story, but in the case of model A, \( R_i = 0 \); \( Mb_i \) = beam end moment of \( i \) story; \( Mu_i \) = unbalance moment of \( i \) joint in the last step that is calculated from actual moment of beam end measured by loading test. The Central Difference Method is utilized for numerical integration of the response analysis. In this testing, when loading of \( n \) step is completed and put forward to \( (n+1) \) step, \( \{x\}_n \) can be calculated from equation (1) while \( \{Mb\}_n+1 \) is necessary for calculating \( \{\theta\}_n+1 \) from equation (2). Nevertheless, it is impossible to proceed loading because \( \{\theta\}_n+1 \) is unknown. Accordingly, the relationship between \( \{\Delta Mb\} \) and \( \{\Delta \theta\} \) need to be predicted. Here, a bilinear model is adopted to predict the restoring of specimen. Unbalance moment due to this prediction error will be dissolved in the next step as shown in equation (2). The NS component recorded at El Centro in 1940 has been used as input earthquake wave, where duration is 10 sec and the input level was magnified to 550 gal.

6. RESULTS OF EARTHQUAKE RESPONSE TESTS

Fig.11 and Fig.12 shows the time histories of displacement, and the results of completely numerical analysis are also plotted in dotted line. In the analysis, a skeleton-shift hysteresis model was used for simulating the
Model A: Unbraced frame

Model B: Braced frame

Fig. 11 Time histories of simulated responses (Split-tee type)

Model A: Unbraced frame

Model B: Braced frame

Fig. 12 Time histories of simulated responses (Angle type)
hysteresis behavior of semi-rigidly connected beam. The parameters are assigned based on the results of quasi-static loading tests, and the influence of pinching of restoring characteristic is not taken into account in the completely numerical analysis. In the case of split-tee type, the maximum displacement response of completely numerical analysis is 20% smaller than the tests. In the case of angle type, the value is over 40% and the unbraced moment frame (model A) in the tests begins to collapse at around 4.5 sec, it looks much different from completely numerical analysis because a significant pinching effect occurs in the hybrid test. In the tests of braced frame (model B), the braces are useful for resisting earthquake loading, and the displacement responses turn to half of the case of unbraced frame.
Fig. 15 Hysteresis loops of model B (Split-tee type)

Fig. 16 Hysteresis loops of model B (Angle type)

Fig. 17 Nodal moment imbalance observed during hybrid test (Split-tee type)
The hysteresis behaviors of joints including beam deformation are shown in Fig. 13 through 16. Slippage of bolt frequently occurs at the stage of loading level is 60% of initial slip resistance, and pinching loop has been formed. The earthquake energy is completely absorbed by deformation of connections in the case of model A. In the case of model B, the braces absorb 30 ~ 40% of whole earthquake energy. The measured restoring characteristics look much different from the bilinear model used as predictor. But the unbalance moment at the node caused by error of prediction is stable as shown in Fig. 17 and 18, and it is kept within a small value except at the moment of bolt slippage.

7. CONCLUDING REMARKS

The following conclusions are drawn from the simulation:

(1) Sub-structuring technique in pseudo-dynamic testing is useful to simulate earthquake response of structural system affected by the local non-linear behaviors like semi-rigid joints.

(2) The local pinching effect at the semi-rigid joints sometimes affects considerably on the global response of the frame, and then it shall be considered properly in the mathematical modeling.

(3) With the presence of moderate non-linearity induced by the semi-rigid joints tested herein, a hybrid test can be performed successfully even with a simple bilinear predictor for unknown specimen resistance, as long as an effective corrective algorithm is employed to remove the moment imbalance at the nodes.

(4) The brace as earthquake resisting element in the semi-rigidly jointed steel frame is useful to absorb earthquake energy and reduce the displacement response of frame.

8. REFERENCES


RECENT ACHIEVEMENTS IN SUBSTRUCTURING ON-LINE PSEUDODYNAMIC TESTS AT IIS

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ABSTRACT

Structural response tests are necessitated to confirm performances of structures based on a design method. The shaking table test is suitable for verifying overall response behavior of the structures, while the on-line or the pseudodynamic test is a powerful tool for large scaled structural model tests. The on-line test is much more utilized by combining the substructuring method in the numerical analysis with the test procedure. The substructuring on-line test technique has high potential particularly to obtain response behavior of buildings with structurally complex frameworks. This paper describes the basic scheme of the substructuring on-line test and recent achievements at Institute of Industrial Science (IIS), University of Tokyo.

KEYWORDS

Earthquake response,On-line test,Pseudodynamic,Substructuring,Shaking table test, Steel frame, Semi-rigid connection,Loading speed

1. INTRODUCTION

1. 1 Brief Description of On-line Pseudodynamic Test

The basic concept of the on-line pseudodynamic test is illustrated in Figure 1. The scheme of the on-line test comprises two phases: The response displacements of the structural system is first computed by the numerical
analysis, and then the actuators are driven to enforce the exact displacements computed above to the test structure for obtaining the current restoring forces, Takanashi et al. (1987).

The on-line pseudodynamic test is superior in several respects to the shaking table test. The most significant advantage of the on-line test is that the loading of the on-line test can be quasistatic and can be halted at any time upon request, allowing close monitoring of detailed local behavior of the tested structure. Moreover, large-scale or even full-scale test structures can be tested, because less actuator capacity is required. In addition, conventional measuring devices used widely in quasistatic tests are sufficient in the on-line tests.

Here taken is a response simulation of a moment resistant frame in which the beams are connected by bolted connections as shown in Figure 2. This is an appropriate example which meets the advantage of the on-line test, because the moment-rotation relationship of the bolted connection is hardly expressed in a simple analytical model, but such a model is not required in the simulation utilizing the on-line pseudodynamic test. In case we need an analytical model like one shown in Figure 3, its adequacy can be verified by the simulation results obtained by the on-line test. The verification was carried out on the timehistory of the response displacement as shown in Figure 4, Takanashi et al. (1981) and Takanashi et al. (1988).
1.2 Fast On-line Test

The on-line test, however, is a static test that cannot examine whether or not the effect of the loading rate affects the material properties of the test structures and accordingly the response is affected. This is a major disadvantage of the on-line test. In order to incorporate more directly the effect of the loading rate into the on-line test, the fast on-line test technique was developed. Two modifications are included in algorithms. First, a dynamic actuator is used instead of the quasistatic actuator so that the test structure can receive dynamic loading. The servo controller is designed so that the velocity of the actuator can be adjusted in accordance with signals sent from the computer. Second, the loading is assigned to be continuous. In contrast to the early stage of the on-line tests, the actuator never pauses even when the test structure arrives at the target displacement in the modified algorithm. The restoring force is measured at the instant when the test structure passes the target displacement, and the structure is further loaded in the same direction. Consequently, a wrong movement occurs when load reversal occurs. This occasional wrong motion, referred to as overshoot, was found to have a secondary effect in usual elastic-plastic response analyses. Precise calculation in the computer and accurate control in the servo system of actuator driving can reduce inaccuracy in the response owing to recent development in the computer and the test machine. To calibrate the effectiveness of the fast on-line test, the dynamic response of a one-story steel frame model shown in Figure 5 was investigated. As shown in Figure (a), the model has two pin-supported rigid columns and a flexible beam. Only the beam part was tested as shown in Figure (b). Using the analogy in the support and loading conditions between the analyzed model and

<table>
<thead>
<tr>
<th>Test</th>
<th>Mass (tonf sec⁻²/cm)</th>
<th>Elastic stiffness (tonf/cm)</th>
<th>Natural period (sec)</th>
<th>Maximum acceleration (cm/sec⁻²)</th>
<th>Time scale</th>
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<td>0.5</td>
<td>320</td>
<td>24.0</td>
</tr>
<tr>
<td>Dynamic test</td>
<td>4.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Time scale=(nominal velocity)/(assigned velocity)

(a) Analytical model  
(b) Test specimen

Figure 5: Simulation model and test specimen
the test beam, the restoring force of the model is estimated. The structural properties of the model are listed in Table 1. Two tests were carried out in different loading rates. In the first test (quasistatic test), the actuator velocity was adjusted to 1/24 of the original velocity measured in the actual ground motion, whereas in the second test (dynamic test) the velocity was set to 1/4.4 of the original. This ratio of 1/4.4 was selected based on the computation speed and the actuator speed capacity. The results including the story shear force as in a time history and the story shear force versus displacement relationship are compared in Figure 6. The correlation between the two response is fairly good, suggesting the fast on-line test algorithm is effective, Takanashi et al (1984). Unfortunately, the real response speed is not attained until now, because of insufficient speed performance of both the data processing and the actuator controlling. The high speed actuator newly purchased at IIS can provide us with possibility of the real time on-line test. Some trials are now underway.

1.3 Substructuring On-line Pseudodynamic Test

There is a limitation in the number of displacements to be controlled, because of the limited numbers of the actuators. Therefore, structural configurations suitable for this earthquake simulation are confined to discrete mass systems such as office typed building frames with stiff concrete slabs. There, the masses of the structural systems are concentrated at the floor levels and the response displacements controlled in the simulation are directly associated with the responses of the masses. In the case that the structural systems, which responses are simulated, become complex, and the structural test capability is limited, we often remove out a part of the structure from the whole for the simulation test. The part removed out may in most cases be a subassemblage which restoring force characteristics is hardly expressed in a simple analytical model and it is much easier to evaluate it in the structural test. The structural tests must be conducted as a part of the response simulation of the whole structure, where the controlled displacements are not always associated with the responses of the masses. This is called the substructuring on-line test, Dermizakis et al (1985).

As illustrated in Figure 1, the discretized equations of motion must be solved in a numerical manner. The equations are written in a matrix form as;

\[
\begin{bmatrix} m \end{bmatrix} \ddot{\mathbf{x}}_i + \begin{bmatrix} c \end{bmatrix} \dot{\mathbf{x}}_i + \begin{bmatrix} \mathbf{F} \end{bmatrix}_i = -\begin{bmatrix} m \end{bmatrix} \ddot{\mathbf{l}}_{gi}
\]

(1)
where \( x, m, c, F, \) and \( \ddot{x} \) are displacement, mass, viscous damping, restoring force, and ground acceleration quantities; the dot(·) denotes differentiation; \( i \) indicates that the equation belongs to the \( i \)-th step operation. The displacement is obtained by the central difference method with \( \Delta t \) as the integration time interval;

\[
\{x\}_{i+1} = \frac{\left[ m \right] \{x\}_i + \left[ c \right] \cdot \Delta t/2 - \left[ m \right] \{x\}_{i-1} - (\Delta t)^2 \left( \{F\} + [m] \{1\} \ddot{x}_g \right)}{\left[ m \right] + \left[ c \right] \cdot \Delta t/2}
\]

(2)

From this expression, one understands that the displacement \( x_{i+1} \) in the next load step can be estimated only by the known quantities until the \( i \)-th step. The displacement \( x_{i+1} \) is sent to the controller of the actuators to find out other quantities at the \( (i+1) \)-th step.

The substructuring on-line tests can be classified into two groups at this stage as the parallel system and the series system: In the parallel system of the substructuring on-line test, the calculated displacements at each step can be directly used to drive the actuators, in spite that accurate displacement control in the test requires some techniques and special schemes, particularly in multi-mass structural systems. The restoring forces at the step are estimated without modification and adjustment from the measured values at the actuators. In the series system of the substructuring on-line test, however, the calculated displacements cannot directly be used for driving actuators. The displacements must be further split into displacements and/or rotations which are imposed to the substructures by the actuators, because of complex structural system. At this moment, however, one don’t know how to split the calculated displacements into the individual substructures, as the tests include plastic tests in addition to elastic tests and the iteration procedure, that is, trial loading and then adjusting is not allowed in the irreversible plastic process. Therefore, we must predict the splitting way. We proposed some prediction methods described later. After the loading, we must compensate the difference between the predicted value and the actual value at the following step. Several test results obtained by the test procedures are described.

2. SOME RESULTS OF SUBSTRUCTURING ON-LINE TESTS

2.1 A Column of a Tall Building (Parallel System)

Columns in lower stories of a tall building are subjected to high axial forces and bi-directional bendings during earthquake vibration. To simulate the response behavior, a simple structural model as in Figure 7 is taken. The test structure is a column specimen which is installed in the test apparatus and exposed to three-dimensional displacements which are accurately controlled according to the required values sent from the computer. The test set up is schematically shown in Figure 8. Typical simulation results are shown in Figure 9(left figures), comparing with the pure numerical results(right figures). In the simulation the waveforms of the N-S component and the E-W component recorded at 1940 El Centro Earthquake were used for the input ground motions in the x-direction and in the y-direction, respectively. The results show to be satisfactory for the simulation use, Chen et al (1993). The displacements calculated in the computer can be directly transmitted to the actuator controller, measuring the reaction forces for the present restoring forces in this simple simulation model.
2.2 Industrial Buildings (Parallel System)

Most industrial buildings have structural configurations different from ordinary office buildings. Roofs are required to cover wide space and their frameworks have irregular shapes due to arrangement of manufacturing layout. Floors are also irregularly shaped, strength and stiffness of which vary in wide range due to the weight of machines and wide opening. Frames are moment resistant and braced in some portions by the X-typed or the K-typed bracing systems. To simulate earthquake response, a structural model shown in Figure 10 is analyzed. The model is presumably composed of four planar frames and three shear floors, and two of the frames are reinforced by X-typed braces. Here, the braced frames are separated into the moment resistant frames and pairs of braces. The simulation parameters are the layout of braced frames and the stiffness of the shear transmission floors. Asymmetrical layout of the braced frames may cause twisting of the floors and high stiffness of the floors may accelerate the twisting. The test was conducted only on the braces. Four braces at most were loaded.
Figure 10: Simulation model of a power plant building

Figure 11: Story shear force of a braced frame

Figure 12: Displacements of frames
simultaneously, following the command on pushing displacements and pulling displacements prescribed by the computed horizontal response displacements at the floor level.

The displacement meters are attached to the brace specimens to measure the axial displacements. The reaction forces sensed by the load cells at the heads of the actuators are converted into the restoring forces of the braced frames. As an example, the results of a simulation is described here. In this simulation, the braced frames are allotted at FRAME 1 and 4 in Figure 10. The floor is flexible and its stiffness is assumed the same as the stiffness of the braced frame. Among the hysteresis loops in Figure 11, only the hysteresis denoted as BRACE is the result of the push-pull test. The hysteresis denoted as MRF followed the assumed analytical model, and the hysteresis denoted as BRACED FRAME is the sum of the BRACE's and MRF's. It is shown that the test results are satisfactorily taken into the simulation and combined with the analytical model. The distributions of frame displacements at the floor level are in Figure 12. The distributions show the displacements at the instant when the displacement of FRAME 4 took the maximum. The solid circles, squares and triangles in the figures show the braced frames. It clearly indicates that the rigid floor makes large twisting if the frames with higher strength were asymmetrically allocated in the plan,Taknashi et al (1996).

2.3 A Subassemblage of a Moment Resistant Frame (Series System)

Next simulation model is a subassemblage removed from a multi-span, multi-story moment resistant frame as shown in Figure 13. Only the first story column was taken as the test column, while the rest of members was assumed elastic during the simulation. It was also assumed that the mass at each story is lumped at the beam-to-column connection and it produces the inertial force horizontally but not the inertial moment (The mass at each story is set to 0.01037 tonf. sec²/cm). Then, the simulation scheme is explained with a T-frame at the first story (Figure 14). The top of the column is swayed with the response displacement, and at the same time is forced to rotated due to the bending of the beams. The response displacement at the (i+1) th step is calculated in the computer, but the rotation angle cannot be found in the computer calculation because the rotation response is disregarded in the simulation. The rotation is governed by the moment balance among the end moments of the beams and the column. And the end moments are dependent on the current state of the flexural rigidities of the members. The column behaves elastically and plastically, changing its flexural rigidity. We, however, cannot know it beforehand so that the increment in the rotation angle for next step loading must be predicted. But the predicted value is not usually correct. Consequently, a small amount of unbalanced moment remains around the connection after the step loading. The unbalanced moment must be vanished by the next step calculation and the loading upon the calculation results. To diminish the unbalanced moment, a proper prediction way must be found. There are some trials; use of the Multi-Spring Model which parameters are determined by the tests on similar structures, use of the well-educated Neural Network or use of a very simple bi-linear model. The effectiveness of each trial was already examined elsewhere, Zavala et al (1996). Figure 16 shows the story shear force versus the story displacement relation. Comparison with the result of the completely numerical simulation has been done. Good agreement was reported and accuracy in the simulation was confirmed. The prediction has done using the Multi-Spring Model there, Zavala et al (1995).

2.4 A Braced Steel Frame with Semi-Rigid Connections (Series System)
It is a key factor that structures can absorb the energy brought by earthquake ground motion in order to survive from severe earthquake. The energy absorption is achieved usually by plastic deformation. In this context the
best way in seismic design is to supply energy absorption devices. Semi-rigid connections made of the split-
etees and angles may be available for this purpose. Structural frames assembled by the semi-rigid connections
are not sufficient in stiffness against horizontal drift. These frames must be stiffened by allocating braces. Our
concern is to find the necessary brace strength which ensures the limited story drift and the ample energy
absorption at the semi-rigid connections as well. The substructuring on-line test was conducted for this study.

Figure 17: Simulation model of a braced MRF

![Simulation model of a braced MRF](image17)

Figure 18: Cantilever with semi-rigid connection

![Cantilever with semi-rigid connection](image18)

Figure 19: End moment versus end rotation relationship

![End moment versus end rotation relationship](image19)

The simulation model is a braced moment resistant frame in which beams are connected to an elastic column
with the semi-rigid connections as shown in Figure 17. The test done is a canti-lever test and the canti-lever
specimen is connected by the semi-rigid joint to evaluate the moment versus rotation relationship. The behavior
of the column and the braces was simulated by the analytical models.

The equation of motion and the associated equilibrium equation are

\[
[M]\{\ddot{x}\} + [K_s]\{x\} + [K_e]\{\theta\} = -[M]\{1\}\ddot{x}_g
\]

(3)

\[
[K_s]\{\Delta x\} + [K_e]\{\Delta \theta\} + \{\Delta M_b\} + \{M_u\} = \{0\}
\]

(4)
in which, \( \{x\} \) = story displacement, \( \{\theta\} \) = beam end rotation angle, \( \{M_\theta\} \) = beam and moment, \( \{M_s\} \) = unbalance moment at the beam-column connection found as the difference between the predicted before the loading and the measured after the loading. Equation 3 is the equation of motion on the elastic column in this simulation. The relationship between \( \Delta M_\theta \) and \( \Delta \theta \) in Equation 4 was predicted after a bilinear hysteretic rule. The typical simulation result is shown in Figure 19, comparing with the pure calculation results, Ohi et al. (1996).

3. CONCLUDING REMARKS

The earthquake response simulations achieved at IIS are outlined. A general scheme for the simulation and the particular scheme for each simulation are summarized. Recent simulations are gradually focussed on the substructuring on-line test, since structural systems to be treated become so complex that the tests on the total structures are almost impossible. Only a part of the structure is tested, while the response behavior of the remains must be covered by the computer analysis. It also meets the requirement from the budget. There are some problems unsolved yet. Particularly, the loading speed is considered to be essential after the Northridge and the Hyogoken-nanbu (Kobe) earthquakes. The fracture problem observed in the earthquakes might be dependent on the loading speed. The fracture must be examined by the tests on large-scaled on full-scaled specimens. The shaking table tests is not a suitable tool for these tests. The substructuring on-line test controlled in the real time may a possible means.

REFERENCES


DESIGN OF STEEL STRUCTURES WITH LRFD USING ADVANCED ANALYSIS

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ABSTRACT

This paper presents a practical advanced analysis method for steel frame design. The method can be used to assess realistically both strength and behavior of a structural system and its individual members in a direct manner. As a result, the method can be used for design without tedious separate member capacity checks, including the calculation of K-factor. The method incorporates the refined plastic-hinge concept for spread of plasticity together with practical modeling of geometric imperfections. The strengths predicted by the proposed method are then compared with those predicted by the exact plastic-zone analysis as well as by the conventional LRFD procedure. The displacement predictions are also compared with those of the plastic-zone solutions. A case study is given for an unbraced frame. The method is recommended for general use.

KEYWORDS
LRFD, steel design, advanced analysis, steel frame, geometric imperfection, stability function

1. INTRODUCTION

In the current AISC-LRFD Specification (1993), first-order elastic analysis or second-order elastic analysis is used to analyze a structural system. In using first-order elastic analysis, the first-order moment is amplified by B₁ and B₂ factors to account for second-order effects. In the Specification, the members are isolated from a structural system, and they are then designed by the member strength curves and interaction equations, which implicitly account for the effects of second-order, inelasticity, residual stresses, and geometric imperfections. In order to account for the influence of a structural system on the strength of individual members, the effective length factor is used. This design approach is marked in Fig. 1 as the indirect analysis and design method. The effective length method generally provides a good design of framed structures. However, several difficulties are associated with the use of the effective length method as follows:

1) The effective length approach cannot accurately account for the interaction between the structural system and its members. This is because the interaction in a large structural system is too complex to be represented by the simple effective length factor K. As a result, this method cannot accurately predict the actual strengths of its framed members.
(2) The effective length method cannot capture the inelastic redistributions of internal forces in a structural system, since the first-order elastic analysis with $B_1$ and $B_2$ factors accounts only for second-order effects but not the inelastic redistribution of internal forces. The effective length method provides a conservative estimation of the ultimate load-carrying capacity of a large structural system.

(3) The effective length method cannot predict the failure modes of a structural system subject to a given load. This is because the LRFD interaction equation does not provide any information about failure modes of a structural system at the factored loads.

(4) The effective length method is not user-friendly for a computer-based design.

(5) The effective length method requires a time-consuming process of separate member capacity checks involving the calculation of K-factors.

With the development of computer technology, two aspects, the stability of separate members, and the stability of the structure as a whole, can be treated rigorously for the determination of the maximum strength of the structures. This design approach is marked in Fig. 1 as the direct analysis and design method. The development of the direct approach to design is called "Advanced Analysis" or more specifically, "Second-Order Inelastic Analysis for Frame Design." In this direct approach, there is no need to compute the effective length factor, since separate member capacity checks encompassed by the specification equations are not required. The purpose of this paper is to present a practical, direct method of steel frame design, using advanced analysis, that will produce almost identical member sizes as those of the LRFD method.

Among various advanced analyses, including plastic-zone, quasi-plastic hinge, elastic-plastic hinge, notional-load plastic-hinge, and refined-plastic hinge methods, the refined plastic hinge method is recommended as a practical advanced analysis, since it retains the efficiency and simplicity of computation and accuracy for practical use. The method is developed by imposing simple modifications on conventional elastic-plastic hinge method.

The practical advanced analysis method described in this paper is limited to two-dimensional braced, and unbraced frames subject to static loads. The spatial behavior of frames is not considered, and lateral torsional buckling is assumed to be prevented by adequate lateral bracing. A compact W-section is assumed so sections can develop full plastic moment capacity without local buckling. The method may be considered an interim analysis/design procedure between the conventional LRFD method widely used now and a more rigorous advanced analysis/design method such as the plastic-zone method to be developed in the future for practical use.

![Figure 1: Analysis and design methods](image)
2. PRACTICAL ADVANCED ANALYSIS

2.1 Stability Function

To capture second-order effects, stability functions are recommended since they lead to large savings in modeling and solution efforts by using one or two elements per a member. The simplified stability functions reported by Chen and Lui (1992) or an alternative may be used.

2.2 Cross-section Plastic Strength

Based on the AISC-LRFD bilinear interaction equations (1993), the cross-section plastic strength may be adopted for both strong and weak-axis bending.

2.3 CRC Tangent Modulus

The CRC tangent modulus concept is employed to account for the gradual yielding effect due to residual stresses along the length of members under axial loads between two plastic hinges. In this concept, the elastic modulus $E$ instead of moment of inertia $I$ is reduced to account for the reduction of the elastic portion of the cross-section since the reduction of elastic modulus is easier to implement than that of moment of inertia for different sections. From Chen and Lui (1992), the CRC $E_i$ is written as:

$$E_i = 1.0E \quad \text{for } P \leq 0.5P_y$$  

$$E_i = 4\frac{P}{P_y^2} (1 - \frac{P}{P_y}) \quad \text{for } P > 0.5P_y$$  

(1a) 

(1b)

2.4 Parabolic Function

The tangent modulus model is suitable for $P/P_y > 0.5$, but it is not sufficient to represent the stiffness degradation for cases with small axial forces and large bending moments. A gradual stiffness degradation of plastic hinge is required to represent the distributed plasticity effects associated with bending actions. We shall introduce the hardening plastic hinge model to represent the gradual transition from elastic stiffness to zero stiffness associated with a fully developed plastic hinge. When the hardening plastic hinges are present at both ends of an element, the incremental force-displacement relationship may be expressed as (Liew, 1992):

$$\begin{bmatrix}
M_A \\
M_B \\
\dot{\rho}
\end{bmatrix} = \begin{bmatrix}
\eta_A [S_1 - S_2^2 \frac{S_2}{S_1} (1 - \eta_B)] & \eta_A \eta_B S_2 & 0 \\
\eta_A \eta_B s_2 & \eta_B [S_1 - S_2^2 \frac{S_2}{S_1} (1 - \eta_A)] & 0 \\
0 & 0 & A/l
\end{bmatrix} \begin{bmatrix}
\theta_A \\
\theta_B \\
\dot{\epsilon}
\end{bmatrix}$$  

(2)

where

$\dot{M}_A, \dot{M}_B, \dot{\rho} =$ incremental end moments and axial force, respectively

$S_1, S_2 =$ stability functions

$E_i =$ tangent modulus

$\eta_A, \eta_B =$ element stiffness parameters
The parameter $\eta$ represents a gradual stiffness reduction associated with flexure at sections. The partial plastification at cross-sections in the end of elements is denoted by $0 < \eta < 1$. The $\eta$ may be assumed to vary according to the parabolic expression:

$$\eta = 1 \quad \text{for } \alpha \leq 0.5$$

$$\eta = 4\alpha (1 - \alpha) \quad \text{for } \alpha > 0.5$$

where $\alpha$ is the force-state parameter obtained from the limit state surface corresponding to the element end:

$$\alpha = \frac{P}{P_y} + \frac{8}{9} \frac{M}{M_p} \quad \text{for } \frac{P}{P_y} \geq \frac{2}{9} \frac{M}{M_p}$$

$$\alpha = \frac{P}{2P_y} + \frac{M}{M_p} \quad \text{for } \frac{P}{P_y} < \frac{2}{9} \frac{M}{M_p}$$

where

$P, M =$ second-order axial force and bending moment at the cross-section

$M_p =$ plastic moment capacity

### 2.5 Geometric Imperfection Methods

Geometric imperfection modeling combined with the CRC tangent modulus model is discussed in what follows. There are three: the explicit imperfection modeling method, the equivalent notional load method, and the further reduced tangent modulus method.

**Explicit imperfection modeling method**

For braced frames, member out-of-straightness, rather than frame out-of-plumbness, needs to be used for geometric imperfections. This is because the $P - \Delta$ effect due to the frame out-of-plumbness is diminished by braces. The ECCS (1991, 1984), AS (1990), and CSA (1994, 1989) Specifications recommend an initial crookedness of column equal to $1/1,000$ times the column length. The AISC Code (1993) recommends the same maximum fabrication tolerance of $L/1,000$ for member out-of-straightness. In this study, a geometric imperfection of $L/1,000$ is adopted.

The ECCS (1991, 1984), AS (1990), and CSA (1994, 1989) Specifications recommend the out-of-straightness varying parabolically with a maximum in-plane deflection at the mid-height. They do not, however, describe how the parabolic imperfection should be modeled in analysis. Ideally, many elements are needed to model the parabolic out-of-straightness of a beam-column member, but it is not practical. In this study, two elements with a maximum initial deflection at the mid-height of a member are found adequate for capturing the imperfection.

The Canadian Standard (1994, 1989) and the AISC Code of Standard Practice (1993) set the limit of erection out-of-plumbness $L_{\Delta}/500$. The maximum erection tolerances in the AISC are limited to 1 in. toward the exterior of buildings and 2 in. toward the interior of buildings less than 20 stories. Considering the maximum permitted average lean of 1.5 in. in the same direction of a story, the geometric imperfection of $L_{\Delta}/500$ can be used for buildings up to 6-stories with each story approximately 10 feet high. For taller buildings, this imperfection value of $L_{\Delta}/500$ is conservative since the accumulated geometric imperfection calculated by $1/500$ times building height is greater than the maximum permitted erection tolerance.
In this study, we shall use $L_i/500$ for the out-of-plumbness without any modification because the system strength is often governed by a weak story which has an out-of-plumbness equal to $L_i/500$ (Maleck et al., 1995) and a constant imperfection has the benefit of simplicity in practical design.

**Equivalent notional load method**

The ECCS (1991, 1984) and the Canadian Standard (1994, 1989) introduced the equivalent load concept which accounted for the geometric imperfections in an unbraced frame, but not in braced frames. The notional load approach for braced frames is also necessary to use the proposed method for braced frames.

For braced frames, an equivalent notional load may be applied at mid-height of a column since the ends of the column are braced. An equivalent notional load factor equal to 0.004 is proposed here, and it is equivalent to the out-of-straightness of $L_i/1,000$.

One drawback of this method for braced frames is that it requires tedious input of notional loads at the center of each column. Another is the axial force in the columns must be known in advance to determine the notional loads before analysis, but these are often difficult to calculate for large structures subject to lateral wind loads. To avoid this difficulty, it is recommended that either the explicit imperfection modeling method or the further reduced tangent modulus method be used.

The geometric imperfections of a frame may be replaced by the equivalent notional lateral loads expressed as a fraction of the gravity loads acting on the story. Herein, the equivalent notional load factor of 0.002 is used. The notional load should be applied laterally at the top of each story. For sway frames subject to combined gravity and lateral loads, the notional loads should be added to the lateral loads.

**Further reduced tangent modulus method**

The idea of using the reduced tangent modulus concept is to further reduce the tangent modulus $E_i$ to account for further stiffness degradation due to geometric imperfections. The degradation of member stiffness due to geometric imperfections may be simulated by an equivalent reduction of member stiffness. This may be achieved by a further reduction of tangent modulus as (Chen and Kim, 1997):

$$E_i' = 4 \frac{P}{P_y} (1 - \frac{P}{P_y}) E \xi_i \quad \text{for} \quad P > 0.5P_y$$  \hspace{1cm} (5a)

$$E_i' = E \xi_i \quad \text{for} \quad P \leq 0.5P_y$$  \hspace{1cm} (5b)

where

$E_i' = \text{reduced } E_i$

$\xi_i = \text{reduction factor for geometric imperfection}$

Herein, the reduction factor of 0.85 is used in the analysis of braced and unbraced frames, and the further reduced tangent modulus curves for the CRC $E_i$ with geometric imperfections are shown in Fig. 2.

The advantage of this method over the other two methods is its convenience for design use, because it eliminates the inconvenience of explicit imperfection modeling or equivalent notional loads. Another benefit of this method is that it does not require the determination of the direction of geometric imperfections, often difficult to determine in a large system. On the other hand, in other two methods, the direction of geometric imperfections must be taken correctly in coincidence with the deflection direction caused by bending moments, otherwise the wrong direction of geometric imperfection in braced frames may help the bending stiffness of columns rather than reduce it.
3. VERIFICATIONS

The practical advanced analysis method will be verified by the use of several benchmark problems available in the literature. Verification studies are carried out by comparing with the plastic-zone solutions as well as the conventional LRFD solutions. The strength predictions and the load-displacement relationships are checked by Chen and Kim (1997) for a wide range of steel frames including axially loaded columns, portal frames, six-story frames, and semi-rigid frames.

3.1 Axially Loaded Columns

The AISC-LRFD column strength curve is used for the calibration since it properly accounts for second-order effects, residual stresses, and geometric imperfections in a practical manner. In this study, the column strength of proposed method is evaluated for columns with slenderness parameters $\lambda_c = (KL/r)\sqrt{F_y/(\pi^2E)}$ varying from 0 to 2, which is equivalent to slenderness ratios $(L/r)$ from 0 to 180 when the yield stress is equal to 36 ksi.
In explicit imperfection modeling, the two-element column is assumed to have an initial geometric imperfection equal to \(L/1000\) at column mid-height. The predicted column strengths are compared with the LRFD curve in Fig. 3. The errors are found to be less than 5% for slenderness ratios up to 140 (or \(\lambda_c\) up to 1.57). This range includes most columns used in engineering practice.

In the equivalent notional load method, notional loads equal to 0.004 times the gravity loads are applied mid-height to the column. The strength predictions are the same as those of the explicit imperfection model (Fig. 3).

In the further reduced tangent modulus method, the reduced tangent modulus factor equal to 0.85 results in an excellent fit to the LRFD column strengths. The errors are less than 5% for columns of all slenderness ratios. These comparisons are shown in Fig. 4.

![Figure 4: Comparison of strength curves for axially loaded pin-ended column (Further reduced tangent modulus method)](image)

3.2 Portal Frame

Kanchanalai (1977) performed extensive analyses of portal and leaning column frames, and developed exact interaction curves based on plastic-zone analyses of simple sway frames. Note that the simple frames are more sensitive in their behavior than the highly redundant frames. His studies formed the basis of the interaction equations in AISC-LRFD design specifications (1993). In his studies, the stress-strain relationship was assumed elastic-perfectly plastic with a 36 ksi yield stress and a 29,000 ksi elastic modulus. The members were assumed to have a maximum compressive residual stress of 0.3F_y. Initial geometric imperfections were not considered, and thus an adjustment of his interaction curves is made to account for this. He further performed experimental work to verify his analysis which covered a wide range of portal and leaning column frames.

In this study, the AISC-LRFD interaction curves are used for strength comparisons. The strength calculations are based on the LeMessurier K-factor method (1977) since it accounts for story buckling, and results in more accurate predictions. The inelastic stiffness reduction factor \(\tau\) (AISC-LRFD, 1993) is used to calculate \(K\) in the LeMessurier's procedure. The resistance factors \(\phi_r\) and \(\phi_e\) in the LRFD equations are taken as 1.0 to obtain the nominal strength. The interaction curves are obtained by the accumulation of a set of moments and axial forces which result in unity on the value of the interaction equation.

When a geometric imperfection of \(L/500\) is used for unbraced frames, most of the strength curves fall within an area bounded by the plastic-zone curves and the LRFD curves. In portal frames, the conservative errors are less than 5%, an improvement on the LRFD error of 11%, and the maximum unconservative error is not more
than 1% shown in Fig. 5. When a notional load factor of 0.002 is used, the strengths predicted by this method are close to those given by explicit imperfection modeling method (Fig. 5). When the reduced tangent modulus factor of 0.85 is used for portal and leaning column frames, the interaction curves generally fall between the plastic-zone and LRFD curves. In portal frames, the conservative error is less than 8% (better than 11% error of the LRFD), and the maximum unconservative error is not more than 5% (Fig. 6).

![Figure 5: Comparison of strength curves for portal frame with $L_c/r_s = 40, G_a = 0$ (Explicit Imperfection modeling method)](image)

![Figure 6: Comparison of strength curves for portal frame with $L_c/r_s = 60, G_a = 0$ (Further reduced tangent modulus method)](image)

3.3 Six-Story Frame

Vogel (1985) presented the load-displacement relationships of a 6-story frame using plastic-zone analysis. Vogel
presented the load-displacement relationship for a 6-story frame using plastic-zone analysis. The frame is shown in Fig. 7. Based on ECCS recommendations, the maximum compressive residual stress is \(0.3F_y\) when the ratio of depth to width \((d/b)\) is greater than 1.2, and is \(0.5F_y\) when the \(d/b\) ratio of is less than 1.2. The stress-strain relationship is elastic-plastic with strain hardening. The geometric imperfections are \(L/450\).

For comparison, the out-of-plumbness of \(L/450\) is used in the explicit modeling method. The notional load factor of 1/450 and the reduced tangent modulus factor of 0.85 are used. The further reduced tangent modulus is equivalent to the geometric imperfection of \(L/500\). Thus, the geometric imperfection of \(L/4500\) is additionally modeled in the further reduced tangent modulus method, where \(L/4500\) is the difference between the Vogel's geometric imperfection of \(L/450\) and the proposed geometric imperfection of \(L/500\).

![Figure 7: Configuration and load condition of Vogel's six-story frame for verification study](image-url)

![Figure 8: Comparison of displacements for Vogel's six-story frame](image-url)
The load-displacement curves given by the proposed method together with the Vogel's plastic-zone analysis are compared in Fig. 8. The errors in strength prediction by the proposed method are less than 1%. Explicit imperfection modeling and the equivalent notional load method under-predict lateral displacements by 3%, and the further reduced tangent modulus method shows a good agreement in displacement with the Vogel's exact solution.

4. ANALYSIS AND DESIGN PRINCIPLES

Analysis and design principles are summarized for the practical application of the advanced analysis method. Step-by-step analysis and design procedures for the method are presented.

4.1 Design Format

Advanced analysis follows the format of Load and Resistance Factor Design. In LRFD, the factored load effect does not exceed the factored nominal resistance of structure. Two safety factors are used: one is applied to loads, the other to resistances. The load and resistance factor design has the format

$$\phi R_n \geq \sum_{i=1}^{m} \gamma_i Q_n$$  \hspace{1cm} (6)

where

- $R_n$ = nominal resistance of the structural member
- $Q_n$ = nominal load effect (e.g., axial force, shear force, bending moment)
- $\phi$ = resistance factor ($\leq 1.0$) (e.g., 0.9 for beams, 0.85 for columns)
- $\gamma_i$ = load factor (usually $> 1.0$) corresponding to $Q_n$ (e.g., $1.4D$ and $1.2D + 1.6L + 0.5S$)
- $I$ = type of load (e.g., $D$ = dead load, $L$ = live load, $S$ = snow load)
- $m$ = number of load type

The main difference between the conventional LRFD method and the advanced analysis method is that the left side of Eq. (6), $(\phi R_n)$ in the LRFD method is the resistance or strength of the component of a structural system, but in the advanced analysis method, it represents the resistance or the load-carrying capacity of the whole structural system.

4.2 Load Combinations

The load combinations in the proposed method are based on the LRFD combinations (1993). Six factored combinations are provided by the LRFD Specification.

4.3 Resistance Factors

The resistance factors $\phi_e$ and $\phi_b$ are built into the analysis program and are thus automatically included in the calculation of the load-carrying capacity. The resistance factors are 0.85 for axial strength and 0.9 for flexural strength corresponding to AISC-LRFD Specification (1993).

4.4 Modeling of Structural Members

Two-element model adequately predicts the strength of a beam member subject to distributed transverse loads. To model a parabolic out-of-straightness in a beam-column, two-element model with a maximum initial deflection at the mid-height of a member adequately captures imperfection effects. It is concluded that practical advanced analysis is computationally efficient.

4.5 Analysis
Analysis is important in the proposed design procedures, since the advanced analysis method captures key behaviors including second-order and inelasticity in its analysis program. Analyses can be simply carried out by executing the program provided by Chen and Kim (1997). This program continues to analyze with increased loads, and stops when a structural system reaches its ultimate state.

4.6 Serviceability Limits

Based on the studies by the Ad Hoc Committee (1986), and by Ellingwood (1989), the deflection limits recommended may be used for general use. At service load levels, no plastic hinges are permitted anywhere in the structure to avoid permanent deformation under service loads.

4.7 Ductility Requirements

Adequate rotation capacity is required for members to develop their full plastic moment capacity. This is achieved when members are adequately braced and their cross-sections are compact. The limits for lateral unbraced lengths and compact sections are described in AISC-LRFD Specification (1993).

5. DESIGN EXAMPLES

5.1 Five-Bay Four-Story AISC Frame

The AISC frame is provided in the 1991 Lecture Series. The building has four stories with a penthouse centered on the top story. Each story is 12 ft high except the first which is 14 ft and the fifth which is 10 ft. There are five bays in the east-west direction spaced at 30 ft. Three bays in the north-south direction have 36 ft exterior bays and 28 ft interior one. The frame has moment resisting exterior supports in the east-west direction and is laterally braced in the north-south direction. All columns other than these are simply supported. Plan view of the frame are shown in Fig. 9.

![Plan view of the AISC frame](image)

Figure 9: Plan view of the AISC frame

5.2 Load Condition

The load condition is determined by BOCA 1990 as follows:

1) Roof Loads
   Dead : 30 psf
Live : 21 psf  
(2) Floor Loads  
  Dead : 68 psf  
  Live : Office; 75 psf  
  Lobby and penthouse; 100 psf  
(3) Cladding : 15 psf  
(4) Wind Load : 80 mph, Exposure C

The following three load combinations govern the member sizes of the frame.  
(1) \[1.2D + 1.6L + 0.5S\]  
(2) \[1.2D + 1.6S + 0.5L\]  
(3) \[1.2D + 1.3W + 0.5(L+S)\]  
where \(D\) = dead load  
\(L\) = live load  
\(S\) = snow load  
\(W\) = wind load  

The live load reduction is considered in the calculation of loads.

5.3 Analyses

The moment frame with leaning columns in the east-west direction, was analyzed using advanced analysis. The yield stress is 36 ksi. The column sizes were changed in every two stories. The analysis and design processes were carried out by the three proposed advanced methods. The geometric imperfection is \(\psi=1/500\) and the equivalent notional load is \(0.002\Sigma P_o\). The further reduced tangent modulus of \(0.85E\), was used.

5.4 Member Sizes

Herein, the load-carrying capacity at the ultimate state rather than the formation of the first plastic hinge was taken as the strength of the structural system. Most member sizes of the frame are governed by the wind load combination. It is the upper story girders that are governed by gravity loads. All three methods result in the same selection of members. These are compared to those given by the conventional LRFD method (Fig. 10) where the sizes are given by Maleck et al. (1995).

Figure 10: Comparison of member sizes of the AISC frame
The proposed procedures generally result in members one or two sizes smaller than the LRFD method could provide. This is because the AISC frame is highly redundant and thus benefits greatly by allowing inelastic moment redistribution to be considered.

5.5 Serviceability

The overall drift of the first-order analysis is calculated as $H/366$ under wind loads of $1.0W$. This frame does not therefore satisfy the drift limit $H/400$. The maximum inter-story drift is $H/357$ which does satisfy the drift limit of $H/300$. The column sizes must be increased to meet the drift requirements. If the interior column sizes in the first and second stories are increased from W14x8 to W16x7 sections, the drift is $H/404$ and thus does not exceed the limit.

6. CONCLUSION

This paper presents a simple, concise, and direct method of designing steel frames, using a practical, advanced technique that produces member sizes very close to those given by the LRFD method. The main advantage of Advanced Analysis is that the laborious and sometimes confused member capacity checks needed to satisfy the AISC-LRFD specifications are avoided. Advanced Analysis captures the limit state strength and stability of a structural system and its component members directly. Since the proposed method strikes a balance between the requirement for realistic representation of actual behavior and failure mode of a structural system and the requirement for simplicity in use without the calculation of $K$-factor, it is recommended for general use.

7. REFERENCES


DEFORMATION AND DUCTILITY DEMANDS IN STEEL MOMENT FRAME STRUCTURES

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ABSTRACT:
Recently observed fractures at welded beam-to-column connections necessitate a re-evaluation of the seismic performance of steel moment frame structures. Assessment of global and local deformation demands imposed by representative ground motions forms part of this re-evaluation. This paper addresses accurate and approximate methods for demand prediction and summarizes salient features of results obtained in recent analytical studies on global, interstory, and element deformation demands.

KEYWORDS
Steel frames, welded connections, seismic demands, ductility, connection fracture, story drift.

1. INTRODUCTION
The 1994 Northridge earthquake has shown that steel structures may experience local failure modes that have not been observed in earlier earthquakes. The attention of the public and the engineering community has been drawn particularly to frequently observed fractures at welded beam-to-column connections. These failures have raised design and detailing issues that have become the focus of professional and research efforts directed towards improvement of the performance of steel moment frame structures in future earthquakes.

The fractures have been attributed to many sources, including design and material issues and a variety of welding problems. To mitigate the immediate and long-term effects of these fractures on seismic safety and design/fabrication practices, efforts are underway in the U.S. to improve the expected performance of moment resisting frame (MRF) structures with welded beam-to-column connections through modifications of seismic code requirements, tighter material specifications, better control of welding procedures and inspection, and development of connection details that will prevent the occurrence of high stresses in critical regions at weldments. The knowledge needed to implement improved design, detailing, and fabrication procedures is being acquired through focused research and development efforts, which are in progress in a multi-institutional effort carried out by the SAC (Structural Engineers Association of California, Applied Technology Council, California Universities for Research in Earthquake Engineering) joint venture and in many government and industry sponsored research projects.

By far the largest of these efforts is the SAC steel project, which is sponsored by FEMA (Federal Emergency Management Agency) and is addressing the following questions:

• How large is the potential of fractures in welded connections?
• How safe are existing steel frame buildings?
These questions are being addressed through a great number of coordinated experimental and analytical studies that address all aspects of the problem, ranging from fracture mechanics issues to global performance issues concerned with behavior of the complete structural system. The system performance issues are critical because strength, stiffness, and configuration of the structural system will control the demands imposed on the connections as well as the consequences connection fractures will have on structural safety. This paper addresses some of the important system performance issues. It focuses on methods for predicting global and local deformation demands and on selected results that have been obtained in recent studies. It must be emphasized that at the time of writing most of the SAC studies on system performance are still in progress and that much more in-depth knowledge will be acquired through work yet to be done.

2. DESIGN ISSUES

Evaluation of deformation demands cannot be separated from design and detailing practices, which are country specific and in the case of the U.S. even region specific. The focus here is on MRF structures with the type of fully restrained beam-to-column connection that has been used extensively in highly seismic regions of the U.S., and on MRFs that have been designed in accordance with present or recent U.S. codes.

2.1 The Standard U.S. Pre-Northridge Connection

Steel MRF structures in highly seismic regions of the U.S. vary widely in configurations, but since about the mid-60ies they vary little in beam-to-column connection details. The standard way of connecting beams to columns is to provide moment transfer through full penetration butt welds between the beam flanges and column flanges (strong axis connections) or continuity plates (weak axis connections), and to provide shear transfer through the beam web connection. The latter connection consists of a vertical plate, which is shop-welded to the column and to which the beam web is either only bolted or bolted plus partially welded.

Fractures at welds have been observed at nominal beam flange stresses that vary from elastic (smaller than yield stress) to stresses associated with relatively large plastic hinge rotations. There are many reasons for this great variability, ranging from vast differences in weld quality to variability in the adverse stress (strain) conditions caused by stress concentrations or by yielding in either the beam, the column, or the joint panel zone. Understanding the reasons for the observed fractures, and the consequences of fractures on seismic
safety, requires knowledge of the deformation demands imposed by earthquakes on all elements present at a beam-to-column joint.

Local stress and deformation demands imposed on weld material, base materials, and heat affected zones around the weld depend strongly on detailing and on the relative strength of the elements present at a joint. The importance of the relative strength issue cannot be overemphasized because its disregard may provide a completely misleading picture of the sources of, and solutions to, the weld fracture problem. There are three types of elements present at a beam-to-column joint: beam(s), column(s), and a joint panel zone. Once inelastic deformations occur, the state of stress and deformation around the weld depends on the element that has yielded. If the beam is the weak element, strain hardening of the beam base material (together with possible stress redistribution due to slippage in a bolted web connection) will likely control the state of stress and strain and the likelihood of fracture. If the column is the weak element and develops a plastic hinge, the strength of the beam becomes irrelevant to the performance of the connection. However, column yielding may have contributed to, or may even have initiated, the significant number of fractures extending across the column flange and sometimes into the column web. If the panel zone yields in shear before either beams or columns reach their strength, the state of stress/strain will be greatly affected by the panel zone distortion. There are good reasons to suspect that fractures may be caused by the strain concentrations occurring in column flanges when the panel zone distortions become large.

The upshot of this short discussion is that the need exists to predict deformation demands in all elements that make up an MRF structure, not just in beams. Inelastic time history analysis will provide this information, but only for specific ground motions. For a general evaluation a broader approach is needed. For regular structures it is feasible to predict first the global structure displacement, then use this quantity to estimate the maximum interstory drift, and finally use story substructures to predict element deformation demands.

2.2 U.S. Code Design Practice

Deformation demands imposed by earthquakes depend strongly on code strength and stiffness requirements. Since these requirements vary greatly between countries, the later presented results need to be interpreted within a code context. In present U.S. practice seismic design is based on elastic behavior concepts and is partially strength based and partially stiffness based. In strength design sufficient strength is provided to resist a relatively small fraction of the elastic strength demands (demands imposed on the structure if it is expected to respond elastically in a design earthquake). For special moment resisting frames this implies that in a design earthquake story ductilities (maximum interstory drift over yield interstory drift) in the order of 8 to 10 could be expected if the structure strength were tuned to code strength requirements. Because of interstory drift limitations (which are rather stringent in U.S. codes), and because of many other reasons associated with mean material strength properties, gravity load effects, and constructibility issues, the actual strength is usually 2 to 3 times as high as required by code strength considerations alone. Thus, in a design earthquake story ductilities of about 2 to 4 can be expected. Recognizing that recent earthquakes have generated much larger ground motions than implied by code design spectra, it must be expected that special moment resisting frames will experience somewhat larger story ductilities.

3. GENERAL DEMAND EVALUATION

3.1 Estimate of Global Structure Displacement

Past studies (Seneviratna and Krawinkler, 1997) have shown that the roof displacement of a frame structure that responds inelastically, \( \delta_{in} \), can be related with reasonable confidence to the spectral displacement of the first mode period SDOF system, \( \delta_{SDOF,el} \), by means of an equation of the type

\[
\delta_{in} = \alpha_1 \alpha_2 \delta_{SDOF,el}
\]  

(1)

where \( \alpha_1 \) relates the roof displacement of the structure responding elastically, \( \delta_{el} \), to the spectral displacement, i.e., \( \alpha_1 = \delta_{el}/\delta_{SDOF,el} \), and \( \alpha_2 \) relates \( \delta_{in} \) to \( \delta_{el} \), i.e., \( \alpha_2 = \delta_{in}/\delta_{el} \). Statistical data on \( \alpha_1 \) and
\( \alpha_2 \), obtained from simulation studies on generic frame structures subjected to 15 ground motion records, are shown in Figs. 2 and 3.

![Figure 2: Ratio of elastic roof displacement to spectral displacement](image1)

![Figure 3: Ratio of inelastic roof displacement to elastic roof displacement](image2)

Figure 2 shows that the scatter in \( \alpha_1 \) is small and that \( \alpha_1 \) can be closely approximated by the elastic first mode participation factor. Figure 3 shows that \( \alpha_2 \) depends on the first mode period and on the extent of inelastic behavior in the structure, which in represented by \( \mu(SDOF) \), the target ductility ratio used in design. It is important to note that, except for short period structures, the roof displacement of the inelastic structure is smaller than that of the elastic structure. The ratio \( \alpha_2 \), as shown in Fig. 3, was also found to be rather insensitive to the plastic hinge mechanism that develops in the structure.

It can be concluded that global displacements of inelastic frame structures can be estimated with good accuracy from spectral information. The caveat is that this conclusion can be drawn only for standard conditions, i.e., for frame structures that exhibit stable hysteresis behavior (i.e., no significant strength deterioration) and no significant P-delta effects (i.e., no effective negative post-yield stiffness), and for structures subjected to "standard" design ground motions (no near-field records or soft soil records). For these special conditions separate studies are performed as part of the SAC effort.

### 3.2 Estimate of Interstory Drift

The interstory drift index is defined as interstory displacement, \( \delta_{ij} \), divided by story height, \( h_i \). Thus, it defines the average rotation angle each beam-column subassembly in a given story will experience. When normalized to the yield drift angle, it defines the interstory ductility ratio \( \mu_{ij} \). The relationship between interstory drift index and the global drift index \( \delta y / h_i \) depends on the extent of inelasticity in the structure, the type of plastic hinge mechanism, and the importance of higher mode effects.

A typical example of the variation of interstory drift index over the height of the structure, together with the measure of global drift, is shown in Fig. 4. This result is obtained from a single time history analysis of a frame structure with a period of 1.22 seconds, in which individual story mechanism are permitted to form by plastic hinging in the column. The drift index varies significantly over the height of the structure, and the maximum interstory drift index (in this case in the first story) is more than twice the global drift index. Mean values of the ratio of maximum interstory drift to global drift for structures in which plastic hinging is permitted only in beams are shown in Fig. 5. More extensive results for other frame types are available in Seneviratna and Krawinkler (1997).

Information of the type presented in Figs. 2 to 5, together with spectral information (acceleration or displacement spectra), can be utilized to derive estimates of global and story deformation demands. Element deformation demands can now be estimated by subjecting representative beam-column subassemblies (usually assumed to extend from midheight to midheight of adjacent stories) to column shear forces and impose a rotation angle equal to the interstory drift index. Clearly, such a process will result only in gross estimates of element demands since simplified assumptions have to be made on boundary conditions for the
beam-column subassemblies. The authors have found that these gross estimates are reasonable unless the structures have significant strength or stiffness irregularities.

For $EQ_{11}$, Damping = 5%, $u_0 = 0\%$, $u_{(SDOF)} = 4$

Interstory Drift

Global Drift

$$\frac{S_j}{h_j} \times 1000$$

Figure 4: Example of variation of interstory drift over height of structure

Figure 5: Mean values for ratio of maximum interstory drift to global drift index

4. ANALYTICAL MODELING FOR DEMAND PREDICTION

The process outlined in the previous section represents the authors' approach to a simplified assessment of global and local deformation demands. In the context of a thorough and accurate evaluation of deformation demands and of the effects of connection fractures on structural performance, much analytical work needs to be performed to support or dispute this simplified approach and to complement the simplified approach with more detailed information on the sensitivity of predictions to the many variables that may affect the seismic response. Such sensitivity studies can only be performed with analytical models that are sufficiently accurate to capture the effects of these variables.

The SAC program is conducting a number of analytical studies that address, amongst others, the effects of the following specific issues on seismic demands:

- Strength variations (due to overstrength, yield strength variations, etc.)
- Relative strength of elements at beam-column joint
- Strength and stiffness of composite floor slabs
- Strength and stiffness of simple (shear) connections
- Structural configuration and redundancy
- P-delta effects
- Stiffness degradation (pinching) and strength deterioration
- Three-dimensional effects
- Consequences of connection fractures on safety of structure

In these analytical studies different analysis tools and analysis methods are employed, with the major objectives being (a) to obtain reliable answers to the posed questions, and (b) to develop a tool kit that will assist design engineers in obtaining reasonable answers in the most efficient manner. From the engineer's perspective this means that the simplest tool that provides a reasonable answer will be by far the most effective one.

4.1 Modeling of Elements

Simple element models are much preferred, provided they will not unduly distort any of the load-deformation characteristics important in seismic response. This implies that, with few exceptions, concentrated plasticity models rather than distributed plasticity models are adequate. In the SAC studies the beam elements are treated as elastic elements (unless plastic hinging is anticipated within the beam proper), and plastification at the beam ends together with connection strength and stiffness characteristics are modeled by means of one or several nonlinear spring(s). The spring strength and stiffness properties need to be adjusted to account for the phenomena itemized in the previous section and of interest in the specific study. Extensive discussions of element modeling issues are presented in SAC (1995).
Summarized here is only the modeling of shear behavior in panel zones, because it is believed to be fundamental to achieving an understanding of local seismic demands at beam column joints but is rarely considered in engineering studies.

The high shear forces that have to be transferred through the panel zones will lead to significant shear distortions, and in many U.S. code designs will lead to joint shear yielding before plastic hinges will develop in beams. The cyclic joint shear force versus shear distortion relationship is characterized by stable and “fat” hysteresis loops, which can be represented approximately by a tri-linear skeleton diagram. The shear strength and stiffness properties of joint panel zones can be modeled as proposed in Krawinkler (1978). In this approach the panel zone shear force demand \( V \) is computed from the unbalanced beam moments at the column faces, \( (M^b + M^c') \), and the column shear force \( V_c \) by the equation

\[
V = \frac{(M^b + M^c')}{0.95d_b} - V_c
\]

where \( d_b \) is the depth of the beam.

The trilinear shear force - shear distortion relationship can be defined by a yield shear force \( V_y \) and the corresponding yield distortion \( \gamma_y \), a plastic shear force \( V_p \), which is associated with a distortion of \( 4\gamma_y \), and a strain hardening stiffness for distortions exceeding \( 4\gamma_y \). The quantities defining this relationship are given in Krawinkler (1978) as follows:

\[
V_y = 0.55F_yd_d t
\]

\[
V_p = V_y \left(1 + \frac{3b_c t_f^2}{d_b d_c} \right) = 0.55F_yd_c \left(1 + \frac{3b_c t_f^2}{d_b d_c} \right)
\]

\[
\gamma_y = F_y/(\sqrt{3} G)
\]

where \( F_y \) is the panel zone yield strength, \( t \) is the panel zone thickness, and \( d_c, b_c, \) and \( t_f \) are the depth, flange width, and flange thickness of the column section.

A model capable of representing all important characteristics of the elements framing into a joint is illustrated in Fig. 6. The joint panel zone is modeled with rigid elements connected with hinges at the four corners. The joint shear strength and stiffness can be modeled by providing a rotational spring in any of the four corners as shown in the figure. The stiffness of this rotational spring, \( K_s \), is given as

\[
K_s = (V/\gamma_d)b_d
\]

The trilinear shear force - distortion relationship can be modeled by providing either one trilinear spring or two bilinear springs. A typical joint response, obtained from a time history analysis, is shown in Fig. 7.

![Figure 6: Analytical modeling of joint area](image)
![Figure 7: Joint shear force - distortion response](image)
4.2 Analysis Methods

Within the SAC program inelastic static and dynamic analyses are being utilized to predict performance. Some emphasis is being placed on an evaluation of the inelastic static (pushover) method, which is becoming a widely used technique in the U.S. for the assessment of seismic performance. In this method the structure is subjected to lateral loads, in either predetermined or adaptive load patterns, and pushed to a target displacement that represents the global displacement the structure is expected to experience in a design earthquake. At this displacement level the force and deformation demands in the individual elements are evaluated by comparing the demands to capacity values, which denote acceptable values associated with specific performance levels (e.g., collapse prevention). In many cases the inelastic pushover analysis provides significantly more relevant information than an elastic analysis, but it must be emphasized that this simplified prediction method may provide misleading results if higher mode effects become important or the structure has more than one weakness. A comprehensive discussion of the feasibility and limitations of the pushover method is provided in Krawinkler (1996).

5. DEMAND PREDICTION STUDIES

SAC (1995) provides detailed information on an extensive series of analytical evaluations of buildings affected by the Northridge earthquake. These studies were carried out in Phase 1 of the SAC steel program. Much more systematic research is being performed now to address the issues identified in Section 4. In this paper only a few salient results of one Phase 1 study and one ongoing Phase 2 study are summarized, with the objective being to point out a few issues of special interest.

5.1 Case Study on 4-Story Northridge Building

This study was concerned with predicting seismic demands for a 4-story MRF structure in which many connections fractured during the Northridge earthquake (Krawinkler and AlAli, 1996-1). Different analysis models were developed to (a) investigate the sensitivity of the results to modeling assumptions, (b) attempt to correlate magnitudes of predicted demands with observed connection fractures, and (c) assess the safety of the structure with fractured connections.

Pushover analyses and dynamic analyses with 9 ground motion records were performed on a 2-D model of that half of the structure that contains the MRF in which most of the fractures were observed. A sketch of this frame, with the locations of fractures indicated with black quarter circles, is shown in Fig. 8. Pushover curves for the following six different models are presented in Fig. 9.

- Benchmark: Model with centerline dimensions, ignoring joint panel zones.
- Model 1: Model that includes finite strength and stiffness of joint panel zones.
- Model 1 (Elas. Jts.): Similar to Model 1, but panel zones with infinite shear strength.
- Model 2: Similar to Model 1, but considering contribution of floor slab and simple connections to strength and stiffness.
- Model 3-1: Similar to Model 2, but considering fractured connections as shown in Fig. 8.
- Model 3-2: Similar to Model 2, but assuming that all connections in MRF have fractured.

As Fig. 9 shows, the structure stiffness and strength vary significantly between the models, but the time history analyses disclosed that global and interstory drifts in the non-fractured models did not vary a great deal. The models with fractured connections exhibit greater drifts, with the maximum interstory drift of Model 3-2 being about twice that of Model 1 (mean of 0.04 versus mean of 0.02).

The importance of considering joint shear behavior (difference between Model 1 and Model 1 (Elas. Jts.)) is illustrated in Fig. 10, which shows the plastic element deformation demands (beam plastic hinge rotations if boxes are located at ends of beams, or panel zone plastic shear distortions if boxes are located at center of joints) from the pushover analysis at a global drift of 0.03. Figure 10(a) provides realistic results based on the limited shear strength of the joint panel zones and shows that all interior panel zones have to undergo large plastic shear distortions whereas most beams at the interior joints remain in the elastic range. It should be noted that in this frame many of the connections at these joints did exhibit fractures after the Northridge
earthquake (see Fig. 8). If yielding in the panel zones is ignored in the analysis, a completely erroneous picture of plastic deformation demands is obtained, as is shown in Fig. 10(b). Here, plastic deformations are concentrated at plastic hinge locations in beams, and considerable plastic rotation demands are indicated.

This example is presented for the following reasons. A global drift of 0.03 is large but not unreasonable in view of the severe ground motions recorded in recent earthquakes. In well designed MRF structures, in which inelastic deformations are distributed over the height of the structure and are not concentrated in a weak story, the plastic deformation demands may be in the range of values shown in Figs. 10(a) and (b). Expectations are that in well designed beams plastic rotations in the order of 0.02 should be sustainable without much deterioration, and that in panel zones a plastic shear distortion in the order of 0.025 should not pose a problem either. However, many weld fractures were observed in this frame, even though the plastic deformation demands in the Northridge earthquake were likely smaller than those shown in Fig. 10(a). [This conclusion is drawn from a series of time history analyses using 9 representative ground motion records.]

Thus, the fractures did occur clearly within the expected range of element deformations, and many of them did occur at locations at which the beams have not yielded but the panel zones have undergone plastic shear distortions. This points out the need for an analysis that accounts adequately for the strength and deformation characteristics of all elements at joints, including panel zones, since large panel zone distortions are a likely cause of connection fractures.

Comparing Figs. 8 and 10(a) it can be seen that large predicted demands correlate reasonably well with the locations of connection fractures. It is observed that connections may fracture when inelastic deformations are concentrated in the joint panel zones or when they are shared between beams and panel zones.

Figure 8: MRF with fracture locations

Figure 9: Base shear vs. roof displacement diagrams

(a) Model with limited panel zone shear strength  (b) Model with infinite panel zone shear strength

Fig. 10 Element plastic deformation demands from pushover analysis, global drift = 0.03
5.2 SAC Studies on System Performance

Several coordinated studies are performed to address the issues identified in Section 4. In all studies a common series of model buildings and ground motion records are being used to obtain results that cover typical designs in three geographic locations (Los Angeles, Seattle, and Boston) which are subjected to ground motions considered typical for these locations.

Nine model buildings have been designed, comprising 3, 9, and 20 story buildings for the three locations. Plan views and elevations of the model buildings are identical for the three regions and are as shown in Fig. 11. The 9 and 20 story buildings have one and two basement levels, respectively. The choice of the lateral load resisting system was left to the design engineers, but turned out to be almost identical for the three locations. The common choice was perimeter framing of the type shown in solid lines in the plan views.

The designs were performed in accordance with 1994 U.S. codes. Most MRF member sizes were controlled by stiffness rather than strength, and some of the Seattle and Boston designs were controlled by wind loads rather than seismic considerations. In the design of the structures the following gravity loads were used (with minor variations between locations and different height structures):

- **Roof dead load**: 3.98 kPa (83 psf)
- **Floor dead load**: 3.64 kPa (76 psf)
- **Partition load**: 0.96 kPa (20 psf)
- **Exterior cladding**: 1.20 kPa (25 psf)
- **Roof penthouse**: 13.89 kPa (290 kips)
- **Floor live load**: 2.40 kPa (50 psf)

The seismically effective weights per unit floor area, including exterior cladding, a roof parapet, and a small penthouse on the roof of the buildings, amount to the following average values [in the U.S. only dead loads are considered as seismically effective]:

- **Roof**: 4.93 kPa (103 psf)
- **Floor**: 4.55 kPa (95 psf)

These values, together with the dimensions of Fig. 11, can be used to compute story masses and weights.

Representative ground motion records have been developed for the three locations, including sets of 20 records with a 10% probability of exceedance in 50 years (10/50-20 records) and a 2% probability of exceedance in 50 years (2/50-20), as well as near-fault and soft soil records. Mean spectra (2% damping) for the 10/50-20 and 2/50-20 Los Angeles record sets are shown in Fig. 12 together with the NEHRP design ground motion spectrum for soil class D. It should be noted that the NEHRP spectrum assumes 5% damping whereas the location specific spectra are for 2% damping. The latter is done because in the dynamic analyses it is assumed that only 2% effective viscous damping is available.

At the time of this writing only limited studies have been carried out with these model buildings and records. Some of the important findings to date are summarized in the next section.
5.3 Story Drift Evaluation for Los Angeles Buildings

The 10/50 and 2/50 record sets have been used to evaluate maximum and cumulative story drift demands for the 3 and 9 story Los Angeles buildings. To preface the presented results, it must be said that the 2/50 set of records contains several near-fault records that impose very high story drift demands. In the records used in this part of the study one of the components is fault-normal and the other is fault-parallel. As will be shown later, the responses in these two directions differ by a huge amount, with the fault-normal component imparting often a multiple of the demand of the fault-parallel component.

In general, the interstory drift response is of either of the types illustrated in Fig. 13. The response shown in Fig. 13(a) is essentially symmetric, with a relatively small maximum amplitude and with little residual drift after the ground motion has stopped. The response shown in Fig. 13(b) exhibits the pulse behavior characteristic for near-fault ground motion response, with a very large amplitude and significant residual drift remaining after the earthquake.

It is necessary to make a clear distinction between drift amplitude and drift range. Maximum amplitude is measured with respect to the undeformed configuration, whereas maximum range is measured from maximum positive to maximum negative amplitude. From the perspective of cumulative damage, the range is believed to be more relevant than the amplitude. From the perspective of dynamic stability (prevention of incremental collapse due to P-delta), the amplitude is more relevant. In the following discussion the emphasis is on maximum and cumulative ranges.

The dynamic analysis results need to be interpreted with the strength and stiffness characteristics of the 3 and 9 story structures in mind. Both structures are symmetric, exhibit a smooth variation of strength and stiffness over the height, and have strength properties that are significantly higher than required by code (because stiffness controls member sizes). The design base shear at the allowable stress level is 0.074W and 0.042W for the 3 and 9 story building, respectively, whereas the first story yield strength is significantly higher as can be seen from the pushover results presented in Fig. 14. The story shear versus story drift results shown in this figure indicate that the P-delta effect for the 3 story building is not important, but that it leads to noticeable negative story stiffnesses for the 9 story building. Moreover, the 9-story pushover indicates that the lower stories are expected to experience large inelastic deformations, whereas the upper stories are expected to remain essentially elastic. The dynamic analyses did not confirm this, which demonstrates one of the shortcomings of the pushover analysis for long period structures in which higher mode effects are important.

The time history results for the two model buildings subjected to the 10/50 and 2/50 record sets have been evaluated for maximum and cumulative response parameters. It was found that for both buildings the top story is the one with the highest drift demands. Figure 15 shows maximum and cumulative responses for the three stories of the 3 story building, using the plastic interstory drift angle range (plastic range of inelastic cycles), $\Delta \theta_p$, as response parameter. The results are presented for each of the 40 records used in this study,
with records 1 to 20 denoting the 20 10/50 records and records 21 to 40 denoting the 20 2/50 records. The predominance of the near-fault records in the 2/50 set is evident, with the fault-normal components (odd numbers) imposing very large demands and the fault-parallel components (subsequent even numbers) imposing much smaller demands.

A statistical evaluation of the data was performed by fitting various probabilistic distributions to the data and using visual inspection to select the best suited distribution. Because of the clear distinction between fault-normal and fault-parallel responses in the 2/50-20 set, the ten fault-normal responses (2/50-10) were evaluated separately. Results from this statistical evaluation are presented in Fig. 16, which shows the 50, 75, and 90 percentile values obtained from the fitted distributions, connected by straight lines.
The plots of maximum plastic range, $\Delta \theta_{\text{pl,max}}$, show that in the 3 story building the plastic deformation demands for the 2/50 records can be very high, particularly if only the ten fault-normal components are considered. The demands for the 10/50-20 record set are more in line with usual expectations. The demands for the 9 story building are much smaller than those for the 3 story building. Comparing cumulative plastic demands to maximum plastic range demands, it is observed that the maximum range contributes about 50% to the cumulative demand, which indicates that the interstory response is dominated by one large excursion.

The results presented here indicate that the interstory drift demands are in line with usual expectations except for the near-fault fault-normal record components present in the 2/50 record set. The importance of this observation needs further study, but it confirms the often expressed concern about near-fault records.

6. CONCLUSIONS

Seismic deformation demands are central to a performance evaluation of structures. For steel MRF structures, which recently have experienced fractures at welded connections, element deformation demands must be evaluated with careful consideration given to the relative strength of the elements framing into beam-to-column joints. In many cases global displacement demands can be predicted with good accuracy from SDOF spectra, interstory drift demands can be estimated, but with less confidence, from global displacement demands, and element deformation demands can be estimated by subjecting beam-column substructures to rotations equal to the interstory drift angle. This simple prediction process breaks down if either severe P-delta effects occur or the input has pulse-type characteristics representative of near-fault ground motion.

ACKNOWLEDGMENTS

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This publication is dedicated to Professor Franz Ziegler, Chair of the Institut für Allgemeine Mechanik of the Technical University of Vienna in Austria, at the occasion of his 60th birthday. For the senior author it was a great pleasure and rewarding experience to have spent a sabbatical leave at Professor Ziegler's institute and have had the opportunity to discuss dynamics with such an eminent leader in applied mechanics. Professor Ziegler has been a gracious host and has become a good friend. Beste Glückwünsche zum Geburtstag, Franz.

REFERENCES

DUCTILITY DEMAND ASSOCIATED WITH SEISMIC INPUT

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ABSTRACT: Damage of steel structures in the Hyogoken-nanbu earthquake and Northridge earthquake revealed the necessity for proper estimate of the ductility of steel-moment frames required to meet severe earthquakes. Originally, steels as materials are equipped with very large inelastic deformation capacities. However, fabricated steel members do not exhibit such a large deformation capacity as the material develops. By equating the total energy input to the total energy absorption capacity, a practical relationship between the ductility, the strength and the seismic input level is made clear.

KEYWORDS: Steel Moment Frame, Beam-to-Column Connection, Fracture, Stress Concentration, Ductility, Seismic Resistance

1. INTRODUCTION

Steel moment-resistant frames composed of rectangular-hollow-section columns and H-shape beams are becoming popular in Japan. By applying RHS-columns, the structural planning becomes simple and the weak-beam type of frame is easily realized. The structural detail, however, is not so simple and raise a problem. The beam-to-column connections are usually

![Fig.1 Beam-to-Column Connection](image-url)
formed by welding as shown in Fig. 1. While the transmission of stresses in flanges of the beam is perfectly made through diaphragm plates, the transmission of stresses in the web of the beam is not perfect, since the stresses must be transmitted through the out-of-plane bending of the flange plate of RHS-columns[1]. This inefficiency in stress-transmission causes the reduction of the inelastic energy absorption capacity of the frame, resulting in the reduction of the seismic resistance of the frame. The 1995 Hyogoken-nanbu earthquake revealed the importance of the fractural mode of failure produced by this cause. In this paper, a realistic estimate of the deformation capacity and the seismic resistance limited by the fracture of beams for weak-beam type moment frames are made.

2. Deformation Capacity of Beams

The load-deformation curve of beams for a typical stress condition for the seismic loading is shown schematically in Fig. 2, in which the curve is simplified by two line segments. When the instability such as local buckling is avoided, the fractural mode of failure appears under the maximum bending moment $M^*$. The fundamental index of the inelastic deformation capacity associated to the energy absorption capacity is defined as

$$\eta_{B} = \frac{W_p}{M_p \theta_p}$$  \hspace{1cm} (1)

where $\eta_{B}$: the cumulative inelastic deformation ratio of the beam
$W_p$: the inelastic energy absorption under the monotonic loading
$M_p$: the full plastic moment of the beam
$\theta_p$: the elastic end slope at $M_p$

$W_p$ is approximately expressed as

$$W_p = \frac{(M_p + M_B) \theta_B}{2}$$  \hspace{1cm} (2)

where $\theta_B$: the maximum end slope
$M_B$ and $M_p$ are deformation by

![Fig.2 M-θ Relationship of Beam](image-url)
where \( \sigma_{M} \): the maximum stress
\( \sigma_{y} \): the yield stress of the material

Using the nondimensional slope, \( k_{p} \), in the inelastic range, \( \eta_{B} \) is obtained as

\[
\eta_{B} = \frac{\left( M_{B} + M_{B} \right) \theta_{B}}{2 M_{p} \theta_{p}} = \frac{\left( \sigma_{M} / \sigma_{y} \right)^{2} - 1}{2 k_{p}}
\]

\( k_{p} = D_{n} / D \): the nondimensionalized slope in the inelastic range
\( D \): the slope in the elastic range
\( D_{n} \): the slope in the inelastic range

When the transmission of the bending moment is smoothly made between the beam and the column, and any instability does not take place, the maximum stress, \( \sigma_{M} \), is limited by the tensile strength of the material, \( \sigma_{B} \). When the transmission of the bending moment in the web of the beam is imperfect, the maximum stress must be reduced to

\[
\sigma_{M} = r \sigma_{B}
\]

\( r \) can be related to the reduction of the effective area of the web as follows.

\[
r = \frac{A_{f}^{*} + r_{w} A_{w}}{4} \quad \frac{A^{*} + A_{w}}{4}
\]

where \( A_{f}^{*} \): the area of one flange of the beam
\( A_{w}^{*} \): the area of the web of the beam

Taking the practical range of \( A_{w}^{*} / A_{f}^{*} \), \( r \) is related to \( r_{w} \) as shown in Table 1. Assuming SN490-steels, \( \eta_{B} \) for each values of \( r \) is also indicated in Table 1. Applied properties are:

\[
\begin{align*}
\sigma_{B} & = 11 \sigma_{B0} \\
\sigma_{y} & = 12 \sigma_{y0} \\
k_{p} & = 0.03 \\
\sigma_{B0} / \sigma_{y0} & = 1.52 \quad \text{for SN490 - steels}
\end{align*}
\]

<table>
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<th>( A_{w} / A_{f} )</th>
<th>( r_{w} )</th>
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</table>
where $\sigma_{Y0}$: the minimum tensile strength prescribed in the standard
$\sigma_{Y0}$: the minimum yield stress prescribed in the standard

Derivation of $r_w$ based on experiments is shown in Appendix.

3. ULTIMATE RESISTANCE OF WEAK-BEAMTYPE MOMENT FRAMAEs IN FRACTURAL MODE OF FAILURE

The earthquake resistant design method shown in the Japanese building code which revised in 1981 is summarised as follows.

a) The buildings should be proportioned on the basis of allowable stress design method under the seismic input which corresponds to $C_o = 0.2$.

b) The building should be equipped with the energy absorption capacity for the seismic input which corresponds to $C_o = 1.0$.

$C_o$ is a coefficient which indicates the level of seismic input, and the simplified design spectra for $C_o = 1.0$ are shown in Fig.3 specifying ground conditions by I, II and III.

Observing the giving and receiving of the energy input, the earthquake resistant design can be clearly formulated[2]. The equilibrium of the energy at the instant when a seismic motion is terminated can be written as

$$W_e + W_p + W_h = E$$

where

$W_e$: the elastic vibration energy
$W_p$: the cumulative inelastic strain energy (damage)
$W_h$: the energy absorption due to damping
$E$: the total energy input due to an earthquake

The total energy input can be assumed to depend only on the total mass of the structure and the fundamental natural period of the structure. Thus the energy spectrum is directly applied to estimate $E$. The energy input attributable for the structural damage $E_D$ can be expressed by the following empirical formula.

$$E_D = E - W_h = \frac{E}{\left(1 + 3h + 12\sqrt{h}\right)^2}$$

where $h$: the damping constant

![Fig.3 Simplified Design Spectra for $C_o=1.0$](image)
$E_D$ is converted to the equivalent velocity $V_D$ by the following definition

$$V_D = \frac{E_D}{\sqrt{2M}}$$

(10)

where $M$ : the total mass of the structure

$V_D$ approximately corresponds to the velocity response spectrum.

The damage of the multi-degree of freedom system equipped with the elastic-perfectly-plastic restoring characteristics can be described as follows.

$$W_p = \gamma_1 W_{p1}$$

(11)

where $\gamma_1 = \frac{W_p}{W_{p1}}$ : the damage distribution factor

$W_{p1}$ : the damage of the first story

$W_{p1}$ is written as

$$W_{p1} = Q_{pl} \delta_{pl} = Q_{pl} \delta_{pl} \eta_i$$

(12)

where $\delta_{pl}$ : the cumulative inelastic deformation of the first story

$\eta_i = W_{p1} / Q_{pl} \delta_{pl}$ : the cumulative inelastic deformation ratio of the first story

$\delta_{pl}$ : the yield deformation of the first story

$Q_{pl}$ : the yield strength of the first story

$W_{p1}$ is written as

$$W_{p1} = \frac{Mg^2 T^2}{4\pi^2} \frac{\alpha_i \eta_i}{\kappa_i}$$

(13)

where $\alpha_i = \frac{Q_{pl}}{Mg}$ : the yield shear force coefficient of the first story

$$\kappa_i = \frac{k_1}{k_{eq}}$$

$k_1$ : the spring constant of the first story

$g$ : the acceleration of gravity

$k_{eq} = \frac{4\pi^2 M}{T^2}$ : the spring constant of the equivalent one mass system

The elastic vibration energy can be written as

$$W_e = \frac{Mg^2 T^2}{4\pi^2} \frac{\alpha_i^2}{2}$$

(14)

Therefore, Eq.(8) is reduced to

$$V_D = gT^2 \frac{\alpha_i \eta_i}{2\pi} \sqrt{1 + 2\eta_i \gamma_1}$$

(15)

For the multi-story frames with uniform distributions of masses and yield deformations, the following approximate expression can be applied.

$$\kappa_i = 0.48 + 0.52N$$

$$\gamma_1 = 1 + 0.64(N-1) p_d^{-n}$$

(16)
where \( N \): the number of story

\( p_d \): a factor which indicates the deviation of yield shear force coefficient distribution from the optimum distribution

\( n \): the exponent which indicates the extent of damage concentration

Reflecting the scatter of yield-point stresses and the distribution of structural members in the actual buildings, \( p_d \) can be expressed as follows.

\[
p_d = 1.185 - 0.0014N
\]

\( \eta_b \) in Eq.(1) can be related to \( \eta_i \) in Eq.(12) as follows

\[
\eta_i = \frac{2 \delta_{y_1} \cdot a_p \cdot \sigma_y \cdot \eta_b}{\delta_{y_1}} \tag{18}
\]

where \( a_b \): the amplification factor due to Bauschinger effect

\( a_p \): the amplification factor due to the plastification of structural components other than beams

\( \delta_{y_1} \): the yield deformation of the first story calculated on the assumption that members other than beams are rigid (see Fig.2 in which \( H \) is the height of story)

The factor of 2 in Eq.(18) corresponds to the assumption that the inelastic deformation takes place with equal amount both in positive and negative directions. Assuming \( \delta_{y_1} / \delta_{y_1} = 1/3, a_b = 2.0 \) and \( a_p = 1.5 \), Eq.(18) is reduced to

\[
\eta_i = 2 \eta_b \tag{19}
\]

As a practical example, the following conditions are taken.

structures: \( \delta_y = \frac{H}{150}, \ H = 400 \text{cm}; \ n = 6 \) (weak beam type)

material: \( \sigma_y = 1.2 \sigma_{y_0}, \ \sigma_y = 1.1 \sigma_B \)

\[
\sigma_{y_0} = 3.3t/cm^2, \ \sigma_{y_0} = 50t/cm^2
\]

specified values for SN490 steels

others: \( h = 0.02 \)

From Fig.3, the shear force coefficient used for the allowable stress design \( \alpha_e \) is read as follows.

For \( T \leq 1.28 \text{sec}, \ \alpha_e = 0.2 \)

For \( T > 1.28 \text{sec}, \ \alpha_e = \frac{0.256}{T} \) \tag{20}

Since the skeletons are designed on the basis of the elastic analysis, the strength which corresponds to \( \alpha_e \) is the elastic limit strength \( Q_{e1} \). The yield strength \( Q_{y1} \) and \( Q_{e1} \) can be roughly related to be

\[
Q_{y1} \geq 1.5Q_{e1} \tag{21}
\]
Therefore, considering also the increase of yield point stress by 20 percents, \( \alpha_i \) can be assumed to be

\[
\alpha_i = 1.5 \times 1.2 \alpha_e = 18 \alpha_e
\]  

(22)

The fundamental natural period \( T \) is written as

\[
T = 2\pi \sqrt{\frac{M}{k_e}} = 2\pi \sqrt{\frac{Mk_i}{k_i}}
\]  

(23)

Knowing \( k_i = \frac{Q_{yi}}{\delta_{yi}} = \alpha_iMg / \delta_{yi} \), \( T \) is reduced to

\[
T = 2\pi \sqrt{\frac{k_i \delta_{yi}}{\alpha_i g}}
\]  

(24)

The seismic resistance which corresponds to the deformation capacity of \( \eta_i \) is shown in relation to \( V_D \) in Table 2. It is clearly seen in Table 2 that the seismic resistance of the moment resistant frame is largely influenced by the efficiency of the stress transmission at the beam-to-column connection. As far as the condition of \( r > 0.85 \) (or \( r_e > 0.5 \)) is satisfied, the steel moment resisting frames designed by the current Japanese building code can resist to the possible maximum level of seismic input, i.e. \( V_D > 300cm/s \).

<table>
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<th>( \beta_i )</th>
<th>( \gamma_i )</th>
<th>( T )</th>
<th>( \alpha_i )</th>
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4. Conclusion

The seismic resistance of the steel moment-resistant frames was estimated in terms of the moment transmission efficiency at the beam-to-column connection. The deformation capacity of the beam is largely influenced by the moment transmission efficiency of the web and its precise estimate is essentially necessary for preventing a premature fractural mode of failure of the weak-beam type moment frame.

REFERENCES


Appendix.

Moment Transmission Efficiency of Web-Plate at the Beam-to-Column Connection

When the moment frame is composed by the rectangular hollow section columns (box-columns) and H-shape beams, the transmission of the moment of beam through web-plate becomes incomplete. This necessarily causes stress concentration on the part of flange plate of beams. Series of experiments were made to make clear quantitatively the moment-transmission capacity at the connection.

In Fig. A1(a), the beam-to-column connection is shown. The forces in the flange plates of beam are smoothly transmitted through diaphragm plates welded to the RHS column. At the connection, the moment transmitted through the web-plate can be expressed by a couple of force, \( T_w \). When the beam-to-column connection is perfectly rigid, the maximum moment transmitted through the web-plate can be reached the following value.

\[
M_w = \frac{h_w^2 t_w \sigma_y}{4} \tag{A1}
\]

where
- \( h_w \) : the depth of the web-plate of beam
- \( t_w \) : the thickness of the web-plate
- \( \sigma_y \) : the tensile strength of the web-plate

Under actual situations, the moment can’t reach the value given by Eq. (A1) because of the flexibility of unstiffened wall of column. In order to obtain the resistance of the web-plate in terms of \( T_w \), a mechanical model by which the resistant force same as \( T_w \) can be simulated is considered as shown in Fig. A1 (b). The beam is cut through its middle line and a half of web-plate is welded on the top of the flange plate. Thus, a symmetrical tension-specimen is formed. When the tension-specimen is pulled, the same resistance of \( T_w \) may develop.

The actual specimen is shown in Fig. A2. The major parameters are the thickness of column wall, \( t_c \), the yield point stress of column, \( \sigma_{yw} \), and the depth of the web-plate of beam, \( h_w \). Referring the practical conditions, all specimens have weld access holes as shown in Fig. A2.
All specimens were torn at the flange-plate with cope. The maximum tensile strength, $P_{\text{max}}$, can be considered to be the sum of the maximum strength of the flange-plate, $P_{f_{\text{max}}}$, and $2T_w$:

$$P_{\text{max}} = P_{f_{\text{max}}} + 2T_w$$  \hspace{1cm} (A2)

$P_{f_{\text{max}}}$ is considered to be:

$$P_{f_{\text{max}}} = A_f \sigma_{nf}$$  \hspace{1cm} (A3)

where $A_f$: the sectional area of the flange.

$\sigma_{nf}$: the tensile strength of the flange.

By knowing $P_{\text{max}}$ experimentally and applying Eqs. (A2) and (A3), $T_w$ can be obtained and is shown in table A1 together with major parameters. $T_w$ was found to be expressed by the following empirical formula.

$$T_w = 0.08 \left[ 1 + 2 \left( \frac{h_w}{B} \right) \right] B t_c \sigma_{yc}$$  \hspace{1cm} (A4)

where $B$: the width of the RHS column

The predicted values of $T_w$ by Eq. (A4) are compared with the test values in table A1. Since, the rigidity of the column wall decreases as the distance from the diaphragm plate becomes larger, the stress distribution over the web-plate at the connection is assumed to be linear as shown in Fig. A3. Then the distance of the coupled force $T_w$ becomes $2h_w / 3$. Therefore, the capacity of moment-transmission of the web-plate is expressed as

$$M_w = \frac{2T_w h_w}{3}$$  \hspace{1cm} (A5)

On the other hand, using the reduction factor of web, $r_w$, in Eq. (A1), $M_w$ is written as

$$M_w = \frac{r_w h_w^2 t_w \sigma_B}{4}$$  \hspace{1cm} (A6)

Substituting Eq. (A4) into Eq. (A5) and comparing with Eq. (A6), $r_w$ is obtained as follows.

$$r_w = 0.21 \left[ 1 + 2 \left( \frac{h_w}{B} \right) \right] \left( \frac{B}{h_w} \right) \left( \frac{\sigma_{yc}}{\sigma_B} \right) \left( \frac{t_c}{t_w} \right)$$  \hspace{1cm} (A7)
Fig. A2 Test Specimen

Fig. A3 Stress Distribution

Table A1 Test Results

<table>
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<tr>
<th>specimen</th>
<th>$\sigma_{yc} (\text{t/cm}^2)$</th>
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<th>$h_w (\text{mm})$</th>
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SEISMIC DESIGN BY PLASTIC ANALYSIS

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ABSTRACT

The purpose of this paper is to briefly illustrate the role of plastic analysis in seismic design of structures. Two examples of the application of plastic design are presented here. In the first example, an existing steel frame designed by elastic method is completely redesigned by using plastic analysis, whereas, in the second example the methodology is applied for the purpose of modifying behavior of the example steel moment frame to form desired yield mechanism. The static and dynamic response results of the modified structure as well as those of the plastic designed frame are compared with those of the original structure designed by conventional elastic method.

KEYWORDS
Seismic design, Plastic analysis and design, Moment frames, Girder web openings

1. INTRODUCTION

It is well recognized that building structures designed by modern code procedures are expected to undergo large cyclic deformations in the inelastic range when subjected to design level severe earthquake ground motions. Nevertheless, most seismic design work around the world at present is carried out by elastic methods using equivalent static design lateral forces. No explicit checks through inelastic analysis are generally required to be made. Therefore, when struck by a severe ground motion, the inelastic activity can be unevenly and widely distributed in the structure resulting in undesirable response and making the repair work after the earthquake much more difficult and costly effort.

An aspect of plastic design that has not received much attention in the past is that structures can be designed to form preselected yield mechanisms at ultimate loads. This has special relevance to seismic design. It is desirable to design structures such that they will behave in a known and predetermined manner during a design earthquake, which essentially means formation of preselected yield mechanism with adequate ductility and strength. The role of plastic analysis in seismic design are illustrated in the following two examples.

2. THE STUDY FRAME

2.1. The Original Frame

A six-story frame is taken as an example of conventionally designed moment frame structures in current practice. The frame is part of the lateral force resisting system of a building located near the epicenter of the January 17, 1994, Northridge Earthquake. The frame suffered significant damage to the welded
moment connections during the earthquake. The building was the subject of an in-depth study by Hart et al. [3]. The bottom story of the frame is below grade with extensive outside and interior basement walls. Therefore, for purposes of this study only the five stories above ground level were considered. A typical three-bay frame along with member sizes is shown in Figure 1. The horizontal drift at design lateral forces satisfied the UBC limits. The UBC design story shear and associated drifts are shown in Table 1.

![Figure 1. Member sizes of the study frame](image)

**TABLE 1**  
**UBC DESIGN STORY SHEARS AND STORY DRIFTS**

<table>
<thead>
<tr>
<th>Story</th>
<th>Story Shear (kips)</th>
<th>Story Drift (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>99.4</td>
<td>0.17</td>
</tr>
<tr>
<td>5</td>
<td>158.0</td>
<td>0.23</td>
</tr>
<tr>
<td>4</td>
<td>202.6</td>
<td>0.20</td>
</tr>
<tr>
<td>3</td>
<td>233.0</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>251.7</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Design Base Shear Coefficient \((V/W) = 0.0884\)

### 2.2 Re-designed Frame by Plastic Method

To illustrate the use of plastic design for seismic loading, the original frame was redesigned by using plastic method to have a selected strong column-weak beam yield mechanism. Conventionally, elastic design method is used by first designing the moment frame to resist the forces prescribed by a building code, UBC for example, and then the member sizes are increased in order to satisfy the drift criteria without explicit check for the yield mechanism of the frame. Unlike in the conventional elastic design, the design of this example frame was done by initially designing the frame to resist a larger load, 3 times the lateral forces prescribed by UBC in this case. This was done to ensure strong column-weak beam behavior as well as to control the drifts by keeping the sizes of the beams and columns in good proportion. The frame in this study was intentionally designed such that the drift limit prescribed by UBC was not satisfied. The drifts could be reduced, if needed, by designing for larger lateral loads such as 4 or 5 times UBC forces. Since the effects from the lateral forces are much larger than the effects from gravity loads, the gravity loads were ignored in the design process.

**Design of Beams**

By selecting a suitable distribution of plastic moment capacity of beams and equating the external work to the internal work, the required beam strength in each story can be determined from:

\[
\sum_{i=1}^{n} F_i h_i = \sum_{i=1}^{n} 2\beta_i M_{pb} + 2M_{pc} \tag{1}
\]

where \(n\) is the number of stories, \(F_i\) is the force at level \(i\), \(h_i\) is the height of level \(i\), \(\beta_i\) is the proportioning factor for beam strength at level \(i\), \(M_{pb}\) is the plastic moment of the reference beam, and \(M_{pc}\) is the plastic moment of columns in the first story. In frames with fixed bases, the value of \(M_{pc}\) must be predetermined.
An appropriate value of $M_p$ should be such that the story mechanism in the first story is prevented. As a first approximation, assuming plastic hinges form at the base and the top of the first story columns, the minimum plastic moment of the first-story columns to prevent this mechanism should be:

$$M_p \geq \frac{1.1Vh_i}{4}$$

(2)

where the factor $V$ is the design base shear, $h_i$ is the height of the first story, and 1.1 is the overstrength factor to account for possible overloading due to strain hardening as explained below. With known value of $M_p$, the required strength of beams can be determined.

**Design of Columns**

In order to ensure a strong column-weak beam yield mechanism, it is important that the columns are designed by assuming that all the beams are fully strain-hardened when the complete mechanism forms. The moment generated by a fully strain-hardened beam must be taken into account by multiplying its nominal plastic moment by a factor called the overstrength factor, $\xi$. By assuming appropriate overstrength factors, generally ranging from 1-1.1, the design moment of the columns can be calculated. In this example, the value of the overstrength factor was taken as 1.02 for the roof beam and 1.05 for the other floor beams. The distribution of moment in the exterior columns of the frame can be found by subjecting the columns to the UBC inverted triangular force distribution, whose magnitude can be found by equating the overturning moment to the moments generated by the floor beams. For this example, the procedure is illustrated in Figure 2. The final member sizes of the plastic designed frame are shown in Figure 3.
2.3. The Modified Frame with Girder Web Openings

The plastic design method can also be used in modifying existing frames to have the desired strong elastic column mechanism. The modification used in this study consisted of creating a rectangular opening in the web near the mid span of each floor girder of the original frame and adding diagonal and vertical web members to provide the desired shear strength. During severe seismic events, the yielding will be limited to the T-shaped chord sections of the opening and the diagonal members only. The frame members outside the openings will remain elastic. The design of openings so that it will behave as intended can be accomplished through a plastic mechanism analysis and the benefits to be gained will be improved structural response and reparability after the earthquake.

The maximum shear strength of the openings in the fully yielded and strain-hardened condition was designed such that the flexural strength ($\phi_0 M_y$) of the end moment connections will not be exceeded. The maximum shear strength of the opening is obtained by multiplying the nominal strength by an overstrength factor which takes into account the expected probable yield strength of the material above the nominal value and increase in strength due to strain-hardening. The overstrength factor was calculated by using the expression suggested by Basha and Goel [1]. Since the overstrength factor is also dependent on the length of the opening and section properties of the T-sections of girder chords at the opening, the design requires some trial and error work. A summary of the design steps is shown in Table 2 and the sizes of member sections in the open segments are given in Table 3 and Figure 4.

<table>
<thead>
<tr>
<th>Beam Size</th>
<th>$\phi_0 M_y$ (kip-in)</th>
<th>V-allowable (kips)</th>
<th>Depth of Chords (in)</th>
<th>Overstrength Factor, $\xi$</th>
<th>V-chord (kips)</th>
<th>Diagonal Members</th>
<th>Vx (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W27x94</td>
<td>10716</td>
<td>71.4</td>
<td>4.00</td>
<td>2.00</td>
<td>29.5</td>
<td>1 1/2x5/8</td>
<td>42.2</td>
</tr>
<tr>
<td>W36x135</td>
<td>19360</td>
<td>129.1</td>
<td>4.25</td>
<td>2.07</td>
<td>41.9</td>
<td>2x3/4</td>
<td>83.0</td>
</tr>
<tr>
<td>W36x150</td>
<td>22226</td>
<td>148.2</td>
<td>4.50</td>
<td>2.08</td>
<td>51.4</td>
<td>1 7/8x7/8</td>
<td>96.3</td>
</tr>
<tr>
<td>W36x210</td>
<td>31708</td>
<td>211.4</td>
<td>4.75</td>
<td>2.00</td>
<td>79.4</td>
<td>2x1</td>
<td>128.0</td>
</tr>
</tbody>
</table>

Notes: 1) All calculations are based on assumed $F_y = 49$ ksi, Span Length $L = 25$ ft, and Length of Opening $L_o = 0.2L = 5$ ft.

2) Overstrength Factor for chords (Basha and Goel 1994)

$$\xi = \frac{0.031 \left( \frac{L - L_o}{L_o} \right) (0.6E_I) + (0.9M_y)}{M_y}$$

3) Shear provided by chords $V_{chord} = \frac{\xi M_y}{L_o/2}$ and by diagonals

4) Shear provided by diagonals $V_d = 1.25 \sin(\theta_v) \left[ Py + 0.3 Pcr \right]$

<table>
<thead>
<tr>
<th>Floor</th>
<th>Beam Size</th>
<th>Opening Length (in)</th>
<th>Depth of Chord Sections (in)</th>
<th>Diagonal Members (inx/in)</th>
<th>Vertical Members</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>W27x94</td>
<td>60</td>
<td>4.00</td>
<td>1 1/2x5/8</td>
<td>2L2x2x3/16</td>
</tr>
<tr>
<td>5</td>
<td>W36x135</td>
<td>60</td>
<td>4.25</td>
<td>2x3/4</td>
<td>2L2x2x3/8</td>
</tr>
<tr>
<td>4</td>
<td>W36x150</td>
<td>60</td>
<td>4.50</td>
<td>1 7/8x7/8</td>
<td>2L2x2x2 1/2x5/16</td>
</tr>
<tr>
<td>3</td>
<td>W36x210</td>
<td>60</td>
<td>4.75</td>
<td>2x1</td>
<td>2L2x2x2 1/2x1/2</td>
</tr>
<tr>
<td>2</td>
<td>W36x210</td>
<td>60</td>
<td>4.75</td>
<td>2x1</td>
<td>2L2x2x2 1/2x1/2</td>
</tr>
</tbody>
</table>
3. DISCUSSION OF INELASTIC ANALYSIS RESULTS

3.1. Inelastic Static "Pushover" Analysis

One-bay, five story models of the original three-bay moment frame, the modified frame with web openings, and the plastic designed frame were prepared for inelastic static ("pushover") analysis. The analysis was carried out by applying lateral forces representing the distribution of UBC design lateral forces. Figure 5 shows the base shear versus roof displacement plot for the three models. Also shown in the figure are the sequences of inelastic activity in the three frames and the 94-UBC design base shear for the structure with the soil factor, S, taken equal to 1.5. The horizontal drift at design lateral forces of the frames satisfied the UBC limits with the exception for the plastic designed frame. All frames possess significant overstrength above the design force level -- six times for the original frame and three times for the modified frame and the plastic designed frame.

Figure 5 shows the sequence of inelastic activity of the three frames under increasing static lateral forces. As can be seen that in the original frame, the first set of plastic hinges to form was at the column base and the yield mechanism was that of story type in the first story, both of which are not considered as good behavior. Early formation of plastic hinges at the column base can mean large ductility demands at a rather critical location and the formation of story mechanism can lead to more serious consequences including collapse in some cases. The plastic designed frame, on the other hand, behaved as designed with the
selected strong column-weak beam mechanism. All plastic hinges were limited only in the beams and at the
column bases which formed at much larger frame drift. Similarly, The modified frame with girder web
openings behaved as intended with inelastic activity essentially limited to the openings in the girders and
minor flexural yielding at the column base forming last as in the plastic designed frame.

\[ (a) \text{Original} \quad (b) \text{Modified} \quad (c) \text{Plastic Design} \]

Legend : ● Plastic Hinge
         - - Yielding and Buckling

Figure 6. Yielding and Mechanism under Static Lateral Forces

3.2. Inelastic Time-history Dynamic Analysis

For the inelastic dynamic analysis, the three models were subjected to the N-S component of the 1978
Miyagi-ken-oki accelerogram with peak ground acceleration equal to 0.4g. This scaled ground motion
represents a ground motion on soft soil whose damped elastic response spectrum matched with the elastic
design spectra specified in the 1994 UBC [1,2]. Results from the inelastic dynamic analysis of the three
frames are presented briefly in Figures 7 through 9. The envelopes of maximum floor displacements of the
three frames are shown in Figure 7a. As can be seen, the floor displacements of both the plastic designed
frame and the modified frame with web openings are consistently smaller than those of the original frame.
The maximum story drifts, shown in Figure 7b are similar, but more significantly, the maximum story
driftsin the first story of the frames designed by plastic method are approximately half of that in the original
frame. This is because a story mechanism in the first story formed in the original frame as seen in Figure
where the inelastic activity in the three frames was shown.

\[ \begin{align*}
a) & \text{Maximum Floor Displacements} \\
b) & \text{Maximum Story Drifts} \\
\end{align*} \]

Figure 7. Response under Miyaki-Ken-Oki Accelerogram (PGA=0.4g)
Figure 8. Inelastic Activity under 1978 Miyaki-Ken-Oki Accelerogram

It can also be noticed from Figure 8 that there is a wide spread distribution of plastic hinges in the original structure with large plastic rotation demand at the column bases. In contrast, the inelastic activity in the plastic design and the modified frame is much more controlled and limited to the designated locations as was intended in the design. Figure 9 shows the time-history of horizontal displacements at the roof levels. Not only are the floor displacements in the modified frame and the plastic designed frame smaller than those in the original structure, but also the excursions with larger amplitude are much fewer in the former. Formation of story mechanism in the first story after ten seconds into the response resulted in larger displacements in later cycles. The results demonstrate that, even though the story drifts under static lateral forces do not satisfy the drift criteria prescribed in UBC as seen in the plastic designed frame, the real response under dynamic loading may be better if the inelastic activity occurs in a control manner following the desired yield mechanism.

4. SUMMARY AND CONCLUSIONS.

Use of plastic analysis in seismic design of structures was briefly discussed in the paper. It was emphasized that it is desirable to design structures such that they will behave in a known and predetermined manner during a design level earthquake. That essentially means designing a structure to form preselected yield
mechanism with adequate strength and ductility. The yield mechanism can be selected to achieve desired structural behavior and performance with considerations of safety, economy, and reparability after the earthquake.

The methodology was applied to an example six story moment frame by redesigning the structure by using plastic design method and also by modifying the original frame by introducing web openings near the mid span of the floor girders. The results of inelastic static "pushover" and dynamic time-history analysis showed that the structure designed by conventional practice behaved in a rather poor manner with wide spread and undesirable distribution of plastic hinges. The plastic designed structure, on the other hand, behaved as intended in a truly strong column-ductile girder combination resulting in better response.

5. REFERENCES


ABSTRACT

An increasing trend in recently published structural design codes is to couple approaches to frame design more closely with functional requirements for the connections. This topic is discussed for the particular example of composite frames designed as non-sway structures. Reference is made to several interlinking pieces of research that have led to a reasonably full understanding of the subject. Their use as the basis for a comprehensive and consistent design approach is reviewed and used to illustrate the sort of research programme that needs to be undertaken before a similarly authoritative view can be formed for composite sway frames that incorporate partial strength and/or semi-rigid connections.

1. INTRODUCTION

Composite construction is widely recognised nowadays as an efficient way of enhancing the structural performance of steelwork by ensuring that, when used in association with concrete, the two materials act as a unit. Such arrangements seek to combine their different properties to provide a more effective overall solution. The best known of these is the simply supported composite beam, in which part of the slab forming a building floor or bridge deck is directly connected to the beam's top flange using shear connectors and bending of the combined cross-section is resisted by compression in the slab and tension in the steel. Thus both materials are used to best advantage, with the inability of the concrete to transmit tension and the weakening effects of buckling due to compression in the steel being avoided.

Following widespread study of the influence of practical forms of steel connection on the behaviour of bare steel frames, pilot studies of the true behaviour of composite beam to column connections and their likely influence on the overall performance of composite frames started to receive increased attention some 10 years ago (1). This followed some earlier, more isolated studies that had indicated the potential for developing significant degrees of continuity through the use of properly detailed composite connections.

For non-sway frames designed to resist static loading the subject has now reached a degree of maturity that has resulted in the development of detailed design procedures for complete frames for both the ultimate and the serviceability limit states.
These approaches give full recognition to the roles played by different facets of the semi-rigid and partial strength nature of the joints in influencing different aspects of the frame's response. To complement frame design, approaches have also been developed to predict the key measures of joint performance for a range of different connection types. This situation will now be reviewed and used as a way of identifying the further studies required for extension to cover sway frames.

2. CONTINUITY AND COMPOSITE CONSTRUCTION

The introduction of continuity into a structural system requires that decisions be taken on the approach used to determine an appropriate set of internal forces for which the various components should be designed. The available choices range from an elastic approach assuming full continuity, through a series of techniques that recognise ductility limitations and utilise varying degrees of moment redistribution to a fully plastic approach. Fig 1 illustrates bending moment distributions for a two-span, uniformly loaded beam corresponding to several different approaches. Whichever of these is used, it is important to realise that each presumes certain matching requirements for the behaviour of the real structure e.g. a continuous elastic analysis requires that joints be capable of transferring the full moment capacity of the members whilst maintaining the original angles between the members essentially unchanged i.e. functioning as "full strength" and "rigid". This concept of more directly linking connection behaviour to the approach used for overall frame analysis and design is a feature of several of the most recently issued structural codes.

The question of compatibility between the actual response of the structure and the method of structural analysis adopted is more acute for composite structures due to the differences in hogging and sagging moment capacities of composite beams. Elastic analysis is clearly inappropriate, whilst simple plastic analysis imposes severe restrictions on the properties of the steel section in order that local buckling does not impair the rotation capacities needed to provide the required amounts of moment redistribution. This leads to the use of more limited moment redistribution linked to the levels of rotation capacity that can readily be achieved as potentially providing the best balance between economy and structural efficiency. Including consideration of the joints - in terms of their moment capacity and rotation capacity - into this process increases the options available to the designer. Moments in hogging regions may be limited through a suitable choice of connection, the rotation capacity needed to redistribute moments into span regions may be provided by the connections thereby eliminating concerns about avoiding local buckling, with the whole process taking due account of the practical difficulties of actually constructing the connection so that economic arrangements are selected (2).

3. ROTATIONAL REQUIREMENTS AND MOMENT REDISTRIBUTION

A method for calculating the rotational requirements of the support sections of continuous beams and frames has recently been developed (3). It recognises contributions from both the elastic and plastic components of member curvature, allows for different member stiffnesses in the cracked and uncracked regions, covers several different patterns of moment and arrives at sets of explicit relationships (4) through carefully appraising a comprehensive series of numerical results for each of the 6 cases of Fig 2. Table 1, which lists the resulting expressions for $\theta_\alpha$, shows that $\theta_\alpha$ is principally dependant on:

- Relative moment ratio $\frac{(M_d - M_p)}{(M_p - M_s)}$
Support to span moment ratio \( \frac{M^1}{M_s} \)

in which

- \( M_p \) = plastic moment of resistance of span section
- \( M_y \) = yield moment of resistance of span section
- \( M_d \) = design value of moment for span section
- \( M^1 \) = support moment at point where \( \theta \) is required

Clearly, the designer has considerable influence over the value of \( \theta \), that must be provided through his selection of \( M^1 \); reducing this below \( M_p \) leads to progressively more readily achievable situations. Table 2 summarises a set of design calculations undertaken as a way of exploring the range of \( \theta \), values likely to be encountered in various practical situations. Note how the requirements drop sharply if \( M^1 \) is reduced from \( M_p \) to 95% of \( M_p \); it should be appreciated that this relaxation requires a considerably smaller reduction in actual load in the beam.

![Distribution of Bending Moment in a Uniform Loaded Two-span Beam](image)

Fig. 1 Distribution of Bending Moment in a Uniform Loaded Two-span Beam
Table I
Equation List for Required Rotation Calculation

<table>
<thead>
<tr>
<th>Load and support conditions</th>
<th>Equations for the required rotation of the support with a moment $M'$ $(R' = M'/M_d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eqn (i) uniformly distributed load with $M' = M''$ with $y = 0.991$</td>
<td>$\theta_r = \left(0.344 - 0.212 R' + 0.561 \left(\frac{M_d - M_y}{M_p - M_y}\right)^2 \frac{1}{\sqrt{1 + R'}}\right) \frac{M_d L}{EI}$</td>
</tr>
<tr>
<td>Eqn (ii) uniformly distributed load with $M'' = 0$ with $y = 0.990$</td>
<td>$\theta_r = \left(0.344 - 0.225 R' + 0.556 \left(\frac{M_d - M_y}{M_p - M_y} \frac{L}{M_p}\right)^2 \frac{1}{\sqrt{1 + R'}}\right) \frac{M_d L}{EI}$</td>
</tr>
<tr>
<td>Eqn (iii) two-point load with $M' = M''$ with $y = 0.985$</td>
<td>$\theta_r = \left(0.344 - 0.211 R' + 1.144 \left(\frac{M_d - M_y}{M_p - M_y} \frac{L}{M_p}\right)^2 \frac{M_s}{M_s (1 + R')}\right) \frac{M_d L}{EI}$</td>
</tr>
<tr>
<td>Eqn (iv) two-point load with $M'' = 0$ with $y = 0.992$</td>
<td>$\theta_r = \left(0.344 - 0.270 R' + 1.091 \left(\frac{M_d - M_y}{M_p - M_y} \frac{L}{M_p}\right)^2 \frac{M_s}{M_s (1 + R')}\right) \frac{1}{EI}$</td>
</tr>
<tr>
<td>Eqn (v) one-point load with $M' = M''$ with $y = 0.987$</td>
<td>$\theta_r = \left(0.255 - 0.300 R' + 0.722 \left(\frac{M_d - M_y}{M_p - M_y} \frac{L}{M_p}\right)^2 \frac{1 - M_s}{M_s (1 + R')}\right) \frac{M_d L}{EI}$</td>
</tr>
<tr>
<td>Eqn (vi) one-point load with $M'' = 0$ with $y = 0.978$</td>
<td>$\theta_r = \left(0.255 - 0.252 R' + 0.610 \left(\frac{M_d - M_y}{M_p - M_y} \frac{L}{M_p}\right)^2 \left(1 - \frac{M_s}{M_d}\right)\right) \frac{M_d L}{EI}$</td>
</tr>
</tbody>
</table>
The most usual arrangements are likely to conform to the ranges: ratio of positive to negative beam moment capacity of 1.1 - 1.5; connection moment capacities of between 30% and 70% of span moment capacity; leading to $M'/M_p$ being within the range 0.2 and 0.7. This gives figures of 30 m Rad and 20 m Rad as indicative safe limits for use with $M_p/M_p$ of 1.0 or 0.95 respectively.
Table 2
Predicted Required Rotations

<table>
<thead>
<tr>
<th>Series</th>
<th>$M_d/M_p$</th>
<th>DMR</th>
<th>$M'/M_d$</th>
<th>Required rotation (mRad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>average</td>
</tr>
<tr>
<td>1-1</td>
<td>1</td>
<td>86%</td>
<td>0.10</td>
<td>29.17</td>
</tr>
<tr>
<td>1-2</td>
<td>1</td>
<td>75%</td>
<td>0.20</td>
<td>27.79</td>
</tr>
<tr>
<td>1-3</td>
<td>1</td>
<td>65%</td>
<td>0.30</td>
<td>26.48</td>
</tr>
<tr>
<td>1-4</td>
<td>1</td>
<td>57%</td>
<td>0.40</td>
<td>25.24</td>
</tr>
<tr>
<td>1-5</td>
<td>1</td>
<td>50%</td>
<td>0.50</td>
<td>24.05</td>
</tr>
<tr>
<td>1-6</td>
<td>1</td>
<td>44%</td>
<td>0.60</td>
<td>22.90</td>
</tr>
<tr>
<td>1-7</td>
<td>1</td>
<td>38%</td>
<td>0.70</td>
<td>21.79</td>
</tr>
<tr>
<td>2-1</td>
<td>0.95</td>
<td>86%</td>
<td>0.10</td>
<td>18.45</td>
</tr>
<tr>
<td>2-2</td>
<td>0.95</td>
<td>75%</td>
<td>0.20</td>
<td>17.53</td>
</tr>
<tr>
<td>2-3</td>
<td>0.95</td>
<td>65%</td>
<td>0.30</td>
<td>16.64</td>
</tr>
<tr>
<td>2-4</td>
<td>0.95</td>
<td>57%</td>
<td>0.40</td>
<td>15.77</td>
</tr>
<tr>
<td>2-5</td>
<td>0.95</td>
<td>50%</td>
<td>0.50</td>
<td>14.92</td>
</tr>
<tr>
<td>2-6</td>
<td>0.95</td>
<td>44%</td>
<td>0.60</td>
<td>14.08</td>
</tr>
<tr>
<td>2-7</td>
<td>0.95</td>
<td>38%</td>
<td>0.70</td>
<td>13.25</td>
</tr>
</tbody>
</table>

Note: 1. The figures shown in the table are the statistical results for 68 beams.
2. The following values are used for the composite beam: $L = 20$, $B_i = 0.175L$, rebar ratio = 1%, $H_c = 130$ mm, $B_i = 0.125L$, $H_d = 46$ mm, $f_{cu} = 30$ N mm$^2$, $f_s = 275$ N mm$^2$.
3. Full shear interaction is assumed.
4. DMR—degree of moment redistribution.

It should be noted from Table 2 that these design requirements are associated with quite significant levels of redistribution of support moments i.e. the design moment diagram is approaching the upper limit of the fully plastic distribution. By re-arranging the results of Table 2 the set of explicit relationships for $M'/M_d$ of Table 3 may be obtained. These permit the direct calculation of the support to span moment ratio for a known level of support rotation capacity $\theta_s$. Alternatively, using a further set of relationships given in ref 4, the results may be expressed directly in terms of the percentage of moment redistribution. Taking $\theta_s$ as 20 m Rad, figures of between 30% and 60% would appear to be readily achievable for the practical range of beam and joint properties indicated previously.
4. ROTATION CAPACITY AND MOMENT CAPACITY

In addition to an appreciation of the demands for certain levels of connection performance that follow directly from the analyses of the previous section, the implementation of a composite frame design methodology that recognises the important role of the connection characteristics requires that these characteristics be capable of calculation in a reasonably straightforward manner. Thus complementary studies have been undertaken to devise methods for the determination of:

- Moment capacity $M_c$
- Rotation capacity $\phi_c$
- Initial rotational stiffness $K_i$

Although the third of these ($K_i$) is not actually required for the ULS calculations, it is needed when attempting to produce reasonably rigorous estimates of serviceability deflections; it is also needed if strength design is undertaken on an elastic basis (5).

The basis for the prediction model for each of $M_c$, $\phi_c$ and $K_i$ is a consistent representation of the force transfers, deformation compatibilities and load-deflection characteristics of the various individual components in the connection identified (for the particular case of an endplate arrangement) in Fig 3. Full details of the design procedures for $M_c$ for endplates (6), finplates and cleats (7), as well as for $\phi_c$ and $K_i$ for endplates (8) are available. The design procedures have been fully validated against all known suitable test results. In addition, the preparation of a comprehensive numerical solution using the nonlinear ABAQUS F.E. package (9), that has itself been carefully verified using detailed experimentally obtained test histories, has enabled several

<table>
<thead>
<tr>
<th>Load and support conditions</th>
<th>Equations for the minimum support-to-span moment ratios ($M_c / M_d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eqn (i) for uniform distributed load with $M = M_c$</td>
<td>$M = 1.0417 + 2.359 \frac{a_c E I}{M_d L} + \sqrt{3.160 - 8.532 \frac{a_c E I}{M_d L} - 5.566 \left( \frac{a_c E I}{M_d L} \right)^2 + 0.423 \left( \frac{M_c - M}{M_d} \right)^2} \left( \frac{M_c - M}{M_d} \right)$</td>
</tr>
<tr>
<td>Eqn (ii) for uniform distributed load with $M' = 0$</td>
<td>$M = 0.464 + 2.222 \frac{a_c E I}{M_d L} + \sqrt{3.971 - 8.856 \frac{a_c E I}{M_d L} - 4.938 \left( \frac{a_c E I}{M_d L} \right)^2 + 5.968 \left( \frac{M_c - M}{M_d} \right)^2} \left( \frac{M_c - M}{M_d} \right)$</td>
</tr>
<tr>
<td>Eqn (iii) for two-point load with $M' = M_c$</td>
<td>$M = 1.330 + 0.733 \frac{a_c E I}{M_d L} - 5.422 \left( \frac{M_c - M}{M_d} \right)^2 \left( \frac{M_c - M}{M_d} \right)$</td>
</tr>
<tr>
<td>Eqn (iv) for two-point load with $M' = 0$</td>
<td>$M = 0.497 - 1.652 \frac{a_c E I}{M_d L} + \sqrt{0.604 - 2.679 \frac{a_c E I}{M_d L} + 3.439 \left( \frac{a_c E I}{M_d L} \right)^2 \left( \frac{M_c - M}{M_d} \right)^2 + 1.347 \left( \frac{M_c - M}{M_d} \right)^2 \left( \frac{M_c - M}{M_d} \right)}$</td>
</tr>
<tr>
<td>Eqn (v) for one-point load with $M' = M_c$</td>
<td>$M = 0.075 + 1.667 \frac{a_c E I}{M_d L} + \sqrt{0.856 - 3.083 \frac{a_c E I}{M_d L} - 2.779 \left( \frac{a_c E I}{M_d L} \right)^2 + 2.409 \left( \frac{M_c - M}{M_d} \right)^2 \left( \frac{M_c - M}{M_d} \right)} \left( \frac{M_c - M}{M_d} \right)$</td>
</tr>
<tr>
<td>Eqn (vi) for one-point load with $M' = 0$</td>
<td>$M = 1.012 - 3.969 \frac{a_c E I}{M_d L} - 2.421 \left( \frac{M_c - M}{M_d} \right)^2 \left( \frac{M_c - M}{M_d} \right)$</td>
</tr>
</tbody>
</table>
additional specific effects to be examined (10, 11). The design procedures are available in the form of explicit relationships that link all the key physical properties of the connection e.g. amount of reinforcement: bolt number, position and strength; beam and column dimensions etc. This has facilitated studies of readily achievable levels of performance as well as the identification of sensible practical limits for use at both the scheme and final stages of frame design.

(a) Joint with load

(b) Free body diagram of the connecting parts

Note: $M_{c2} = \eta M$

Figure 3
Non-symmetrically Loaded Composite Flush Endplate Connection with Internal Forces
5. DESIGN IMPLICATIONS

The procedures set out above provide the designer with significantly enhanced options since the inclusion of connection properties $M$ and $\Phi$, allow a wider range of design moment patterns to be selected. More importantly, achievable connection properties can first be decided upon and the most appropriate degree of moment redistribution consistent with these selected - rather than try to configure connections to deliver impossibly high levels of performance. Some indication of this increased choice is provided by the example of Fig 4. Five different design options have been selected - 3 traditional approaches plus 2 partial strength alternatives - and the key results are given in Table 4.

![Figure 4](image)

**Figure 4**
Two-span Sub-frame

**Table 4**
Summary of Design Options for Example of Fig 4

<table>
<thead>
<tr>
<th></th>
<th>$M_{1e}$ (kN.m)</th>
<th>$M_{1c}$ (kN.m)</th>
<th>$M_{2c}$ (kN.m)</th>
<th>$M_{2e}$ (kN.m)</th>
<th>$\theta_{1cpl}$ (m Rad)</th>
<th>$\theta_{2cpl}$ (m Rad)</th>
<th>$q$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic continuous</td>
<td>103</td>
<td>324</td>
<td>334</td>
<td>457</td>
<td>0</td>
<td>0</td>
<td>53.2</td>
</tr>
<tr>
<td>Simply supported</td>
<td>273</td>
<td>0</td>
<td>614</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>60.7</td>
</tr>
<tr>
<td>Plastic continuous</td>
<td>481</td>
<td>350</td>
<td>614</td>
<td>457</td>
<td>18.4</td>
<td>24.3</td>
<td>81.7 (143.2)</td>
</tr>
<tr>
<td>Partial Strength</td>
<td>481</td>
<td>200</td>
<td>614</td>
<td>320</td>
<td>21.8</td>
<td>27.6</td>
<td>75.7 (128.2)</td>
</tr>
<tr>
<td>Quasi-plastic</td>
<td>457</td>
<td>200</td>
<td>584</td>
<td>320</td>
<td>15.8</td>
<td>19.7</td>
<td>72.6 (122.8)</td>
</tr>
</tbody>
</table>
Although the fully plastic option gives the highest load carrying capacity, this is accompanied by demands for full strength connections and significant rotation capacity within the beam cross-section. The two partial strength options show how for either a 7% or an 11% reduction in load more easily achievable connections may be employed. Both arrangements provide substantially more capacity than the conventional simply supported approach.

6. SERVICEABILITY DESIGN

Spurred by the almost universal introduction of limit states design codes, structural research has concentrated heavily on developing better understanding of the ultimate limit state. However, many practical design situations are controlled by the need to limit deflections at working load levels, a requirement that is often accentuated in composite construction with its more efficient use of the combined cross-section to provide moment capacity. Utilisation of joint stiffness when assessing serviceability deflections provides a potentially helpful means of alleviating this situation. Once again, the emphasis is on recognising and using an effect that is present and not necessarily on the making of changes to the form of construction.

Analyses based on the use of a sub-frame approach (5) have shown how the reduction in mid-span beam deflection due to varying degrees of rotational end restraint may be simply obtained from the design chart of Fig 5. This uses two parameters: $\beta$ the ratio of $I$ for the composite section to $I$ for the bare steel beam and $R$ a measure of relative stiffness calculated from a knowledge of connection stiffness $K$, and beam and column properties. Trial calculations suggest that practical arrangements will normally give $R$-value in excess of 5 as indicated by Fig 6, for which the curves become sensibly horizontal. Noting that deflection calculations are normally only required on the basis of a demonstration that a limit is not exceeded i.e. great accuracy is not normally required, a reasonable assumption that end restraint reduces beam deflections to one third of the figure calculated assuming simple supports appears appropriate. For those cases where this simple rule indicates a lack of compliance with the design limit, the full procedure of ref 5 may be used to obtain a more accurate estimate.

![Figure 5](image)

**Figure 5**

Midspan Deflection Ratio $\frac{\Delta_w}{\Delta_{w0}}$ vs. Connection-to-beam Stiffness Ratio $R$ (for UDL)
MOMENT REDISTRIBUTION AND JOINT DETAILING ISSUES

7. SWAY FRAMES

All of the foregoing relates to structures designed on the basis that some form of bracing system will be provided to resist horizontal loading so that the main framing need only be designed for gravity loading. Studies of bare steel frames (13) have, however, clearly demonstrated that partial strength/semi-rigid joints may also be used with advantage in place of more costly full strength/rigid connections in sway frames. The principle is, of course, rather different: for the sway frame a reduced level of structural performance is being proposed.

In order to facilitate the development of a comprehensive and soundly based design methodology of the type outlined above for the non-sway case, it would appear that a considerable amount of basic study is required. Comparatively little work has so far been undertaken to investigate the particular issues that complicate the situation for sway frames. In particular, the following (that to some extent parallel the development for non-sway frames) require consideration:

- Tests on composite connections under reversed bending i.e. slab in comparison.
- Extension of F.E. modelling (9) of composite connections to include behaviour under reversed bending.
- Consideration of the influence of angular movement due to sway on connection properties.
- Development of design models to predict the main connection characteristics under reversed loading i.e. the equivalent of $M_0$, $K_1$, and $\phi_s$ for the non-sway case.
- Development of a prediction method for required connection rotations under realistic loading regimes.
- Extension of F.E. modelling (14) of complete frame response to cover sway deformations.

Whilst these tasks appear relatively straightforward in principle, experience with the non-sway investigations suggests that their resolution will require considerable ingenuity.

8. CONCLUSIONS

The behaviour of non-sway frames with semi-rigid and/or partial strength connection has been reviewed in the context of several recently completed research studies. Their use as the basis for a
full design treatment has been explained. The research tasks that must be completed before a similarly authoritative approach can be derived for sway frames have been identified.

9. ACKNOWLEDGEMENTS

Several individuals have contributed to the work at Nottingham on which much of the content of this paper is based. These include: Drs B Ahmed, B S Choo, T Q Li, and Y Xiao, Professor Y Mei and Mr J Hensman. The group have also benefited from exchanges with Professor D Anderson of Warwick University, Drs R M Lawson and G Couchman of the SCI, Dr C Gibbons of Ove Arup and Dr D B Moore of BRE. Financial assistance has been provided by: BRE, DTI, SCI and SERC.

REFERENCES

4 PLATES AND PLATED STRUCTURES
ULTIMATE STRENGTH OF BIAXIALLY LOADED PLATES

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ABSTRACT

An approximate and simple method to predict the post-buckling and collapse behaviour of plates loaded axially along two mutually perpendicular directions is presented. Effects of initial imperfection, plate slenderness, aspect ratio, boundary conditions and varying rates of strain in the two directions on the load-carrying capacity are accounted for in the method. Model tests on biaxially compressed square plates on a purpose-built test rig are described. Results obtained from the experiments are compared with the corresponding theoretical values using the proposed method in order to ascertain the accuracy of the method. Additional comparisons with nonlinear finite element results are also presented. Interaction curves for square plates subjected to stresses in the longitudinal and transverse directions, applicable to typical boundary conditions, are presented.

KEYWORDS

Plates, in-plane load, biaxial load, energy method, ultimate load, experiment, interaction curves.

1. INTRODUCTION

Plate elements in structures such as double bottoms of ship hulls, box-girder bridges, dock gates and offshore platforms are subjected to biaxial in-plane loading. A typical double bottom in a ship hull consists of inner and outer platings connected by thin webs which run longitudinally and transversely. The longitudinal compressive stresses in the hull plates are introduced due to the longitudinal hogging of the entire ship; transverse stresses are a consequence of bending of hull under the action of hydrostatic pressure from below. A similar state of stress occurs in certain box-girder flange plates. These plate panels carry longitudinal in-plane compression caused by the sagging moment in the box-girder and transverse in-plane compression consequent on these panels acting as the compression flanges of the cross girder.

Information available in published literature on the behaviour of plates subjected to biaxial compression is limited to a few publications (Becker et al., 1970; Dowling, 1974; Smith, 1975; Davidson et al, 1989). Both numerical and experimental investigations have been carried out at the Imperial College, London during the later part of the 1970s and early part of the 1980s to study the behaviour of ships' plating under combined action of lateral loading and biaxial compression (Dowling et al., 1979, Dier and Dowling, 1980, 1984, Dier, 1987). Biaxially applied in-plane compressive and tensile forces were also considered and the effects of aspect ratio, slenderness
and initial imperfections (consisting of geometric deformations and residual stresses) were investigated. Design methods employing interaction curves were proposed for longitudinally stiffened steel plates under biaxial compression that occur in steel decks of cable-stayed and arch bridges (Fruta et al., 1988; Kitada et al., 1990; Fruta et al., 1991). An approximate and simple method to evaluate the strength of such plates has been proposed earlier by the authors (Narayanan and Shanmugam, 1983). The method is based on the Energy Concept developed earlier for uniaxially loaded plates (Horne and Narayanan, 1976; Narayanan and Shanmugam, 1980).

The present paper is concerned with some results of biaxially loaded plates obtained using the method. Plates with simple support and clamped conditions on all sides are considered in this study. An experimental investigation carried out to study the elastic and ultimate load behaviour of square plates subjected to biaxial loading is described. The test specimens have been analysed using the proposed method and the results thus obtained are compared with the corresponding observed strengths. In addition, the accuracy of the method is also ascertained by comparing the results with nonlinear finite element results (ABAQUS, 1991) and the method proposed by Coombs (1975). The parameters studied include plate aspect ratio (a/b), initial imperfection of the plate, ratio of strain increment in the x and y directions (εx / εy) and plate slenderness (b/t).

2. ASSUMPTIONS

Plate panels that occur commonly in bridge and marine structures are supported on all four sides and generally they do not have slenderness (b/t) values in excess of 70; the computations have, however, been continued for values beyond 70. In carrying out the analysis it is assumed that (i) the material of the plate is homogeneous, isotropic, elastic and thereafter perfectly plastic, strain hardening effect is neglected; (ii) all edges of the plate are held straight in plane and out-of-plane but are free from restraining or applied moments; (iii) membrane shear stresses on planes parallel to the edges of the plate panel are zero; (iv) the number and length of half-waves of buckling are the same in post-buckled stages as at incipient buckling; the final buckled shape in the post-buckling stage is identical with the infinitely small buckles developed at incipient buckling; (v) the second order membrane strains are assumed to be entirely dependent upon the out-of-plane displacements (although this assumption violates equilibrium and compatibility conditions locally, the integrated effects of this violation would be negligible. The results obtained from the proposed analysis are satisfactory), (vi) the magnitude of displacements in the x and y directions are very small in comparison with transverse deflection and are neglected in calculating the energy of the system.

3. ANALYSIS

The plate shown in Figure 1 can be assumed to have an initial imperfection given by

\[ w_0 = A_0 f(y)g(x) \]

in which \( A_0 \) is a coefficient defining the amplitude of the initial imperfection. \( f(y) \) and \( g(x) \) are deflection functions in terms of \( y \) and \( x \), respectively, which are chosen in the form of trigonometric functions to suit the conditions of support along the longitudinal and transverse edges. \( A_0 \), the initial imperfection coefficient, can be related to the maximum value of the initial imperfection, \( \delta_0 \), in each of the cases considered by satisfying the boundary conditions of the plates. Due to the action of applied loads, the deflected shape of the plate is assumed to be described by the equation

\[ w = Af(y)g(x) \]

in which \( A \) is a coefficient defining the deflection amplitude. Suitable functions for \( f(y) \) and \( g(x) \) can be assumed such that they satisfy the boundary conditions of the plate and the analysis can be carried out by using the energy concept. The analysis involves non-linear equations which cannot be solved to obtain closed form equations; numerical techniques can, therefore, be adopted to obtain expressions for \( \sigma_x \) and \( \sigma_y \). Expressions thus derived
for typical boundary conditions are as follows.

**Case 1: Simply supported on all four edges**

It can be assumed for this boundary condition that the functions \( f(y) \) and \( g(x) \) can be chosen as

\[
g(x) = \sin \frac{p\pi x}{a}, \quad f(y) = \sin \frac{q\pi y}{b}\]

in which \( p \) and \( q \) are the number of half-wave lengths in the \( x \) and \( y \) directions, respectively; \( a \) and \( b \) are the length and width of the plate, respectively. The deflection function will, therefore, take the form

\[
w = A \sin \frac{p\pi x}{a} \sin \frac{q\pi y}{b}\]

The total stresses acting in the \( x \) direction is the sum of the stress \( (\sigma_x) \) in the \( x \) direction due to the applied strain and the induced stress \( (\sigma_{xa}) \) due to loading in the \( y \) direction, i.e.

\[
\sigma_x = \sigma_{xa} + \frac{(q\pi)^2 E A_0^2 (m^2 - 1)}{8b^2} \cos \frac{2q\pi y}{b}
\]

\[
\sigma_x = \frac{(p\pi)^2 E A_0^2 (m^2 - 1)}{8a^2} \cos \frac{2p\pi x}{a}
\]

The energy method can be applied to the plate and, \( \sigma_{xa} \) and \( \sigma_{ya} \) can be determined by solving numerically the resulting equations given below:

\[
p^2 \sigma_x d\varepsilon_x + \frac{q^2}{\alpha} d\varepsilon_y = \frac{(p^2 \alpha^2 + q^2)^2}{\alpha} \left( \frac{t}{b} \right) \frac{2}{E} \frac{\sigma_x}{m^2} + \frac{3\pi^2}{8} \left( \frac{A_0}{b} \right)^2 \left( \frac{\alpha^4 p^4 + q^4}{\alpha} \right) m dm
\]

\[
- \frac{v}{E} \left\{ \frac{\alpha p^2}{a} \int_0^a \sigma_y dx + \frac{q^2}{ab} \int_0^b \sigma_{xa} dx \right\}
\]

\[
d\sigma_{xa} = E \left\{ d\varepsilon_x - \frac{(p\pi)^2 A_0^2 m dm}{4a^2} + \frac{v}{aE} \int_0^a \sigma_y dx \right\}
\]
where \( m = A/A_0 \), \( \alpha = b/a \), \( \epsilon_x \) and \( \epsilon_y \) are the strain in the \( x \) and \( y \) directions, respectively and 
\( \sigma_e = \pi^2 E / 12 (1 - \nu^2) \).

Case 2: Clamped on all four edges

Considering an approximately square plate with all its four edges fixed, and loaded along each of them, we may assume only a single half-wave buckled mode, i.e. \( q = p = 1 \). The deflection function for a buckled mode of a half-wave is

\[
w = A \left( 1 - \cos \frac{2\pi x}{a} \right) \left( 1 - \cos \frac{2\pi y}{b} \right).
\]

The general equations for strain and stress are given by:

\[
\alpha \, \epsilon_x + \frac{1}{\alpha} \epsilon_y = (3\alpha^2 + 3 + 2\alpha) \left( \frac{t}{b} \right)^2 \frac{\epsilon_x \, dm}{\epsilon_y \, m^2} + \frac{35}{6} \pi^2 \left( \frac{A_0}{b} \right)^2 \left( \alpha^2 + \frac{1}{\alpha} \right) \frac{m \, dm}{\alpha} - \frac{\nu}{E} \left[ \frac{\alpha}{a} \int_{0}^{a} \sigma_{yx} \, dx + \frac{1}{b} \int_{0}^{b} \sigma_{xx} \, dy \right]
\]

\[
\sigma_{xx} = E \left\{ \epsilon_x - \frac{3\pi^2 A_0^2 \, m \, dm}{a^2} \right\} + \frac{\nu}{aE} \int_{0}^{a} \sigma_{yx} \, dx \right\}
\]

\[
\sigma_{yy} = E \left\{ \epsilon_y - \frac{3\pi^2 A_0^2 \, m \, dm}{b^2} \right\} + \frac{\nu}{bE} \int_{0}^{b} \sigma_{xx} \, dy \right\}
\]

By solving numerically the three equations given above for \( \sigma_{xx} \) and \( \sigma_{yy} \), the stresses \( \sigma_x \) and \( \sigma_y \) at any section can be obtained from

\[
\sigma_x = \sigma_{xx} + (m^2 - 1) \frac{\pi^2 E A_0^2}{a^2} \left\{ \frac{3}{2} \left( 1 - \cos \frac{2\pi y}{b} \right) \right\}^2
\]

\[
\sigma_y = \sigma_{yy} + (m^2 - 1) \frac{\pi^2 E A_0^2}{b^2} \left\{ \frac{3}{2} \left( 1 - \cos \frac{2\pi x}{a} \right) \right\}^2
\]
Case 3: Clamped on two opposite edges with other two opposite edges simply supported

Consider an approximately square plate simply supported at \( x = 0, \ x = a \), clamped at \( y = 0, \ y = b \). We may assume only a single half-wave buckled mode, i.e. \( q = p = 1 \). The deflection function for a buckled mode of a half-wave would be:

\[
w = A \sin \frac{\pi x}{a} \left( 1 - \cos \frac{2 \pi y}{b} \right)
\]

The general equations for strain and stress are given by:

\[
\frac{3}{4} \alpha \frac{d\varepsilon_x}{dx} + \frac{1}{\alpha} \frac{d\varepsilon_y}{dy} = (3\alpha^3 + \frac{16}{\alpha} + 8\alpha) \left( \frac{\varepsilon_x}{2} \right) \frac{d\sigma_x}{dm} + \frac{\pi^2}{E} \left( \frac{A_0}{b} \right)^2 \left( \frac{35}{32} \alpha^3 + \frac{3}{2\alpha} \right) m dm
\]

\[
\sigma_{xa} = E \left\{ \frac{d\varepsilon_x}{dx} - \frac{3\pi^2 A_0^2 m dm}{4a^2} \right\} + \frac{\nu}{aE} \int_0^a \frac{d\sigma_{xa}}{dx} dx
\]

\[
\sigma_{ya} = E \left\{ \frac{d\varepsilon_y}{dy} - \frac{\pi^2 A_0^2 m dm}{b^2} \right\} + \frac{\nu}{bE} \int_0^b \frac{d\sigma_{ya}}{dy} dy
\]

By solving numerically the three equations given above for \( \sigma_{xa} \) and \( \sigma_{ya} \), the stresses \( \sigma_x \) and \( \sigma_y \) at any section can be obtained from

\[
\sigma_x = \sigma_{xa} + (m^2 - 1) \frac{\pi^2 E A_0^2}{4a^2} \left\{ \frac{3}{2} \left( 1 - \cos \frac{2\pi y}{b} \right) \right\}^2
\]

\[
\sigma_y = \sigma_{ya} + (m^2 - 1) \frac{\pi^2 E A_0^2}{2b^2} \cos \frac{2\pi x}{a}
\]

Similar equations can be obtained for plates with any other boundary conditions. In this paper, results for approximately square plates having all sides simply supported, or all sides clamped or two opposite edges clamped with two other opposite edges simply supported are presented.

4. STRESS CRITERION

As the state of stress in the plate is biaxial, rather than uniaxial, it is necessary to define a limiting stress criterion which incorporates the two stress components acting on an element of the plate material. This criterion must be adequate to determine that the plate element has yielded. It is usual to employ the following semi-empirical stress criterion proposed by Von Mises for ductile materials.
An element is considered to yield when the equivalent stress reaches the yield stress \( \sigma_y \) of the material in
uniaxial tension (i.e. \( \sigma_{eq} = \sigma_y \)). The analysis presented in the paper is, however, based on the Energy method
with an assumed deflection surface; such an analysis is known to give an upper bound solution. To allow
approximately for this, a similar but conservative yield criterion proposed by Haig and Beltrami (Timoshenko,
1960) has been adopted in this analysis:

\[
\sigma_{eq} = (\sigma_x + \sigma_y - 2v \sigma_x \sigma_y)^{1/2}
\]

5. EXPERIMENTAL INVESTIGATION

In order to obtain verification of the accuracy of the proposed method and to examine more directly the collapse
and post-peak behaviour of the plates, an experimental programme was undertaken by Chow (1983) in
collaboration with the authors.

5.1 Test Rig

A test rig which is capable of testing individual plates in uniaxial compression or biaxial compression was
fabricated and the details of the rig is shown in Figure 2. It was designed to apply a maximum axial load of 90
kN in two perpendicular directions \( (x \text{ and } y) \) in the horizontal plane on specimens up to 300 mm wide and 300
mm long. The rig was desk-mounted and self-contained and could be used for testing small-scale models in a
laboratory. No strong floors or auxiliary frame works were needed. The rig consisted of two main channels \( (a) \)
of size 178 mm x 89 mm in cross-section and 800 mm long, which were fastened on the base channels \( (C) \) of size
76 mm x 38 mm in cross-section and 900 mm long by sixteen 10 mm diameter black bolts. The webs of channels
\( (A) \) were reinforced with plates 840 mm x 180 mm x 10 mm thick. The test specimen was loaded longitudinally
through bearing blocks \( (M \text{ and } N) \) containing sets of needle bearings to provide simply supported edges. The
"simply supported" loaded condition was obtained by using a number of slotted semi-circular bars \( (Z) \) in half
circle needle bearings which were mounted on to the bearing block \( (E) \) which were bolted to the two loading
channels \( (B \text{ and } D) \). The longitudinal test load in the \( x \) direction was applied to the loading channel \( (B) \) by one
45 kN jack \( (Q2) \), operated manually by a piston pump to give controlled longitudinal deformation under
longitudinal compression. For plates which were expected to have strengths in excess of 45 kN, two identical
jacks connected to a single piston pump to give equal longitudinal deformation were used. The loading channel
\( (D) \) at the opposite end took the reaction from the plate specimen under test.

Channels \( (B \text{ and } D) \) were free to slide between two high strength horizontal bars \( (F) \), which had one end rigidly
fixed to the channels \( (A) \) and the other end rigidly fixed on the column \( (G) \). (The bars \( F \) were not continuous
from channel \( (B) \) and channel \( (D) \), so that a free space for mounting a fixture is available for biaxial loading).
When tests on fixed ended mode were required, the needle bearing sets were removed, and the slotted semi-
circular supporting bars \( (Z) \) were bolted to bearing blocks \( (E) \). A uniform application of displacement to channel
\( (B) \) enabled the plate specimen to be tested in a simply-supported or fixed mode. Additional attachments are
required for the loading in \( y \) direction in the case of biaxial loading. The force applied was taken by two thick
plates \( (K \text{ and } I) \) which were fastened on to the 15 m thick base plate \( (J) \) fixed on to the base channels \( (C) \), at any
desired position appropriate to the size of plate specimens tested.

The transverse edges of the specimen under test were also loaded through bearing blocks \( (N, M) \) similar to \( (E) \).
Bearing block \( (N) \) was bolted to a solid loading block \( (O) \) which was free to slide on sets of needle bearings for
a limited length of 20 mm. This assembly was mounted on a pair of rails \( (T) \) to ensure vertical stability of the
assembled components. The load in the \( y \) direction was applied to solid block \( (O) \) by a 45 kN jack \( (Q1) \). For
biaxial compression with applied load \( P_x \) and \( P_y \), the jacks \( (Q1) \) and \( (Q2) \) were connected to the same pump.
Two separate pumps were used when \( P_x \) was different from \( P_y \). The plate \( (I) \) rigidly mounted to the base \( (J) \) took
the reaction from the plate panel under loading in \( y \) direction. The loads applied were measured by two load cells
(P1) and (P2) positioned between the rams of jacks and the channel (B) or the block (O). The jacks had cylindrical end bearings (R) to ensure central loading.

Figure 2: Details of the Test Rig

5.2 Instrumentation

Axial shortening of the specimens was measured by a pair of horizontal transducers (L). The vertical (out-of-plane) displacements of the plate under test, at various points were measured using linear-displacement transducers. A single transducer was mounted at the centre of the panel to obtain deflection measurements at that location. The transducers were connected to digital voltmeters which have been directly calibrated to give an accuracy of displacement measurement correct to 0.001mm.

5.3 Test Specimens

The test specimens were made from mild steel plate with slenderness ratios (b/t) varying from 30 to 100. The ends and sides of the plates were machined to ensure squareness and straightness of the edges. The test rig could accommodate a range of plate thickness up to 2 mm and any width of plate up to 300 mm. Various (b/t) ratios were obtained by having plates of various thicknesses. The details of all the test specimens are summarised in Table 1 in which the overall size of the plates, thickness and the respective yield stress determined from separate coupon tests are listed. Before the test, a measure of the initial flatness of the plate was obtained by using a transducer which was mounted on a bar and could be moved along the length of the plate. Initial plate
imperfections thus measured are also given in the table.

5.4 Testing Procedure

A test panel thus positioned, was initially loaded up to about one-third of its critical load, then the load was released. This operation was repeated several times to ensure that the loading edges were properly in contact with the edges of the specimen. These trials of loading and unloading also acted as a check on the proper functioning of instruments. The test specimen was then subjected to incremental loading starting from zero using small increments of load and the test was continued even after the peak load was surpassed. The applied loading in the \( x \) and \( y \) directions viz. \( P_x \) and \( P_y \) were kept equal and it was maintained throughout entire loading cycle. The load, the out-of-plane deflection and the end-shortening measurements corresponding to each increment of load were carefully noted.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Plate size ( a = b )</th>
<th>Thickness, ( t ) (mm)</th>
<th>( b/t )</th>
<th>Measured Imperfection, ( \delta_0 ) (mm)</th>
<th>( \delta_0/t )</th>
<th>Yield Stress, ( \sigma_{ys} )</th>
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<td>42.32</td>
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6. RESULTS AND DISCUSSION

Very little theoretical work has been reported on biaxially loaded plates and fewer experimental results have been published. In this paper, the results obtained by the proposed method are compared first with the experimental and finite element results for test specimens and then with those presented by Coombs (1975) who used a dynamic relaxation finite difference approach to solve the large deflection plate equations for plates simply supported on all the four edges. The experimental and analytical values of ultimate strength for the test specimens described in Section 5 are summarised in Table 2. One set of analytical values has been obtained by using the proposed method whilst the other set determined by nonlinear finite element modelling. Comparison of each of the two sets with the corresponding experimental values is also given in the Table. Generally good agreement between the experimental and analytical values is observed. For plates with simply supported edges, the mean values of the ratio between the analytical and experimental results are 1.004 and 0.938, respectively, for the proposed method and the finite element method. In the case of the clamped plates the corresponding mean values are, respectively, 0.970 and 0.965. The predictions by the proposed method is even better than that by the finite element method.
The proposed method and the finite element formulation were also used to analyse simply supported square plates having plate slenderness varying from 30 to 60. These plates were first investigated by Coombs (1975) to validate his method in which he has used a dynamic relaxation finite difference approach to solve the large deflection plate equations. Each of the plates was analysed under different strain ratios, viz \( \varepsilon_x / \varepsilon_y = 1.0, 2.0 \) and 3.0. A set of predicted values of mean stress at collapse in the \( x \) and \( y \) directions and the corresponding values from the finite element method and Coombs method for various \( b/t \) ratios, applied strain ratios and selected degrees of initial imperfection for the square plates are tabulated in Table 3. It will be seen that for all the \( b/t \) ratios tabulated, the predictions are sufficiently accurate thereby demonstrating the capability of this theoretical method.

7. STRENGTH OF BIAXILY LOADED PLATES

The buckling stiffness and ultimate load behaviour of biaxially loaded plates are influenced by complicating factors such as plate slenderness, aspect ratio, initial geometric imperfection and boundary conditions. Comparisons of the results (Table 4) obtained on square plates and plates with aspect ratio \( (a/b) = 3 \) having the same initial bows, slenderness ratio of \( b/t = 60.0 \) and all edges simply supported, show that for equal average strains applied in the orthogonal directions, the longitudinal strengths of the rectangular plate with a single half-wave initial bow are greater than those for a square plate with the same magnitude of initial deformation. The transverse peak strengths, however, are in the reverse order. This is also true when the applied strain ratio is doubled. If the initial imperfections for the above rectangular plate is assumed to have three half-waves longitudinally, the strength of the plate in both the long and short direction is identical to the strength of the square plate; this is
obviously due to the fact that each square portion of the 3:1 plate has the same dimension and loading conditions as the square plate, provided the edges are constrained to remain straight and have a constant displacement applied along each edge.

The interaction curves for square plates, simply supported or clamped along the four edges or simply supported along two parallel edges whilst the other two parallel sides are clamped are presented in Figures 3, 4 and 5, respectively. The initial imperfection of magnitude given by \( \delta \) has been assumed for plates with simply supported edges while the corresponding value for plates with clamped edges has been

\[
\delta = \frac{0.145 (b/t)}{ \sqrt{\frac{E_y}{\gamma}}}
\]

\( \delta \) is plotted against the transverse mean stress at collapse \( \sigma_y \) for selected plate slenderness \( b/t \) ratios, so that the corresponding longitudinal and transverse stresses can be obtained directly from the curves. The abscissae and the ordinates are plotted in a non-dimensional scale, by dividing the mean stresses \( \sigma_x \) and \( \sigma_y \) by the yield stress \( E_y \). The \( b/t \) ratios ranged from 30 to 80, and are within the practical limits. Two sets of curves for each of the plate slenderness ratios are plotted in Figures 3, 4 and 5, one corresponding to those obtained by the proposed method and the other obtained by the finite element package. The two curves in each category can be seen to lie close to each other showing a good agreement.

### TABLE 3

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<th>b (mm)</th>
<th>t (mm)</th>
<th>( b/t )</th>
<th>( \delta_0 ) (mm)</th>
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<th>Authors</th>
<th>ABAQUS</th>
<th>Coombs</th>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>( \sigma_x / \sigma_y )</td>
<td>( \sigma_x / \sigma_y )</td>
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### TABLE 4

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<th>3:1 Plate</th>
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<td>( \sigma_y / \sigma_y )</td>
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<td>0.49</td>
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<td>3.00</td>
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<td>0.75</td>
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<td>0.32</td>
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</table>

\( E = 205,000 \text{ N/mm}^2, \sigma_y = 245 \text{ N/mm}^2, \quad q = 1, \quad a = 720 \text{ mm}, \quad b = 240 \text{ mm}, \quad b/t = 60 \)
agreement, within the acceptable range of accuracy, between the results predicted by the two analyses.

8. CONCLUSIONS

An approximate method to predict the post-buckling and collapse behaviour of plates subjected to biaxial loading is presented. The analysis is based on the energy method and accounts for the effect of initial imperfections and the ratios of applied strain in the two directions. An experimental investigation that has been carried out to investigate the behaviour of square plates subjected to biaxial loading is described. By comparing with alternative methods, the theoretical approach is shown to be adequate to predict the ultimate compressive strengths of biaxially loaded plates. Interaction curves have been presented for plates with simply supported or clamped edges and a wide range of plate slenderness ratios have been considered.

Figure 3: Interaction Diagram for Simply-Supported Plates

Figure 4: Interaction Diagram for Clamped Plates

Figure 5: Interaction Diagram for Clamped - Simply Supported Plates
9. REFERENCES


NOTATION

The following symbols are used in this paper:

- $A$ = A coefficient governing the deflection amplitude
- $A_o$ = A coefficient governing the initial deflection amplitude
- $a, b$ = Length and width of the plate in the $x$ and $y$ directions
- $E$ = Modulus of elasticity of steel
- $f(y)$ = A function of $y$ defining the deflected shape
- $g(x)$ = A function of $y$ defining the deflected shape
- $m$ = Magnification ratio, $A/A_o$
- $p, q$ = Number of half-wave lengths in the $x$ and $y$ directions
- $t$ = Thickness of plate
- $w$ = Deflection at any point $(x,y)$ of the plate
- $w_o$ = Initial imperfection at any point $(x,y)$ of the plate
- $v$ = Poisson’s ratio
- $\delta_o$ = Maximum value of initial plate imperfection
- $\alpha$ = aspect ratio of plate, $b/a$
- $\varepsilon_x$ = Strain in the $x$ direction
- $\varepsilon_y$ = Strain in the $y$ direction
- $\sigma_{eq}$ = Equivalent stress
- $\sigma_x$ = Longitudinal stress given by $\sigma_{xx} + \sigma_{xy}$
- $\sigma_{xx}$ = Applied stress in the $x$ direction
- $\sigma_y$ = Transverse stress given by $\sigma_{yx} + \sigma_{yy}$
- $\sigma_{yy}$ = Applied stress in the $y$ direction
- $\sigma_{ys}$ = Yield stress of steel
INFLUENCE OF WELDING ON THE STABILITY OF ALUMINIUM THIN PLATES

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ABSTRACT

The effect of welding on the local buckling of aluminium thin-walled sections is focused in this paper. With regard to a round-house type law material and by considering different hardening parameters as well as plate slenderness, the effect of welding in terms of geometrical imperfections, residual stresses and heat affected zones are analysed by means of a finite element analysis. The obtained results emphasise the influence of welding on the strength of slender plates, which has been only partially evidenced by experimental analyses. Besides, the comparison with the new EC9 approach allows to check the reliability of the design curves with respect to such a phenomenon.

KEYWORDS

Aluminium alloys, local buckling, plates, thin-walled sections, welding, geometrical imperfections, residual stresses, heat-affected zones, finite element analysis, codification.

1. INTRODUCTION

The increased use of thin-gauge metal sections in structural applications led to a progressive renewal of technical codes in this field. As it is well known, the response of such sections is strongly affected by local instability phenomena, which arise in the compressed parts. Usually, account is taken for this aspect by considering the cross-section as a set of elementary plates and, therefore, studying the buckling and post-buckling behaviour of the single element on the basis of the theory of plate stability. The onset and the evolution of such phenomena are strictly connected to the stress-strain relationship of material, which may be of a sharp-knee type (like in mild steel) or of a round-house type (like aluminium alloys, high-strength steel and stainless steel). The particular hardening features of the latter can play a significant role, mainly in the post-critical behaviour of the plate elements which the section is made of.

In order to take into account the local instability phenomena in the compressed plate elements, different approaches, such as reduced thickness, effective width and reduced limiting stress are available. In each case, every law given by technical codes to reduce cross-section or limiting stress can be essentially referred to the well known Winter formulation, by means of an appropriate choice of a generalised imperfection parameter, in which both geometrical and mechanical imperfection are contemplated, BS 8118 (1991).

This paper is in particular devoted to analyse the influence of welding on the buckling behaviour of slender
aluminium plates. The available experimental results evidence, in fact, the strong reducing in collapse load due to welding process. Aiming to verify the European code approach, a numerical model able to account for such effects has been set up. With regard to round-house type law of material and by considering different hardening parameters as well as plate slenderness, the effect of welding in terms of geometrical imperfections, residual stresses and heat affected zones is analysed and discussed also in relation to the present code approach.

2. THE BEHAVIOUR OF THIN PLATES IN COMPRESSION

When the local instability of compressed plate elements of a slender round-house type material section is investigated, the approaches used for steel should be properly modified.

For plates which buckle in the elastic range, the critical load depends on the ratio of geometrical dimensions, the edge conditions as well as mechanical characteristics of the basic material. As it is well known, the elastic buckling stress can be calculated through the Euler formula:

\[ \sigma_{cr, E} = \frac{\pi^2 \cdot E}{h^2} \]

in which \( \lambda_p = \sqrt{\frac{12(1-\nu^2)}{k}} \cdot \frac{b}{t} \) is the equivalent slenderness ratio, \( E \) is initial modulus of elasticity, \( \nu \) is the Poisson's ratio, \( k \) is a buckling coefficient depending on the boundary conditions and plate geometry, \( b \) is the width and \( t \) the thickness of plate.

For a buckling load which falls in the elastic-plastic range, the critical stress can be put into the following simplified form:

\[ \sigma_{cr} = \eta \cdot \sigma_{cr, E} \] (2)

where the non dimensional factor \( \eta \) is essentially a function of the shape of the stress-strain material curve. Different formulas for \( \eta \) parameter have been proposed in the field of plate and shell stability. They are all based on the combination of initial, secant and tangent elastic moduli of the material. The differences connected to the different formulations of \( \eta \) are always significant and are more evident in the case of strain-hardening material, Landolfo and Mazzolani (1997).

Going to post-buckling behaviour of aluminium plates, it is known that, in this phase, the in-plane stress distribution becomes non-uniform, with higher values of stress at the edges respect to the internal zones. Owing to this redistribution, plates are able to provide an ultimate strength generally different from the buckling load. For plates that buckle in the inelastic range the two values are very close, but for thin plates the average stress at failure \( \sigma_c \) is much higher than the buckling stress \( \sigma_{cr} \). Von Karman (1932) introduced the concept of effective width in order to analyse such a behaviour. This approach consists in substituting the actual non-uniform stress distribution with a constant value \( \sigma_{max} \) acting on a reduced plate width \( b_{eff} \) defined as the one for which \( \sigma_{cr} = \sigma_{max} \). In the case of elastic-perfectly plastic materials, the maximum stress of which is limited to the yielding value \( f_y \), the normalised ultimate plate strength \( \overline{\sigma_c} = \sigma_c / f_y \) is therefore proportional to \( b_{eff} / b \), i.e. to \( 1/\overline{\lambda} \), where:

\[ \overline{\lambda} = \lambda_p / \sqrt{\pi^2 E / f_y} \] (3)

is the plate normalised slenderness.

For what concerns round-house type materials, experimental tests on slender plates have shown the strong influence of hardening features on the buckling behaviour, particularly in the range of low slenderness, where the post-critical resources are much more significant, Mofflin and Dwight (1984). The Von Karman
methodology can be extended to such material, provided that an iterative procedure, which follows step-by-step the increase of strain and stress, is performed, Ghersi and Landolfo (1996). To describe the stress-strain relationship of continuous hardening material, the Ramberg-Osgood model is often assumed. It can be expressed in the form:

\[ \varepsilon = \frac{\sigma}{E} + \varepsilon_0 \left( \frac{\sigma}{f_{\varepsilon_0}} \right)^n \]

where \( f_{\varepsilon_0} \) is the conventional limit of elasticity and \( \varepsilon_0 \) the residual deformation corresponding to \( f_{\varepsilon_0} \) (usually \( f_{\varepsilon_0} \) is given by the 0.2% offset proof stress and \( \varepsilon_{0.2} = 0.002 \)), while the exponent \( n \) interprets the shape of the curve and characterises the hardening of the material. For each strain value, the stress can be obtained and consequently, for a given \( \eta \) formulation, the effective width evaluated by imposing \( \sigma = \sigma_\eta \). The ultimate strength can be then defined as the value which corresponds to a maximum or, if the strength is always increasing, to a limit value of strain (usually the one corresponding to \( f_{\varepsilon_0} \)). The results obtained using such a model have emphasised the need to distinguish the post-critical behaviour of aluminium slender sections, among materials with different values of hardening, Landolfo and Mazzolani (1996). As a consequence, two main families have been identified in the Eurocode 9 (1996): non-heat-treated (NHT) and heat-treated (HT) alloys, for which two different design curves have been stated.

3. THE INFLUENCE OF IMPERFECTIONS

Besides to the shape of material stress-strain relationship, the load bearing capacity of metal plates, is influenced by unavoidable imperfections produced during the fabrication process, Mazzolani (1994). In particular, account must be taken for geometrical (plate out-of-flatness) as well as mechanical (residual stress distributions and inhomogeneous distribution of mechanical properties) imperfections. In aluminium alloy structural elements, imperfections due to welding process are actually important. In particular, the increasing of out-of-flatness, the introduction of residual stress and the softening of material within the heat affected zones (HAZ) must be considered. All these factors have been assumed to justify the great scatter experimentally observed between welded and unwelded plate strength for both NTH and HT alloys, Mofflin and Dwight (1984).

Residual stresses are self-equilibrated internal stresses present in metal members caused by thermal or mechanical processes inducing non-uniform plastic deformations. Experimental and theoretical analysis have shown that they are always negligible in extruded profiles thanks to the high thermal diffusion factor of aluminium alloys, Mazzolani (1994). On the contrary, residual stresses represent a mechanical imperfection which can not be neglected in welded profiles. The welding process produces in fact a very strong concentrated heat input, whose consequence is the generation of high tension stresses close the welding zone, whereas equilibrating compression stresses arise further from the weld. The residual stress distribution is usually regular, depending on the shape of cross-section as well as on the type and the position of the welding. In each case, the highest tension stress tends to the yield value of the parent material, without reaching it. Despite such effect is lower than in steel, it can not be neglected as in extruded profiles in determining the bearing capacity of members.

As far as inhomogeneous distribution of mechanical properties is concerned, also in this case, differently from steel, the effect due to extrusion process are not significant with regard to load bearing capacity of the members and can be completely ignored. Much more important is instead the effect of welding. The heat input leads, in fact, to a decrease in the elastic material close to the weld, resulting in a variation of strength along the cross section of the profile. Experimental analyses carried in USA by Hill et al (1962) have evidenced that the extension of such a zone is equal to about 25 mm on each side of the weld and the reduction of the limit stress of the parent material is between 33 and 50 %. Further analyses have stated that the distribution depend upon the heat treatment and the above value are related only to HT alloys, while for NHT alloys a strength reduction of about 10 % can be expected.
The strength of a structure being affected by its imperfections demand to the code a model able to correctly account for such effects. Since the random variation occurring in the phenomenon, the approach can not be deterministic. The effects of imperfections should be therefore studied by means of an appropriate analytical or numerical model able to include such an effect in order to develop appropriate parametric studies.

4. THE CODIFIED APPROACH

The present version of Eurocode 9 represents, nowadays, the most up-to-dated document for evaluating the ultimate strength of aluminium plates, which can also be extended to other round-house type materials.

First of all a classification of cross-sections has been provided according to the slenderness of their parts, which are divided into internal or outstand elements. Distinction is then made between NTH and HT alloys as well as between welded and unwelded profiles. On the basis of this distinction, three design curves have been assumed, corresponding to the following cases: Curve A for unwelded plates in heat-treated alloy, Curve B for welded plates in heat-treated alloy and unwelded plates in non heat-treated alloy and Curve C for welded plates in non heat-treated alloy.

The design curves provided by EC9 are shown in Fig.1 for welded or unwelded internal and outstand elements. It can be observed that the buckling curves for outstand non-symmetrical elements is limited by the branch \( \rho_e = (\beta / \epsilon)^2 \), which corresponds to their critical stress.

\[
\rho_c = \frac{\delta_1}{\beta / \epsilon} - \frac{\delta_2}{(\beta / \epsilon)^2}
\]

being \( \beta \) a slenderness parameter depends both on \( h/t \) ratio and stress gradient for the element concerned and \( \epsilon = (250 / f_{0.2})^{0.5} \); \( \delta_1 \) and \( \delta_2 \) are two numerical coefficients whose values are reported in Table 1 together with the limit value of the \( \beta / \epsilon \) corresponding to \( \rho_e = 1 \).
By introducing the normalised slenderness $\bar{\lambda}$ as a function of $\beta\epsilon$ factor, it is possible to express such curves through a unified approach in which no distinction is made between internal and outstand elements. In fact, by considering that the buckling coefficients are $k=4$ and $k=0.425$ for internal and outstand elements respectively, the following relationships between $\bar{\lambda}$ and $\beta\epsilon$ can be derived:

$$\bar{\lambda} = 0.52\sqrt{250 / E \cdot \beta / \epsilon}$$  \hspace{1cm} (6)

$$\bar{\lambda} = 1.61\sqrt{250 / E \cdot \beta / \epsilon}$$  \hspace{1cm} (7)

As a consequence, the EC9 design curves can be synthesised as a function of $\bar{\lambda}$ by the following equation:

$$\rho_c = \omega_1 \left( 1 - \omega_2 \frac{\bar{\lambda}}{\bar{\lambda}_0} \right)$$  \hspace{1cm} (8)

being $\omega_1$ and $\omega_2$ two numerical coefficients.

Unfortunately, starting from the expressions of the EC9 design curves, the values of $\omega_1$ and $\omega_2$ coefficients obtained by using (6) and (7) are slightly different, depending on internal or outstand element formulations are assumed. Such discordance is due to the fact that in the EC9, in order to get more simplified expressions for design curves, integer values of $\delta_1$ and $\delta_2$ coefficients have been adopted, leading to the differences above mentioned. For this reason, the values of $\omega_1$ and $\omega_2$ coefficients corresponding to the original unified design curves proposed by Landolfo and Mazzolani (1996) on which the EC9 design curves have been based on, are reported in table 1 together with the limit value of the normalised slenderness corresponding to $\rho_c=1$.

**TABLE 1**

**NUMERICAL COEFFICIENTS OF DESIGN CURVES**

<table>
<thead>
<tr>
<th>CURVE</th>
<th>CODIFIED APPROACH</th>
<th>UNIFIED APPROACH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Internal elements</td>
<td>Outstand elements</td>
</tr>
<tr>
<td></td>
<td>$\delta_1$</td>
<td>$\delta_2$</td>
</tr>
<tr>
<td>A</td>
<td>32</td>
<td>220</td>
</tr>
<tr>
<td>B</td>
<td>29</td>
<td>198</td>
</tr>
<tr>
<td>C</td>
<td>25</td>
<td>150</td>
</tr>
</tbody>
</table>

For the first curve, such relationship is coincident with the Winter formulation, which is assumed in the American and European codes on cold-formed steel section for determining the effective width ratio. For the other ones, a similar structure is kept, practically by assuming appropriate equivalent reductive factors in the Winter formulation.

In the present Eurocode 9, therefore, in order to account for the effects of imperfections on the buckling and post-buckling behaviour of slender section, distinction is made between welded and unwelded profiles. The influence of welding is estimated by accounting globally all the effects in a higher value of generalised imperfection factor, which reduces the buckling curves respect to extruded profile ones. Such a reduction is practically the same for both NHT and HT alloys and is not substantially affected by plate slenderness parameter.

**5. INVESTIGATION ON THE INFLUENCE OF WELDING**

**5.1 Numerical Analysis**

Aiming to better investigate on these phenomena, the influence of welding has been studied by means of non-linear finite element analyses. The developed model takes account for both material strain hardening and plate large deflection. Besides, it considers the actual behaviour of shell elements under deformation in terms of
The analyses have been performed on simply supported aluminium imperfect plates in simple compression, with aspect ratio equal to $a/b=0.8$ and with reference to values of slenderness $\bar{\lambda}$ varying in the range 0.75-3. Two different materials have been considered. For both of them, the stress-strain relationship has been expressed through a Ramberg-Osgood law. The first one corresponds to a classic not heat treated alloy, with $n=10$ and $f_{0.2}=100$ N/mm$^2$, while the second one to a treated alloy, being $n=25$ and $f_{0.2}=250$ N/mm$^2$, Lai and Nethercot (1992). An initial geometrical imperfection in the form of a double sinusoidal deflection has been assigned. Such a form, with maximum deflection at the centre of the plate (Fig. 2a), is, in fact, nearly affine to the first eigenmode and corresponds to the maximum amplification of out-of-plane displacements. Two values of the initial deflection amplitude $w_0$ have been fixed: $w_0=b/200$ and $w_0=b/1000$ to characterise welded and unwelded plates, respectively.

For what concerns mechanical imperfections, in order to emphasise the effects of welding, both residual stresses and heat affected zones have been introduced. The models chosen to schematise such imperfections are represented in Figs 2b,c and d. In particular, with reference to a longitudinal weld (along unloaded edges), a constant distribution of tension residual stresses $\sigma_r^+=0.75f_{0.2}$ has been assumed in edge zones with $b_r=25$ mm together with constant compression residual stresses in the remaining central part of the plate, equilibrating the previous ones.

As far as mechanical properties adjacent to welds are concerned, in accordance with experimental relevance and code prescriptions, to express the softening of the material, a reducing factor for the stress-strain curve $k_{HAZ}=0.9$ and $k_{HAZ}=0.6$ has been considered, for NHT and HT alloys, respectively (Fig. 2c). An extent of HAZ from the plate longitudinal edges equal to $b_{HAZ}=25$ mm and therefore $b_{HAZ}=b_r$ has been instead assumed for both NHT and HT alloys (Fig.2d).

The results of numerical analysis are reported in Figs 3a,b,c,d for both material and welding conditions. Such results are expressed in terms of normalised plate ultimate strength as a function of normalised slenderness, where the plate strength is defined as the value corresponding to the attainment of the peek in the load-shortening curve, or to the attainment of a maximum deformation in plate elements equal to the conventional elastic limit ($E_{0.2}$).
WELDING EFFECT ON STABILITY OF ALUMINIUM THIN PLATES

In the same figures, the codified design curves as well as the available experimental results obtained by Mofflin and Dwight (1984) are also reported. The latter concern simply supported individual aluminium plates loaded in uniaxial compression made of two alloys: 6082-TF fully-heat-treated material and 5083-M non-heat-treated material. The 6082 alloy (Fig.3a,b) had a remarkably stable shape of stress-strain curve (n=25-28) while the 5083 alloy (Fig.3c,d) was much more variable (n=8-18).

5.2 Interpretation of the results

The comparison between experimental results and the codified approach evidences as the design curves fit well the experimental data both for welded and unwelded plates even if leading to a more conservative evaluation of plate strength for high values of normalised slenderness in the case of welded elements.

On the other side, the comparison between the trends of numerical and experimental results shows a good agreement and therefore corroborate the reliability of the adopted numerical model. In particular, the difference between welded and unwelded ultimate load points out as the effects of welding are quite important, leading to a decrease of strength until to about 15%.

In order to assess the influence of welding on the plate strength in compression, the percentage scatter:

$$\Delta\% = \left[ \frac{\sigma_{c,\text{uw}} - \sigma_{c,\text{w}}}{\sigma_{c,\text{uw}}} \right] \times 100$$

between corresponding unwelded ($\sigma_{c,\text{uw}}$) and welded ($\sigma_{c,\text{w}}$) ultimate load have been diagrammed as a function of the normalised slenderness, for both experimental and numerical results. As far as the experimental results are concerned (Fig.4a), a reduction of strength, with an average value of about 10%, has been usually observed for both tested alloys. In particular, despite the great dispersion of experimental results, such an effect seems to be independent of both plate slenderness and material properties. On the contrary the numerical data, which are reported in Fig.4b, demonstrate that the influence of welding is not the same,
depending on the mechanical features of the alloy, providing a lower percentage reduction of strength in case of HT alloys. Besides, a diminishing of such an effect, as the plate slenderness increases, can be observed.

6. CONCLUSIVE REMARKS AND FURTHER DEVELOPMENTS

The performed analyses on the influence of welding on the strength of slender aluminium plates give rise to several important considerations. First of all, the comparison between available experimental results and the EC9 design curves A, B and C evidences as the latter fit quite well the effects due to welding process. Even if leading to a conservative evaluation of plate strength for high values of slenderness in case of welded elements, nowadays, the codified approach appears adequate in predicting the collapse load of welded elements for both HT and NHT alloys.

On the other side, numerical results have confirmed the important influence of welding on the slender plate strength, as it has been emphasised experimentally. Nevertheless, they clearly show a reduction of such an effect with the increasing of the plate slenderness as well as the dependence on the hardening features of the alloy. In order to better interpret the variation of welding influence with the hardening features of the material as well as the plate slenderness, further analyses should be performed. Aiming to verify the reduction of welded aluminium plates with reference to a number of slender sections, by varying residual stress distribution as well as the characterisation of heat affected zones, a more exhaustive parametric study should be therefore undertaken.

REFERENCES

A DESIGN STUDY OF THE PATCH LOAD INSTABILITY PHENOMENON

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³ Civil Engineering Department, PUC-RIO, Structural Engineering Department, State University of Rio de Janeiro, UERJ, Brazil

ABSTRACT

State of the art investigations in the patch load instability phenomenon still did not produce an efficient formula for the prediction of this local buckling. In the present work a non linear finite element program was used to model the phenomenon achieving good results in terms of the ultimate load, collapse configuration and web's stress distributions. An extensive study was also made comparing the finite element results with critical patch load formulas developed by investigators like: Bergfelt (1979); Roberts et al (1988) and others. Relevant results were found in terms of the influence of parameters like the thickness of the web, and others on the collapse load of the girder.

KEYWORDS


1. INTRODUCTION

The natural search for more efficient structural systems leads to solutions using steel as the main structural element. The optimal structural solution is also directly related to the material and the labour costs involved in manufacture and assembly. Concentrated loads applied to the flange of main beams, so called patch loads, can lead to an instability phenomenon that could increase the global cost of the structure. One of the solutions to overcome this problem is the use of transversal stiffeners welded to the beam's web. This solution presents several drawbacks, specially when assembly and crane girder loads are taken into account. Another solution, not cost effective, to solve the local buckling problem is to increase the web's thickness until it can sustain the applied load.

The current state of the art investigations in the patch load phenomenon still did not produce an efficient formula for the prediction of this local buckling, Vellasco, et al (1993). Many theoretical and experimental
investigations were accomplished but all the formulas proposed include a 20% error margin when compared to experimental and practical cases.

The modelling of this phenomenon, Souza, et al (1996) can be executed with the aid of the finite element method taking into account the material and geometric non-linearity’s associated with the patch load problem due to the plastic local buckling involved, Bergfelt (1979).

Good results in terms of critical loads, deformed shapes and web’s stress distribution were found with the aid of a non-linear finite element program, Saloof, Andrade et al (1985). These results were then compared to theoretical and experimental investigations performed by Bergfelt (1979), Roberts et al (1988) and others. Relevant results were also found in terms of the influence of parameters like the thickness of the web, the web’s height and others on the buckling load of the girder.

2. LITERATURE SURVEY

The main objective of this section, consist in reporting a brief survey of some relevant works done in the patch load phenomenon. Zetlin (1955), studied the buckling of rectangular plates by the energy method. The results were then set against experimental data and some faults were detected which were attributed to the fact that the post-buckling strength of the plate was not taken into account in the analysis. Dubas et al (1990), conducted a large number of tests to study the patch load phenomenon. He arrived at Eqn. 1, based on a mechanism solution compared against the tests results.

\[
P_{cr} = 0.11 t_{w}^{2} \left( E \sigma_{w} \left( c + 2t_{f} \right) / t_{w} \right) + \sqrt{\frac{b_{t} t_{f}}{100r_{i}^{2}}} \sqrt{\frac{E \sigma_{f} t_{w}^{5} \left( c + 2t_{f} \right)}{1}}
\]  

(1)

One of the most active investigators in the patch load field was Bergfelt (1979). He concluded that the girder had a large post-critical reserve under patch loading. Subsequently, he described the failure process of a beam subjected to a patch load in three major phases. The first comprised the path until some yielding was detected in the surface of the web. The second phase ran to the moment where some small folds could be detected in the web and the last phase went to the point of failure. The collapse could be achieved by a overall buckling of the web or by a localised zone under the load where bigger folds and high stress concentrations were present. Based on the test results he proposed the use of Eqn. 2.

\[
P_{cr} = 0.8 t_{w}^{2} \sqrt{E \sigma_{w} \left( t_{i} / t_{w} \right)} f(\Theta) \quad \text{where: } t_{i} = t_{r} \frac{b_{t}}{25t_{f}}
\]  

(2)

Roberts et al (1988), created a method to predict the ultimate load resistance based on the plastic hinge mechanism. Using the theorem of virtual work they developed Eqn. 3:

\[
P_{cr} = 0.77 t_{w}^{2} \left( 0.9 + 1.5c / h \right) \sqrt{E \sigma_{w} t_{f} / t_{w}}
\]  

(3)

In 1977, Drdacky et al (1977), described another series of experiments with approximately 170 tests. He discovered that a smooth transition occurred between the stages of pre and post-buckling and proposed Eqn. 4, to predict the failure load:

\[
P_{cr} = 0.55 t_{w} \left( 0.9t_{w} + c t_{w} / h \right) \sqrt{E \sigma_{w} t_{f} / t_{w}}
\]  

(4)

Some eighty references have been found to a major international effort on patch load effects, Vellasco et al (1993). Painstaking experimental studies have been combined with a variety of analytical and numerical
approaches to derive (and later to criticise) the design formulations in the national design codes of many countries.

3. FINITE ELEMENT ANALYSIS

The geometry of the finite element model, Souza (1995) consisted of a simply supported beam subjected to a load uniformly distributed along a short length $c$. This model was composed of 260 elements and 290 nodes. The flanges were modelled with 40 elements (4 vs. 10). The web was composed of 120 elements (10 vs. 12). Transversal stiffeners were modelled with 24 elements (12 vs. 2). The geometry of the model was completed with flange stiffeners to simulate the load distribution length, figure 1.

![Finite element mesh adopted](image)

The finite element used, called Semiloof, Andrade, et al (1985), was developed for thin shell applications. A non-linear geometric and material analysis was performed using Von Mises yielding criterion and full Newton Raphson iterative scheme as the solution method. The boundary conditions were established in order to represent the actual constraint conditions described in the experimental results obtained by Bergfelt (1979).

In order to validate the finite element model five beams tested by Bergfelt (1979), were simulated and compared to the experimental results. The material and geometric characteristics of these beams are presented in table 1.

<table>
<thead>
<tr>
<th>Girder</th>
<th>$b$(mm)</th>
<th>$h$(mm)</th>
<th>$t_w$(mm)</th>
<th>$b_f$(mm)</th>
<th>$t_f$(mm)</th>
<th>$c$(mm)</th>
<th>$\sigma_y$(MPa)</th>
<th>$\sigma_t$(MPa)</th>
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</thead>
<tbody>
<tr>
<td>b14</td>
<td>2400</td>
<td>400</td>
<td>2</td>
<td>100</td>
<td>8</td>
<td>180</td>
<td>294</td>
<td>294</td>
</tr>
<tr>
<td>R01</td>
<td>800</td>
<td>800</td>
<td>2</td>
<td>300</td>
<td>15</td>
<td>40</td>
<td>266</td>
<td>295</td>
</tr>
<tr>
<td>R03</td>
<td>800</td>
<td>800</td>
<td>2</td>
<td>120</td>
<td>5</td>
<td>40</td>
<td>266</td>
<td>285</td>
</tr>
<tr>
<td>b08</td>
<td>800</td>
<td>800</td>
<td>2</td>
<td>120</td>
<td>5</td>
<td>40</td>
<td>285</td>
<td>290</td>
</tr>
<tr>
<td>b83</td>
<td>800</td>
<td>800</td>
<td>3</td>
<td>250</td>
<td>12</td>
<td>40</td>
<td>328</td>
<td>298</td>
</tr>
</tbody>
</table>

Imperfections Study

Unfortunately no data concerning the beam's initial imperfections was documented in Berfelt's work. A study of the influence of the initial imperfections over the critical collapse load of the beams was performed.
Beam b14 described previously, was simulated using different amplitude and positions of initial imperfections. In this investigation, maximum imperfection's amplitude of: \( h/1600 \), \( h/800 \), \( h/650 \) and \( h/560 \), applied at node point 140, 0.2\( h \) away from the top flange were adopted. The influence of the initial imperfection's amplitude over the collapse load is represented in figure 2. It is observed that this influence is negligible (minimum and maximum values for the ultimate load ranged from 91.5% to 92.7%.

![Figure 2: Load versus lateral displacement, beam b14](image)

The position of the maximum amplitude of the initial imperfection was also investigated. The imperfection's application point was situated at: 0.25 \( h \), 0.20 \( h \), 0.15 \( h \), 0.10 \( h \) (nodes: 139, 140, 141, 142 respectively) away from the top flange, figure 3. The ultimate load varied 90% to 92.5% proving that for this beam the influence of the location of the maximum imperfection was also insignificant.

![Figure 3: Load versus lateral displacement, beam b14](image)
Considering these results, a standard imperfection of $h/500$ applied at a node $d = 0.2h$ far from the beam's top flange was used. Four more beams were simulated to calibrate the model. They were beams: R01, R03, b08, b83. The behaviour of beams R01, R03, b08 and b83 in terms of load versus lateral displacement of web's nodes: 140, 141, 142 e 143 (0.20 h, 0.15 h, 0.10 h, 0.05 h away from the superior flange) is presented in figures 4 to 7.

Figure 4: Load versus lateral displacement, beam R01

Figure 5: Load versus lateral displacement, beam R03
The buckling configuration associated with the principal web’s stress distribution for beam R01 and b83 is represented in figures 8 and 9. In figure 8 the loading stages presented corresponded to 83.81%, 83.85%, 105.3% and 103.3% (unloading path) of the experimental collapse load. In figure 9 the loading stages presented corresponded to 40.9%, 93.6%, 93.8% and 91.9% (unloading path) of the experimental collapse load. An inspection of the stress evolution associated with the deformed shape reveals that the onset of web buckling has begun at a load levels corresponding to 83% and 93% (beam R01 and b83) of the experimental collapse load. This is confirmed with the aid of figures 4 and 7. All the other beams simulated had similar deformed shapes and web’s stress distribution.
Design Study of the Patch Load Instability Phenomenon

Figure 8: Principal Stress Distribution Evolution, Beam R01

Figure 9: Principal Stress Distribution Evolution, Beam b83
A collapse load comparison was obtained by the use of the formulas proposed by: Dubas et al (1990), Roberts et al (1988), Bergfelt (1979), Drdacky et al (1977), the finite element simulations and the experimental results, table 2. The finite element analysis margin of error ranged from 0.01% to 14.06 with an average of 6.86%.

TABLE 2
COLLAPSE LOAD COMPARISON

<table>
<thead>
<tr>
<th>Tests/Researchers</th>
<th>Roberts</th>
<th>Dubas</th>
<th>Drdacky</th>
<th>Bergfelt</th>
<th>F. E. Analysis</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.9226</td>
<td>0.7276</td>
<td>0.8880</td>
<td>0.9520</td>
<td>0.9126</td>
</tr>
<tr>
<td>R01</td>
<td>1.0335</td>
<td>0.4269</td>
<td>0.5071</td>
<td>0.6915</td>
<td>1.0532</td>
</tr>
<tr>
<td>R03</td>
<td>1.1352</td>
<td>0.5673</td>
<td>0.6510</td>
<td>1.0082</td>
<td>1.1406</td>
</tr>
<tr>
<td>B08</td>
<td>0.9848</td>
<td>0.4616</td>
<td>0.5334</td>
<td>0.8382</td>
<td>1.0154</td>
</tr>
<tr>
<td>B83</td>
<td>0.9195</td>
<td>0.4532</td>
<td>0.6461</td>
<td>1.0137</td>
<td>0.9384</td>
</tr>
</tbody>
</table>

4. INFLUENCE OF GEOMETRIC AND MATERIAL PARAMETERS OVER THE COLLAPSE LOAD

In order to investigate the influence of geometric and material parameters in relation to the critical patch load a set of experimental results produced by Bergfelt (1979), Roberts et al (1988) and others was analysed. First some of the most important parameters were investigated in respect to the experimental collapse load.

According to the authors that have investigated the patch load phenomenon the most important geometric parameter is the thickness of the web. Its influence over the collapse load can be observed in figure 10 where its possible to observe that the collapse load is almost directly proportional to the square of the thickness of the web.

The influence of the web’s yield stress is represented in figure 11 where it seems that the collapse load is proportional to the square root of the mentioned yield stress.
Figures 12 to 15 depict the influence of the length of the applied load, \(a/h\) ratio, \(b_l/2t_f\) ratio and \(h/t_w\) ratio over the collapse load. Unfortunately, these graphs did not indicate a clear relation between these parameters and the ultimate load. Despite this fact, these graphs are presented having in mind the objective of gathering information in terms of range of parameters covered by the experimental work. For example, figure 14 shows a large concentration of test results with very stocky or very slender flanges (\(b_l/2t_f\) less than 8 and \(b_l/2t_f\) greater than 11), on the other hand, only a few number of test results falls in the practical intermediate range.

The plastic load for all the tests with the loaded length \(c\) greater or equal than 100mm is represented in figure 16. From that graph it is possible to notice a great number of test near the diagonal line that represents points that had its experimental load equal to the plastic load. Some other points fall below this line giving a clear indication that a premature buckling happened at a load level inferior than the plastic load.
Figure 13: Influence of the a/h ratio over the collapse load

Figure 14: Influence of the b/2t ratio over the collapse load

Figure 15: Influence of the h/t ratio over the collapse load
5. PATCH LOAD PREDICTION FORMULAE EVALUATION

In this section an evaluation of equations proposed by Dubas et al (1990), Bergfelt (1979), Roberts et al (1988) and Drdacky et al (1977), will be presented through a comparison with experimental results. The performance of Eqn. 1, Dubas et al (1990), and Eqn. 4, Drdacky et al (1977), is represented in figures 17 and 18. From that graphs it is possible to state that the values relative to Eqn. 1 are concentrated in the region $0.30 \leq P/P_{\text{exp}} \leq 0.60$ while for Eqn. 4 around $0.60 \leq P/P_{\text{exp}} \leq 0.80$. The results of the use of those equations seems to be conservative due to the fact that the ratio $P/P_{\text{exp}}$ is always less than 1.0 being inferior limits for the patch load buckling phenomenon.
The behaviour of Eqn. 2, proposed by Bergfelt (1979) is depicted in figure 19 below. A concentration of the results in a region delimited by $0.80 \leq \frac{P}{P_{\text{exp}}} \leq 1.20$ is visible. It is also possible to notice that this Equation can produce unconservative results being by that a superior limit for the local buckling studied.

The behaviour of Eqn. 3, proposed by Roberts et al (1988) is shown in figure 20. A concentration of the results in a region delimited by $0.80 \leq \frac{P}{P_{\text{exp}}} \leq 1.20$ is visible. The use of that equation produced a large number of results for the ratio $\frac{P}{P_{\text{exp}}}$ around 1.0. This is a strong indication that this equation can produce a good estimation for the critical patch load investigated.

Finally, figure 21 presents a summary of the performance of all the equations studied plus the performance of the finite element analysis described previously. A large dispersion of the results indicate that there is still no definite solution for the prediction of the critical buckling load.
6. CONCLUSIONS

Good results in terms of ultimate loads, deformed shapes and web's stress distribution were found with the aid of a non-linear finite element program, Saloof, Andrade, et al (1985). The amplitude and position of the initial imperfections over the collapse load was negligible. These results were then compared to theoretical and experimental investigations performed by Bergfelt (1979), Roberts et al (1988) and others. These comparisons validated the structural behaviour of the model and guarantee the consistency of the proposed solution. The finite element model achieved good results when compared to other formulas for the collapse loads being close to the results of the most accurate formula proposed by Roberts et al (1988). It was also possible to conclude that the collapse load is directly proportional to the square of the thickness of the web and to the square root of the web's yield stress.

The main conclusion that can be reached is that the question of patch loading is still to some extent an open one, one which continues to attract the attention of many research teams world-wide. Many theoretical and experimental investigations were performed but all the formulas proposed included a 20% margin of error when compared to experimental and practical cases.
If this difference is to be reduced, a significant set of experiments, backed by state of the art numerical analysis including the effects of initial imperfections, eccentricity, and residual stresses will still be needed. It is also possible to suggest an extension of the finite element analysis in order to generate a parametric study that might give rise to improved design rules. Another interesting possibility to model the phenomenon is achieved with the use of neural networks. This technique well backed with experimental and finite element results could prove to be one breakthrough for predicting the critical patch loads.

LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>vertical stiffener’s distance;</td>
</tr>
<tr>
<td>b_f</td>
<td>width of the flange;</td>
</tr>
<tr>
<td>c</td>
<td>load distribution length;</td>
</tr>
<tr>
<td>h</td>
<td>beam’s height;</td>
</tr>
<tr>
<td>E</td>
<td>Young’s Modulus;</td>
</tr>
<tr>
<td>f(θ)</td>
<td>correction factor for patch load;</td>
</tr>
<tr>
<td>P</td>
<td>critical collapse load;</td>
</tr>
<tr>
<td>P_exp</td>
<td>experimental collapse load;</td>
</tr>
<tr>
<td>t_f</td>
<td>thickness of the flange;</td>
</tr>
<tr>
<td>t_w</td>
<td>thickness of the web;</td>
</tr>
<tr>
<td>σ_f</td>
<td>flange’s yield stress;</td>
</tr>
<tr>
<td>σ_w</td>
<td>web’s yield stress</td>
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</tbody>
</table>

ACKNOWLEDGEMENTS

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5 SHELLS
EVALUATION OF COLLAPSE LOADS OF RETICULATED DOMES UNDER SEISMIC MOTIONS

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ABSTRACT

An evaluation of collapse loads of high- and low-rise reticulated domes under self-weight (gravity load) and earthquake motions in relation with static ultimate strength and corresponding vertical critical displacement, by carrying out elastic plastic dynamic response analysis, is presented. The dynamic time history response analysis considers both geometric and material nonlinearities based on incremental equation of motion with average acceleration method, Newmark beta-scheme with small time increments. Collapse acceleration for high-rise domes under horizontal ground motions can be evaluated by the base shear coefficient and critical member strength. For low-rise domes under vertical and horizontal ground motions, collapse acceleration can be related with static critical displacement and maximum elastic vertical displacement response at a specific node in domes.

KEYWORDS

reticulated dome, elastic-perfectly plastic, seismic motions, dynamic time history response analysis, Newmark beta-scheme, collapse load, base shear, maximum elastic response, collapse acceleration

1. INTRODUCTION

Spatial reticulated structures to be built in seismic areas of earthquake prone countries must be designed for the effects imposed on them by strong earthquake motions, in addition to their design for usual loads. In contrast to research for dynamic response behaviors of double-layer grids (DLG), Kato et al (1986), Holzer (1984) and Malla and Serrette (1996), the studies into dynamic collapse behavior of single-layer reticulated domes (SRD) have been scarce, Mukaiyama et al (1992) and Kato et al (1996). The DLG can sometimes endure both the dead load and the earthquake motions due to the redundancy of the other members with internal force redistribution, Karamanos (1993) and Malla and Serrette (1996).

On the other hand, SRD belongs to non-dissipative structures. The SRD exhibits a static collapse
behavior under dead loading which is often represented with elastic plastic bucklings of nodes and members in a local area over dome surface. Then the dynamic collapse often starts from such a local area in critical conditions with development of yieldings or bucklings as well as large vertical displacements in specific nodes, Mukaiyama *et al.* (1992) and Kato *et al.* (1996).

This paper investigates the dynamic collapse of SRD by elastic plastic dynamic response analysis applied to the high-rise and low-rise domes, with pre-assigned "static safety factors" $\nu_s$ against static collapse loads, under earthquake motion time histories. It is also discussed how the ultimate member strength as a function of the slenderness parameter may be evaluated the dynamic collapse acceleration on the basis of maximum acceleration response and maximum displacement response through the linear elastic response analysis.

2. COLLAPSE ACCELERATION AND EARTHQUAKE RESISTANT CAPACITY

2.1 Procedure 1 (Maximum Acceleration Response, $S_a$)

The diagonal members nearest to the supports in a high-rise SRD are simultaneously affected by its vertical dead load (gravity load) and the horizontal seismic force. The fundamental mode with natural period $T_1$ is supposed to be dominant and the participation factor in horizontal components of motions to this mode is significant in contrast with it in vertical components which can be calculated by an eigenvalue (free vibration) analysis for the high-rise SRD, Mukaiyama *et al.* (1992). Examples for fundamental vibration behavior of high-rise domes, having dome half-open angle $\phi$ of 60, 90 and 120 degs., are given as in Fig.A of Appendix. The characteristic horizontal excitation was found for each fundamental natural periods corresponding eigenmodes as shown in Fig.A(b), except in case rather shallower domes. Also from the distribution of linear acceleration response under ELCENTRO-NS component of ground motions with gravity load, horizontal response accelerations at each nodes were larger than these in vertical directions. However in case of $\phi = 60^\circ$ and $120^\circ$ relative amounts of vertical motion contribution are appeared.

The distribution of the story shear force $Q_i$ and the story shear coefficient $C_i$ in the vertical direction are obtainable on the basis of the base shear coefficient $C_0$ derived from the normalized acceleration response spectrum $S_a(T_1, A_k, h)$ with a standard peak acceleration $A_k$ of input ground motions and a characteristic damping factor $h$ assigned to SRD through linear response analysis.

The design axial force $N_d$ in constituent members of SRD, in particular, the critical member prone to yield, is first determined by using $N_V=$the axial force under gravity load and $N_H=$the axial force under horizontal seismic force $Q_d$ in case of $C_0=1.0$, as below.

$$N_d = N_V + N_H \times C_0$$

(1)

Next, keeping the gravity load constant, linear buckling loads $Q_{cr}^{lin}$ and elasto-plastic buckling loads $Q_{cr}^{pl}$ are obtained by the linear buckling (eigenvalue) analysis and the geometric/material nonlinear analysis, respectively. As a result, corresponding axial forces of the member, $N_{cr}^{lin}$ and $N_{cr}^{pl}$, can be given as below.

$$N_{cr}^{lin} = N_V + N_H \times \frac{Q_{cr}^{lin}}{Q_d}, \quad N_{cr}^{pl} = N_V + N_H \times \frac{Q_{cr}^{pl}}{Q_d}$$

(2)

From $N_{cr}^{lin}$ with the yield axial force $N_p$ of the member, the normalized slenderness ratio $\lambda$ defined by Eqn(3) can also be given.

$$\lambda = \sqrt{\frac{N_p}{N_{cr}^{lin}}}$$

(3)
shown in Fig.1. Since for analysis of large span reticulated domes where axial forces are rather large, it is preferable considering the effect of axial force on the member bending stiffness with the aid of stability function to the three dimensional geometric stiffness matrix, Mukaiyama et al (1992) and Kato et al (1996). Quadratic terms in the translational displacements are used to compute the axial force in the member. The geometric nonlinear term can be given by incremental equilibrium formulation through neglecting over the third-order terms of incremental translations. This formulation involves elastic member bucklings as well as bowing effect of members, a kind of second-order elastic analysis.

The mechanical behavior of the elastic plastic elements is assumed as elastic before yielding, and after that the axial force $N$ and bending moments $M_y$, $M_z$ without torsion around the $x$ axis referring to Fig.1, are assumed to flow plastically on the yield surface $f$ defined as below.

$$ f = \left( \frac{N}{N_p} \right)^2 + \left( \frac{M_y}{M_p} \right)^2 + \left( \frac{M_z}{M_p} \right)^2 $$

where $N_p$=the yield axial force and $M_p$=the fully plastic moments about $y$ and $z$ axes.

In the elastic plastic state the total displacement increment is separated in an elastic part and a plastic part. Incremental equilibrium for elastic plastic springs lumped in an "equivalent" beam element can be obtained from the plastic-hinge approach. Then the elastic plastic stiffness matrix for an "equivalent" beam element with hinges is described by modifying the elastic spring stiffness which is the component of condensed tangent stiffness matrix, Kato et al (1996).

However the joint connections located at the both ends of each member are modelled by elastic springs, assumed not to yield. Also a dimensionless bending spring constant $\kappa$ defined as the ratio of joints rotational rigidity $K_B$ to bending rigidity of member $EI_p/\ell_0$, namely $\kappa = K_B\ell_0/EI_p$. Thus the yielding is governed by the strength of steel pipes. The yield stress of the steel pipes $\sigma_y$ and the Young's modulus $E$ are assumed as 235N/mm$^2$ and 206kN/mm$^2$, respectively.

### 3.2 Method of Analysis

For the static collapse analysis, the static ultimate load $P_u$ of the dome with pin-supported boundaries under dead load (gravity load) $P_d$ is calculated by elastic plastic analysis including material yield and geometric nonlinearity as described before. Here $P_u$ is assumed to be given by $P_u/\nu_\alpha$, where $P_u$ means the static load capacity per node for domes. Referring to an update Lagrangian approach, numerical calculations are based on an incremental load and displacement control scheme with Newton-Raphson method.

For the dynamic collapse analysis, equation of motion during incremental step within a small time interval $\Delta t$ is given by decomposing incremental tangent stiffness matrix $[K^I]$ at step $i$ into linear stiffness $[K^L]$ and nonlinear term $[K^N] = [K^I] - [K^L]$ as below, by placing the nonlinear terms on the right-hand side of the equations of equilibrium and treat them as additional loads (pseudo loads), Stricklin et al (1971), Robertson (1984) and Jun et al (1989).

$$ [M]\{\dot{z}_{i+1}\} + [C]\{\ddot{z}_{i+1}\} + [K^L]\{\Delta z\}_L = -[M][\alpha]\{\ddot{U}_{i+1}\} + [K^N]\{\Delta z\}_R - ([K^L]\{\Delta z\}_R + \{f_i\} - \{P_d\}) \quad (9) $$

where $[M]$=the mass matrix, $[C]$=the damping matrix, $\{f_i\}$=the restoring force vector of member at step $i$, $\{z_{i+1}\}$ =the displacement vector at step $i + 1$, $\{\ddot{U}_{i+1}\}$=the input acceleration having three directional components at step $i + 1$ with $[\alpha]$=the acceleration influence matrix (=unit matrix for each component), $\{P_d\}$=the load vector of gravity load. $\{\Delta z\}_R$=the incremental displacement vector can be given by a Taylor series expansion at the time step $i$ as given,

$$ \{\Delta z\}_R = \Delta t \cdot \{\ddot{z}_i\} + 0.5 \times (\Delta t)^2 \cdot \{\dddot{z}_i\} \quad (10) $$
Finally, the maximum input acceleration $A_{cr}$ of earthquake motions which may cause a specific member in SRD to equivalent static collapse of overall structure due to reaching the critical member strength to $N^{pl}_{cr}$, is estimated by static collapse analysis with the base shear coefficient $C_0$ and the standard level of input acceleration $A_h$ (gal) as follows.

$$A_{cr} = \frac{1}{C_0} \times \frac{N^{pl}_{cr} - N_V}{N_H} \times A_h \quad (4)$$

### 2.2 Procedure 2 (Maximum Displacement Response, $S_d$)

One can obtain the maximum elastic vertical displacement response $S_d(A_h)$ at a specific node in a low-rise (shallow) SRD by linear elastic response analysis under actual ground motions with a standard level of maximum acceleration $A_h$. Examples for fundamental vibration behavior of low-rise domes were given as in Fig. B of Appendix. The eigen vibration modes for each domes having $\phi$ of 30, 45 and 60 degs. appear in vertical excitations for shallower domes under horizontal motions. An apparent fundamental natural period which can be determined by characteristic participation factor even to dominant horizontal excitation modes. It is remarked that the vibration behavior in both directions seems to be important to investigate dynamic collapse of low-rise domes as mentioned before in case of $\phi = 60^\circ$, for example.

Here a specific node is the node in SRD which displaces significantly in vertical direction under the static collapse load $P^{pl}_{cr}$. The static critical vertical displacement $d_{cr}^e$ at $P^{pl}_{cr}$ is assumed to relate with $S_d(A_h)$ and vertical displacement $d_e$ under gravity load $P_d$ as follows.

$$d_{cr}^e = d_e + \frac{A_p}{A_h} \times S_d(A_h) \quad (5)$$

where $A_p$ is the peak acceleration which cause the vertical displacement response at a specific node just to $d_{cr}^e$. When the different values of static safety factors $\nu_s$ can be defined as $\nu_s = \frac{P^{pl}_{cr}/P_d}$, the corresponding critical displacements can be approximately given by $d_{cr}^e = \nu_s \times d_e$, respectively. Thus the static safety factors $\nu_s$ above determine different $P_d$ values with the same $P^{pl}_{cr}$ for the SRD according to Eqn(5). From this relation and Eqn(5), the peak acceleration $A_p$ can be obtained as follows.

$$A_p = \frac{(\nu_s - 1) \cdot d_e}{S_d(A_h)} \times A_h \quad (6)$$

Finally the collapse acceleration $A_f^*$ corresponding to the maximum input ground motions resulted in sudden divergent vertical displacement $d_{max}$ much larger than $d_{cr}^e$ at a specific node is assumed to obtain from $A_p$ with the characteristic factor $\alpha_d$ as below.

$$A_f^* = \alpha_d \times A_p = \alpha_d \times \left( \frac{(\nu_s - 1) \cdot d_e}{S_d(A_h)} \right) \times A_h \quad (7)$$

where the characteristic factor $\alpha_d$ means some margin of the peak acceleration which may contribute to an increase in vertical displacement from $d_{cr}^e$ to $d_{max}$.

### 3. ANALYTICAL MODELS AND NUMERICAL METHODS

#### 3.1 Constituent Member Model

The model for members in SRD is assumed to be consisted of a general beam-column element with three elastic-perfect plastic elements at both the ends and the midspan in tubular steel pipes, as
Then the average acceleration method of Newmark beta-scheme with $\gamma$, $\beta$ operators for successive numerical integration to solve Eqn(9) can be applied to estimate $\{\Delta x\}_L$ during the small time increment $\Delta t$ as follows, Bathe(1982).

$$\{\Delta x\}_L = ([K]^L_i + \frac{\gamma}{\beta \Delta t}[C] + \frac{1}{\beta (\Delta t)^2}[M])^{-1}[-[M][\alpha]\{\tilde{U}^{i+1}\}]\{\Delta x\}_R$$

$$- [K]^L_i\{\Delta x\}_R - \{f_i\} + \{P_d\} + [M] < \frac{1}{2\beta} - 1\{\tilde{z}_i\} + \frac{1}{\beta \Delta t}\{\hat{z}_i\}$$

$$+ \frac{\gamma}{\beta (\Delta t)^2}\{x_i\} > [+C] < (\frac{\gamma}{2\beta} - 1)\Delta t\{\tilde{z}_i\} + (\frac{\gamma}{\beta} - 1)\{\hat{z}_i\} + \frac{\gamma}{\beta \Delta t}\{x_i\}]$$

Here Eqn(11) derived from the results at the current state $i$ with Eqn(10) are used to get $\{\tilde{z}_{i+1}\}$ and $\{\hat{z}_{i+1}\}$ at the end of the time step. However the time increment $\Delta t$ is constant but suitably small interval, and the operators $\gamma, \beta$ are chosen as 0.5 and 0.25, respectively. The term $(\star)^{-1}$ in Eqn(11) becomes constant and the result obtained through so-called LU decomposition of equation once can be successively used.

In this study, the mass matrix is composed as a lumped mass at each nodes which can be derived from the dead load $P_d$ dividing by the gravity acceleration, due to the present member (springs-in-series) model with lumped elastic plastic springs in members and formulations above. Consistent mass formalism may lead to be discontinuous against the component of nodal rotations. Also the generalised Rayleigh damping matrix in case of low-rise domes or the damping matrix proportional to stiffness matrix in case of high-rise domes is used with the damping factor of 2% assumed for whole the eigenmodes in SRD.

3.3 Analytical Models and Parameters

In order to discuss the Procedure 1 for high-rise domes under horizontal ground motions, the hemispherical configuration with dome half-open angle $\phi_0$ = 90° of span $L$ =60m and rise $H$ =30m as shown in Fig.2 is used. All the nine models for rigid-jointed pipe elements with each gravity loads $w$ and the first natural periods $T_1$ of the domes as summarized in Table 1 were used previously, Mukaiyama et al (1992), to get the distributions of story shear coefficient based on the base shear coefficient $C_0$ subjected to actual ground motions with a standard level of input acceleration $A_s$=100gal, ELCENTRO-NS(1940), TAFT-EW(1952) and MIYAGI-NS(1978). The base shear coefficients $C_0$ derived from linear elastic response analysis reported previously, Mukaiyama et al(1992), were from 0.86 to 0.88 in the average.
Nine models of the pipes of average length=4.6 meters with different diameters (ϕ=216, 165, 114 mm) and thickness (3.5, 4.5, 5.8 mm) as well as with κ=0.1, 3.0 and 1000.0 under gravity load 100.0kgt/m²(980N/m²) which corresponds to P₉=18kN/m², and input horizontal acceleration component of ELCENTRO NS are used to obtain the collapse acceleration Aₑ by elastic plastic dynamic response analysis and to compare the result with the estimated Aₑ by Eqn(4). At that time, the small time increment is set to Δt=0.002sec and the duration time 8.0sec of strong motions is used. From 0.0 to 1.0sec, the dome is loaded only by gravity load by using the critical damping to reduce dynamic effects, after that, from 1.0 to 8.0sec, the dome starts to be subjected to horizontal seismic motions. The dynamic elastic plastic response analysis by gradually increasing the level of Aₑ can provide the collapse acceleration as the minimum of peak value of input acceleration which causes the dynamic behavior of dome to be divergent.

To discuss the Procedure 2 for low-rise domes under actual ground motions having horizontal and vertical components of accelerations, the dome configuration as shown in Fig.3 with dome half-open angle φ₀ = 40°, the ratio of rise H to span L equal to 0.182, having rigid-jointed 930 members with length =10m and 331 nodes is illustrated. The dome has 20 beam-column members with length 10m and slenderness ratio 60, along three meridional lines. The member strength parameters, Nₚ and Mₚ are 3.46×10⁶kN and 5.21×10⁶kNmm. The static elasto-plastic analysis was first applied to obtain the critical displacement dₑ at Node 23 corresponding to Pₑ. The results obtained is shown as in Fig.4. The safety factors are assumed as νₑ=2, 3 and 4 for the static collapse load Pₑ=523kN/node.

Figure 3 Low-rise dome model
From the relation $\nu_s = \frac{P^i_{cr}}{P_d}$, the mentioned each dead load per node $P_d$ equals to 261.4kN, 174.3kN and 130.7kN, respectively.

Next to compare the estimated collapse acceleration $A_s^*$ by Eqn(7) with the result obtained from dynamic collapse response analysis, three ground motions aforementioned and KOBE(1995)EW-UD are used. The vertical peak accelerations of each motion are however assumed to be a half the horizontal components level $A_k$. In this study, the horizontal input ground motions are assumed to be applied directly to nodes at boundary supports in parallel with the X axis in the dome. In the analysis, despite of different durations of input motions for TAFT and ELCENTRO with 10sec, MIYAGI-OKI with 20sec and KOBE with 12sec, the time increment $\Delta t$ is set to 0.005sec. From 0.0 to 2.0sec, the domes are only under dead load $P_d$ slowly increased by using critical damping to reduce dynamic effects. Then the domes start to be subjected to horizontal and vertical motions simultaneously.

![Figure 4 Static collapse behavior](image)

![Figure 5 Axial strength as a function of slenderness $A$](image)
4. DYNAMIC RESPONSE BEHAVIORS AND DISCUSSIONS

4.1 High-Rise Domes (Procedure 1)

Figure 5 shows the results obtained from static collapse analyses for nine domes in relation between axial force ratio \( \frac{N_{ef}}{N_p} \) according to Eqn(2) and normalized slenderness ratio \( \Lambda \) defined by Eqn(3) with Eqn(2). In this model the member denoted by "a" in Fig.2 is the one that is subjected to the maximum axial force in the dome. The results are along or slightly lower than the solid lines, \( \frac{N_{ef}}{N_p} = 1 \) and \( LB \) or \( \frac{N_{ef}}{N_p} = \frac{1}{\Lambda^2} \), in this figure, and these are also upper than the modified Dunkerley curve with dot-dashed line, MDF.

Next one example for the model of P165(165.2 x 4.5) with \( \kappa = 3.0 \), is depicted as in Fig.6 the displacement response at the top of the dome by dynamic collapse analysis. The natural period \( T_1 \) is 0.317 sec. The upper shows the horizontal displacement time history and the lower shows the vertical displacement time history, under each the maximum input horizontal acceleration levels \( A_{max} = 860 \) gal and 880 gal, respectively. The level of \( A_{max} \) denoted by solid line represents that the dome has collapsed. From this figure, one can find that the dome has experienced residual deformation in vertical direction due to start yieldings of members located nearby support boundaries after almost 5.0 sec (denoted by \( t_5 \)), then at 6.0 sec or \( t_6 \) the vertical residual displacement increases according to progressive elastic plastic bucklings of diagonal members at the lowermost story of the dome.

![Progressive buckling of other members](image)

![Buckling of member "a"](image)

**Figure 6 Displacement response history**
Finally comparisons between the estimated collapse acceleration $A_{cr}$ due to elastic plastic buckling in the critical member defined by Eqn(4) and the dynamic collapse acceleration $A_f$ is illustrated in relation of the ratio $\nu_f = A_f/A_{cr}$ with $\Lambda$ as shown in Fig.7. From this figure, one can find the values of $\nu_f$ is larger than 1.0 for all models. $A_f$ is greater than $A_{cr}$ and the $\nu_f$ converges almost to 1.0 for $\Lambda=1.0$.

4.2 Low-Rise Domes (Procedure 2)

The vertical displacement response of the Node 23 and the axial force response of the Member 63 under TAFT earthquake motions in case for $\nu_f=2$ are, for example, shown in Fig.8 and Fig.9, respectively. Here the small time increment of 0.005sec is used for both TAFT EW-UD and KOBE EW-UD of which the durations of input motions are 10sec and 12sec, respectively. Also the natural period $T_1=0.916$sec, however much higher eigenmode, e.g. 67th mode with the eigenperiod of 0.493sec, may sensitively participate into the vertical motions.

When the dome is subjected to $A_{max}=200$gal denoted by thin dashed line, the dome behaves elastically with rather small displacement than $d_{cr}^{st}$. At $A_{max}=500$gal, the dome is however displaced downward with exceeding $d_{cr}^{st}$ and the Member 63 yields at $t=6.2$sec as shown in Fig.9. Then the vertical displacement gradually diverges downward, in addition, due to dead loading which affects to the node displacement increase as well as yielding of adjoining members. After $t=10$sec, sudden collapse occurs as shown in Fig.8.
Also an effect of static safety factor \( \nu_s \) on the vertical displacement response of the dome under KOBE EW-UD is illustrated as shown in Fig.10. When \( \nu_s = 2 \), the dome behaves elastically up to \( A_{\text{max}} = 100 \text{gal} \) within vertical displacement of \( d_{\text{cr}}^v \). Then around \( A_{\text{max}} = 260 \text{gal} \) the gradual increase in vertical downward displacement over 40cm may lead to yield of the Member 63 section. On the other hand, the dome with \( \nu_s = 3 \) shows a critical vibration in vertical displacement with average level of \( d_{\text{cr}}^v \) as shown in Fig.10.

Fig.11, for example, shows the relationships between the horizontal peak input acceleration \( A_{\text{max}} \) and the maximum vertical displacement response \( d_{\text{max}} \) at the Node 23 until dynamic collapse with each value of \( \nu_s = 2, 3 \), and 4 under TAFT and KOBE earthquake motions, respectively. From this figure, if \( \nu_s \) is fixed, however, the maximum acceleration \( A_{\text{max}} \) which makes \( d_{\text{max}} \) exceed \( d_{\text{cr}}^v \) varies significantly due to difference in dynamic response under each ground motions. Also from the figure the characteristic factor \( \alpha_d \) in Eqn(7) in order to estimate the collapse acceleration \( A_t^* \) can be eval-
EVALUATION OF COLLAPSE LOADS OF RETICULATED DOMES

Figure 11 Horizontal peak acceleration and maximum vertical displacement

Figure 12 Estimation as a function of static safety factor

5. CONCLUSIONS

When the domes with high-rise and low-rise configurations are investigated their dynamic collapse through the Procedure 1 based on the base shear concept and 2 based on the maximum displacement response, respectively, it is remarked that a specific member behaves different from the ordinary framework with braced columns, due to initial compression under gravity load in addition to significant vertical vibration participation, leading to divergence in displacement response over a local portion, where the members may reach to yield, even in case of low-rise domes.

Dynamic collapse of SRD roofs under gravity load and seismic force could be estimated effectively, not only (1) by the Procedure 1 using strength interaction and member strength formula, but (2) by the Procedure 2 evaluating collapse acceleration under actual ground motions.
Appendix

Fundamental vibration characteristics of high- and low-rise reticulated domes

Here $R$, $H$ and $\phi$ are radius of curvature, rise height and dome half-open angle, respectively, with dome span $L$. $T_1$ means natural period of 1-st mode, but of 8-th mode in case of $\phi = 60^\circ$ which corresponds to one of components with largest participation to excitation into $X$ direction.

$A_{\text{max}}^{\text{gal}}$ means maximum acceleration response. Also distributions are depicted at the time $t$(sec) which base shear factors for both horizontal and vertical directions become to be maximum. Here blacked circles and empty circles are sign of acceleration response for negative and positive components, respectively.

Equivalent static seismic loads are obtained from multiplication of maximum acceleration response by mass. Also $N_{\text{max}}$ means maximum axial force response and $C_0$ is base shear factor in horizontal component at the time $t$(sec).
EVALUATION OF COLLAPSE LOADS OF RETICULATED DOMES

Figure A High-rise domes (ELCENTRO NS with gravity load of 980N/m²)

- $\phi = 60^\circ$, mode 37
  - $T_i(37) = 0.329$ sec

- $\phi = 90^\circ$, mode 37
  - $T_i(37) = 0.274$ sec

- $\phi = 120^\circ$, mode 37
  - $T_i(37) = 0.248$ sec

(c) Acceleration response distribution

\[ T_i(37 - th) = 0.329 \text{ sec} \]

\[ T_i(30 - th) = 0.274 \text{ sec} \]
(a) Configuration

\( \phi = 60^\circ, \) 8-th mode

\( T_1(8-th) = 0.282 \text{sec} \)

(b) Eigenmode

\( (-238 \text{gal}) \phi = 30^\circ, t = 2.61 \text{sec} \)

\( (-323 \text{gal}) \phi = 45^\circ, t = 2.54 \text{sec} \)

(c) Distribution of "equivalent" static seismic load

Figure B Low-rise domes (ELCENTRO NS with gravity load of 980N/m²)
**Table A Estimation of collapse accelerations $A_f^*$ by Eqn(7)**

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<td>2.52</td>
<td>287</td>
</tr>
<tr>
<td>3</td>
<td>41.5</td>
<td>39.3</td>
<td>211</td>
<td>2.56</td>
<td>2.16</td>
<td>456</td>
</tr>
<tr>
<td>4</td>
<td>31.0</td>
<td>22.3</td>
<td>417</td>
<td>2.76</td>
<td>2.04</td>
<td>851</td>
</tr>
</tbody>
</table>

Note: $d_v$=vertical displacement under gravity load, $S_d(A_k)$=maximum elastic vertical displacement response under acceleration level $A_k$, $A_p$=peak acceleration by Eqn(6), $\alpha_d$=characteristic factor.

**References**


PLASTIC BUCKLING OF TRANSITION RINGBEAMS IN STEEL SILOS AND TANKS

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ABSTRACT

An annular plate ringbeam at the transition junction of a uniformly-supported steel silo (or tank) is subject to a large circumferential compressive force which is derived from the radial component of the meridional tension in the conical hopper. Under this compressive force, the ringbeam may fail in one of several possible modes, including plastic out-of-plane buckling which may be viewed as the result of interaction between elastic buckling and yielding (or plastic collapse). This paper first presents an investigation into the effects of various factors on the plastic buckling behaviour and strength of this ringbeam. This leads to a modified modelling approach and the identification of the dimensionless ringbeam size parameter as the only parameter which influences the form of interaction between elastic buckling and plastic collapse. An approximation for the plastic buckling strength is then developed using the modified structural model.

KEYWORDS

Buckling, Ringbeams, Rings, Silos, Tanks, Shells, Stability, Design

1. INTRODUCTION

A typical elevated steel silo consists of a cylindrical vessel above a conical hopper which is then supported on a skirt or a number of columns (Figure 1). The point of intersection of the cylinder and hopper is called the transition. The skirt below the transition may be long, even extending to ground level. Elevated silos supported on a deep skirt or many columns (typically more than 12) may be deemed to be uniformly supported. A ring is generally provided at the transition junction. This ring is a main structural member (in contrast to relatively light ring stiffeners on shell walls) and is often referred to as a ringbeam. A common form of ringbeams is an annular plate (Figure 1). The structural form of elevated liquid storage tanks is similar to that of steel silos, so the work described here is relevant to their design as well.

The ringbeam at the transition junction of a uniformly-supported steel silo is subject to a large circumferential compressive force which is derived from the radial component of the meridional tension in the conical hopper. Under this compressive force, the ringbeam may fail by axisymmetric plastic collapse when it is stocky (i.e. the plate width-to-thickness ratio B/T is small), by elastic buckling when it is slender and by plastic buckling when it is of intermediate slenderness. Theoretically speaking, both in-plane flexural buckling or out-of-plane flexural torsional buckling may be critical. In-plane buckling is the only form of buckling recognized in earlier design guides.
However, in-plane buckling appears to be almost always prevented by the membrane stiffness of the cone and the cylinder (Jumikis and Rotter, 1983). For most practical cases, out-of-plane buckling is the dominant mode. Out-of-plane buckling failure usually involves twisting deformations of the annular plate about the point of attachment (Figure 2) and the buckling deformations assume many waves around the circumference.

Figure 1: Typical form of elevated steel silo
Figure 2: Ringbeam buckling mode

Elastic out-of-plane buckling of annular plate ringbeams was first studied by Jumikis and Rotter (1983) and later by Sharma et al. (1987). Simple design equations from Jumikis and Rotter (1983) have been included in a design guide (Trahair et al., 1983). Plastic collapse of the transition ringbeam (junction) has been studied by Rotter (1987) and Teng and Rotter (1991a). Again a simple design equation has been developed. A recent junction collapse failure in a liquid storage tank was reported and investigated by Guggenberger (1997). Plastic buckling of annular plate ringbeams was examined by Rotter (1987), Teng and Rotter (1991b) and Greiner (1991), but no simple design method has been proposed. Consequently, the plastic buckling strength of annular plate ringbeams is being investigated in a current project, of which the work described here forms part. The aim of this paper is to examine the effects of various factors on the plastic buckling behaviour and strength of this ringbeam, and to develop a simple plastic buckling strength approximation.

2. STRUCTURAL MODELLING AND ANALYSIS

Finite element results presented in this paper were obtained using program NEPAS for the non-symmetric bifurcation buckling analysis of axisymmetric shells developed by the author. The formulation implemented in this program has been described in detail elsewhere (Teng and Rotter, 1989). The program first carries out an axisymmetric elastic-plastic large deflection (or small deflection) analysis with the material modelled according to the J2 flow theory of plasticity, and then performs a non-symmetric bifurcation buckling analysis based on one of the following three plasticity options: J2 flow theory, J2 deformation theory, and modified J2 flow theory which is the J2 flow theory with the shear modulus replaced by that predicted by the J2 deformation theory. The program has been shown to give accurate predictions by extensive comparison with existing theoretical and experimental results (Teng and Rotter, 1989) and has been applied widely to study collapse and buckling problems in shells of revolution. The material of the transition junction was assumed to have properties typical of steel: a Young’s modulus of $2 \times 10^5$ MPa, a Poisson’s ratio of 0.3, and to exhibit an elastic-perfectly plastic behaviour.
The transition junction investigated includes the complete hopper so that the loading on the transition junction from the hopper is realistically simulated (Figure 3). The structure is subject to a uniform internal pressure $p$ with a friction drag of $\mu p$. The frictional coefficient $\mu$ of the stored material was assumed to be 0.5. This simple loading pattern is sufficient as the strength of the transition junction depends chiefly on the meridional tension in the conical hopper, and within the failure zone, the pressure is sensibly uniform. The strength of the structure will be characterized using the pressure $p$. Generalization of the results obtained using this simple loading pattern for application to a junction under a more realistic loading pattern will be dealt with later.

The structure is supported only at the bottom of the skirt which was allowed radial displacements only during prebuckling deformations and meridional rotations only during bifurcation buckling. The length modelled for both the skirt and the cylindrical silo wall was 2 times their linear elastic meridional bending half wave length. Because the buckling or collapse mode of the transition junction is very localized, the support condition for the skirt has little influence as long as failure is at the transition. All shell components (cylinder, skirt and hopper) of the junction have the same thickness $t$, with the ringbeam having a cross-section of $B \times T$ (Figure 3). The hopper has an apex half angle of 40°. This structure with an $R/t$ ratio of 500, was used in the investigations of Rotter (1987) and Teng and Rotter (1991b). New insight will be gained here by studying the effects of various factors on the plastic buckling behaviour and strength. These effects were not examined in previous studies (Rotter, 1987; Teng and Rotter, 1991b; Greiner, 1991).

3. EFFECT OF PLASTICITY THEORIES

For plastic bifurcation analysis of plates and shells, the paradox still remains that the analytically more rigorous flow theory usually produces bifurcation buckling loads which agree less closely with experimental results and are higher than those from a deformation theory analysis (Teng, 1996). It is generally accepted that a bifurcation analysis based on the deformation theory provides a more realistic assessment of the buckling strength. Previous results (Rotter, 1987; Teng and Rotter, 1991b; Greiner, 1991) were based on the flow theory or modified flow theory which is the flow theory with the shear modulus predicted by the deformation theory (Teng and Rotter, 1989), so the effect of using deformation theory is examined here for the described intersection with a dimensionless ring area $BT/t^2 = 40$ and a yield stress $\sigma_y = 250$ MPa (Figure 4). As the plate width-to-thickness ratio $B/T$ increases, the failure mode of the ringbeam changes from plastic collapse, through plastic buckling, to elastic buckling. In general, yielding is first attained in the shell segments at the
point of intersection and this yielding reduces the restraint of the shell walls to the ringbeam. Relatively slender ringbeams fail by buckling with yielding in the shell segments only, while stocky ringbeams buckle after the ring cross-section has also yielded. The dividing B/T ratio between these two regions is about 26, which explains the shapes of the strength curves (Figure 4). This aspect has been discussed in Teng and Rotter (1991b). The effect of using a different plasticity theory is quite important for stocky ringbeams which buckle after yielding is reached in the ringbeam cross-section (Figure 4). It is interesting to note that the deformation theory and the modified flow theory lead to similar results for this problem, justifying to some extent the usefulness of the modified flow theory. The flow theory predicts that failure is by axisymmetric collapse when B/T is 20 or smaller, and a sudden jump in failure strength around a B/T value of 20. The other two plasticity modelling options predict that axisymmetric collapse is critical only for B/T = 10 among the B/T values examined here. They do not predict a jump in strength as the failure mode changes from axisymmetric collapse to plastic buckling.

When the plastic buckling strengths are compared with the elastic buckling strengths, it can be easily seen that a large reduction in buckling strength occurs due to yielding. At B/T = 10, the critical elastic buckling mode was found to be the buckling of the skirt instead of the ringbeam. This points to the fact that if the ringbeam size is sufficiently large, the axial force in the skirt and also in some cases in the cylindrical silo wall can be so large that they interact with junction failures. In practical design, it is desirable to avoid this kind of interaction by proportioning the structure so that either axial compression in the skirt or circumferential compression at the transition junction dominates the critical failure mode.

![Figure 4: Effect of plasticity modelling](image)

The results from the deformation theory are seen to be conservative compared to those of the flow theory and are known to approximate well test results of shells (Teng, 1996). In the absence of better information, the deformation theory results provide a good basis for developing a design strength approximation for the plastic buckling of annular plate ringbeams at silo transition junctions. From here on, only results from the deformation theory will be discussed.

4. **EFFECT OF PRE-BUCKLING LARGE DEFLECTIONS**

Jumikis and Rotter (1983) based their design proposal for elastic buckling of annular plate ringbeams on the linear elastic buckling strength. The effect of large deflections was investigated for a single geometry and was
found to be very small in Jumikis (1987). Teng and Rotter (1991a) examined the effect of large deflections carefully on the plastic collapse strength of transition junctions and found the effect to be significant only when the axial force becomes large in the skirt compared to the circumferential compression at the junction and when the cone is very shallow in which case the effect of large deflections is stiffening. The effect of pre-buckling large deflections on elastic as well as plastic buckling strengths is examined here for ringbeams with a dimensionless area \( BT/t^2 = 40 \) and a yield stress of 250 MPa (Figure 5). In general, pre-buckling large deflections are seen to lead to a small amount of stiffening for the buckling strength of these ringbeams (Figure 5). When material yielding is considered, the ringbeam fail by plastic collapse at \( B/T = 10 \) and its strength is slightly reduced due to the effect of large deflections. For elastic buckling, the effect of large deflections leads to some reduction in strength at low \( B/T \) values for which the destabilizing effect of the axial compression in the skirt becomes important (Figure 5). If very shallow hoppers are used, the effect of pre-buckling large deflections is also expected to be important. However, very shallow hoppers are not common in practice. In an investigation of the plastic buckling strength of this ringbeam, it is thus not important whether the effect of pre-buckling large deflections is included in the analysis or not. As mentioned earlier, the case of very high axial compression in the skirt should be avoided in practice by choosing an appropriate skirt thickness.

5. EFFECT OF YIELD STRESS

The effect of yield stress is now examined. Results for ringbeams with a dimensionless area \( BT/t^2 = 40 \) are shown in Figure 6 for a yield stress of 250 MPa and 350 MPa respectively. The horizontal axis used in Figure 6 is the slenderness parameter of the ringbeam, defined as

\[
\lambda = \sqrt{\frac{p_p}{p_e}}
\]  

(1)

where \( p_p \) is the plastic collapse pressure and \( p_e \) is the elastic buckling pressure of the transition ringbeam (junction) respectively. The vertical axis represents the failure pressure \( p_f \) of the ringbeam normalized by the axisymmetric plastic collapse strength \( p_p \). As expected, this dimensionless strength curve shows little variation with the yield stress. The small differences between the two curves at low values of the slenderness parameter are probably due to the high axial forces in the skirt. This point will be confirmed later using a modified structural model.
6. EFFECT OF RINGBEAM SIZE

Dimensionless plots as mentioned above are given for junctions with ringbeams of three different sizes: $BT/t^2 = 20$, 40 and 160 in Figure 7. The effect of ringbeam size is seen to be important for lower values of the slenderness parameter $\lambda$ for which buckling occurs after yielding has been reached in the ringbeam cross-section. It may thus be suggested that the ultimate failure strength $p_\tau$ be formulated in the following manner:

$$p_\tau = f(k, \lambda)p_p \quad (2)$$
where $p_p$ is the plastic collapse pressure, $f(k, \lambda)$ is a function of both the slenderness parameter $\lambda$ and the parameter $k$ which represents the effect of the ringbeam size on the shape of the strength curve. For a given structure, the elastic buckling pressure $p_e$ and the plastic collapse pressure $p_p$ can be found easily using existing formulae (Rotter, 1983; Jurukis and Rotter, 1983; Teng and Rotter, 1991a), so the slenderness parameter can be evaluated. A suitable dimensionless ringbeam size parameter is expected to be the ratio between the ringbeam cross-section area $BT$ and the effective compression area $A_{ps}$ of the shell segments in resisting the circumferential compressive force at plastic collapse as defined by Teng and Rotter (1991a):

$$k = BT / A_{ps}$$

Once this expectation is confirmed, a set of strength curves corresponding to different values of $k$ can be generated and the function $f(k, \lambda)$ defined.

### 7. MODIFIED MODELLING APPROACH

The above results suggest that a modified modelling approach as discussed below should be adopted to develop a strength approximation for plastic buckling of annular plate ringbeams at steel silo transition junctions.

(a) A structural model consisting of a ringbeam on a cone-cylinder junction should be used in a parametric study to establish the function $f(k, \lambda)$. That is, the skirt of the structural model of Figure 3 should be omitted and a vertical support be provided at the point of intersection. The omission of the skirt has the advantage that interaction between a high axial compressive force in the skirt and the circumferential compression at the junction is avoided, so the obtained results relate to junction collapse or buckling only. The generality of the results so obtained is not lost as the important ringbeam size parameter can still be varied. This model also represents some real cases where a skirt is not present and the transition junction sits instead on a large number of closely-spaced discrete vertical supports. The modified structural model is shown in Figure 8.

(b) For the modified structural model suggested above, if large deflection effects are considered, the load deflection curve, with load as vertical axis, may keep rising, although only slowly when extensive plastification has taken place at the junction. To avoid the difficulty and tedium associated with defining the collapse load of a geometrically stiffening structure (Teng, 1994), the axisymmetric collapse strength of the modified structural model should be determined using a small deflection plastic analysis. Such an analysis leads to a collapse load corresponding to the classical limit load. The plastic buckling
load may also be determined without considering the effect of prebuckling large deflections. This is desirable as the plastic buckling loads need to be normalized by the plastic collapse load for a given junction geometry to establish the function \( f(k, \lambda) \) in Eqn 2. The effect of pre-buckling large deflections is small and its omission leads to conservative results for the modified structural model.

(c) The deformation theory should be used in the analysis as it leads to results lower than those from the flow theory, and is known to approximate well experimental results in many cases.

8. DEVELOPMENT OF A PLASTIC BUCKLING STRENGTH APPROXIMATION

Using the modified structure model (Figure 8), it can be demonstrated without any doubt that the plot of the dimensionless strength \( p/p_p \) versus the dimensionless slenderness parameter \( \lambda \) does not depend on the yield stress at all (Figure 9). This confirms that the small variations between the plots shown in Figure 6 are due to instability effects in the skirt.

To demonstrate that the parameter \( k \) defined by Eqn 3 is the key ringbeam size parameter, the plastic buckling strengths of a large number of junctions covering \( R/t = 250, 500, 750, t_c/t_c = 0.5, 1, 2 \) and \( \alpha = 20^\circ, 40^\circ, 60^\circ \) were obtained. Figure 10 shows conclusively that, as long as \( k \) is kept constant, the form of interaction between elastic buckling and plastic collapse is the same. This is a very important conclusion: the form of interaction between elastic buckling and plastic collapse is dependent on a single parameter so its effect can be easily captured.

To generate a plastic buckling strength approximation, plastic buckling results were obtained for a transition junction (Figure 8) with the following geometric parameters: \( R/t = 500, \alpha = 40^\circ, t_c = t_s = t \). The ringbeam areas were varied to arrive at desired values of the dimensionless ringbeam size parameter \( k \). In the region where the interaction between elastic buckling and plastic collapse is the strongest \((0.5 < \lambda < 1.2)\), the lowest strength curve is that given by the smallest \( k \) value. Very small ringbeams are not used in practice, so a \( k \) value of 0.25 may be viewed as a lower bound to practical \( k \) values. For design use, a multiple-curves approximation is possible. For example, three lower bound curves may be easily defined, corresponding to the following three ranges of \( k \) values: (a) \( 0.25 < k < 1 \), (b) \( 1 \leq k < 2 \) and (c) \( k \geq 2 \). Due to the present lack of information on the effect of geometric imperfections and residual stresses, it is felt that
at this stage, the simpler single-curve approach is preferred. The following equations are seen to provide a lower bound approximation to the plastic buckling region and a close approximation overall:

\[
\frac{p_L}{p_p} = 1 - 0.3\lambda^{0.5}, \quad 0 < \lambda < 1.62 \quad \text{(Plastic Buckling Region)}
\]

(4)

\[
\frac{p_L}{p_p} = \frac{1}{\lambda}, \quad \lambda > 1.62 \quad \text{(Elastic Buckling Region)}
\]

(5)

**Figure 10:** Strengths of ringbeams of the same dimensionless size

**Figure 11:** Approximation for plastic buckling strength
The division into the two regions (elastic buckling and plastic buckling) is based on the strength behaviour, so the elastic buckling region covers purely elastic buckling failures as well as nearly elastic buckling failures occurring after limited yielding which does not reduce the buckling strength appreciably. The above two equations give the same value at the transition point of $\lambda = 1.62$ between the two regions and also have a similar (but not identical) slope at this point. They are thus satisfactory as a design proposal. Appropriate load and resistance factors should be used with Eqns 4 and 5 in a limit state design formulation.

9. OTHER LOADING PATTERNS AND APPLICATION IN DESIGN

Although the results obtained so far are for a specific loading pattern, generalization of the results for application to junctions under other loading patterns is easy as the pressure used here can be related to the equivalent circumferential compressive force at the junction (Teng and Rotter, 1991a). For a given geometry, the value of this force at failure is largely independent of details in the loading pattern. Therefore, once the function $f(k, \lambda)$ is defined, the equivalent circumferential compressive force at failure can be related to its value at plastic collapse through the same function $f(k, \lambda)$ with $\lambda$ being defined using the values of this circumferential compressive force at plastic collapse and at elastic buckling. For any given loading condition, this circumferential compressive force can be calculated without difficulty and checked to avoid failure (Teng, 1997). For a general pattern of internal pressure with or without accompanying frictional drag, the ultimate failure strength in terms of the circumferential compressive force $F_t$ is thus given by:

$$F_t = f(k, \lambda)F_p$$

(6)

$$\frac{F_t}{F_p} = 1 - 0.33 \lambda^{1.5}, \quad 0 < \lambda < 1.62$$

(Plastic Buckling Region)

(7)

$$\frac{F_t}{F_p} = \frac{1}{\lambda^5}, \quad \lambda > 1.62$$

(Elastic Buckling Region)

(8)

with $\lambda$ being defined by

$$\lambda = \sqrt{F_p / F_t}$$

(9)

where $F_t$ and $F_p$ are the elastic buckling strength and plastic collapse strength in terms of the equivalent circumferential compressive force. A comprehensive description of how the above equations can be applied to design transition junction ringbeams utilizing previous results on effective section analysis (Rotter, 1983), elastic buckling (Jumikis and Rotter, 1983) and plastic collapse (Teng and Rotter, 1991a) together with the results discussed here can be found in Teng (1997) where a numerical example is also given.

It is worth noting that the effects of imperfections and residual stresses have not been considered in this study. Little information is currently available on either residual stresses or initial imperfections in these ringbeams. They are probably best assessed by experiments whose results can then be used to gauge the safety margin of a theoretically-based design proposal. Experiments have been planned and will be conducted in the near future as part of a major research project on Stability and Strength of Steel Silo Transition Junctions supported by the Hong Kong Research Grants Council.

10. COMMENTS ON BUCKLING MODES

In the formulation of NEPAS (Teng and Rotter, 1989), the buckling load is found when the minimum eigenvalue among possible circumferential buckling wave numbers is equal to 1. The numerical results presented above are for the out-of-plane buckling of ringbeams with $B/T$ ratios not less than 10. For a transition junction (Figure 8) with $R/t = 500$, $\alpha = 40^\circ$, $\rho = t$, $BT/t^2 = 40$ and $B/T = 7$, it was found that a pressure $p$ of 0.1671 MPa (together with a frictional drag $\mu p$, where $\mu = 0.5$) leads to a minimum eigenvalue of 1 for a wave number $n = 383$ (at least
PLASTIC BUCKLING OF TRANSITION RINGBEAMS

within the checked range of \( n = 1 \text{ to } 500 \). The meridional buckling mode assumes an unusual pattern, and within the annular plate, it involves only relative in-plane movements between nodes. Corresponding to this load level, a local minimum was also found at \( n = 42 \) (Figure 12). When the pressure was increased to \( p = 0.1892 \) MPa at which the eigenvalue is 1 for \( n = 42 \), the variation of eigenvalue with \( n \) shows a monotonic decrease from \( n = 0 \) to \( n = 372 \) at which there is a minimum (Figure 12). Not even a local minimum could be identified which corresponds to the out-of-plane buckling mode. The same junction with \( B/T = 10 \) has a local minimum in eigenvalue at \( n = 34 \) with a buckling pressure \( p = 0.1842 \) MPa corresponding to the out-of-plane buckling mode, although the eigenvalue becomes less than 1 when \( n \) is equal to 120 and above. The eigenmode for \( n = 120 \) assumes an unusual pattern. A minimum eigenvalue of 1 was found for \( n = 568 \) corresponding to a buckling pressure of 0.1646 MPa. If the higher mode can be considered as physically unrealistic, the out-of-plane buckling mode at \( n_{cr} = 34 \) can be taken as the critical mode. The issue of very high modes needs further examination, and some experimental verification of this behaviour would be very useful. It is expected that at these very high wave numbers, the present thin shell model is inaccurate.

![Graph showing eigenvalue variation with wave number](image)

**Figure 12**: Variation of eigenvalue with circumferential wave number \( n \)

11. ACKNOWLEDGMENT

The work described in this paper forms part of the project Stability and Strength of Steel Silo Transition Junctions. The author wishes to thank the Hong Kong Research Grants Council for its financial support.

12. REFERENCES


6 CONNECTIONS
BEHAVIOUR OF STEEL BEAM-TO-COLUMN JOINTS UNDER CYCLIC REVERSAL LOADING: AN EXPERIMENTAL STUDY

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ABSTRACT

An experimental study was carried out to investigate the low-cycle fatigue behaviour of beam-to-column joints. Several full-scale specimens were tested, in a multi-specimen testing program, using constant amplitude displacement histories, to develop a cumulative damage model. This model is based on $S$-$N$ line approach and, although proposed here for low-cycle fatigue, was derived and is valid also for high cycle fatigue. Possible definitions of parameter $S$, as well as, failure criteria for the definition of $N$, are compared. The influence of the definition of parameter $S$ on the value of the slope of the $S$-$N$ line is discussed. A statistical method for the assessment of the $S$-$N$ lines, based on a given probability of failure with reference to suitable levels of safety and reliability of the structural joints, is also presented.

KEYWORDS

Joints, connections, experimental behaviour, fatigue, damage, failure, design codes, S-N curves.

1. INTRODUCTION

Selection of the energy dissipation mechanisms appears to be a fundamental requisite for anisismic design of steel structures as it allows the combination of the stable hysteretic response of the whole system, with the possibility to control reliably the main behavioural parameters influencing the structural behaviour under seismic input. Hysteretic behaviour of individual frame components (i.e, members and joints) plays a relevant role in the definition of the global response of the structure. Therefore, in addition to material ductility, overall and local buckling as well as low cycle fatigue phenomena are important factors which can significantly affect the seismic performance of the structural system.

Traditional design approaches for steel structures, based on simple and rigid frame models, concentrate the dissipation of the energy associated with the earthquake, respectively in bracing cantilever systems or in beam-to-columns joints, CEN (1994). Moreover, the semi-continuous frame model, which already proved its efficiency under static loading, Bjorhovde et al (1991), may also be a convenient options for seismic design if joints are considered part of the dissipation systems, Nader et al (1991), Elnashai et al (1994).
The paper presents the main results of a jointed research project, between the Universities of Lisbon (Portugal), Milan (Italy) and Trento (Italy) with the general scope to define suitable and accurate criteria for low-cyclic fatigue design (i.e., fracture resistant seismic design) of rigid as well as semi-continuous steel frames. A limited number of structural steel joints were identified and they were fabricated and tested in a multi-specimen program in order to establish classes of low-cycle fatigue resistance for connections, similar to those existing for structural details under high cycle fatigue, CEN (1992).

2. DAMAGE ASSESSMENT MODELS

It is the author's opinion that a modern methodology to assess the low-cycle fatigue endurance of civil engineering structures should adopt parameters related to the global structural behaviour, such as displacements, rotations, bending moments, etc. Taking this into account, the S-N line approach may be adopted, considering as strain parameters related with the global structural ductility (e.g., interstorey drifts or joint rotations).

The fatigue failure prediction function, used by the S-N line approach, can be expressed by the following equation:

\[ N \cdot S^m = K \]  \hspace{1cm} (1)

where \( N \) is the number of cycles to failure at the constant stress (strain) range \( S \). The non-dimensional constant \( m \) and the dimensional parameter \( K \) depend on both the typology and the mechanical properties of the considered steel component. In the Log-Log domain Eqn. (1) can be re-written as:

\[ \log(N) = \log(K) - m \log(S) \]  \hspace{1cm} (2)

Eqn. (2) represents a straight line (so called S-N line) with a slope equal to \(-1/m\) called fatigue resistance line, which identify the safe and unsafe regions (Figure 1).

![Figure 1: Fatigue resistance line in the Log(S)-Log(N) scale.](image)

In order to apply Eqn. (1), both parameters \( S \) and the number of cycles to failure \( N \) should be defined, by a consistent re-elaboration of test data in accordance with the basic assumption of the selected damage model. The number of cycles to failure \( N \) can be identified on the basis of the adopted failure criterion, while \( S \) can be defined with reference to the definitions given in the literature by various authors. By statistical re-analysis of such data plotted in a Log(S) - Log(N) scale, is than possible to define both the slope \((-1/m)\) of the S-N line and the constant \( K \). Of course such value might be different for various definitions of parameters \( S \) and \( N \).
2.1 Definition of the Strain Range (S)

Some of the most relevant proposals available in literature for the definition of the strain range $S$ are shortly presented.

**Krawinkler & Zohrei proposal**

The concept to connect the parameter $(S)$ of the fatigue failure prediction functions with the global structural ductility was originally proposed by Krawinkler et al. (1983). They proposed a relationship between the fatigue endurance and the plastic portion of the generalised displacement component, which can be expressed as:

$$N\left(\Delta \delta _{pl}\right)^{n_{cr}} = K_{cr}$$

where $\Delta \delta _{pl}$ represents the plastic portion of the deformation range.

**Ballio & Castiglioni proposal**

Ballio et al. (1995) proposed an unified approach for the design of steel structures under low- and/or high-cycle fatigue, which is based on global displacement parameters instead of local deformation parameters. The fundamental hypothesis is the validity of the following equation:

$$\Delta \varepsilon _{y} = \frac{\Delta v}{v_y} = \frac{\Delta \phi}{\phi_y}$$

where $\varepsilon$ represents the strain, $v$ the displacement, $\phi$ the rotation (or the curvature), $\Delta$ the range of variation in a cycle and the subscript $y$ identifies the yielding of the material ($\varepsilon_y$) as well as the conventional yielding with reference to the generalised displacement ($v_y, \phi_y$) assumed as the test control parameter.

The following parameter can be defined:

$$\Delta \sigma^* = E \Delta \varepsilon = \frac{E \Delta v}{v_y} \varepsilon_y = \frac{\Delta v}{v_y} \sigma(F_y) = \frac{\Delta \phi}{\phi_y} \sigma(F_y) = \frac{\Delta \delta}{\delta_y} \sigma(F_y)$$

The term $\Delta \sigma^*$ represents the effective stress range, associated with the real strain range $\Delta \varepsilon$ in an ideal member made of an indefinitely linear elastic material, $\delta$ is the generalised displacement and $E$ is the Young's modulus. In the case of high-cycle fatigue (i.e. under cycles in the elastic range), $\Delta \sigma^*$ coincides with the actual stress range $\Delta \sigma$. Using the $S$-$N$ curves and assuming the effective stress range $\Delta \sigma^*$ as $S$, Eqn. (1) can be re-written as:

$$N\left(\frac{\Delta \delta}{\delta_y} \sigma(F_y)\right)^{w_{cr}} = K_{cr}$$

**Feldmann, Sedlacek, Weynand & Kuck proposal**

On the basis of an extensive finite element analysis, Feldmann (1994), Sedlacek et al. (1995) and Kuck (1994) investigated the behaviour of beam-to-column joints under constant amplitude cyclic loading. It
appeared that the relationship between the plastic strain, $\varepsilon_{pl}$, at the hot spot (strain at the relevant place where first crack occurs) and the plastic rotation ($\phi_{pl}$) is linear, i.e.:

$$\frac{\varepsilon_{2,pl}}{\varepsilon_{1,pl}} = \frac{\phi_{2,pl}}{\phi_{1,pl}}$$

(7)

where subscripts 1 and 2 refer to two different loading steps. This linear relationship, that is somehow similar to Eqn. (4), can be re-written in terms of the plastic portion $\delta_{pl}$ of the generalized displacement component $\delta$ and, when plotted in a Log-Log scale, plots as a "Wöhler" line. Hence, it is possible to define an effective plastic stress range $\Delta\sigma_{pl}^*$:

$$\Delta\sigma_{pl}^* = \frac{\Delta\delta_{pl}}{\delta_y} \sigma(F_s)$$

(8)

that can be assumed as parameter $S$ and Eqn. (1) can be re-written as:

$$N \left( \frac{\Delta\delta_{pl}}{\delta_y} \sigma(F_s) \right)^{\alpha_{max}} = K_{ISCX}$$

(9)

**Bernuzzi, Castiglioni & Calado proposal**

Based on the re-elaboration of an extensive experimental data of beam-to-column joints as well as of beams and beam-columns, Castiglioni et al (1996) and Bernuzzi et al (1997), they proposed to use the total interstorey drift (i.e., the total displacement range) $\Delta u$ as parameter $S$, and consequently Eqn. (1) can be re-written, in the form:

$$N.\Delta u^{\alpha_{max}} = K_{BTV}$$

(10)

### 2.2 Definition of the Fatigue Endurance ($N$)

For the prediction of the low-cycle fatigue endurance of steel structural components, it seems convenient to adopt a failure criterion based on parameters associated with the response of the component (i.e. stiffness, strength or dissipated energy). Two failure criteria, which have been proposed, Castiglioni et al (1996), are in the following reviewed.

**Calado, Azevedo & Castiglioni criterion ($N_\alpha=0.5$)**

These authors, Calado et al (1989, 1995, 1996), developed a failure criterion of general validity for structural steel components under both constant and variable amplitude loading histories that can be written in the following form:

$$\frac{\eta_f}{\eta_0} \leq \alpha$$

(11)

In this equation $\eta_f$ represents the ratio between the absorbed energy of the considered component at the last cycle before collapse and the energy that might be absorbed in the same cycle if it had an elastic-perfectly plastic behaviour while $\eta_0$ represents the same ratio but with reference to the first cycle in plastic range. The value of $\alpha$, which depends on several factors (such as type of the joint and the steel grade of the component)
should be determined by fitting the experimental results. As it is particularly interesting to identify \( \alpha \) \textit{a priori}, in order to define a unified failure criterion, a value of \( \alpha = 0.5 \) is recommended for a satisfactory and conservative appraisal of the fatigue life. In the case of variable amplitude loading histories the same criterion remains valid but should be applied on semi-cycles, which can be defined in plastic range as the part of the hysteresis loop under positive or negative loads or as two subsequent load reversal points.

\textit{Bernuzzi, Calado & Castiglioni criterion (}N_{\\text{AWL}}\text{)}

In this criterion, valid only for constant amplitude loading, Bernuzzi \textit{et al} (1997), it is assumed that the drop of the hysteretic energy dissipation is the main parameter for the definition of the low-cycle fatigue endurance. In particular, focusing attention only on the cycles performed at the displacement range \( \Delta u_s \), the relative energy drop, \( \Delta W_r \), can be defined as:

\[
\Delta W_r = \left( \frac{W_{\text{dis}} - W_i}{W_{\text{dis}}} \right)
\]

where \( W_{\text{dis}} \) represents the dissipated energy in the first cycle at the assumed displacement range while \( W_i \) is the energy associated with the \( i \)-th cycle at the same displacement range. Failure is assumed to occur when the energy drop is evident, i.e. the generic point of co-ordinates \( n_k, \Delta W_{rk} \), is not on the straight line.

2.3 Definition of \( m \)

The parameter \( m \) identifies the slope of the line interpreting, in a \textit{Log-Log} scale, the relationship between number of cycles to failure \( (N) \) and the stress (strain) range \( (S) \).

Focusing the attention on \( m \), some discrepancies can be found in the literature among the proposal of various authors. In particular it can be noticed that Ballio & Castiglioni suggested to adopt a value of \( m_{BC}=3 \) like Bernuzzi \textit{et al} \( m_{BCC}=3 \); on the contrary, Krawinkler & Zohrei proposed an exponent \( m_{KZ}=2 \). The Feldmann approach tried to re-write the Coffin-Mason equation in terms of global deformation parameters, proposing a value \( m_{FSHK}=2 \) in agreement with Krawinkler & Zohrei. However, both Feldmann and Krawinkler adopted the plastic range of the assumed parameters, while Ballio adopted the total cyclic excursion (i.e. elastic and plastic).

Hence, it seems that the value of the exponent \( m \) depends on the definition of \( S \). In order to clarify the influence of the definition of parameter \( S \) on the value of the exponent \( m \), this last parameter was assessed for possible definitions of \( S \).

In order to assess the exponent \( m \), the data obtained in tests with cycles of constant amplitude should be plotted in terms of number of cycles to failure \( (N) \) and stress (strain) range \( (S) \) in a \textit{Log-Log} diagram and fitted by a straight line. The slope \( m \) of the best fitting line can be considered the exponent to be adopted in Eqn. (1).

3. DEFINITION OF THE DESIGN S-N LINES

According to the limit state design method, the parameter governing the design should be defined on the basis of a statistical analysis, allowing to make reference to a value associated with a given probability of failure \( P_f \) (or of survival, \( 1 - P_f \)). Such a probability should be defined with reference to suitable levels of safety and reliability of the structure and its components. Hence, the \( S-N \) lines should be defined with reference to a given probability of failure \( P_f \). In this research, the procedure proposed in the JWG XIII-XV - Fatigue Recommendations (1994) was used and is briefly summarised herein.
Scope of the procedure is the definition of the value Log($K_d$) such that the line with a slope (-1/m), and intersecting the x-axis at Log($K$), is associated with a probability of 5% that test data plot below it. In the following, reference is made to the set of values Log($K$) representing the intersections with the N axis of the S-N lines, each one having a constant slope (-1/m), and passing for every single test data point.

The basic formula to determine the design value of the random variable $X$=$\text{Log}(K)$ can be assumed as:

$$X_d = \mu - \phi^{-1}(\alpha) \cdot \sigma$$

(13)

where $X_d$ is the design value, $\mu$ is the mean value of $X$, $\sigma$ is the standard deviation of $X$, $\phi$ is the normal law of probability and $\alpha$ is the probability of survival. Generally it is assumed that $\mu$ is distributed according to the t-Student model and the standard deviation $\sigma^2$ follows the chi-square law ($\chi^2$). As $\sigma$ and $\mu$ are unknown values, their estimated values from tests, $\mu$ and $\sigma$ respectively, should be associated with $\beta$, confidence level. The design value of $X$ can be expressed as:

$$X_d = \mu - \sigma \cdot \left[ t(\beta, n-1) - \frac{\phi^{-1}(\alpha)}{\sqrt{n}} \cdot \frac{n-1}{\chi^2(1-\beta/2 \cdot n-1)} \right]$$

(14)

According to Eurocode 3, CEN (1992), the confidence range should be 75% of 95% probability of survival on the Log(N) axis. Hence, by assuming $\alpha$ and $\beta$ respectively equal to 0.95 and 0.75, Eqn (14) can be re-written as:

$$X_d = \mu - \sigma \cdot \gamma$$

(15)

where, in the case of limited amount of data, the value of $\gamma$ can be obtained directly from Table 1 on the basis of the number of available data ($n$).

TABLE 1

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<thead>
<tr>
<th>n</th>
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<th>5</th>
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4. THE EXPERIMENTAL PROGRAMME AND THE TESTS

An experimental study was recently carried out at the Technical University of Lisbon on steel beam-to-column joints, Calado et al (1996). In order to minimise size effects, specimens were as close as possible to full size. They consisted of a beam attached to a column by means of different connection details, representing frequent solutions adopted in the European steel construction practice for beam-to-column joints, Figure 2.

The following typologies of beam-to-column joints were considered and tested: bolted web and flanges cleats (BCC1), extended end plate (BCC2), bolted flange plates (welded to the column) with web cleats (BCC3), welded flanges with bolted web cleats (BCC4) and welded joints (BCC5 and BCC6). Bolts used in all specimens were M16 grade 8.8; in the case of flange plates with web cleats specimens and for extended end plate connections, bolts were preloaded according to EC3 provisions, CEN (1992). All welds were full
penetration butt welds. Several specimens were fabricated and tested for each joint, according to a multi-
specimen testing program, consisting of different cyclic loading histories with both constant and variable

![Diagram of beam-to-column connections](image)

In Table 2 are presented the relevant parameters of each specimen ($f_t, F_r$, and $\delta$), the values of the various
definition of $(S)$ ($\Delta\delta$, $\Delta\delta_{ph}$, $\Delta\sigma_{pl}$, and $\Delta\sigma$) and the values of the fatigue endurance $(N)$ ($N_{tot}$, $N_{avr}$, and

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<tr>
<th>TESTS</th>
<th>$f_t$</th>
<th>$F_r$</th>
<th>$v_i$</th>
<th>$\Delta s$</th>
<th>$\Delta\delta_{ph}$</th>
<th>$\Delta\sigma_{pl}$</th>
<th>$\Delta\sigma$</th>
<th>$\sigma_{avr}$</th>
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Figure 2: Schematic of beam-to-column connections specimens.
Four types of failure were observed during the tests: joint failure (JF), failure in the base material of the beam (BBM), weld failure (WF) and panel mechanism (PM).

The behaviour of web and flange cleats connections (Figure 3) is characterised by large slippage between the beam flange and the angle leg due to the ovalization of holes, which increases with the number of cycles although the displacement amplitude is constant.

![Figure 3: Hysteresis loops and failure mode of a web and flange cleats connection.](image)

Extended end plate connections (Figure 4) are characterised by regular hysteresis loops without any slippage and with regular deterioration of both the absorbed energy and the maximum moment at the end of each cycle.

![Figure 4: Hysteresis loops and failure mode of an extended end plate connection.](image)

Bolted flange plates with web cleats connections (Figure 5) exhibit slippage between flange plates and the beam flange due to the ovalization of the holes, but this phenomena has less importance when compared with web and flanges cleats connections.
Figure 5: Hysteresis loops and failure mode of a bolted flange plates with web cleats connection.

The cyclic behaviour of specimens with welded flanges and web cleats (Figure 6) is characterised by high elasticity and great regularity in the shape of the loops with gradual deterioration of energy absorption capacity. Large plastic deformations developed in the panel zone of the column. In most cases, failure was due to cracking at toe of welds connecting the beam flanges to the column ones.

Figure 6: Hysteresis loops and failure mode of a welded flanges and web cleats connection.

The welded connections (Figure 7) are characterised by regular hysteresis loops and a very good capacity for energy dissipation; note that deterioration in terms of strength, stiffness and energy absorption capacity is very limited and stable up to collapse.
5. LOW-CYCLE FATIGUE PERFORMANCE

The previously mentioned test data were statistically re-elaborated, in accordance with the described procedures for the assessment of the stress (strain) range ($S$) and the number of cycles to failure ($N$). Concerning the fatigue endurance, assessed by the two mentioned methodologies ($N_{a=0.5}$ and $N_{dW}$), (Table 2), a good accuracy was observed between the values obtained with the proposed criterion and the actual number of cycles to complete failure ($N_{TOT}$).

For all the specimens tested, the Bernuzzi et al criterion ($N_{dW}$) exhibits a mean value error of 9% while the Calado et al criterion ($N_{a=0.5}$) shows 12%, allowing to conclude that both criteria are valid for the assessment of the fatigue endurance of steel components under constant amplitude loading.

However, it should be noticed that the Calado et al criterion has a general validity in both cyclic and random loading conditions; furthermore, the value of $\alpha=0.50$ represents a unified failure parameter, independently of the structural detail.

In order to assess the slope $m$ of the $S-N$ lines in the Log-Log scale for each typology of beam-to-column connections, and to investigate the influence of the fatigue endurance definition ($N_{a=0.5}$ and $N_{dW}$) on such slope, all experimental data were re-elaborated according to the previously mentioned proposals of definition of $S$. Their values are presented in Table 3.

It can be noticed that the definition of the fatigue endurance ($N_{a=0.5}$ and $N_{dW}$) has a limited influence on the variation of the slope $m$ and on $\log(K)$. However, these two parameters are very dependent of the definition of $S$. Values of $\log(K)$ and $m$ parameters obtained with Krawinkler et al and Feldmann et al approaches are very similar to each other. Also the results obtained with the Ballio et al and Bernuzzi et al approaches give values for the previous mentioned parameters, very similar to each other, although different from those obtained by means of Krawinkler et al and Feldmann et al methods.

In fact, the slope $m$ of the line best fitting in a $\log(S)$-$\log(N)$ plot the test data, if assessed with Krawinkler et al and Feldmann et al methods is nearly 2 while a value of approximately 3 is obtained with Ballio et al
and Bernuzzi et al approach. This is due to the fact that both Krawinkler et al and Feldmann et al approaches adopted plastic range of $S$, while Ballio et al and Bernuzzi et al adopted the total cyclic excursion (i.e. elastic and plastic). Figure 8 shows $S$-$N$ lines for one typology of beam-to-column connections presented in Table 3.

### Table 3

**Log($K$) and $m$ coefficients for beam-to-column connections**

<table>
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<tr>
<th>Tests</th>
<th>$N_{Aw}$</th>
<th>$N_{0.5}$</th>
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<td>B</td>
</tr>
<tr>
<td></td>
<td>$\log(K)$</td>
<td>$m$</td>
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<tr>
<td>c</td>
<td>1.66</td>
<td>0.40</td>
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</table>

**Criterion for the strain range definition:**

A: Krawinkler & Zohrei
B: Ballio & Castiglioni
C: Feldmann et al
D: Bernuzzi et al

This evidence can be considered to be a general conclusion, as it can be seen directly from Table 3, which summarises the results in terms of slope $m$ and Log($K$) for all test data.

In order to define $S$-$N$ lines to be adopted for design and damage assessment, for the types of connections considered in this study, the statistical procedure previously presented in section 3 was applied. This is done hereafter, assuming a slope $m=2$ in the case of definition of $S$ according to Krawinkler and Feldmann approaches, while $m=3$ is assumed in the case of Ballio and Bernuzzi definition of $S$.

Table 4 and Figure 9 summarises the results of such a statistical re-analysis of the test data, and give the expressions for the design S-N lines for the considered structural details.
Figure 8: Evaluation of the S-N line of beam-to-column connections on the basis of $N_{d}=0.50$ and considering $S$ defined by Krawinkler, Ballio, Feldmann and Bernuzzi proposals.

Figure 9: Evaluation of the S-N line of beam-to-column connections on the basis of $N_{d}=0.50$ and considering $S$ defined by (a) Krawinkler with $m=2$, (b) Ballio with $m=3$, (c) Feldmann with $m=2$ and (d) Bernuzzi with $m=3$ approaches.
BEHAVIOUR OF STEEL BEAM-TO-COLUMN JOINTS

TABLE 4

$\log(K)$ COEFFICIENTS FOR BEAM-TO-COLUMN CONNECTIONS WITH $m=2$ OR $m=3$

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<th>TESTS</th>
<th>$N_{awp}$</th>
<th>$N_{a=0.5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>$\log(K)$</td>
<td>$\log(K)$</td>
</tr>
<tr>
<td>BCC1</td>
<td>a</td>
<td>4.93</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>1.86</td>
</tr>
<tr>
<td>BCC2</td>
<td>c</td>
<td>5.51</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>5.06</td>
</tr>
<tr>
<td>BCC4</td>
<td>e</td>
<td>4.44</td>
</tr>
<tr>
<td>BCC5</td>
<td>f</td>
<td>4.46</td>
</tr>
<tr>
<td>BCC6</td>
<td>g</td>
<td>4.16</td>
</tr>
</tbody>
</table>

CRITERION FOR THE STRAIN RANGE DEFINITION:

A: KRAWINKLER & ZOHREI (m=2)
B: BALLIO & CASTIGLIONI (m=3)
C: FELDMANN ET AL. (m=2)
D: BERNUZZI ET AL. (m=3)

6. CONCLUSIONS

A major issue of this research is the assessment of fatigue strength category of steel connections, i.e., the appropriate $S$-$N$ curve to be associated with each type of structural detail. These categories can be developed by means of extensive experimental research or by numerical modeling. Such models should, however, be calibrated using test results.

One of the scopes of the research was also to investigate the cause of the discrepancies that were found in the proposed values for exponent $m$, presented by various authors in the literature.

The re-elaboration of the experimental data allows to conclude that the assessment of the slope $m$ of the $S$-$N$ lines is very dependent on the definition of the parameter $S$. In fact, if the parameter $S$ is based on the plastic range, like in Krawinkler et al and Feldmann et al, a value of $m=2$ applies, while a value of $m=3$ is assessed if the total excursion (i.e., elastic and plastic) is used, as proposed by Ballio et al and Bernuzzi et al.
7. ACKNOWLEDGEMENTS

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8. REFERENCES


A DESIGN MODEL FOR BOLTED COMPOSITE SEMI-RIGID CONNECTIONS

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ABSTRACT

This work presents a design model for bolted composite semi-rigid connections based on simple force paths. The continuity of the structural system is assured with the aid of longitudinal reinforcement bars located at the concrete slab. The stiffness of the connection can be divided in three major components. The first part is related to the stiffness of the bottom angle. The second part is due to the stiffness of web angles and finally, the last part takes into account the shear connectors and its interaction with the reinforcement bars. The design model can cover the cases in which the bending moment of the beam is transmitted to external or intermediate columns. An experimental investigation on perfobond rib shear connector resistance is presented. Finally, the test results are compared to a prediction formula available in the literature to access its validity.

KEYWORDS

Composite Steel Structures, Composite Semi-Rigid Bolted Connections, Steel Connections, Experimental Analysis, Perfobond Rib Shear Connector, Composite Structural Systems.

1. INTRODUCTION

The conventional structural design methods for steel connections tend to assume an idealistic behaviour far from the real response of the structural system. These methods, in order to simplify the analysis, consider the connections as being simple (hinge) or fixed. A literature survey on semi-rigid steel connections has pointed out that one of the most fundamental design tools is the moment vs. rotation curve. With the aid of this curve it is possible to make a rational design improving the structural behaviour and reducing the global cost of the structure.

The composite action present in negative moment regions is still lacking some fundamental answers. Even in the current detailing practice, it is common to specify some reinforcement bars in the negative moment region to avoid concrete cracking but the extra resistance provided by its use it is not considered in the connection design.
This was the main motivation for this work which presents a design model for bolted composite semi-rigid connections. These beam-column connections are composed of angles connecting the upper part of the beam’s web and the bottom flange of the beam to the column’s flange. The continuity of the structural system is guaranteed by the use of longitudinal reinforcement bars.

2. COMPOSITE SEMI-RIGID CONNECTIONS LITERATURE SURVEY

One of the first attempts to investigate the structural behaviour of semi-rigid connections date back to 1917 Lorenz et al (1993), while the inclusion of the composite action in those connections was studied in the sixties. Later on, the investigations focused on using the composite connections as an alternative to rigid steel connection design, which conducted to high reinforcement ratios.

In the eighties an alternative to the rigid design, treating the composite connection as being semi-rigid was adopted, leading to economic and efficient solutions for simple portal frames. The investigations developed a design and detailing process for simple and easy to assemble connections with moderate reinforcement ratios Narayanan (1989). These investigations indicated that the composite semi-rigid connections can produce an economic and safe design to withstand gravitational and lateral loads in buildings up to ten floors.

A definitive analysis of the structural behaviour of the composite semi-rigid connections should take into account the tridimensional nature of the problem. This fact is attenuated by the presence of a continuous concrete slab that gives the system the necessary stiffness to prevent out of plane and torsional effects. Considering all the facts exposed previously the need for more full scale portal frame experiments become imperative.

3. THE PERFOBOND SHEAR CONNECTOR

A Perfobond rib shear connector was first used by Leonhardt and associates, in the third Caroni bridge, Venezuela, to overcome fatigue problems. This shear connector is made from a rectangular plate with holes as indicated in figure 1. The mechanical behaviour of this shear connector is dependent on the concrete slab in which it is embedded. As the concrete gets harder dowels are formed through the shear plate holes. These dowels contribute to the horizontal shear resistance and also prevents the slip and vertical separation between the beam and the concrete slab. The structural performance of this shear connector is improved by the use of reinforcement bars through its holes.

This type of shear connectors presents one extra advantage when compared to shear studs. The Perfobond Rib requires simple equipments for its assembly, fact that was corroborated by full scale tests. Push-out test in which the numbers of holes and the plate’s height varied in relation to the slab thickness led to an optimal shape for the Perfobond Oguejiofor et al (1994). The Perfobond shear connector adopted in this study has the optimal range dimensions described previously. The only difference was the plate’s height that was modified to be in accordance to the slab thickness adopted.

![Figure 1: The perfobond rib shear connector](image-url)
3.1 The Experimental Program

In order to access and compare the perfobond shear resistance formulas and tests results, (EB and ED tests, table 1), proposed by Oguejiofor et al (1994), an additional experimental program was conducted. The program consisted of eight push-out tests (PB tests) with material and geometry characteristics defined in table 2. The perfobond specimens, 375 mm x 80 mm x 12.7 mm were welded to I 203 mm x 27.4 kg/m sections, 720 mm long, photo 1, and encased in two concrete slabs 720 mm x 720mm x 100 mm. A reinforcement bar square mesh of $\phi$ 4.76 mm @ 172 mm was also used in all the tests. The values $n_1$ and $n_2$ presented in tables 1 and 2 represents, for each side, the number of reinforcement bars inside all the perfobond rib holes and the total number of transversal reinforcement bars used.

### TABLE 1
GEOMETRIC AND MATERIAL CHARACTERISTICS USED IN OGUEJOFOR ET AL TESTS

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Concrete Slab</th>
<th>Perfobond Rib</th>
<th>Reinforcement Bars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_c$</td>
<td>H (mm)</td>
<td>L (mm)</td>
</tr>
<tr>
<td>EB 03</td>
<td>20.91</td>
<td>152</td>
<td>712</td>
</tr>
<tr>
<td>ED 03</td>
<td>24.82</td>
<td>152</td>
<td>712</td>
</tr>
<tr>
<td>EB 07</td>
<td>20.91</td>
<td>152</td>
<td>712</td>
</tr>
<tr>
<td>ED 07</td>
<td>24.82</td>
<td>152</td>
<td>712</td>
</tr>
</tbody>
</table>

To enable a better result comparison tests PB 01 and PB 02 were made similar to previous ED 03 and EB 03. Due to the same reason tests PB 04, PB 06 and PB 07 were made similar to previous ED 07 and EB 07. The only difference was the absence in EB 03 and EB 07 of a wiring mesh layer near the slab's external side. This mesh was used in some tests to reduce and control the concrete cracking. The geometrical detailing of the concrete slab and perfobond rib shear connectors adopted in the tests as well as the reinforcement bars numbers and location are depicted in figure 2. Specimens PB 05 and PB 08 were tested to identify the real shear contribution of the reinforcement bars and concrete dowels present and formed in the perfobond holes. They were made with a slot (256 mm x 20 mm x 100 mm) present in the concrete slabs from the edge of the shear connector to the concrete slab base, photo 2. Figures 3 to 5 presents the load versus slip curves for all the tests performed. A typical failure configuration for the push-out series is presented in photo3.
Photo 1: Perfobond rib shear connector geometry

Photo 2: Perfobond rib push out test layout

Figure 2: Ferreira et al test series layout

Figure 3.a: Load versus slip curves for PB 01, PB 02 and PB 03 tests

Figure 3.b: Load versus slip curves for PB 04, PB 06 and PB 07 tests
Figure 4: Load versus slip curves for PB 05 and PB 08 tests

Figure 5: Load versus slip curves for all Ferreira et al. tests

Photo 3: Typical failure configuration for the push-out test.
3.2 The Oguejiofor and Hosain perfobond shear resistance formula

The perfobond shear resistance formula, Eqn. 1, proposed by Oguejiofor et al (1994) is divided into three terms. These independent terms take into account the contribution to the shear resistance of the concrete slab, the reinforcement bar used inside the perfobond rib holes and the concrete dowels, respectively.

\[ q_n = 0.590 A_n \sqrt{f'_c} + 1.233 A_n f_s + 2.871 n D^2 \sqrt{f'_c} \]  

To introduce the limit state design concept a shear design resistance \( R \) developed by each perfobond rib shear connector can be defined as:

\[ R = \phi q_n \]

In this paper a resistance factor \( \phi = 1.0 \) is adopted to enable a comparison between the predicted and experimental results. Tables 3 and 4 to follow presents an evaluation of Eqn. 1 through a comparison with Oguejiofor et al and Ferreira et al tests where the contribution of each of the three terms that influence the shear connector resistance is identified.

In these tables \( P_{\text{pred}} \) and \( P_{\text{exp}} \) represents the nominal resistance of the shear connector evaluated with the aid of Eqn. 1 and the experimental result respectively. In order to access Eqn. 1 range of domain and associated error the ratio \( P_{\text{exp}} \) divided by \( P_{\text{pred}} \) defined as \( p \), is evaluated and depicted in figures 6 to 9.

### TABLE 3
EVALUATION OF EQN. 1 THROUGH OGUEJIOFOR ET AL TESTS

<table>
<thead>
<tr>
<th>Prototype</th>
<th>( f_c ) (MPa)</th>
<th>( f_s ) (MPa)</th>
<th>Slab (kN)</th>
<th>Dowels (kN)</th>
<th>Bars (kN)</th>
<th>( P_{\text{pred}} ) (kN)</th>
<th>( P_{\text{exp}} ) (kN)</th>
<th>( P_{\text{exp}}/P_{\text{pred}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>EB 03</td>
<td>20.91</td>
<td>426.6</td>
<td>179.4</td>
<td>98.5</td>
<td>0.0</td>
<td>277.8</td>
<td>274.0</td>
<td>0.99</td>
</tr>
<tr>
<td>ED 03</td>
<td>24.82</td>
<td>406.4</td>
<td>195.4</td>
<td>107.3</td>
<td>0.0</td>
<td>302.7</td>
<td>343.8</td>
<td>1.14</td>
</tr>
<tr>
<td>EB 07</td>
<td>20.91</td>
<td>426.4</td>
<td>179.4</td>
<td>98.5</td>
<td>123.9</td>
<td>401.7</td>
<td>393.6</td>
<td>0.98</td>
</tr>
<tr>
<td>ED 07</td>
<td>24.82</td>
<td>406.4</td>
<td>195.4</td>
<td>107.3</td>
<td>118.1</td>
<td>420.8</td>
<td>580.9</td>
<td>1.38</td>
</tr>
</tbody>
</table>

### TABLE 4
EVALUATION OF EQN. 1 THROUGH FERREIRA ET AL TESTS

<table>
<thead>
<tr>
<th>Prototype</th>
<th>( f_c ) (MPa)</th>
<th>( f_s ) (MPa)</th>
<th>Slab (kN)</th>
<th>Dowels (kN)</th>
<th>Bars (kN)</th>
<th>( P_{\text{pred}} ) (kN)</th>
<th>( P_{\text{exp}} ) (kN)</th>
<th>( P_{\text{exp}}/P_{\text{pred}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB 01</td>
<td>16.72</td>
<td>494.4</td>
<td>116.7</td>
<td>95.2</td>
<td>0.0</td>
<td>211.9</td>
<td>294.2</td>
<td>1.39</td>
</tr>
<tr>
<td>PB 02</td>
<td>15.92</td>
<td>494.4</td>
<td>113.9</td>
<td>92.9</td>
<td>0.0</td>
<td>206.8</td>
<td>291.5</td>
<td>1.41</td>
</tr>
<tr>
<td>PB 03</td>
<td>15.49</td>
<td>494.4</td>
<td>112.3</td>
<td>91.7</td>
<td>47.9</td>
<td>251.9</td>
<td>300.1</td>
<td>1.19</td>
</tr>
<tr>
<td>PB 04</td>
<td>13.21</td>
<td>494.4</td>
<td>103.7</td>
<td>84.6</td>
<td>143.6</td>
<td>332.0</td>
<td>371.4</td>
<td>1.12</td>
</tr>
<tr>
<td>PB 06</td>
<td>9.00</td>
<td>494.4</td>
<td>85.6</td>
<td>69.9</td>
<td>143.6</td>
<td>299.1</td>
<td>278.0</td>
<td>0.93</td>
</tr>
<tr>
<td>PB 07</td>
<td>8.40</td>
<td>494.4</td>
<td>82.7</td>
<td>67.5</td>
<td>143.6</td>
<td>293.9</td>
<td>274.4</td>
<td>0.93</td>
</tr>
<tr>
<td>PB 08</td>
<td>11.85</td>
<td>494.4</td>
<td>0.0</td>
<td>80.2</td>
<td>143.6</td>
<td>223.8</td>
<td>220.1</td>
<td>0.98</td>
</tr>
<tr>
<td>PB 09</td>
<td>9.96</td>
<td>494.4</td>
<td>0.0</td>
<td>73.5</td>
<td>143.6</td>
<td>217.1</td>
<td>207.4</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Specimens without reinforcement bars inside the perfobond holes, see figure 7, PB 01 and PB 02 possesses a 40% increase in the experimental load when compared to the predicted values. This was caused by the fact that Eqn. 1 does not take into account the additional resistance provided by a reinforcement mesh used in these specimens. Prototype EB 03 did not have this mesh and had a 1% error. On the other hand specimen ED 03 had the mesh and only overestimated the collapse load by 14%.
The influence of the reinforcement bars set in the perfobond rib holes is shown in figure 8. Specimens PB 04, PB 06 and PB 07 possess a mean value for $p$ equal to 0.99, indicating that the reinforcement mesh influence on this specimens is negligible. On the other side test ED 07 shows a 38% overestimation of the experimental load. These two facts leads to the conclusion that the influence of the reinforcement mesh over the shear connector’s resistance is less significant for specimens with transversal reinforcement bars inside the perfobond rib holes.

4. A DESIGN MODEL FOR COMPOSITE BEAMS IN NEGATIVE MOMENT REGIONS

The composite beam design in negative moment regions adopted was first proposed in Johnson (1975) and refined in the present study. An economic solution is achieved when the plastic neutral axis is located inside the beam’s top flange or very close to it. In the following deduction, figure 10, the plastic neutral axis is located close to the top flange. A simple modification can extend the use of this method when the plastic neutral axis is located on the top flange. By a longitudinal equilibrium the moment resisted by the reinforcement bars is given by:

$$\Delta M = M_{ps} - \phi M_p = F_p (d_w - d_t) = 2 f_{yr} t_w (d_w - d_t)$$

(3)

after some more equilibrium and geometrical considerations:

$$d_w^2 - 2(d - d_t)d_w + \frac{\Delta M}{f_{yr} t_w} = 0$$

(4)

using Eqn. 3 and 4 the reinforcement area ($A_r$) can be obtained:

$$A_r = \frac{\Delta M}{(d - d_t - \frac{d_w}{2}) f_{yr}}$$

(5)

5. A DESIGN MODEL FOR COMPOSITE SEMI-RIGID CONNECTIONS

This section describes the proposed design model for composite semi-rigid connections. The model is based on the classical beam theory and, to guarantee this structural behaviour, Perfobond Rib and “T” rib shear connectors (described in detail in section 6.2) are employed. One of the main functions of these shear connectors is to ensure that a mechanical bond between the steel section and the reinforcement bars. They can also provide a necessary path to transmit the developed forces to the column and serves as means to anchor the reinforcement bars.
A design model for two types of composite semi-rigid connections to be used in external and intermediate columns, figures 11 and 12 will be presented. The proposed design model was developed for connections in the major column axis, but some modifications can validate its use for connections in the minor column axis as will be commented later.

The proposed model was based in an exponential model developed by Lorenz et al (1993), figure 13. This model can predict the initial stiffness and the plastic moment capacity of semi-rigid connection by a simple mechanism developed in the connecting angles. In this mechanism, the centre of rotation is assumed to be located in a point on the outstanding leg of the sited angle, figure 13. The initial stiffness of the connection, Eqn. 6, and the plastic moment capacity of the connection, Eqn. 7, are found by the sum of the individual initial stiffness or plastic moment capacity of the web angles, the sited angle and the “T” rib connector respectively. A complete development of the first and second terms is present in Lorenz et al (1993). The initial stiffness of the “T” rib connector can be obtained in a similar procedure used for the web angles stiffness, figure 14. This procedure is valid if the axial stiffness of the reinforcement bars is greater than the stiffness of the “T” rib connector.
Fig. 13: A design model for semi-rigid connections, after Lorenz, Robert F. et al (1993).

Fig. 14: The "T" rib shear connector

\[ K = K_{It} + K_{nw} + K_u \]  \hspace{1cm} (6)

in which:

\[ K_{nw} = \frac{6EI_u(d_j)^2}{gs^2}, \quad K_u = \frac{4EI_u}{l_s}, \quad K_{It} = \frac{6EI_u(d_j)^2}{gs^2} \]

\[ M_{pl} = M_{plT} + M_{plw} + M_{plb} \]  \hspace{1cm} (7)

where:

\[ M_{plT} = 2V_{plT}d_i \] ; \hspace{1cm} \[ M_{plb} = \frac{f_yd_r(t_r)^2}{4} \] ;

\[ V_{plT} = \frac{l_r}{4}(V_{pl} + 2V_{plm} + V_{plb}) \] and \hspace{1cm} \[ V_0 = \frac{f_yd_r}{2} \]

The terms \( V_{plb} \), \( V_{plm} \) and \( V_{pl} \) are obtained with:

\[ V_{plb}^4 + \frac{E_yd_rV_0^3}{t_r}V_{plb} - V_0^4 = 0 \]  \hspace{1cm} (8)

The expression for the plastic moment capacity of the web angles, \( M_{plw} \), can be obtained in a similar way to capacity of the "T" rib connector, \( M_{plT} \) just using the geometric characteristics of the web angles defined in figure 11. The use of the proposed model in intermediate columns connections is possible with a few modifications. To calculate the initial stiffness and plastic moment capacity of the composite semi-rigid connection Eqn. 6 and 7 can be used. The only modification is the substitution of the terms related to the "T" rib contribution for terms considering the reinforcement bars. This assumption is valid if the reinforcement bars are adequately anchored in the Perforbond rib shear connectors present along the beam’s span.

The use of the proposed model for composite semi-rigid connections along the column minor axis is feasible in principle but some important issues have to be pointed out. Full scale portal frame tests were conducted at the University of Sheffield by Gibbons et al (1996) and concluded that one of the major problems associated with that type of connection is associated with the local instability of the relatively thin webs used. It is still
necessary an investigation for the development of an economic solution for stiffening that region. Up to the present moment some welded connections solutions Blodget (1966) can be modified to be used in bolted connections along the column minor axis. This is currently being investigated by the authors that expect to publish some of these results in the near future.

Another aspect currently being investigated is an extension of the proposed model as a part of an iterative process. In this procedure, with an assumed moment capacity it is possible to calculate, by means of a moment rotation curve, the associated rotation. Assuming a linear relationship between the rotation and the associated strains, the deformations in the steel section and in the reinforcement bars can be evaluated. These deformations can be associated with a stress block diagram distribution. This stress distribution produces an updated moment capacity. Finally, this updated moment resistance can be compared with the initially estimated capacity. This procedure could be repeated up to a certain acceptable margin of error is obtained.

6. **PERFOBOND RIB AND "T" RIB SHEAR CONNECTORS DESIGN**

This section describes a structural design method for the Perfolon Rib and the "T" Rib shear connectors, Figures 1 and 14.

**6.1 The Perfolon Rib Connector**

A parametric study on the Perfolon rib connector structural behaviour Oguejiofor et al (1994), suggested that the optimal shape should have three 50mm holes spaced at least with 2.25 times this diameter. The holes should also be located closer to the top flange face of the beam in order to improve the performance of the concrete dowels. The thickness of the plate is fixed after the number and the diameter of the reinforcement bars that passes into each hole is established (generally two bars to easy up the assembly). A series of verifications for the completion of the design is suggested.

i) The maximum plate height should be limited in function of the concrete slab thickness allowing some concrete cover (around 15mm).

ii) The reinforcement bars tensile force may be conservatively determined by its shear capacity.

iii) The thickness of the plate should be designed taking into account the bearing resistance of the plate, the tension capacity of the plate at the sections AA and BB, and the shear capacity at the CC and DD sections, figure 1.

iv) The welds should be designed according to the assumed capacity of the reinforcement bars.

**6.2 The "T" Rib Connector**

The "T" rib connector, photo 4, should be designed not to be affected by prying action in order not generate a premature loss of connection stiffness. The "T" rib resistance is also limited by the column web capacity for mechanical and economical reasons. Some design steps follows.
i) The height of the "T" rib should set according to the slab thickness. A concrete cover and some clearance from the beam's top flange around 10mm should be provided.

ii) The same design checks used in 6.1 should be adopted for the design of the "T" web thickness. A good choice is the adoption of the same thickness of the column's web.

iii) The "T" flange thickness design is controlled by the horizontal distance from the centre of the bolt to the medium line of the "T" web. This distance should be as little as possible to avoid amplification of the prying action, Canadian Institute of Steel Construction (1995). A good choice for lower rise buildings is to use the same thickness of the column's flange.

7. A COMPOSITE SEMI-RIGID CONNECTION DESIGN EXAMPLE

To illustrate the proposed model a design of a composite semi-rigid connection will be presented. The beam and column sections used were S250x38 and WWF-300X66 respectively.

7.1 The "T" Rib Design

The final geometric dimensions adopted are present in figure 14. The design process can be divided in two steps:

i) The "T" rib is designed as a function of the thickness of the column flange and column web to avoid prying action. Using the Canadian Institute of Steel Construction the "T" can withstand a force, $F_t$, equal to 76.40 kN. This force leads to:

$$M_{RT} = F_t \cdot d_s = 76.40 \text{ kN} \times 0.328 \text{ m} = 25.06 \text{ kN.m}$$

ii) The initial stiffness and the plastic moment capacity of the "T" rib can be calculated in a similar way used in the web's angles design. Tables 5 and 6 depicts the results.

Consequently, the maximum moment capacity of the "T" connector is 25.06 kN.m.

<table>
<thead>
<tr>
<th>TABLE 5</th>
<th>INITIAL CONNECTION STIFFNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1$ (mm)</td>
<td>$g_2$ (mm)</td>
</tr>
<tr>
<td>T</td>
<td>24.4</td>
</tr>
<tr>
<td>Web</td>
<td>-</td>
</tr>
<tr>
<td>Seat</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 6</th>
<th>PLASTIC MOMENT CAPACITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_y$ (kN/cm$^2$)</td>
<td>$V_0$ (kN/cm)</td>
</tr>
<tr>
<td>T</td>
<td>25.0</td>
</tr>
<tr>
<td>Web</td>
<td>25.0</td>
</tr>
<tr>
<td>Seat</td>
<td>25.0</td>
</tr>
</tbody>
</table>
7.2 The Web Angle Design

The first design step is to fix the sum of the web angle thickness to be as close as possible to the beam’s web thickness. The length of the angles should be delimited to the beam’s top flange up to half of the beam’s height in order to generate the necessary lever arm. Some design step follows:

i) The beam’s web thickness should be checked for bearing. A325 M16 bolts were used and conducted to a bearing resistance equal to 60kN. When two bolts are considered it leads to:

\[ M_{nw} = B \cdot d = (60 \text{ kN} \times 2) \times 0.173 \text{ m} = 20.76 \text{ kN.m} \]

ii) The next step is the evaluation of the initial stiffness and the plastic moment capacity. The angles adopted were L101x101x6.4 with 124mm length. Results are shown on tables 5 and 6.

The plastic moment capacity of the web angle is 13.75kNm, indicating that a thicker angle should be adopted to raise its resistance up to 20.76kNm, the beam’s web plastic moment resistance.

7.3 The Sited Angle Design

The sited connection has a small influence on the initial stiffness and on the plastic moment capacity of the connection as it can be observed in tables 5 and 6. One extra design check should be done to find out if a stiffener on the column web close to the sited angle position is necessary.

7.4 Connection Moment Capacity

The maximum moment resisted by the connection is given by the sum of the individual resistance of the “T”, the web angles and the sited angle:

\[ M_T = 25.06 + 13.75 + 0.52 = 39.33 \text{ kN.m} \]

8. FINAL CONSIDERATIONS

This work presented a model for composite semi-rigid connection design. One of the major contributions of the proposed model is the development and use of Perfobond Rib and “T” Rib shear connectors to guarantee the transmission of the force developed in the reinforcement bars to the column’s flange. This approach solves the continuity problem for connections at external and intermediate building columns. Some modifications in the model can adapt its use for connections in the column minor axis. A rational approach for the composite semi-rigid connection design is also depicted.

The influence of the reinforcement mesh over the shear connector’s resistance is less significant for specimens with transversal reinforcement bars inside the perfobond rib holes. The accuracy of Eqn. 1 second and third terms (the reinforcement bars used inside the perfobond holes and the concrete dowels contribution) is confirmed through figure 9 where the margin of error has fallen to a level less than 5%. The experimental results described in this work suggests a preliminary resistance factor \( \phi = 0.65 \) for use in design Eqn. 2 to evaluate the factored shear resistance of the perfobond rib connectors. This affirmative will be verified with the aid of full-scale composite beam tests with perfobond rib and “T” rib shear connectors that are currently being tested in the Structures Laboratory of PUC-RIO.

ACKNOWLEDGEMENTS

The authors would like to thank Mr. Luciano R. O. de Lima, for the help given in the preparation of this paper. Thanks are also due to UFRR, CAPES and FINEP for the financial support for this research project.
LIST OF SYMBOLS

\[ D \text{ (mm)} \] Diameter of the perfobond rib hole;
\[ d, d_1, d_{sc}, d_w \] Distances defined in figures 10 and 11;
\[ d_1, d_2, d_3, d_s \] Distances defined in figures 10 and 11;
\[ d_b \] Diameter of reinforcement bars.
\[ f_c \text{ (MPa)} \] Nominal compressive strength of concrete;
\[ f_y \text{ (MPa)} \] Nominal yield strength of reinforcement bars;
\[ f_{yd} \] Specified yield strength of reinforcement bars;
\[ f_{yr} \] Specified yield strength of the steel section;
\[ h_e \] Concrete slab thickness;
\[ g_e, g_1, g_3, g_y \] Distances defined in figure 11;
\[ I_s, I_T, I_w \] Sited angle, "T" and web angle lengths;
\[ n \] Number of rib holes;
\[ n_1 \] Number of reinforcement bars inside all the perfobond rib holes;
\[ n_2 \] Total number of transversal reinforcement bars used;
\[ q_n \] Nominal resistance for the perfobond rib shear connector;
\[ t_s, t_T, t_w, t_f \] Sited angle, "T" and web angle and beam/column flange thickness;
\[ y \] Distance defined in figure 11;
\[ A_{cc} \text{ (mm}^2\) Shear area of concrete per connector bars;
\[ A_r \] Area of reinforcement bars;
\[ A_t \text{ (mm}^2\) Area of transverse reinforcement
\[ E \] Young's modulus;
\[ I_s, I_T, I_w \] Sited angle, "T" and web angle moments of inertia;
\[ K_i \] Initial connection stiffness;
\[ K_{iS}, K_{iT}, K_{iw} \] Sited angle, "T" and web angle initial stiffness;
\[ M_p \] Plastic moment capacity;
\[ M_{ph}, M_{phT}, M_{phw} \] Sited angle, "T" and web angle plastic moment capacity;
\[ R \] Project resistance for the perfobond rib shear connector;
\[ V_{ph}, V_{phT}, V_{phw} \] Sited angle, "T" and web angle plastic shear capacity;
\[ \Delta M \] Reinforcement bars moment capacity;
\[ \phi \] Resistance factor;

REFERENCES


7 BRIDGE AND BRIDGE PIERS
FAILUE MECHANISM OF STEEL BOX PIERs UNDER CYCLIC LOADING

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ABSTRACT

In the Hyogo-ken Nanbu Earthquake, a few steel box piers collapsed, while some ones suffered failures only such as local buckling of stiffened plate. Normally, steel bridge piers are filled with concrete at the lower portion because of the protection for the vehicle collision. In this earthquake most of the damages occurred at non-concrete filled portions. In the present experiment using non-concrete-filled and concrete-filled pier specimens, these specimens were submitted to three cycles of loadings with a certain amplitude of a lateral displacement and thereby the amplitude was stepped up. As a result, the failure mechanism of steel bridge piers was clarified.

KEYWORDS

Steel box piers, Cyclic loading, Concrete-filled, Non-concrete-filled, Buckling, Crack, Failure mechanism, Hysteresis loop

1. INTRODUCTION

There are many cyclic loading tests of concrete-filled pier specimens and non-concrete-filled ones using small-size specimens, but such tests would be too small to estimate the size effect properly [Usami et al. (1994) and Suzuki et al. (1995) as examples].

The present experiment of ours features successful reproduction of the real structural and welding details. We performed cyclic loading tests using one-third specimens of a real bridge pier, so that the failure mechanism of concrete-filled portions and non-concrete-filled ones under a repeated large displacement could be revealed.
2. EXPERIMENTAL PROCEDURE

2.1 Design and Fabrication of Specimens

Figure 1 shows the configuration of the specimens. There are two types of specimens: concrete-filled (ACON) specimen and non-concrete-filled (A) specimen. Both are the same structure except presence of concrete filling or none and difference in the detail of base diaphragm. The steel employed is SM490Y (nominal yield stress = 353MPa, measured yield stress = 435MPa). In the following, the measured yield stress is used and if the nominal yield stress is used, it is indicated by the numeral in ( ). The measured strength of concrete used is 14.7MPa.

As indicated in Figure 1 (b), the portion investigated ranges from the base plate upward to an extent of 1700mm. The parameters in this portion are $R_R = 0.523 (0.473)$, $\gamma_1/\gamma_1^* = 1.16$, $R_h = 0.756 (0.679)$, where the nominal value is nearly equal to the optimum value in allowable stress design method. The parameters are as follows:

$$R_R \text{ or } R_h = \frac{b}{t} \left\{ \frac{12(1-\nu^2)}{\pi k} \right\} \sqrt{\frac{\sigma_y}{E}}$$

in which $R_R$ = width-thickness ratio parameter of unstiffened plate supported simply by two stiffeners in flange; $R_h$ = width-thickness ratio parameter of longitudinal stiffener; $b$ = flange width (800mm) or stiffener width (60mm); $\sigma_y$ = yield stress; $E$ = Young's modulus; $\nu$ = Poisson ratio; $k$ = buckling coefficient of flange (4.0n) or stiffener (0.43); $\gamma_1^*$ = optimum value of flexural rigidity of one stiffener, $\gamma_1^*$ = optimum value of flexural rigidity of stiffener obtained from the linear buckling theory, $n$ = number of panels divided by longitudinal stiffeners.

ACON specimen has its investigated portion filled with concrete and welding with studs. S flange and N flange shown in Figure 1 (a) are a compressive flange and a tensile one in the first half of a cycle respectively. In ACON specimen the connections of stiffener and intermediate diaphragm or base diaphragm have the same detail as in the real pier (Figure 2 (a) and (b)). On the other hand A specimen assumes a portion upward of the concrete-filled portion of the real pier, therefore its detail of the connection of stiffener and base diaphragm is designed the same as that of the intermediate diaphragm. In the other portion beyond the investigated portion, the diaphragm spacing is designed short in order to make it resistant to buckling.

2.2 Experimental Method

The specimen pier base was anchored to the reaction wall and a constant axial load was applied to the top (Photograph 1). As shown in Figure 3, the displacement was controlled in consideration of the slip and rotation of the pier base such that the displacement $\delta$ toward the perpendicular axis of the base plate was stepped up equal to $1.5 \delta$, two times that, three times that, four times that, and so on. However, in the case of ACON specimen, application of $0.75 \times \delta$ was also done. With respective amplitudes, 3 cycles of load application were repeated. Here, $\delta$ = elastic analysis displacement at the loading position when maximum stress becomes yield stress.

Following a fixed cycle of loading, the residual deformation and cracking in the stiffened plate and the stiffener were investigated from inside and outside of the box.

3. EXPERIMENTAL RESULT
FAILURE MECHANISM OF STEEL BOX PIERS

Figure 1: Configuration of Specimens

Figure 2: Detail of Welding
Photograph 1: Setup of Specimen

Figure 3: Loading Pattern
3.1 Concrete-filled Pier Specimen

Figure 4 shows a load-displacement hysteresis curve. Displacement $\delta$ means a corrected displacement and not equal to the controlled that in test. At the end of 1st cycle of $2.8 \delta y$, fatigue cracks invisible to the naked eyes were discovered from outside of specimen at the toe of base weld near the box corner by magnetic particle inspection method, while at the end of 2nd cycle of $4.3 \delta y$, a local buckling happened at the flange. Up to 1st cycle of $5.2 \delta y$, a decline in the strength was hardly observed but thereafter cracks abruptly propagated, while at the same time the strength dropped suddenly.

![Figure 4: Load - Displacement Hysteresis of ACON Specimen](image)

Figure 5 indicates the relation of the load or the crack length of the flange versus the maximum displacement in each cycle. The load and displacement took 1/2 of the total amplitude. Figure 5 (a) shows the crack propagation in N flange at the end of 1st cycle of $5.2 \delta y$, that is, just when the crack abruptly begins to propagate. Initially the crack propagates along the weld toe and then deviates from the weld toe. In both sides of N flange the crack length "a" is almost equal to about 8% of b/2, i.e., half the flange width. The cracking in S flange behaved virtually the same as N flange. Fracture came at 3rd cycle of $5.2 \delta y$ in S flange and at 1st cycle of $6.4 \delta y$ in N flange. As seen from Figure 5 (c), the strength decline in this specimen is caused not by the local buckling, but by propagation of low-cycle fatigue crack. One of the reason is supposed that the restraint effect of concrete to local buckling also increased by the studs (see Photograph 2).
Figure 5: Displacement vs. Load and Crack Length (ACON Specimen)

Figure 6: Distribution of Strain Range on Flange Surface in Vicinity of Weld at Base Plate at the End of 3rd Cycle of 2.1 $\delta y$ (ACON Specimen)
Photograph 2: Local Buckling of N Flange in A Specimen

(a) at the End of 1st Cycle of $5.2 \delta y$

(b) at the End of 3rd Cycle of $5.2 \delta y$
Figure 6 shows the distribution of the total amplitude of strains in N flange near the base weld, just before cracking starts, that is at 3rd cycle of 2.1 $\delta$, it is seen how high the stress concentrates at the weld toe close to the box corner. This is the reason that cracking originates from the corner.

### 3.2 Pier Specimen with No Concrete Filling

Figure 7 shows a load-displacement hysteresis curve of A specimen. It is noted how a local buckling happens in the flange and the stiffener at 1st cycle of 2.1 $\delta$, and the load slightly decreases in consequence. Photograph 3 illustrates the state at the end of this cycle. The flange exhibits an overall residual deformation in the area between the base plate and the first intermediate diaphragm; the stiffener exhibits a residual deformation due to a local buckling near the base plate. Thereafter the strength begins to decline with a repeated loading at the same amplitude and as the result of an increased amplitude and repeated cycle it drops suddenly. At 3rd cycle of 3.0 $\delta$, when the load is considerably decreased, a cracking is observed at the corner weld inside of the box and at 1st cycle of 4.0 $\delta$, the crack penetrates to the surface, but it never spreads at a stroke. Photograph 4 is a photo of the crack of the corner weld after test finishing.

![Buckling detecting at 1st cycle of 2.1$\delta$](image1)

![Crack penetrating at 1st cycle of 4.0$\delta$](image2)

![Crack detecting at 3rd cycle of 3.0$\delta$](image3)

**Figure 7: Load - Displacement Hysteresis of A Specimen**

Figure 8 shows the relation of the load, the maximum residual deformation "d" of the flange and the maximum residual deformation "dr" of the stiffener corresponding to the maximum displacement at each cycle. As for the load, reference is made also in the data on ACON specimen. It is noted as compared with ACON specimen with concrete filling that the strength and ductility of A specimen are considerably low. It is recognized that maximum residual deformation of flange and that of stiffener is similar to load decline after reaching the maximum strength.

### 4. CONCLUSIONS

Specimens were prepared such that the parameters of stiffened plate and stiffener in bridge pier became
FAILURE MECHANISM OF STEEL BOX PIERS

Photograph 3: Residual Deformation of N Flange in A Specimen at 1st Cycle of 2.1 $\delta_y$

Photograph 4: Crack of Box Corner in S Flange of A Specimen after Test Finishing
approximately equal to their optimum values in the allowable stress design method.

They were submitted to cyclic lateral loading in the case of concrete-filling and no filling and in consequence the following conclusions are reached:

(1) In the portion filled with concrete, unlike the portion non-filled, local buckling hardly happens in the stiffened plate and both strength and ductility are considerably higher. When the local buckling does not happen in the stiffened plate or when the load decrease due to local buckling is delayed as compared with the crack propagation in the base weld, the crack initiates from the box corner at the base weld toe where the stress concentration is high and in consequence the flange ruptures.

(2) In the portion with no concrete filling, the stiffened plate and stiffener suffer a local buckling at about 2.0 \( \delta \), and thereafter with repeated loading and an increase in the displacement amplitude the strength drops suddenly. In the case of a satisfactory corner weld, only when the strength considerably drops, the cracking reaches so serious that it can penetrate the plate.

REFERENCES


NONLINEAR DYNAMIC RESPONSE OF
THIN CIRCULAR STEEL TUBES

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ABSTRACT
Thin-walled circular steel tubes used as cantilever bridge piers suffer damage in the form of local buckling during severe earthquakes. In the present study, a combined experimental and analytical approach is used to clarify their cyclic behaviour. Monotonic, cyclic and pseudodynamic tests performed on thin-walled circular tubes modeling bridge piers are described. An evolutionary-degrading (E-D) hysteretic model based on a comprehensive damage index is presented which gives accurate simulations of the seismic response of circular steel tubes even after the inception of local buckling. The strength and ductility values necessary for using the hysteretic model are estimated for tubes with different radius-thickness ratio parameters and slenderness ratio parameters.

KEYWORDS
Thin-walled, Circular Steel Tubes, Bridge Piers, Cyclic Test, Pseudodynamic Test, Local Buckling, Strength and Ductility, Radius-thickness Ratio, Slenderness Ratio, Hysteretic Model.

1. INTRODUCTION
Thin-walled circular steel tubes are widely used as columns in buildings and as bridge piers. During a severe seismic event, the tubes are subjected to cyclic lateral loads in addition to the constant axial load of the supported structure. This leads to their damage in the form of local buckling and consequent collapse of the structure. Such failures were observed in steel bridge piers during the Hyogoken Nanbu Earthquake near Kobe City on January 17, 1995. Therefore, understanding of their inelastic cyclic behaviour is important to prevent collapse under earthquake loading. The present study aims to clarify their behaviour by means of a combined analytical and experimental approach.

Although circular steel tubes as bracing members subjected to axial loads have been widely studied, studies on their behaviour under bending are scant. One of the earliest and widely known studies on the inelastic bending of circular steel tubes is the work by Schilling (1965). The study deals at length with the phenomenon of local buckling under monotonic loading and forms the basis for most of the
present codes including the Japanese Code for bridge design (Japan Road Association, 1990). Based on test results, it sets an upper limit to the radius-to-thickness ratio so as to ensure that the tubes will reach the full plastic moment without premature local buckling. Later Sherman (1976) carried out a number of monotonic tests on circular steel tubes and as a result proposed a revision of the buckling parameter.

Studies on the cyclic behaviour of circular steel tubes include the one by Tsuji and Nakatsuka (1986) who tested relatively thick tubes under constant axial load and cyclic lateral load. They described in detail the formation of ring-type local buckling and the consequent degradation of strength and stiffness. More recently Mizutani et al. (1996) reported tests on thin-walled cantilever tubes under similar loading conditions. The tubes exhibited multi-faceted local buckling following the initial ring-type buckling. They found that the ductility of the tube increased with a reduction in the radius-to-thickness ratio.

In the present study, a combined experimental and analytical approach is used to clarify the inelastic cyclic response of thin-walled tubes used as bridge piers. The information obtained from the experiments is used to modify an E-D hysteretic model, developed elsewhere for hollow box sections so as to enable the prediction of the response of tubes. Finally help of the finite element method is taken to recommend suitable guidelines for the evaluation of the parameters of the hysteretic model. This will facilitate the accurate determination of the response with very little computational effort.

2. OUTLINE OF TESTS

To study the behaviour of thin-walled steel tubes under cyclic loading, tests were carried out on four identical specimens. The specimens were 1/8-th scaled models of the prototypes. The two parameters which inhibit local and global buckling in these tubes are the radius-thickness ratio parameter \( R_t \) and the column slenderness ratio parameter \( \lambda \). These are given by

\[
R_t = \frac{R \sigma_y}{t E \sqrt{3(1 - \nu^2)}} \tag{1}
\]

\[
\lambda = \frac{2h}{r \pi \sqrt{E \sigma_y}} \tag{2}
\]

where, \( R \) = the outer radius of the tube; \( t \) = the thickness of the tube; \( h \) = the height of the tube; \( r \) = the radius of gyration; \( \sigma_y \) = the yield stress; \( E \) = the Young’s modulus and \( \nu \) = Poisson’s ratio. The values of \( R_t \) specified for practical piers by the Japan Road Association (1990) is about 0.10 while the practical range of \( \lambda \) is from 0.2 to 0.4.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Specimen</th>
<th>Test Type</th>
<th>( R_t )</th>
<th>( \lambda )</th>
<th>( \sigma_y/P_y )</th>
<th>Accelerogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P10-35-M</td>
<td>Monotonic</td>
<td>0.10</td>
<td>0.35</td>
<td>0.181</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>P10-35-C</td>
<td>Cyclic</td>
<td>0.10</td>
<td>0.35</td>
<td>0.181</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>P10-35-1</td>
<td>Pseudodynamic</td>
<td>0.10</td>
<td>0.35</td>
<td>0.181</td>
<td>1.2*G.T.I</td>
</tr>
<tr>
<td>4</td>
<td>P10-35-3</td>
<td>Pseudodynamic</td>
<td>0.10</td>
<td>0.35</td>
<td>0.131</td>
<td>1.1*G.T.III</td>
</tr>
</tbody>
</table>

The specimens were tested as cantilever columns under a constant axial load \( P \) and cyclic lateral load \( H \) applied at the free end. The test program is summarized in Table 1 where \( \sigma_y \) indicates the squash load. Test No.1 was a monotonic test while Test No.2 was an incremental cyclic test where the amplitude of cycling was stepped up by \( \delta \) after each complete cycle. These two tests were carried out quasi-statically.

Tests with earthquake loading were carried out pseudodynamically by idealizing the system as a single-degree-of-freedom system (Usami and Kumar, 1996). The earthquake accelerograms used were those specified by the Public Works Research Institute (PWRI) for Ground Type I (hard) and Ground Type III
NONLINEAR DYNAMIC RESPONSE OF THIN STEEL TUBES

Amplified versions of these accelerograms were used, the amplification factors being chosen by judgement so as to inflict significant damage without collapse. In the case of the pseudodynamic tests, the axial loads were calculated based on the expected horizontal force given by the seismic coefficient of the Japan Road Association code and column interaction equations (Usami and Kumar, 1996). The axial load ratio corresponding to Ground Type I was used for the monotonic and cyclic tests. The specimen parameters are given in Table 2 while the material properties are given in Table 3. In Table 2, $L_d$ and $t_d$ indicate the spacing and thickness of the diaphragms respectively while in Table 3 $\epsilon^y$ is the yield strain, $\epsilon^t$ is the strain at the onset of strain hardening and $E^t$ is the initial strain hardening modulus.

<table>
<thead>
<tr>
<th>No.</th>
<th>Specimen</th>
<th>$R$ (mm)</th>
<th>$t$ (mm)</th>
<th>$h$ (mm)</th>
<th>$L_d$ (mm)</th>
<th>$t_d$ (mm)</th>
<th>$R_t$</th>
<th>$\bar{\lambda}$</th>
<th>$H_y$ (tf)</th>
<th>$\delta_y$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P10-35-M</td>
<td>203</td>
<td>6.35</td>
<td>1800</td>
<td>100</td>
<td>6.0</td>
<td>0.10</td>
<td>0.35</td>
<td>13.6</td>
<td>8.7</td>
</tr>
<tr>
<td>2</td>
<td>P10-35-C</td>
<td>203</td>
<td>6.35</td>
<td>1800</td>
<td>100</td>
<td>6.0</td>
<td>0.10</td>
<td>0.35</td>
<td>13.6</td>
<td>8.7</td>
</tr>
<tr>
<td>3</td>
<td>P10-35-1</td>
<td>203</td>
<td>6.35</td>
<td>1800</td>
<td>100</td>
<td>6.0</td>
<td>0.10</td>
<td>0.35</td>
<td>13.6</td>
<td>8.7</td>
</tr>
<tr>
<td>4</td>
<td>P10-35-3</td>
<td>203</td>
<td>6.35</td>
<td>1800</td>
<td>100</td>
<td>6.0</td>
<td>0.10</td>
<td>0.35</td>
<td>14.4</td>
<td>9.2</td>
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</table>

The specimen was bolted to the base and the axial load $P$ was applied by means of a hydraulic jack to simulate the weight of the superstructure while the cyclic lateral load $H$ was applied by means of a mechanical actuator to simulate the earthquake loading. Displacement transducers were used on either side so as to monitor any twisting that might take place due to unsymmetric buckling. The displacement of the top of the specimen $\delta$ was taken as the average of that recorded by both of the transducers. Correction for sliding and rotation of the base was also done during the experiments.

In the monotonic test (Test No. 1), the strength increased beyond the yield strength $H_y$ upto $1.49H_y$ corresponding to a displacement of $3.44\delta_y$. Beyond this point, it started decreasing due to the formation of local buckling on the compression side at a distance of 100 mm from the base. The strength dropped to $H_y$ at a displacement of $7.5\delta_y$. The local buckling at this point was significant and is shown in Figure 1(a).

![Figure 1: Local Buckling of Circular Tubes: (a) P10-35-M and (b) P10-35-C](image)
In the cyclic test (Test No. 2), local buckling became visible on the compression side, earlier than that in the monotonic test, at $-3\delta_y$ due to low-cycle fatigue. Upon reverse loading, local buckling developed on the opposite side and the two buckles merged gradually to produce a ring-type buckling. This gave rise to rounded hysteretic loops with sloping reloading branches as shown in Figure 3. The maximum strength attained was $1.55H_y$ at a displacement of $3\delta_y$. The strength at peak displacement, also known as the residual strength, dropped to $H_y$ at $5\delta_y$. The state of the specimen at this point is shown in Figure 1(b).

In the pseudodynamic test using amplified Ground Type I accelerogram (Test No. 3), a large displacement of $5.3\delta_y$ occurred on one side causing significant local buckling. The specimen did not return to the initial position subsequently and had considerable residual displacement at the end of the test. The local buckling in the other pseudodynamic test using amplified Ground Type III accelerogram (Test No. 4) was small. The hysteretic loops and the response-time histories for the pseudodynamic tests (Test Nos. 3 and 4) are shown later in Figure 4 and Figure 5 respectively. It was interesting to note that in both these tests, after the formation of local buckling some hysteretic energy was dissipated even at loads below the yield level.

### 3. ANALYTICAL SIMULATION

#### 3.1 The Damage Index

To simulate the test results analytically, a damage index based evolutionary-degrading (E-D) hysteretic model similar to the one developed for box columns (Kumar and Usami, 1996a) is used. The model is based on a comprehensive damage index which takes into account the maximum deformation experienced, the hysteretic energy dissipated as well as the effect of the loading history (Kumar and Usami, 1996b). The damage index is given by

$$D = (1 - \beta) \sum_{j=1}^{N} \left( \frac{\delta_{\max,j} - \delta_y}{\delta_u - \delta_y} \right)^e + \beta \sum_{i=1}^{N} \left( \frac{E_i}{H_y(\delta_u - \delta_y)} \right)^c$$

where, $\delta_y$ = the yield displacement; $\delta_u$ = the ultimate displacement under monotonic loading expressed as $\mu\delta_y$; $\delta_{\max,j}$ = the maximum displacement produced for the j-th time; $E_i$ = the energy dissipated in the i-th half-cycle (defined to be the interval between consecutive zero load points); $H_y$ = the predicted minimum of yield, local buckling and instability loads; $N$ = the total number of half-cycles and; $\beta$ and $c$ are empirical constants. The number $N_1$ indicates the number of half-cycles producing maximum deformation $\delta_{\max,j}$ for the first time such that it exceeds the previous value by $\delta_y$, i.e., $\delta_{\max,j} > \delta_{\max,j-1} + \delta_y$. Collapse is assumed to occur when the damage index $D$ becomes unity.

#### 3.2 The Hysteretic Model

It was observed from the results of the cyclic test (Test No. 2) that a trilinear model consisting of an elastic limb, a strain hardening limb and a plastic limb would be adequate to simulate the response of circular tubes. It was also observed that a perfectly plastic limb would be inadequate and a sloping limb would be required. To avoid the specification of the slope, the plastic limb was directed towards the point where the monotonic curve would intersect the displacement axis (see Figure 2). The degradation of strength and stiffness were achieved by evaluating the damage index at the end of each half-cycle or
when the maximum deformation exceeded its previous value by $\delta_y$. The maximum strength $H_{\text{max}}$ and the elastic stiffness $K$ were then obtained as (Kumar and Usami, 1996a)

$$H_{\text{max}} = H_{\text{in}} \cdot e^{-\left[\ln\left(H_{\text{in}}/H_y\right)\right]D}$$

and

$$K = K_E \cdot e^{-\left[\ln\left(H_{\text{in}}/H_y\right)\right]D}$$

where $H_{\text{in}}$ and $K_E$ = the initial (corresponding to zero damage) strength and stiffness respectively.

$$E_1 = \text{Area}(OBCD)$$

$$E_2 = \text{Area}(DEFG)$$

Figure 2: Hysteretic Model

### 3.3 Simulation Results

From the monotonic test result, the ductility $\mu$ was obtained as 7.5 for an axial load ratio $P/P_y$ of 0.181. Also by substituting the maximum load $H_{\text{max}}$ and the corresponding damage index in Eqn. 4, $H_{\text{in}}$ was calculated. The damage index constants $c$ and $\beta$ were taken as 2.0 and 0.11 respectively. The simulation of the cyclic test was obtained as shown in Figure 3. It can be seen that the simulation agrees well with test results.

Figure 3 Simulation of Cyclic Test Results for P10-35-C Specimen
Next, the hysteretic model is used to simulate the seismic response of thin circular tubes. The simulation parameters are shown in Table 4 where m is the mass. The simulation obtained for Test No. 3 is shown in Figure 4. In the case of Test No. 4, the axial load used was different from that of the monotonic test. Therefore the monotonic ductility $\delta_u/\delta_y$ and the initial strength $H_{in}$ need to be corrected using a spring-rigid bar model (Kumar and Usami, 1996a). Using suffixes 1 and 2 to denote the values corresponding to $P_1/P_y = 0.181$ and $P_2/P_y = 0.131$ the revised values for $\delta_u$ and $H_{in}$ can be obtained as

$$H_{max,2} = H_{max,1} + \frac{(P_1 - P_2)\delta'}{h}$$  \hspace{1cm} (6)

and

$$\delta_{u,2} = (P_1/P_2)\delta_{u,1}$$  \hspace{1cm} (7)

where, $\delta'$ is the displacement corresponding to $H_{max,1}$. The corrected values of the two quantities are shown in Table 4. The simulation of Test No. 4 obtained using the hysteretic model is shown in Figure 5. It can be seen that the hysteretic model simulates the pseudodynamic test results adequately.

### TABLE 4

<table>
<thead>
<tr>
<th>No.</th>
<th>Specimen</th>
<th>$K_E$</th>
<th>m</th>
<th>$H_{in}/H_y$</th>
<th>$P/P_y$</th>
<th>$\delta_u/\delta_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P10-35-M</td>
<td>1.565</td>
<td>-</td>
<td>1.58</td>
<td>0.181</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>P10-35-C</td>
<td>1.558</td>
<td>-</td>
<td>1.58</td>
<td>0.181</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>P10-35-1</td>
<td>1.566</td>
<td>3.606</td>
<td>1.58</td>
<td>0.181</td>
<td>7.5</td>
</tr>
<tr>
<td>4</td>
<td>P10-35-3</td>
<td>1.561</td>
<td>2.610</td>
<td>1.62</td>
<td>0.131</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Figure 4 Pseudodynamic Test Simulation: 1.2$^*$G.T.I on P10-35 Specimen (Test No. 3)

### 4. STRENGTH AND DUCTILITY OF CIRCULAR TUBES

To facilitate the use of the hysteretic model for various circular steel tubes, a finite element analysis was carried out using the program ADINA (1992). A simple bilinear stress-strain relationship with a strain...
hardening modulus of 1% of the elastic modulus $E$ was used for the material. The details of the Finite Element Analysis including the deformed mesh are shown in Figure 6. The accuracy of the analysis was first verified by predicting the load-displacement relationship for P10-35-M Specimen (Test No. 1) (see Figure 7).

Next, the ductility and strength were evaluated for various $R_t$ and $\bar{\lambda}$ values. While the former was varied from 0.08 to 0.14 at an interval of 0.02, values of the latter ranged from 0.1 to 0.5. A total of 30 cases were investigated and the ductility values obtained were plotted against a combination parameter $R_t\sqrt{\bar{\lambda}}$ as shown in Figure 7. By fitting a curve to the data an equation was obtained which would enable a quick determination of the ductility. The results also indicated that the initial strength $H_{in}$ may be taken as 1.6 times the yield strength $H_y$.

$$\sigma_y = 304 \text{ MPa}$$
$$E = 2.06 \times 10^5 \text{ MPa}$$
$$E_{st} = E / 100$$
$$\nu = 0.3$$

**Figure 5** Pseudodynamic Test Simulation: 1.1*G.T.III on P10-35 Specimen (Test No. 4)

**Figure 6** Details of Finite Element Analysis
5. CONCLUSIONS

The inelastic cyclic behaviour of thin circular steel tubes modeling bridge piers was studied by means of monotonic, cyclic and pseudodynamic experiments. Using the test results an evolutionary-degrading (E-D) hysteretic model was developed. The hysteretic model is based on a comprehensive damage index which takes into account the maximum deformation, the hysteretic energy dissipated and the effect of the loading history. The model was used to simulate the cyclic and pseudodynamic test results. The simulations were found to agree well with experimental results even after the inception of local buckling. Therefore it is concluded that the model can be used to predict the inelastic seismic response of thin circular steel tubes used as bridge piers. By means of a finite element analysis, simple formulas for the strength and ductility as functions of the radius-to-thickness ratio parameter and the column slenderness ratio parameter were proposed. This will facilitate the use of the hysteretic model for estimating the nonlinear dynamic response of practical piers.

\[ \ln \mu = -2.47 \ln (R_t \lambda^{0.5}) - 5.17 \]
\[ P/P_y = 0.2 \]

Figure 7 Ductility of Circular Steel Tubes

REFERENCES

DUCTILITY IMPROVEMENT OF STEEL COLUMNS WITH CORNER PLATES BASED ON LARGE-SCALE CYCLIC LOADING TESTS

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ABSTRACT

Steel bridge columns, manufactured by the welding together of thin plates, are widely used in the urban area in Japan, since the sections of them are smaller than those of reinforced concrete columns. However, in the 1995 Hyogo-ken Nanbu earthquake, there were examples of buckling of stiffened plates or fracture of corner welding. Ductile behavior is required for steel columns against a large earthquake, preventing from brittle failures such as a fracture of the corner between a flange plate and a web plate. In this paper, cyclic loading tests carried out on steel columns with sections of 900 mm in a flange width and 900 mm in a web height are described. Test results show that the corner plates are effective and the ductility of steel columns with the corner plates increases.

KEYWORDS

Steel Column, Ductility, Seismic Design, Cyclic Loading Test, Corner Plate, Axial Load Ratio, Column Slenderness Ratio, Flange Plate Slenderness Parameter

1. INTRODUCTION

Steel bridge columns, manufactured by the welding together of thin plates, are widely used in the urban area in Japan, since the sections of them are smaller than those of reinforced concrete columns. However, in the 1995 Hyogo-ken Nanbu earthquake, there were examples of buckling of stiffened plates or fracture of welding, see Figure 1. After the earthquake, "Guide Specifications for Reconstruction and Repair of
Highway Bridges Which Suffered Damages due to the Hyogo-ken Nanbu Earthquake" was issued by the Ministry of Construction on February 27, 1995. In this specifications, it was specified that dynamic strength and ductility should be checked not only for reinforced concrete columns but also for steel columns and that steel piers with concrete in-fill should be used tentatively. This approach of concrete in-fill is certainly one of the most effective and practical methods to improve the ductility of steel columns. However, one of the demerits is that it also increase the flexural capacity and weight of columns, which will be imposed on anchors and foundations. Most of steel columns are used in the urban area so that retrofit work including anchors and foundations must be difficult to do.

With this point of view, an intensive research has been carried out in the Public Works Research Institute to develop the seismic retrofit method for rectangular steel columns to increase the ductility without increase of much flexural capacity. One of the candidates is to implement the corner plate at the four corners between webs and flanges of rectangular steel columns. In this paper, basic concepts why the corner plates are considered to be effective are introduced firstly. Secondly, the results of the cyclic loading tests of steel column models with different axial load ratio and with different slenderness ratio are described. Lastly, design recommendations for the retrofit of rectangular steel columns with corner plates are discussed.

2. BASIC CONCEPT

The research activity for improving the seismic performance of steel bridge columns were carried out by Kawashima et al (1992) or by the committee of steel structures in Japan Society of Civil Engineering(1994). The general countermeasures are as follows,

1) To improve structural parameters related to buckling strength
   (To decrease the width-thickness ratio of stiffened plates, to decrease the diameter-wall thickness ratio of steel pipe columns, to increase the stiffness or to reduce the spacing of the diaphragms)
2) To fill steel columns with concrete or to combine the steel columns with concrete into composite structures

The latter method has been used on many steel columns after Hyogo-ken Nanbu Earthquake for the reconstruction and retrofit of damaged highway bridges. However, most of existing steel columns are
located in the urban area such that concrete-in-fill method is not a good solution since the retrofit work of anchors may be needed. Consequently, the following requirements are considered to be necessary for the retrofit method for rectangular steel columns.
- Avoid failure modes
- Improve ductility
- Less increase in strength (Less increase in seismic force to anchors)
- Easy retrofit work on site

Figure 2 illustrates the image of the seismic retrofit method with corner plates on four corners for rectangular steel columns. Figure 3 shows the behavior after the local buckling. Four corners are strengthened with plates to prevent cracking or fracture after local buckling. The concept is that even after the local buckling of the stiffened plates at the maximum load, the steel column can sustain its flexural capacity in a similar mechanism as a rigid frame structure consisting of the four corners acting as columns and stiffened panels acting as bracings.

Figure 2: Image of Seismic Retrofit Method

Figure 3: Expected Behavior after Local Buckling
3. CYCLIC LOADING TESTS

3.1 Test Setup

Cyclic loading tests were carried out to examine the seismic behavior of as-built and retrofitted steel columns in a different slenderness ratio and in a different axial load ratio. TABLE 1 gives the column slenderness ratio \( \lambda \), axial load ratio and flange plate slenderness parameter \( R_f \) of each specimen based on the measured yield strength.

<table>
<thead>
<tr>
<th>TEST SPECIMEN</th>
<th>h (mm)</th>
<th>( \lambda )</th>
<th>Axial Load Ratio</th>
<th>Rf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen No.1</td>
<td>3423</td>
<td>0.272</td>
<td>0.125</td>
<td>0.611</td>
</tr>
<tr>
<td>Specimen No.2</td>
<td>3423</td>
<td>0.259</td>
<td>0.128</td>
<td>0.581</td>
</tr>
<tr>
<td>Specimen No.3</td>
<td>3423</td>
<td>0.258</td>
<td>0.154</td>
<td>0.581</td>
</tr>
<tr>
<td>Specimen No.4</td>
<td>5000</td>
<td>0.393</td>
<td>0.118</td>
<td>0.566</td>
</tr>
<tr>
<td>Specimen No.5</td>
<td>5000</td>
<td>0.393</td>
<td>0.118</td>
<td>0.566</td>
</tr>
</tbody>
</table>

The axial load ratio means the ratio of the applied axial load to measured yield stress. The column slenderness ratio \( \lambda \) is calculated by Eqn. 1, as follows,

\[
\lambda = \frac{2h}{r} \frac{1}{\pi} \sqrt{\frac{\sigma_y}{E}}
\]

where \( h \) is a distance from the bottom of the column to the point of application of the horizontal load, \( r \) is a radius of gyration of the cross section, \( \sigma_y \) is a measured yield stress and \( E \) is a Young's modulus.

The values of structural parameters of stiffened plates were determined by referring to those for existing steel columns designed by the conventional design. Flange plate slenderness parameter \( R_f \) is 0.611 for the specimen No.1 and 0.566 for the specimen No.4 and No.5. Flange plate slenderness parameter \( R_f \) is calculated by Eqn. 2.

\[
R_f = \frac{b}{t} \sqrt{\frac{\sigma_y 12(1-\mu^2)}{E \pi^4 k_p}}
\]

where \( b \) is a flange width, \( t \) is a flange plate thickness, \( \mu \) is Poisson's ratio and \( k_p \) is a buckling coefficient of flange specified in the "Design Specifications for Highway Bridges, Part II Steel Bridges" (1996).

Figure 4 shows the dimensions of specimen No.1, No.2, No.4 and No.5. The specimen No.3 has the same dimension as the specimen No.2, but acting axial load differs. The sections of all specimens are 900 mm by 900 mm. The distance from the bottom of the column to the point of application of the horizontal load is 3423 mm for the specimen No.1, No.2 and No.3 while the distance is 5000 mm for the specimen No.4 and No.5. The specimen No.1 and the specimen No.4 are simulating the as-built steel columns, but having different column slenderness ratio. The specimen No.2 is simulating the retrofitted steel column with corner plates at the four corners and the seismic behavior will be compared with the specimen No.1. The
specimen No.5 is also simulating the retrofitted steel column with corner plates at the four corners and the seismic behavior will be compared with the specimen No.4 without the corner plates.

The specimen was set in the horizontal position, as shown in Photo 1. The simulated seismic load was applied using a hydraulic actuator acting against the reaction wall or the rigid frame. To simulate the selfweight of the superstructure and the column, an additional axial load was applied by means of an axial load apparatus. The axial load ratio was determined 0.15Py based on the nominal yield stress. A displacement for the cyclic loading tests \( \delta_c \) was determined by a calculated yield load when section yielding first occurred based on the calculated section properties and sizes. It should be noted that the section of corner plates was not included in the section properties. Testing was generally carried out with a fixed number of a cycle to displacements of \( 1\delta_c, 2\delta_c, 3\delta_c, 4\delta_c \), and so on, as illustrated in Figure 5. The displacement at the top of the specimen was corrected to take account of the effects of the horizontal displacement and the rotation of the footing.

Figure 4: Test Specimen
Photo 1: Test Setup (No.4)

Figure 5: Test Sequences

3.2 Test Results

Comparing No.4 with No.5 specimen, No.4 specimen, which is representing as-built steel column, showed the maximum horizontal load at 3δ, and the horizontal load decreased gradually. The progressed out-plane deformation due to local buckling was observed in the flanges and webs at 3δ. The fracture occurred at the corner between the web and the flange at 6δ, and the test terminated at this stage. The No.5 specimen, which representing the retrofitted steel column with corner plate at the corner, showed the maximum horizontal load at 4δ, and the horizontal load decreased gradually. The local buckling was observed at the flanges at 3δ. The fracture occurred at bottom of the column at 5δ, where is a different point compared with No.4 specimen. The test terminated at 7δ, because of the serious deformation of panels and degradation of the flexural strength.

Figure 6 shows the horizontal load-horizontal displacement hysteresis curve for five specimens. It can be said that the No.2 and No.3 specimens are much more ductile than No.1. Also the No.5 specimen with the...
Figure 6: Horizontal Load - Horizontal Displacement Hysteresis Curve
corner plates has much ductility than No.4 without the corner plates. With regard to the effects of the axial load ratio, the degradation of flexural strength after the maximum horizontal load of No.3 is more severe than that of No.2.

Here, to examine the effects of the column slenderness ratio, the hysteresis curve was compared between No. and No.5 specimen. The maximum horizontal load was applied at the $5\delta_0$ for the No.2 specimen while the maximum horizontal load was applied at the $4\delta_0$ for the No.5 specimen. From this observation, it can be said that the bigger the column slenderness ratio becomes, the less ductility the steel column will have.

Figure 7 describes the definition of the absorbed energy for a cycle. Based on this definition, the absorbed energy of the each specimen was calculated, see Figure 8. Clearly, the retrofitted steel column with corner plates, which are No.2, No.3 and No.5, showed better behavior to absorb the energy during their inelastic cycles.
4. DESIGN RECOMMENDATIONS

To take into account of the effects of the axial load ratio \( P/\bar{P_y} \), the column slenderness ratio \( \lambda \) and the flange plate slenderness parameter \( R_f \), \( \xi \) is defined by Eqn. 3.

\[
\xi = \frac{P}{\bar{P_y}} \times \lambda \times R_f
\]  

The displacement ductility factor \( \mu = \frac{\delta}{\delta_0} \), versus \( \xi \) for the five specimens are plotted in Figure 9, where \( \delta \) is defined as the displacement at the 97% of the ultimate horizontal load. In other words, \( \delta \) is a value defined as the displacement where the horizontal load hardly drop down. The experimental data is still few, but the line in the figure can be proposed to predict the displacement ductility factor \( \mu \). In the range of \( \xi < 0.020 \), the displacement ductility factor \( \mu \) can be 5.5. In the range of \( \xi < 0.025 \), the displacement ductility factor \( \mu \) can be 3.5. Between \( \xi = 0.020 \) and \( \xi = 0.025 \), \( \mu \) can be calculated by a linear interpolation. Regarding the as-built steel columns, the displacement ductility factor \( \mu \) can be taken as 2.5 by the results of No.1 and No.4 specimens.

![Figure 9: Displacement Ductility Factor \( \mu \) versus \( \xi \)](image)

5. CONCLUSIONS

The purpose of this research is to study the effectiveness of the retrofit method to implement the corner plates at the four corners of steel columns. Based on the observations of the experiments and the findings from the test results, the following conclusions were reached.

1. The retrofit method was proposed that the corner plates are implemented at the four corners of steel columns. This method is considered to be a better idea, because it increases few flexural strength of columns so that the horizontal load, which the anchors are imposed on, will not increase.

2. The cyclic loading tests were carried out to examine the effectiveness of the corner plates at the four corners of steel columns. The test results showed that the corner plates were effective, but the axial load ratio, the column slenderness ratio and the flange plate slenderness parameter also affect the
displacement ductility of steel columns.

3. For design recommendations, the simple equation was proposed to predict the displacement ductility factor of the as-built rectangular steel columns and the retrofitted steel columns with corner plates.

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ABSTRACT
This paper deals with the elasto-plastic behaviour of columns with variable cross-section subjected to horizontal cyclic load. Attention is paid to the two or more possible collapse mechanisms for the columns. The instability phenomenon of columns under cyclic loading is illustrated through numerical and experimental investigations. This phenomenon is caused by the combination of collapse mechanisms. The characteristics of the corresponding load versus displacement curves are investigated and cyclic effects are summarized for the general framed structures.

KEYWORDS
column, variable cross-section, cyclic loading, instability, collapse mechanism, cyclic effects

1. INTRODUCTION
Identification of the collapse mechanism is of great importance in estimating the inelastic earthquake performance of steel framed structures. According to the plastic theory, the collapse mechanism of frames can generally be decided by the number of fully-plastically deformed portions and the combination of their locations. In the case of frames under monotonic loading, it is well known that there is the possibility to follow the different combinations of paths with increasing sway displacement.

In the case of frames under cyclic loading, several kinds of horizontal load ($H$) versus sway displacement ($\delta$) curves have hitherto been confirmed and classified by Igarashi and Matsui et al. (1970), Wakabayashi and Matsui (1972), and Matsui and Mitani (1979). It is clear that decreases in the load carrying capacities of frames are often caused by local and torsional buckling.

However, these characteristics are normally obtained from numerical and experimental studies which presume that the collapse mechanism does not change under cyclic loading. Very little is known about the variation of collapse mechanisms of frames with cyclic loading. Uetani and Nakamura et al. (1983,1993) have pointed out...
an important factor, apart from local and torsional buckling, that causes a decrease in the strength of isolated columns as well as multistory frames. This phenomenon results from the fact that the column deformation is much affected by the coupled action of residual deformation and vertical loads, and leads to the occurrence of a new collapse mechanism which is not governed by the rigid plastic theory.

The purpose of this paper is to discuss the combination of collapse mechanisms for columns under alternating horizontal load by rearranging the previous studies (1992, 1993). Simple cantilever columns with variable cross-section are used to represent the inelastic behaviour of general framed structures with two or more collapse mechanisms. Firstly, the remarkable change in hysteresis $H - \delta$ loops of columns is explained through the numerical simulation based on the plastic zone theory. Secondly, this phenomenon is confirmed by an experiment. Test results demonstrate the reason why the collapse mechanisms of columns change under cyclic loading. Finally, the variation of hysteresis loops with the change in collapse mechanisms is analyzed by the rigid plastic theory and the cyclic effects which cannot be predicted under monotonically increasing loading are summarized for the general framed structures.

2. CHANGE IN COLLAPSE MECHANISMS OF COLUMNS WITH VARIABLE CROSS-SECTION

2.1 Numerical and experimental models of columns with variable cross-section

The cantilever columns with two cross-sections Sec.1 and Sec.2, as shown in Figure 1, are utilized in order to simulate the variation of collapse mechanisms. The cross-sectional properties change abruptly at the location of $x = kh$. Table 1 shows the dimensions and properties of the numerical model CN and experimental models, CE1 to CE4. The yield load ratio $H_{y1}/H_{y2}$ means which of Sec.1 and Sec.2 reaches initially inelastic range. The columns CN and CE2 are designed such that the initial yielding occurs at the bottom of Sec.1 and Sec.2 almost at the same stage. The experimental models have the same combination of Sec.1 and Sec.2 but only the location of junction between them is different. It is then noted that the bending and overall buckling strengths of columns become larger with an increase in the length of Sec.1.

In the numerical study (1992), the geometrical and material nonlinearities of columns are taken into account by the updated-Lagrangian method and plastic zone theory, respectively. Vertical load $P$ is kept constant while horizontal load $H$ is applied at the column top. The stress-strain relation is taken as bi-linear with kinematic strain hardening, in which the plastic tangent modulus is assumed to be 1% of the elastic modulus ($=2.05 \times 10^5$ MPa).
TABLE 1
DIMENSION AND PROPERTIES OF COLUMNS WITH VARIABLE CROSS-SECTION

<table>
<thead>
<tr>
<th>Columns</th>
<th>Items</th>
<th>Column height h (cm)</th>
<th>Column width and depth (cm)</th>
<th>Thickness (cm)</th>
<th>Yield point (MPa)</th>
<th>Vertical load PN2</th>
<th>Yield load ratio H3/H2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical model</td>
<td>CN-0.4</td>
<td>900</td>
<td>75</td>
<td>75</td>
<td>2.95</td>
<td>2.20</td>
<td>353</td>
</tr>
<tr>
<td>Experimental</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>CE1-0.21</td>
<td></td>
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<td>CE2-0.31</td>
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<td>CE3-0.41</td>
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<tr>
<td>CE4-0.51</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Notes: 1) h, k, B, ( t_1 ), and P: See Figure 1. 2) N2 is the squash force of Sec.2. 3) ( H_3 ) and ( H_2 ): Horizontal loads corresponding to initial yield of Sec 1 and Sec 2, respectively.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Plastic collapse mechanisms of column with variable cross-section

Figure 3: Monotonic elasto-plastic behaviour of column with variable cross-section

In the experimental study (1993), the boundary conditions of the test columns were a little different from those of the numerical model. At the top of the columns, the horizontal displacement was fixed by a pin shoe with a roller, and at the base, the vertical displacement and rotation were restrained by the roller shoe with a sole plate of thickness 100mm. Then, the column was tested up to failure by imposing the alternating horizontal displacement at the base and vertical load at the top.

For the columns with two cross-sections, there exist two plastic collapse mechanisms, \( V \) and \( Y \), as shown in Figure 2, in which the plastic hinge is formed at the bottom of Sec.1 and Sec.2, respectively. The equilibrium of external and internal work for each mechanism leads to the following equations relating the vertical load \( P \), horizontal load \( H \) and sway displacement \( \delta \);

\[
H = \frac{M_{p1}}{h} - \frac{P \delta}{h} \quad \text{(for mode } V \text{)}
\]

\[
H = \frac{M_{p2}}{h_2} - \frac{P \delta}{h_2} \quad \text{(for mode } Y \text{)}
\]

where \( M_{pi} \) is the fully plastic moment of Sec.\( i \).

The relationship between these solutions and elasto-plastic column behaviour under increasing horizontal loading can be sketched as Figure 3. The lines \( V \) and \( Y \) are helpful to understand the change in collapse mechanisms in the advanced inelastic range. The more general relationship between the two collapse mechanisms and elasto-plastic hysteretic behaviour of columns is discussed below.
2.2 Numerical study on inelastic behaviour of columns under alternating horizontal load

Figure 4 shows the sway displacement modes at the displacement reversals and the load $H$ - displacement $\delta$ curve obtained from the numerical model. The maximum values of positive and negative displacements $\delta_+$ and $\delta_-$ are set as 0.027$h$ and -0.011$h$, respectively (See Figure 1(b)).

It can be seen from Figure 4(a) that the column bends significantly above the junction of the cross-sections as if the modes $V$ and $Y$ were combined. This is the same kind of displacement mode as observed by Uetani and Nakamura (1983) for columns with equivalent cross-section. Figure 4(b) shows that, in the positive region of load $H$, each slope between the peak loads and displacement reversals in the first few cycles is almost equal to that of mode $V$. For an increasing number of cycles, however, the peak loads decrease and the corresponding slopes approach the line $Y$. On the other hand, in the negative region of $H$, the slope after the peak loads equals that of mechanism line $V$ for each hysteresis loop. The resistance load, then, increases more and more with an increase in cycle number. This behaviour is opposite to that in the positive region of $H$.

It is almost certain that the column shows two different collapse mechanisms in one hysteresis loop. This phenomenon is an instability phenomenon under cyclic loading, because the hysteresis loops shift and become smaller with an increase in the number of cycles without remaining in a steady state.

2.3 Experiment on collapse mechanisms of columns with variable cross-section

Figure 5 shows the $H$ - $\delta$ curves of the test columns, CE1 and CE2 with $k=0.21$ and 0.31, respectively. The initial tip displacement amplitude was taken as $2\delta$, in which $\delta$ is the sway displacement corresponding to initial yielding of the column. The displacement was increased by $\delta$ every five cycles up to $5\delta$.

The column CE1 shows the very stable behaviour such that the $H$ - $\delta$ loops become large with an increase in the tip displacement and cyclic numbers. This is due to strain hardening at the lower portion of Sec.2 (See Table 2). On the other hand, when $\delta_+\delta_-=5$ in the results of column CE2, the same elasto-plastic behaviour can be seen as for the numerical results. In the positive region of load $H$, the collapse mechanism is almost mode $V$,
Figure 5: Relationship between horizontal load $H$ and displacement $\delta$ of test columns CE1 and CE2

TABLE 2
ELASTO-PLASTIC COLLAPSE MODES OF TEST COLUMNS

<table>
<thead>
<tr>
<th>Test columns</th>
<th>Ratio of initial yield loads $H_{y1}/H_{y2}$</th>
<th>Collapse mode</th>
<th>Classification of collapse modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE1-0.21</td>
<td>1.21</td>
<td>Y</td>
<td>$\delta_{yy}$ $\delta_{max}$</td>
</tr>
<tr>
<td>CE2-0.31</td>
<td>1.06</td>
<td>S</td>
<td>$\delta_{yy}$ $\delta_{max}$</td>
</tr>
<tr>
<td>CE3-0.41</td>
<td>0.89</td>
<td>V</td>
<td>$\delta_{yy}$ $\delta_{max}$</td>
</tr>
<tr>
<td>CE4-0.51</td>
<td>0.76</td>
<td>V</td>
<td>$\delta_{yy}$ $\delta_{max}$</td>
</tr>
</tbody>
</table>

Notes; $\delta_{max}$: Maximum tip displacement applied to columns during test ($=5\delta_{y}$).

while in the negative region of $H$, the mechanism corresponds to the mode $V$ in the first and second loops, and thereafter becomes close to mode $Y$.

The collapse modes of test columns at each final stage, as summarized in Table 2, can be classified into three modes, $Y$, $S$ and $V$. The columns CE1, CE3 and CE4 with more than 10% difference between yield resistance forces $H_{y1}$ and $H_{y2}$ show modes $Y$ or $V$ according to the fundamental plastic theory.

Photo. 1 shows the pictures of the columns CE1 and CE4 at each final stage and illustrates the variation of collapse modes with an increase in cyclic numbers and tip displacement amplitude. These columns demonstrate the collapse mechanisms, $Y$ and $V$. The plastic deformation is restricted to the bottom of Sec.2 and Sec1 for CE1 and CE4, respectively, because there is more than 10% difference between the resistance loads of mechanisms $V$ and $Y$.

Photo. 2 also shows the results of the column CE2 with collapse mode $S$, which is neither $Y$ nor $V$. The first stage $A$ shows the situation before testing. From the stages $B$ to $I$, the following observations can be made with regard to sway displacement modes of the column;
1) Stages $B$ and $C$: The displacement modes are almost symmetrical for $\delta = \pm 3\delta_{y}$.
2) Stage $D$: The plastic deformation at the bottom of Sec.1 is related to mode $V$.
3) Stage $E$: Sec.2 bends clearly (mode $Y$) and the residual bending deformation appears at the base due to the last stage $D$. The plasticly deformed portion differs from $D$.
4) Stage $F$: Sec.1 deforms more in the plastic range (mode $V$) than stage $D$. Then, the plate buckling occurred in the flange near the base. The residual deformation is observed at the bottom of Sec.2 due to the last
5) Stage $G$: The plastic bending deformation is largely concentrated at Sec.2 (mode $Y$). The residual deformation can distinctly be viewed in Sec.1 due to the local buckling at stage $F$ and the displacement mode approaches "S".

6) Stage $H$: The local buckling of the flange in Sec.1 becomes predominant compared with stage $F$ and a length of about $h/10$ from the junction of the cross-sections bends significantly. The deformed column takes on an "S" shape.

Photograph 1: Sway displacement modes of test columns CE1 and CE4

Photograph 2: Variation of sway displacement modes of test column CE2-0.31
7) Stage I: The local buckling wave is observed at the bottom of Sec.1 and Sec.2 in the right and left hand sides of this picture, respectively.

From this description of the failure process for the column CE2, it is seen that the $P\delta$ moments applied to the bottom of Sec.1 and Sec.2 greatly influence the location of plastic deformation and eliminate alternately the difference between the resistance loads of mechanisms $V$ and $Y$ every half cycle.

3. RIGID PLASTIC ANALYSIS OF COLUMNS WITH VARIABLE CROSS-SECTION AND DISCUSSION ON CYCLIC EFFECTS

3.1 Rigid plastic mechanisms of column considering residual deformation

The reason why the $H$ - $\delta$ loops shift and become smaller is investigated by applying the rigid plastic theory. Figure 6(a) illustrates the $n$-th $H$ - $\delta$ loop consisting of the peak loads, $n_c$ and $(n+1)_c$, and displacement reversals, $n_b$ and $n_d$. The corresponding collapse mechanisms are idealized in Figures 6(b) to (e). The equilibrium condition for each stage gives the following equations:

Stage $n_b$: $H_b = \frac{M_{p1}}{h_2} - \frac{P\delta_{ub}}{h_2} - \frac{P\delta_{mc}}{n_2}$

Stage $n_c$: $H_c = \frac{M_{p1}}{h} - \frac{P\delta_{p1}}{h} + \frac{P\delta_{uc}}{h}$

Stage $n_d$: $H_c = \frac{M_{c1}}{h} - \frac{P\delta_{uc}}{h}$

Figure 6: Collapse mechanisms of column including residual deformation
Figure 7: Comparison of $H_b$ and $H_d$ by Eqs. (3) and (5), respectively, with those of numerical results of column CN-0.4

$$\text{Stage } (n+1)_s : \quad H_s = \frac{M_{p_2}}{h_2} - \frac{P\delta_{p_2}}{h_2} - \frac{P(\delta_{m_2} - \delta_{u_2})}{h_2} \quad (6)$$

where

- $\delta_{u_2}$ and $\delta_{m_2}$ : tip displacement amplitudes in the negative and positive regions, respectively.
- $\delta_{m_2}$ and $\delta_{u_2}$ : residual displacements at the junction of cross-sections $(x=kh)$, which correspond to the stages $(n-1)_m$ and $n_m$, respectively.
- $\delta_{p_2}$ and $\delta_{p_2}'$ : tip displacements from the stages $n_m$ to $n_a$ and from $n_d$ to $(n+1)_s$, respectively.

3.2 Comparison of numerical results with mechanism loads including residual deformation

The horizontal loads $H_b$ and $H_d$ at the reversal points, $n_b$ and $n_d$, are compared with the numerical results described in 2.2, in which $M_{p_1}$ is the original plastic moment obtained from Table 1 and the residual deformation $\delta_{m_1}$ is extracted from the numerical results.

As revealed by this figure, Eq. (3) can closely approximate $H_b$ from the numerical analysis, but the prediction of $H_d$ from Eq. (5) differs considerably from the numerical results in accordance with the increase in $N$. The discrepancy in $H_d$ can be attributed to the evaluation of the resistance moment $M_{p_1}$ of Sec.1 without considering the influence of strain hardening. In fact, when $M_{p_1}$ is replaced by the value of resistance moment including strain hardening at the base, $H_d$ given by Eq. (5), as shown by the broken line, coincides well with the numerical results.

As a consequence, the variation of hysteresis loops with the change in collapse mechanisms can be characterized as follows:

1) In the case where the plastic deformation occurs near the bottom of Sec. 2 (mode Y), the influence of strain hardening is negligible, but the overturning $P\delta$ moment is much affected by the residual deformation near the bottom of Sec.2 and it makes the peak loads decrease. This effect corresponds to the term enclosed by the dotted line in Eq. (3).

2) When the plastic deformation occurs at the base of Sec.1 (mode V), the load carrying capacity increases because of the increasing resistance moment $M_{p_1}$ due to strain hardening. This implies that the mechanism line $V$ moves so as to strengthen the column.

3) Combining 1) with 2), the centre of the $H - \delta$ loops moves in the direction of mode V and the size of the loops reduces because the decrease in strength caused by the $P\delta$ effect for mode Y is greater than the increase in strength for V (See Figure 7).

It should be noted that the variation of collapse mechanisms is not concerned with the ultimate strength such
3.3 Cyclic effects on stability of framed structures

Now consider more general framed structures with two or more collapse mechanisms. When two collapse mechanisms are recognized within the sway response, one may be considered the strong mechanism and the other weak as illustrated in Figure 8(a). If these modes appear alternately due to the form of the response under cyclic loading, the inelastic behaviour of the structure will involve cyclic instability as shown in Figure 8(b).

The factors which initiate the combination of collapse mechanisms can be identified as follows:
1) Extensive propagation of plastic zones along the frame members due to cyclic bending under uniaxial compressive force.
2) Cumulative residual deformation due to plastic deformation and plate buckling.
3) Variation of overturning moment applied to the two or more plastically deformed portions.

These factors are the cyclic effects due to cyclic loading. When these factors eliminate the difference between the horizontal resistance loads corresponding to collapse mechanisms, the structures have the possibility to assume the unstable state. It is possible that other unknown effects are present within frames, because of the many combinations of collapse mechanisms including local buckling. There is, therefore, a need to further study the relationship between the inelastic behaviour of structures and cyclic effects.

4. CONCLUSIONS

The elasto-plastic behaviour of columns with variable cross-section under cyclic loading was studied in this paper. The main conclusions can be drawn as follows:

1) The combination of the strong and weak mechanisms in one hysteresis loop results in an instability phenomenon for the column with variable cross-section.
2) The column strength decreases in the region of the weak mechanism due to the $P\delta$ effect and increases in the region of the strong mechanism due to strain hardening. The resulting load - displacement loops shift and become smaller.
3) This phenomenon is caused by the combination of collapse mechanisms due to the cyclic effects which cannot be predicted from the inelastic behaviour of columns under increasing horizontal load.
4) It is necessary to perform further studies concerning the collapse mechanisms of general framed structures.
and to investigate the possibility of probable combinations of collapse mechanisms by cyclic effects.

ACKNOWLEDGMENT

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REFERENCES


INELASTIC BUCKLING OF STEEL PIPE PIERS IN SEVERE EARTHQUAKES

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ABSTRACT: In the Great Hanshin Earthquake of 1995, several steel piers were severely damaged. In some of these piers, it was found that local buckling did not occur at the base of the pier, but at the middle of pier's height. It was thought that not only horizontal earthquake motion but also vertical earthquake motion should be taken into consideration to reveal the behavior of actual structures. The purpose of this study is to simulate the behavior of these steel pipe piers using a nonlinear dynamic analysis program. As a result, it is found that the location of local buckling is to some extent influenced by vertical earthquake motion.

KEYWORDS
steel pipe pier, local buckling, earthquake, nonlinear analysis, impact load, dynamic response

1. INTRODUCTION

In the Great Hanshin Earthquake of 1995, civil engineering structures were severely damaged. Several steel piers in Kobe route No. 3 of Hanshin expressway collapsed. In steel pipe piers, a typical local buckling pattern of elephant foot bulge was observed at the welded joint of two steel pipes with different thicknesses and at the base part of the piers (Itoh (1995), Usami et al. (1995), Saizuka et al. (1995)). It has been suggested that not only horizontal earthquake motion but also vertical earthquake motion should be taken into consideration to reveal the behavior of actual structures.

The purpose of this study is to simulate the behavior of these steel pipe piers using the nonlinear dynamic analysis program LS-DYNA3D. In these analyses, vertical impact load, horizontal (north-south) seismic wave, and vertical seismic wave were introduced.

2. DETAILS AND DAMAGE SITUATION OF ACTUAL PIERS

In this study, two steel pipe piers, P-584N and P-584S, near the Matsubara intersection of the Hanshin expressway were considered as objects of the analyses. Figures 1 and 2 show the shape and collapse mode of the piers, P-584N and P-584S, respectively. In this area, the elevated bridge was separated into eastbound
Figure 1: Shape and collapse mode of pier P-584N

Figure 2: Shape and collapse mode of pier P-584S
and westbound bridges. The piers P-584N and P-584S belong to these eastbound and westbound bridges, respectively. These piers are single column pipe piers of about 15 m height, having filled-concrete at the base part. These piers have several diaphragms, but no longitudinal stiffener. The parameters of these piers are shown in Table 1. Dead loads are 482 tf (4,724 kN), and 422 tf (4,135 kN) for piers P-584N and P-584S, respectively.

### Table 1
PARAMETERS OF PIERS

<table>
<thead>
<tr>
<th></th>
<th>P-584N</th>
<th>P-584S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (mm)</td>
<td>16,481</td>
<td>16,435</td>
</tr>
<tr>
<td>Diameter (mm)</td>
<td>2,200</td>
<td>2,200</td>
</tr>
<tr>
<td>Thickness (mm) [Height (mm)]</td>
<td>25[0-8,000]</td>
<td>28[0-8,000]</td>
</tr>
<tr>
<td></td>
<td>19[8,000-16,481]</td>
<td>21[8,000-12,000]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19[12,000-16,435]</td>
</tr>
<tr>
<td>Yield stress (MPa)</td>
<td>313.6[0-4,000]</td>
<td>313.6[0-4,000]</td>
</tr>
<tr>
<td></td>
<td>235.2[4,000-16,481]</td>
<td>235.2[4,000-16,435]</td>
</tr>
<tr>
<td>Yield bending load, H&lt;sub&gt;v&lt;/sub&gt; (kN)</td>
<td>1,798</td>
<td>1,612</td>
</tr>
<tr>
<td>Yield displacement, δ&lt;sub&gt;y&lt;/sub&gt; (mm)</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
<td>Period of pier (sec.)</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>Dead load, P (kN)</td>
<td>4,135</td>
<td>4,724</td>
</tr>
<tr>
<td>Yield vertical load, P&lt;sub&gt;y&lt;/sub&gt; (kN)</td>
<td>30,600</td>
<td>30,600</td>
</tr>
<tr>
<td>P/P&lt;sub&gt;y&lt;/sub&gt;</td>
<td>0.14</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 2 shows the parameters at the buckling points. These pipe piers were locally buckled with cracks at about the middle of their heights near the welded joint of two sections with different thicknesses. Radius-to-thickness ratio parameter R<sub>t</sub> in Table 2 is given according to reference by Mizutani et al. (1996):

\[
R_t = \frac{\sigma_r}{\sigma_{el}} = \frac{R}{t} \frac{\sigma_{el}}{E} \sqrt{\frac{1}{3(1-\nu^2)}} \tag{1}
\]

in which

- \(\sigma_r\) : Yield stress;
- \(\sigma_{el}\) : Elastic local buckling strength of pipe;
- \(R\) : Radius;
- \(t\) : Thickness;
- \(E\) : Young's modulus; and
- \(\nu\) : Poisson's ratio.

### 3. ANALYSIS MODELS

#### 3.1 Outline of Models

The fundamental shapes of analysis models are shown in Figure 3. The finite elements of the models are generated using a 4 nodes shell element to express the cylindrical shape of a pipe. In the actual piers, the two steel pipes with different thicknesses are welded using tapered sections so that the change in the thickness is made smooth. This tapered shape is reproduced in the models.
### TABLE 2
PARAMETERS AT BUCKLING POINTS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>P-584N</th>
<th>P-584S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height from base (mm)</td>
<td>8,000</td>
<td>8,000</td>
</tr>
<tr>
<td>Radius, R (mm)</td>
<td>1,100</td>
<td>1,100</td>
</tr>
<tr>
<td>Thickness, t (mm)</td>
<td>19</td>
<td>21</td>
</tr>
<tr>
<td>Radius-to-thickness ratio, R/t</td>
<td>57.9</td>
<td>52.4</td>
</tr>
<tr>
<td>Steel grade</td>
<td>SS41</td>
<td>SS41</td>
</tr>
<tr>
<td>Radius-to-thickness ratio parameter, R</td>
<td>0.109</td>
<td>0.0989</td>
</tr>
</tbody>
</table>

#### Figure 3: Fundamental shapes of analysis models

**3.2 Material Properties**

The constitutive equation of steel for this analysis uses isotropic hardening rule. Relationship between stress and strain uses perfectly elasto-plastic rule. The nominal yield stress shown in Table 1 is used in the analysis. The effect of strain rate is not considered even in Case 1, because it usually gives stronger results.

The constitutive low of concrete filled at the base of the pier uses Material type No.5 (soil/clashable foam) prepared by LS-DYNA3D, and the yield condition uses Drucker-Prager's criterion for this analysis. Yield stresses for compression and tension are 14.7 MPa and 1.61 MPa, respectively.
3.3 Mesh Size and Time Step

The basic mesh size of analysis models is 20 cm for the edges of the 4 nodes square shell elements. Smaller mesh size is used where large deformation is anticipated because of local buckling, i.e. at the welded joint of two sections with different thicknesses and at the base of the pier. The time step of analysis depends on the mesh size, and it is decided by the program automatically [5]. In these analyses, using Sun Workstation S4/20L, the time step and CPU time are $8.4 \times 10^{-6}$ seconds and about 92 hours, respectively.

4. ANALYSES

The analyses are classified into two groups. The first group includes the analyses of the effect of vertical impact load (Case 1). It is assumed that the girder hits the pier because of the falling of the shoes. The second group includes the analyses of the effect of seismic motions (Case 2 and Case 3). The seismic waves are introduced at the base of the piers, and the response is observed.

In Case 1, the steel weight representing the dead load of the girder is introduced as impact load on the pier. In Case 2 and Case 3, this dead load is introduced statically.

4.1 Analysis of Vertical Impact Load (Case 1)

In this case, it is assumed that the girder hits the pier because of the falling of the shoes. The falling height is assumed from 10 cm to 50 cm by 10 cm step. The interface between the falling weight and the pier is slide-void-friction type.

4.2 Analysis of Seismic Motions (Case 2 and Case 3)

In these cases, the seismic waves are introduced at the base of the piers, and the response is observed. In Case 2, only horizontal (north-south) seismic wave is introduced. In Case 3, horizontal (north-south) and vertical seismic waves are introduced simultaneously. The seismic waves used in these analyses were measured at the Japan Railway Takatori station. Figure 4 shows the seismic wave used in the analyses.

![Seismic waves](image)

Figure 4: Introduced seismic waves
5. ANALYSES RESULTS AND SEVERAL CONSIDERATIONS

5.1 Analysis of Vertical Impact Load (Case 1)

In the case of the falling height of less than 40 cm, the local buckling does not occur. In the case of the height of 40 cm and over, elephant foot bulge local buckling occurred. The position of local buckling is about 8 m and 12 m height from the base of the piers P-584N and P-584S, respectively. The thickness at buckling point is 19 mm. Figure 5 shows the deformation shape in Case 1.

The height of shoe is from 20 cm to 30 cm, so even if the shoe falls, it is inconceivable that the girder jumps up to more than 40 cm. Furthermore, the position of buckling was different from that of the actual buckling in the case of P-584S. Therefore, it can be concluded that the local buckling cannot be caused by only the vertical impact load caused by the earthquake motion.

![Figure 5: Deformation mode in Case 1](image)

5.2 Analyses of Seismic Motion (Case 2 and Case 3)

In Case 2 of applying north-south horizontal seismic wave only, the local buckling occurs at about 4 m height from the base of the pier in both piers. This position is welded joint of different steel types.

In Case 3 of applying horizontal and vertical seismic waves, the local buckling occurs at about 8 m height from the base of the pier in both piers. This position is welded joint of two sections with different thicknesses, and it is the same position of the actual buckling. Figures 6 and 7 show the deformation mode and response of horizontal displacement of Case 2 and Case 3 of P-584N and P-584S, respectively. Figure 8 shows the yield areas of Pier P-584N.
Figure 6: Deformation mode and response of horizontal displacement of pier P-584N
Figure 7: Deformation mode and response of horizontal displacement of pier P-584S

(a) Behavior of Case 2

(b) Behavior of Case 3

(c) Response of horizontal displacement

(d) Shortening of piers
5.3 Examination for the Position of Buckling

Figure 9 shows yield bending moments taking the dead load into account. Resisting bending moment is given like stairs shape. Slanting lines represent the bending moments in case of horizontal loads working on the top of piers. From this figure, it is found that if piers are under the effect of horizontal seismic motion, the position of 4 m height from the base of the piers is the weakest point for bending.

6. CONCLUSIONS

Based on the results of the analyses, the following conclusions were found:

(1) In the case of vertical impact load (Case 1), when the falling height is less than that of the shoe, the local buckling pattern of elephant foot bulge was not observed. Furthermore, even the buckling occurred when the falling height is over 40 cm, the position of buckling was different from that of the actual buckling. Therefore, it can be concluded that the local buckling can not be caused by only the vertical impact load caused by earthquake motion.
(2) In the case of horizontal seismic motion (Case 2), and the case of horizontal and vertical seismic motions (Case 3), the local buckling occurred at the same position of the actual buckling.

(3) Based on the previous two conclusions, it is concluded that the position of local buckling is to some extent influenced by the vertical seismic motions.

REFERENCES


ON THE DYNAMIC BEHAVIOR OF COMPOSITE BOX GIRDER EMBEDDED WITH VISCOELASTIC LAYER

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ABSTRACT:

In this research a traditional steel box girder was redesigned and combined with the viscoelastic material to upgrade the dynamic performance for the structure. A finite element formulation was utilized for the girder combined with the viscoelastic material and the associated material model, which was able to describe the material behavior accurately was also introduced. A nonlinear dynamic analysis in the time domain was carried out for the girder when subjected to the dynamic loading. The responses for the box girder incorporated with the viscoelastic material were then compared to the one without the viscoelastic effect.

KEYWORDS

Composite beam, composite girder, viscoelastic material, vibration mitigation, steel structure, dynamic analysis.

1. INTRODUCTION

When structures are subjected to dynamic loading tremendous amounts of energy are input into the structural system usually. In order to mitigate the vibration and then avoid serious damage, the viscoelastic material, which has substantial energy absorption ability, was incorporated in the structural system. One of the cases is the sandwich beam with viscoelastic core in between the stiff layers, which was first introduced by William Swallow (1939) as early as 1939. Since then a number of investigations on the design formulation for the three-layer damped sandwich beam have been carried out by many researchers such as Kerwin (1959), Ungar and Kerwin (1962), Di Taranto (1965), Di Taranto and Blasingame (1966) and Mead and Markus (1969). In these series of studies the analytical formulations for the sandwich beam were good for the elastic system with linear damping generally in the high frequency range.

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Lately due to the intensive occurrence of major earthquakes, which have caused innumerable damages on buildings, bridges and other civil structures, a viscoelastic material which is suitable for the low frequency range of the civil building system, was developed. As presented in the results of experimental testing for this viscoelastic material carried out by Mahmoodi (1972), Bergman and Hanson (1986), Lin et al (1988) and Chang et al (1991), the nonlinear behavior was observed in the stress-strain relationship, particularly during the relatively higher frequency loading test. As was observed in the experimental results, the stress was not only dependent on the loading frequency but also on the strain ratio, especially when the degradation of the material properties was realized. Therefore, it is not suitable to use the traditional complex model to describe this nonlinear behavior for the viscoelastic material. To overcome this problem an analytical material model for this viscoelastic damper, which can accurately describe the mechanical behavior, was developed recently by Lee and Tsai (1992) and modified to account for the temperature effect by Lee and Tsai (1994). By using this model, evaluations have been made for some typical structural systems which were incorporated with the damping devices such as the analysis for the jointed structures by Tsai and Lee (1992), the analysis on the high-rise buildings by Tsai and Lee (1993a), and the analysis on the bridges also by Tsai and Lee (1993b), and good results in dynamic performances were obtained.

As we know, the steel material has been the main structural material for the high-rise buildings for a long time due to its high strength, good ductility and affordable price compared to other engineering material. However, to survive in a major earthquake and free from plastic deformations in minor earthquakes additional capacity for energy absorption could be very helpful for the steel structural systems. Therefore, it is the purpose of this study to combine this nonlinear viscoelastic material to the traditional box girder or similar steel beam system to improve its energy absorption capacity. A redesigned composite box girder system associated with an analytical method was developed in this study and subsequently the nonlinear dynamic behavior in the time domain when subjected to a variety of loading and spans was evaluated and discussed.

2. GENERAL THEORY OF VISCOELASTIC MATERIAL AND BOX GIRDER

2.1 Analytical Model for Viscoelastic Material

In order to adequately predict the behavior of a structural material subjected to dynamic loading, an analytical model must be capable of representing the typical material characteristics and adequately describing the dynamic behavior. Based on the molecular theory and the fractional derivative model, a nonlinear analytic model was derived and modified by Lee and Tsai (1992, 1994). This model was also calibrated by using the available experimental results and showed a good agreement with the actual material behavior. The constitutional formula at time step $n\Delta t$, for the linear variation of the strain between two time steps, $(n-1)\Delta t$ and $n\Delta t$, is presented as

$$\tau(n\Delta t) = \left[ G' + \frac{G''(\Delta t)^{-\alpha}}{(1-\alpha)\Gamma(1-\alpha)} \right] \gamma(n\Delta t) + \tau_p(n\Delta t), \quad 0 < \alpha < 1, \quad (1)$$

where $\tau$ and $\gamma$ are the stress and strain of the material; $G'$ and $G''$ represent the shear modulus corresponding to the storage and the loss energy, respectively, and $\Gamma(1-\alpha)$ is the gamma function. The previous time effect of the strain, $\tau_p(n\Delta t)$, is given by

$$\tau_p(n\Delta t) = \frac{G''(\Delta t)^{-\alpha}}{(1-\alpha)\Gamma(1-\alpha)} \left( W_0^* \gamma(0) + \sum_{i=1}^{n-1} W_i^* \gamma(i\Delta t) \right) \quad (2)$$

where $W_0^*$ and $W_i^*$ are functions corresponding to time step $n$ and presented as
A typical force-displacement relationship representing the mechanical behavior of the viscoelastic damper is shown in Figure 1, where (a) represents the experimental data and (b) represents the analytical simulation from the model. It is found that the energy was dissipated significantly during each cycle of hysteretic motion as encompassed in each loop, and this phenomenon was predicted accurately by the analytical model.

Figure 1: Typical force-displacement relationship for the viscoelastic material
(a) Experimental testing results (b) Analytical simulated results
(after Lee and Tsai 1994)

2.2 Composite Girder with Viscoelastic Layer

The steel box girder was redesigned by combining the viscoelastic layer on the top and bottom plate as was shown in Figure 2, where the similar design for the I-beam system and other combinations such as the sleeved box-girder, the composite I-beam system, and the I-box combination were also shown. For the analysis this composite steel box girder system was simulated by a five-layer sandwich beam. An illustration for the definition of the force and displacement used in the following sections was also shown in Figure 3, where the top, middle and the bottom layers were steel materials while two layers of viscoelastic material were combined in between. Following the theories developed by Liaw and Little (1967), Kao and Ross (1968) and Khatua and Cheung (1973) for the multilayer sandwich beam, a multilayer sandwich beam with viscoelastic material in the shear layer was modified and rederived by Lee (1997) when the analytical constitutive model for the viscoelastic material was adopted.
In the discrete time domain at time step $n\Delta t$ the relationship between the nodal force vector and the strain vector is given in matrix form by

$$\sigma(n\Delta t) = D\varepsilon(n\Delta t) + \sigma_p(n\Delta t),$$  \hspace{1cm} (5)

where the nodal force vector and the strain vector are represented as
COMPOSITE BOX GIRDER EMBEDDED WITH VISCOELASTIC LAYER

\[
\begin{bmatrix}
M_1(n\Delta t) \\
N_1(n\Delta t) \\
V_1(n\Delta t) \\
M_2(n\Delta t) \\
N_2(n\Delta t) \\
V_2(n\Delta t) \\
M_3(n\Delta t) \\
N_3(n\Delta t)
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
-dw^2(n\Delta t) / dx^2 \\
du_x(n\Delta t) / dx \\
\gamma_1(n\Delta t) \\
-dw^2(n\Delta t) / dx^2 \\
du_x(n\Delta t) / dx \\
\gamma_1(n\Delta t) \\
-dw^2(n\Delta t) / dx^2 \\
du_x(n\Delta t) / dx
\end{bmatrix}
\]

(6) and (7)

\[
\sigma(n\Delta t) = \begin{bmatrix}
D_{11} = E_i I_i \\
D_{22} = E_i b_i h_i \\
D_{33} = \left( G_1 + \frac{G_2 \Delta t}{\Gamma(2-\alpha)} \right) b_i h_i \\
D_{44} = E_2 I_2 \\
D_{55} = E_2 b_2 t_2 \\
D_{66} = \left( G_2 + \frac{G_3 \Delta t}{\Gamma(2-\alpha)} \right) b_2 h_2 \\
D_{77} = E_3 I_3 \\
D_{88} = E_3 b_3 t_3
\end{bmatrix}
\]

(8)

where \( E_i \) and \( I_i \) are the elastic modulus and the moment of inertia of the i-th stiff layer respectively while \( G_j \) and \( G_j \) are the shear modulus corresponding to the storage and the loss energy for the j-th viscoelastic layer.

The force vector corresponding to the previous effect of the shear induced by the viscoelastic material is given by

\[
\sigma_s(n\Delta t) = \begin{bmatrix}
0 & 0 & 0 & V_{1p}(n\Delta t) & 0 & 0 & V_{2p}(n\Delta t) & 0 & 0
\end{bmatrix}^T
\]

(9)

where \( V_{1p} \) and \( V_{2p} \) can be obtained through Eqn. (2)

\[
V_{kp}(n\Delta t) = \frac{h_k b_k G_1 \Delta t^{-\alpha}}{(1-\alpha)\Gamma(1-\alpha)} \left( W_0^n Y_k(0) + \sum_{i=1}^{n-1} W_i^n Y_k(i\Delta t) \right), \quad k = 1 \sim 2
\]

(10)

3.FINITE ELEMENT FORMULATION OF COMPOSITE GIRDER

A typical element for the box girder incorporated with viscoelastic material is presented here. The strains are related to the displacements as

\[
\varepsilon(t) = LU(t)
\]

(11)

where \( L \) is an operational matrix and \( U \) is the displacement field vector, that can be further related to the nodal
displacements $X(t)$ as

$$U(t) = NX(t)$$

(12)

where $N$ is the interpolation function matrix. In terms of the nodal displacements at time step $n\Delta t$ in the discrete time domain the strains become

$$\varepsilon(n\Delta t) = LU(n\Delta t) = LNX(n\Delta t) = Bx(n\Delta t)$$

(13)

Substitution of Equations (9) and (13) into Eqn. (5) and using the virtual work principle, leads to the stiffness matrix and the previous time effect vector for the finite element formulation as

$$K^* = \int_{L} B^T DBdx$$

(14)

and

$$F^*_p(n\Delta t) = \int_{L} B^T \sigma_p(n\Delta t)dx$$

(15)

In the discrete time domain at time step $n\Delta t$, taking into account of the previous time effect for the viscoelastic material in the shear layers, the dynamic equation of motion for the beam element with mass $M^*$, damping coefficient $C^*$, and stiffness $K^*$, subjected to loading $P^*(n\Delta t)$ can be written as

$$M^*\ddot{X}(n\Delta t) + C^*\dot{X}(n\Delta t) + K^*X(n\Delta t) = P^*(n\Delta t) - F^*_p(n\Delta t)$$

(16)

where the mass matrix is

$$M^* = \int_{L} N^T mNdx$$

(17)

and the damping matrix $C^*$ may be obtained through the linear combination of mass matrix and stiffness matrix or solely from the mass matrix as customary application due to the uncertainty of the system damping. Now having the equations of motion and the forces exerted on the structural system ready, the analysis can be carried out by using the step-by-step integration schemes for the nonlinear structural system.

4. NUMERICAL RESULTS AND DISCUSSION

In the numerical analysis, a typical box girder having viscoelastic layer embedded under the top and the bottom plate was analyzed for the time domain behavior. The boundary conditions at both ends of the girder were assumed to be simply supported. The transverse constraint was applied to all layers at both supported ends while the axial constraint was applied to the middle layer at one end, and to the top and bottom plates at the other end. However, the span of the girder was varied such that the dominant frequency of the girder was within the frequency range for normal high-rise buildings. The loading was assumed to be concentrated and exerted on the middle point of the girder. The magnitude of the loading was assumed to be 1 lb. relative to the length (inches) of the girder. The loading type adopted in the analysis included the transient impulse and the random type loading similar to earthquakes.

The box girder adopted for the analysis is 10 in wide and 10 in high, and the other geometrical dimensions are:
1 in thick for top and bottom stiffener plate; 3/16" in web and 1 1/8" in flange plate, and the viscoelastic layer is 0.1 in thick. The material properties were assumed to be constant such as the elastic modulus $E_s = 30 \times 10^6$ psi for the steel plates and the shear modulus $G_s = 5 \times 10^3$ psi for the core layers. In order to have an original stiffness compatible to the beam of which the nonlinear viscoelastic behavior for the cores was ignored, the coefficient correlated to the shear modulus of the viscoelastic material is given to be $G_s / 2$. To be able to reflect the damping effect that is solely resulted from the viscoelastic material the system damping was ignored in the analysis. The analysis was focused on the response of the transverse displacement, velocity, acceleration and also the relative axial displacement between the top and the middle stiff layers induced by the input loading, and the effect of response reduction when the viscoelastic damping effect was taken into account. The results were obtained by carrying out the calculation for the coupled MDOF nonlinear system. The first loading is a transient impulse loading, and the second is a random type similar to the earthquake.

### 4.1 Transient Loading Response

In the impulse analysis the loading was assumed to be suddenly exerted on the middle point of the composite girder, of which both ends were simply supported. The loading duration was assumed to be 0.01 second. Figure 4, 5 and Figure 6 showed the comparison on the time domain response of the transverse displacement, velocity and the acceleration respectively when the dominant frequency of the girder was about 0.32 Hz equivalent to a 32-story high-rise steel building, when the viscosity was not taken into account. It is observed from the analytical results that the responses for each case were gradually decayed due to the nonlinear viscosity damping effect.

![Figure 4: Transverse displacement response for transient loading (0.32 Hz for girder)](image)

![Figure 5: Transverse velocity response for transient loading (0.32 Hz for girder)](image)
Figure 6: Transverse acceleration response for transient loading (0.32 Hz for girder)

Figure 7 and Figure 8 also showed the comparison for the response of the transverse displacement and velocity for the composite girder of which the original dominant frequency is about 0.85 Hz, equivalent to the frequency of the first mode for a 12-story high-rise steel building. It also showed a gradual decay on both the displacement and the velocity responses, while the decaying is more significant compared to the girder with a lower dominant frequency.

Figure 7: Transverse displacement response for transient loading (0.85 Hz for girder)

Figure 8: Transverse velocity response for transient loading (0.85 Hz for girder)

Figure 9 showed the relative axial displacement response between the top and the middle stiff layers for the same composite girder, where consistent with the transverse responses the axial response also gradually decayed.
4.2 Random Type Loading Response

In the random loading analysis a time history similar to 1940 El Centro earthquake as shown in Figure 10 was used for the input loading.

Figure 10: Time history in N-S component of 1940 El Centro earthquake

Figure 11 and Figure 12 showed the comparison of the response on the displacement and acceleration respectively when the girder frequency was about 0.85 Hz. Again, as indicated in the results a reduction on
both of the displacement and acceleration responses was observed when the viscoelastic material was incorporated to the beam system.

Figure 12: Transverse acceleration response for earthquake loading (0.85 Hz for girder)

Figure 13: Transverse displacement response for earthquake loading (1.22 Hz for girder)

Figure 14: Transverse acceleration response for earthquake loading (1.22 Hz for girder)

Figure 13 and Figure 14 are the comparisons on the response of the displacement and the acceleration for the girder with dominant frequency of 1.22 Hz while Figure 15 showed the relative axial displacement response for the same girder subjected to a earthquake-like loading. A more significant reduction effect was observed in this composite girder with higher dominant frequency.
CONCLUDING REMARKS

According to the numerical analytical analysis for the box-girder combined with viscoelastic layer when subjected to the transient loading or random earthquakes, general reduction on the responses of the displacement and acceleration was observed. According to the transverse displacement response the early-time amplitude drop is about 30% compared to the girder with a similar stiffness but without viscosity.

The damping factor estimated from the logarithmic decrement method is about 5.20% between the first and the second peak and dropped to 4.50% between the second and the third peak. When the natural frequency of the first mode for the girder is lower, the response reduction effect seems to be less significant, whereas when the girder frequency is higher the reduction effect is more obvious during the same time interval. For the earthquake loading analysis in the early loading stage the motion for both the displacement and the acceleration was almost the same as that without damping layer, but in the later loading stage both responses showed significant reduction in the amplitude.

REFERENCES


8 EVALUATION AND RETROFIT OF DAMAGED STRUCTURES
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POST-EARTHQUAKE ANALYSIS ON DAMAGE TO STEEL BEAM-TO-COLUMN CONNECTIONS OBSERVED IN THE 1995 HYOGOKEN-NANBU EARTHQUAKE

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ABSTRACT

This paper presents a partial review of the post-Kobe research activities on the fracture of steel welded beam-to-column connections and a few examples of dynamic response analysis conducted for steel building frames with fractures at beam ends. Material tests conducted on the steel of the fractured beams indicated that the steel should have experienced strains far beyond yielding. Dynamic response analyses showed that fractures at beam ends could affect story drifts significantly. Correlation between observed damage and results of the analyses is discussed.

KEYWORDS

Hyogoken-Nanbu Earthquake, Steel Buildings, Damage, Beam-to-Column Connections, Numerical Analysis

1. INTRODUCTION

The magnitude 7.2 Hyogoken-Nanbu (colloquially called Kobe) Earthquake wreaked havoc throughout Kobe and surrounding regions and destroyed or damaged a large percentage of the building infrastructure. Modern steel buildings sustained serious damage for the first time in the history of large earthquakes in Japan. Damage varied in both degree and location. The primary locations of damage were identified as columns, beams, braces, column bases, and beam-to-column connections. Details of the damage are presented elsewhere [for example, AIJ (1995), AIJ-Kinki (1995), Nakashima et al (1997)].

Of the known damage, the fracture of welded beam-to-column connections has been considered most serious in terms of the number of instances of damage and the difficulty in identifying the causes. Fractures were mostly brittle and occurred at weld metals, heat-affected zones, base metals (initiated from the toe of weld access holes), and diaphragm plates. For many instances of such fractures, the following observations had been made: (1) residual story drift was not significant; (2) damage to interior and exterior finishes was minimal; (3) fractures occurred mostly at beam bottom flanges; (4) significant yielding, plastification and local buckling of
beam bottom flanges were observed, indicating that the beams dissipated some energy before fracture; and (5) such plastification occurred only in the beams whose adjoining columns remained almost elastic.

Since the Kobe Earthquake, the following two questions have been constantly addressed: (1) When (with how much plastic rotation capacity) beam-to-column connections fractured? and (2) How much plastic rotation capacity would have been needed to prevent them from failing during the earthquake? Detailed investigation and analysis of damaged buildings, such as material tests of fractured steels, element tests of damaged connections, and dynamic response analyses of damaged buildings, are no doubt useful for answering these questions. Unfortunately, such investigation have been found difficult because, among other reasons: (1) no strong motion was recorded in the most severely shaken areas, so that motion input to damaged buildings remain unknown; (2) no steel buildings were instrumented with seismographs, so that the damaged buildings' behavior is not known; and (3) a majority of damaged beam-to-column connections were repaired immediately after the earthquake, so that detailed information on damage was rather scarce.

Although circumstances made it difficult to study what really had happened to damaged steel buildings and their beam-to-column connections, various research efforts are underway to this end using all available resources. This paper presents a partial review of post-Kobe research activities on the damage to beam-to-column connections and introduces the writers' dynamic response analyses of steel building frames that involve fractures at beam-to-column connections.

2. REVIEW OF POST-KOBE STUDY ON DAMAGE TO BEAM-TO-COLUMN CONNECTIONS

2.1 Material and Element Behavior

The material properties of the steels that fractured at beam-to-column connections were analyzed [Sugimoto and Takahashi (1996a), Ohbayashi et al (1996), Okada et al (1996), Hashida and Toyoda (1996)]. Tests included material coupon tests, hardness tests, and Charpy V-notch tests. The results showed that steel in the vicinity of a fractured section had significant hardening, by about 30 to 50% [Sugimoto and Takahashi (1996a)] or by about 35% [Ohbayashi et al (1996)], and reduction in absorbed energy [down to 10-odd % according to Sugimoto and Takahashi (1996a)], as compared to the same steel not experiencing yielding. Tensile coupon tests suggested that steel in the vicinity of a fractured section might have experienced a strain of about 3.6% [Okada et al (1996)] in one and about 10% [Hashida and Toyoda (1996)] in another. All of the investigations noted that fractured beams should have sustained strains far beyond yielding.

Sugimoto and Takahashi (1996b) tested full-size beam-to-column connections in which beams extracted from undamaged portions of a damaged building were used as specimens. They loaded the specimens cyclically using a dynamic actuator, and observed that the specimens fractured in a cycle of 1/50 in the beam rotation. The material toughness of the tested beams was found to be large, and fracture was initiated from the toe of a weld access hole.

2.2 Dynamic Response Analysis of Damaged Buildings with Damage to Beam-to-Column Connections

Several groups of researchers carried out detailed dynamic response analysis of buildings that had exhibited fractures at beam-to-column connections [Makibe et al (1996), Hasegawa (1996), Komono et al (1996), Kaneda et al (1996), Terada et al (1997), Hisatoku et al (1996)]. For the input motion, recorded ground motions were used in some cases [Makibe et al (1996), Hasegawa (1996)], while in other cases ground motions specific to the site were developed and used [Komono et al (1996), Kaneda et al (1996), Terada et al (1997), Hisatoku et al (1996)]. Most of the analyses assumed the fixed base condition (meaning no interaction with the ground considered), while in a few other cases [Kaneda et al (1996), Terada et al (1997)] soil-structure interaction was taken into account either by modeling the ground into many layers or by introducing equivalent ground stiffness and viscous damping. These investigations indicated that: (1) input motions exerted to the buildings that sustained severe damage should have been much larger than those considered in contemporary seismic design; (2) the maximum story drifts sustained might have been more than 1/50 in the drift angle; and (3) the stories showing serious observed damage corresponded well with the stories for which analysis showed large plastic deformation and/or large input energy. Some [Hasegawa (1996)] suggested that panel deformation could have
been significant and rigid-node assumption may have mislead about the response. It was noted that the soil structure interaction effect would somewhat reduce the motion exerted into the building [for example, by 16% in Kaneda et al (1996)], as well as the maximum acceleration response, but the maximum story drift might or might not decrease.

2.3 Effect of Fracture on Frame Behavior

All of the analysis mentioned above included dynamic response analyses in which plastification of each member was explicitly taken into account, using, for instance, the general plastic hinge method. None of them, however, considered possible cracking and fracture at beam-to-column connections, although they stated that such cracking and fracture would most likely alter the behavior. Kuwamura and Sato (1996) was the first to analytically examine the earthquake response behavior of buildings involving member fracture, in which brittle fracture of columns made of cold-formed square tube sections was considered. They pointed out that fracture of one column tends to trigger sequential fractures in other columns located in the same story and that the random nature of fracture deformation of individual columns should be considered in evaluating the responses.

After the Kobe earthquake, analyses that consider fracture at beam-to-column connections began extensively [Uetani and Tagawa (1996), Ono et al (1996), Sato and Kuwamura (1996), Kusaka et al (1997)]. Among those, the study by Ono et al (1996) presented the most detailed analysis, in which process of dissipation of energy released suddenly by the fracture of a connection was accurately traced through the high frequency vibration of the fractured beam. In other studies [Uetani and Tagawa (1996), Kusaka et al (1997)], the imbalance in the flexural moment that occurs at fracture was resolved using static redistribution of the moment. All analyses that considered fracture indicated that: (1) fracture could alter the earthquake response (like the maximum story shear, maximum story drift, and residual drift) significantly; (2) the earthquake response is very sensitive to the limit rotation at which fracture is taken to occur, with the failure pattern changing significantly in accordance with the selection of limit rotations; and (3) sequential fractures from one connection to another are characteristic of the failure pattern. It should be noted that the analyses were conducted for idealized building models rather than for buildings that were damaged in the Kobe Earthquake.

3. EARTHQUAKE RESPONSE OF STEEL FRAMES INVOLVING FRACTURE

3.1 Examples of Earthquake Responses of Steel Frames Involving Fracture

The following data were obtained from inelastic dynamic response analyses conducted by the writers in order to estimate the degree of plastic rotations that damaged beam-to-column connections might have sustained in the Kobe earthquake. Three building models were analyzed: a five story, one bay steel moment frame; a six story, three bay steel moment frame; and a seven story, five bay steel moment frame (Fig.1 and Table 1).

Simple plastic hinge method was employed to analyze the buildings. One spring was inserted at each end of a member, and it was assumed that the hinge's behavior was rigid-plastic. Unlike the conventional plastic hinge, a limit plastic rotation was specified to simulate the fracture at the bottom flange. Once the plastic rotation reaches the specified limit plastic rotation during the rotation in which the bottom flange is subject to tension, the hinge stiffness degrades (meaning a negative slope), and the hinge behaves as a slip model for subsequent loading (Fig.2). On the other hand, under the rotation in which the bottom flange is in compression, the hinge behaves only rigid-plastically. The direct integration method was employed to obtain the dynamic response, and two acceleration histories were used for input motion. One was the N-S component of the acceleration histories recorded at the Japan Meteorological Agency (JAM) Kobe, which showed one of the largest ground motions recorded at the Kobe Earthquake and has been used popularly in post-Kobe analysis. Fig.3(a) shows its elastic pseudo acceleration response spectrum, together with those of the Sylmar and Newhall records observed in the 1994 Northridge Earthquake. The other acceleration history adopted was a synthesized motion estimated by Hayashi and Kawase (1996). This history is designated as Point-B-20 in the original paper, and it is the one estimated for the center of downtown Kobe. Its elastic pseudo-acceleration response [Fig.3(b)] is significantly larger than those of the JAM Kobe records.
Figure 1: Steel frame models used in analysis

TABLE 1
MEMBER PROPERTIES FOR STEEL FRAME MODELS USED IN ANALYSIS

<table>
<thead>
<tr>
<th>Member</th>
<th>5 story 1 bay frame</th>
<th>6 story 3 bay frame</th>
<th>7 story 5 bay frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member</td>
<td>M_p(kN x m)</td>
<td>I(mm^4)</td>
<td>M_p(kN x m)</td>
</tr>
<tr>
<td>C1</td>
<td>950</td>
<td>774,000,000</td>
<td>1,480</td>
</tr>
<tr>
<td>B1</td>
<td>816</td>
<td>907,000,000</td>
<td>941</td>
</tr>
<tr>
<td>B2</td>
<td>665</td>
<td>744,000,000</td>
<td>1,190</td>
</tr>
<tr>
<td>C2</td>
<td>735</td>
<td>561,000,000</td>
<td>931</td>
</tr>
<tr>
<td>B3</td>
<td>627</td>
<td>687,000,000</td>
<td>627</td>
</tr>
<tr>
<td>B4</td>
<td>451</td>
<td>419,000,000</td>
<td>451</td>
</tr>
</tbody>
</table>

Figure 2: Hysteresis model for plastic rotation spring involving beam bottom flange fracture

Figure 3: Pseudo acceleration elastic response spectra of ground motions used in analysis
## TABLE 2
RESULTS OF RESPONSES OBTAINED FROM ANALYSIS

<table>
<thead>
<tr>
<th>Input Motion</th>
<th>Limit Rotation (rad.)</th>
<th>Input Energy (m/sec)</th>
<th>Max. Story Drift</th>
<th>Max. Residual Story Drift</th>
<th>Max. Plastic Rotation (rad.)</th>
<th>Max. Cumulative Plastic Rotation (rad.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JMA Kobe</td>
<td>infinite</td>
<td>2.84</td>
<td>1/52</td>
<td>1/153</td>
<td>0.012</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>3.02</td>
<td>1/55</td>
<td>1/407</td>
<td>0.021</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>2.45</td>
<td>1/51</td>
<td>1/620</td>
<td>0.026</td>
<td>0.67</td>
</tr>
<tr>
<td>JMA Kobe</td>
<td>infinite</td>
<td>2.91</td>
<td>1/41</td>
<td>1/190</td>
<td>0.018</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>2.72</td>
<td>1/41</td>
<td>1/258</td>
<td>0.031</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>2.79</td>
<td>1/25</td>
<td>1/41</td>
<td>0.047</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>infinite</td>
<td>4.41</td>
<td>1/21</td>
<td>1/50</td>
<td>0.041</td>
<td>0.25</td>
</tr>
<tr>
<td>Point-B-20</td>
<td>0.01</td>
<td>3.91</td>
<td>1/18</td>
<td>1/48</td>
<td>0.061</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>3.64</td>
<td>1/14</td>
<td>1/46</td>
<td>0.082</td>
<td>0.87</td>
</tr>
<tr>
<td>JMA Kobe</td>
<td>infinite</td>
<td>2.56</td>
<td>1/51</td>
<td>1/190</td>
<td>0.014</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>2.53</td>
<td>1/46</td>
<td>1/130</td>
<td>0.027</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>2.49</td>
<td>1/31</td>
<td>1/327</td>
<td>0.036</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>infinite</td>
<td>4</td>
<td>1/28</td>
<td>1/388</td>
<td>0.029</td>
<td>0.26</td>
</tr>
<tr>
<td>Point-B-20</td>
<td>0.01</td>
<td>3.49</td>
<td>1/20</td>
<td>1/55</td>
<td>0.056</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>3.22</td>
<td>1/21</td>
<td>1/47</td>
<td>0.055</td>
<td>0.57</td>
</tr>
</tbody>
</table>

**Figure 4** Examples of response time histories and story shear vs. story deflection relationships
The analyzed buildings had 0.62 sec (the five story frame), 0.86 sec (the six story frame) and 1.05 sec (the seven story frame) for the elastic fundamental natural period. These buildings were hypothetical, but their dimensions closely followed steel buildings actually constructed in Kobe. [For detailed dimensions, see Minami et al (1996).] For each ground motion, three limit plastic rotations (same values for all beam members) were assigned: i.e., infinite, 0.01 rad. and 0.004 rad. The first was adopted to see how much plastic rotation capacity would be needed if the buildings responded without beam fracture; the second represented a fair amount of plastic rotation capacity [Kozai Club (1993)]; and the third represented a rather small plastic rotation capacity.

The results obtained are summarized in Table 2, and two displacement response histories and two story shear versus story displacement relationships, both obtained for the seven story, five bay steel moment frame, are shown in Fig.4 A summary of the results is as follows.

(1) The total input energies to the buildings (assuming no fracture) were 2.56-2.91 m/sec (JMA Kobe) and 4.0-4.41 m/sec (Point-B-20) in terms of the equivalent velocity. This significant difference suggests that the earthquake power and damage potential is much larger in Point-B-20 than in JMA Kobe. When early beam fracture was considered (0.004 rad. for the limit rotation), the total input energies were 2.45-2.79 m/sec for JMA Kobe, indicating that the total input energy remained relatively unchanged despite the beam fracture. On the other hand, Point-B-20 showed a reduction of the energy by about 20% (down to 3.22-3.64 m/sec), most likely the result of a significant drop of the elastic input energy of Point-B-20 in the period range greater than 1.0 sec [Fig.3(b)], combined with a large period elongation of the buildings caused by beam fracture.

(2) When beam fracture was assumed not to occur, the maximum plastic rotation attained at the beam ends was approximately twice as large in Point-B-20 as in JMA Kobe, which also indicates the significant difference in damage potential between the two motions.

(3) The maximum plastic rotations required for beam-ends not to fracture were 0.012-0.018 rad. for JMA Kobe, which may be attainable for beam-to-column connections designed and fabricated with the present practice [Kozai Club (1993)]. On the other hand, plastic rotations required for Point-B-20 were much larger (up to 0.029-0.041 rad.) Such large rotations may not be easy to achieve even using the modern practice.

(4) The ratios of cumulative to maximum plastic rotations (without fracture) ranged from 6 to 10. These ratios were similar to those obtained for conventionally used input motions, such as the 1940 El Centro record [Nakashima et al (1996)].

(5) Early beam fracture indeed increased maximum story drifts, maximum plastic rotations, and particularly maximum cumulative plastic rotations. The reduction in stiffness caused by fracture was also evident, and the period of vibration was enlarged significantly [from 2.5 times (JMA Kobe) to 4 times (Point-B-20) the period of vibration that was achieved assuming no fracture).

(6) Residual story drifts were rather random: greater than 1/50 in some cases but almost zero (smaller than 1/400) in other cases.

3.2 Discussion

It is notable that the results here, as well as the results obtained by other researchers (see Sections 2.2 and 2.3), rather contradicted the observed damage. The largest mystery is that many buildings with significant damage to beam-to-column connections exhibited minimal damage to interior and exterior nonstructural elements and small residual lateral drifts as mentioned in Section 1, whereas most analyses resulted in large maximum story drifts (more than 1/50 even without fracture consideration).

A few possible questions are: (1) Could nonstructural elements accommodate such large deformation without serious damage? (2) Were the motions input to the buildings much smaller than those considered in the analyses? and (3) Were the damaged buildings much stronger than the models used in the analyses? As for the first question, it has been recognized that nonstructural elements cannot accept 1/50 of story drifts without damage. For the second question, effective forces input to buildings could decrease if soil-structure interaction were taken into account, but the degree of reduction was reported to be in a range of at most 20%. Such reduction is most likely not sufficient to lessen the response to a level of minimal nonstructural damage. As for the third question, some possible candidates to increase the strength are: full composite action with RC floor slabs,
resistance of nonstructural elements, and three dimensional (orthogonal) effects in the building resistance. One conclusion drawn from the above statement is such that post-Kobe dynamic response analyses are not fully convincing yet in terms of reproduction of the observed damage.

4. CONCLUSION

This paper presented a partial review of post-Kobe research activities on the fracture of beam-to-column connections in modern steel buildings and introduced a few examples of dynamic response analyses conducted for steel building frames including fracture. A summary of this paper is as follows:

(1) Material tests conducted on the steels of fractured beams indicated that fracture should have occurred after experiencing significant plastic strains.

(2) Dynamic response analyses of damaged buildings suggested that earthquake forces input to the buildings having serious damage to beam-to-column connections should have been significantly larger than those specified in the present seismic design.

(3) Fractures at beam-to-column connections could increase lateral drifts significantly, say, to more than 1/50 in the story drift angle. The results, however, do not necessarily correspond to the observed damage, because in many cases damage to nonstructural elements and residual story drifts remained minimal. More refinement of analyses is required to interpret the observed damage.

References


SHEAR STRENGTH OF DAMAGED HIGH STRENGTH STEEL BRIDGES

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ABSTRACT

Experimental and theoretical studies of the residual shear resistance of damaged maraging steel web panels are described. Failure of the damaged web panels was sudden and explosive and occurred due to brittle fracture of the web material. Theoretical predictions of the residual shear resistance of damaged girders, based on the Cardiff tension-field theory, are compared with test results. The proposed theoretical methods, which are based on an assumed plastic collapse mechanism, were found to be unrepresentative of the mode of failure of the web panels.

KEYWORDS

Shear strength, Steel plate girders, Damage, Residual strength, Maraging steel, Cardiff tension-field theory, Plastic collapse mechanism, Testing, Discrete hole, Pepper-pot damage.

1. INTRODUCTION

The characteristics required in modern transportable bridges are mobility and rapid deployment. In the design of such transportable bridges, a compromise between lightness, load capacity and span length must be reached. Since strength and lightness are paramount, the high strength to weight ratios possessed by maraging steel are ideally suited for the design of transportable bridges. Transportable bridge girders are particularly susceptible to damage due to frequent handling or when they are used in hazardous conditions. In situations where it is essential that vehicles are allowed to cross the bridge but immediate repair of the damaged girders is not possible, guidelines are required to enable engineers to assess whether the damaged girders are capable of carrying the imposed loads. Hence, the development of simple analytical methods that can be used to quickly evaluate the residual resistance of damaged plate girders in adverse conditions is very important.

The residual shear resistance of damaged plate girders depends upon the extent and form of damage inflicted on the web panels and the boundary members (Lee 1994). With regard to the web panels, two types of damage are generally considered; (a) a large single discrete hole and (b) small holes scattered randomly (pepper-pot) over the entire area of the web panel. Suitable analytical methods for evaluating the residual strength of steel girders containing a large single discrete hole in the web panel have already been proposed by Narayanan and Der Avanessian (1984), and for aluminium girders containing scattered holes in the web panel by Lee et al (1995). Both methods are based on the well established Cardiff tension-field theory for

An extensive program of experimental and theoretical research into the residual shear strength of damaged steel and aluminium plate girders has been in progress at the Cardiff School of Engineering for the past ten years. Herein, available test results from a series of static tests on damaged maraging steel plate girders (Lee and Davies 1995a and 1995b) are reviewed and analysed, with particular emphasis on the mode of failure and residual strength. Theoretical predictions of the residual shear resistance of damaged maraging steel plate girders are compared with experimental results. The objective of the research is to develop simple and practical methods, that may be used under difficult field conditions, to estimate the residual strength of damaged maraging steel girders.

2. TESTS ON DAMAGED PLATE GIRDERS

2.1 Details of Test Specimens

A series of static tests was conducted on two tapering web panels, which were taken from a maraging steel transportable bridge (Lee and Davies 1995a and 1995b). Fabrication details of the test girders are presented in Figures 1 and 2. The test girders consisted of two main web panels. Substantial stiffeners were bolted to the test girders under the loading point and to form a rigid end post at one of the supports. The other end was on actual cross-girder support. The original single-sided transverse web stiffener on the test panel was removed, whereas those on the adjacent panel were kept.

![Fig 1 Loading arrangement and dimensions of test panel]
The girders were fabricated from 18% Ni 90 ton maraging and aged for 10 hrs @ 460°C. Two tensile tests were conducted on material taken from the web plate. The average uniaxial yield stress and modulus of elasticity $E$ were found to be 1629 N/mm$^2$ and 195 kN/mm$^2$ respectively. The tests indicated that the material has low ductility when compared to normal structural mild steel.

![Wine glass flange](image1)

![Box flange](image2)

Note: All plates 4.76mm thick unless otherwise noted

Fig. 2 Flange dimensions of test panel

### 2.2 Details of Damage

Two types of damage were investigated. Test girder M1 contained a single 210mm square hole, cut from the tension diagonal of the web panel, the exact location of which is shown in Figure 3. Test girder M2 contained a large number of small holes, ranging in size from 200mm$^2$ to 2500mm$^2$. The location of the holes is shown in Figure 4. The severity of the worst pepper pot damage, expressed as a percentage of the web depth, was 49%.

All the holes were cut from the web panels using a plasma cutter. The edges of the holes in girder M2 were left jagged to simulate the nature of real damage, while the edges of the square hole in girder M1 were ground flat.

### 2.3 Support, Loading and Instrumentation

The girders were tested using a 3000 kN self-straining test frame. Each girder was simply supported on notched rollers, which allowed rotation but no horizontal movement, and subjected to a central reaction point load. The loading arrangement is shown in Figure 1. The applied load was measured by a pressure transducer incorporated in the hydraulic jack loading system. The central in-plane deflection and out-of-
plane deflection at the centre of the web panel were measured by transducers. The outputs of the pressure
transducer and the central in-plane deflection transducer were fed into an X-Y plotter, so that the load-
deflection curve could be monitored automatically during the test.

The test panels were instrumented with strain gauges, as shown in Figures 3 and 4. Web strains were
measured using 45° rosette strain gauges. The gauges were placed in pairs, one on each side of the web, so
that the bending and membrane components of strain could be deduced from the readings. The surface
strains along the bottom tension box flange were measured using linear strain gauges.

![Fig. 3 Instrumentation and damage details of test panel M1 containing a single discrete hole](image1)

![Fig. 4 Instrumentation and damage details of test panel M2 containing pepper-pot damage](image2)
2.4 Test Procedure

At the start of each test, the girder was subjected to a pre-load of 200 kN, to seat the supports and load reaction point. The load was then removed and the strain gauges initialised (set to zero). The girder was then subjected to a static test, in which the load was applied in increments using deflection control, until failure occurred. At selected load increments, the girder was unloaded back to zero, so that the extent of permanent deformation could be assessed. Residual strain gauge and deflection readings were taken at the end of the test.

3. TEST RESULTS

3.1 Load versus Central In-Plane Deflection

The load versus central in-plane deflection for test girders M1 and M2 are shown in Figure 5. Apart from the initial bedding-in of the supports and central reaction point, the graphs show an approximate linear behaviour up to failure. It is worth noting that girder M2 (pepper pot damage) is slightly stiffer than girder M1 (single discrete hole). At the loading and unloading cycles, little permanent plastic deformation was apparent. The load-deflection curves for both tests shows no failure plateau, thus giving little warning of failure.

![Graph showing load versus central in-plane deflection for test girders M1, M2, intact girder, pepper pot damage, and single discrete hole.](image)

Fig. 5 Load versus central in-plane deflection
3.2 Failure Modes

The collapse of both test girders was sudden and explosive. The web panel failure load for girders M1 and M2 were 593 kN and 1157 kN respectively. For girder M1, failure was caused by the rapid formation of two through thickness web fractures, which initiated at the corners of the compression diagonal of the hole, as shown in Figure 6, where stress concentration was highest. A separate crack also formed along the wine glass flange-web boundary. This crack was not connected to the other fracture lines and its formation is believed to have been caused by the explosive collapse of the test panel. For girder M2, failure was triggered by cracks which formed at the jagged edges of the pepper-pot holes which developed rapidly into extensive fractures, as shown in Figure 7. The initiation of these cracks was probably caused by high stress concentrations at the jagged edges of the holes.

Fig. 6 Extent and location of fractures in test panel M1

Fig. 7 Extent and location of fractures in test panel M2
The fracture surface of the cracks were set at 45° to the web plate and no appreciable signs of ductility were found. The fracture surface of the boundary crack in girder M1 exhibited a cup and cone failure surface.

3.3 Web Strains and Stresses

Extensive strain and stress details have been presented elsewhere (Lee and Davies 1995a and 1995b) and only a brief summary is reproduced herein. For test girder M1, the highest surface strains occurred adjacent to the corners of the web hole. Away from the web hole, an approximately uniform shear stress distribution was observed in the web panel. For test girder M2, since the distribution of web strains and stresses were influenced by the presence of the holes, no firm conclusions can be drawn from the results. Values of the principal strains for both girders M1 and M2, before and after the unloading cycles, were approximately the same at all strain gauge positions.

4. RESIDUAL SHEAR STRENGTH OF DAMAGED GIRDERS

4.1 Background

Perforated steel plates are extensively used in civil engineering construction as designers frequently find it necessary to introduce openings in the webs of plate girders; these openings are there to provide access for inspection and maintenance or to allow the passage of services. In the present context, such openings may result from damage sustained in service.

The structural behaviour of steel plate girders with slender webs containing discrete web openings has been investigated by several researchers (Narayanan and Der Avanessian 1984, Narayanan and Rockey 1981 and Lee 1987). However, the method proposed by Narayanan and Der Avanessian (1984) has been widely accepted for predicting the shear resistance of web panels containing discrete holes. A method for determining the residual strength of aluminium girders containing pepper-pot damage in the web panel has been proposed by Lee (1994) and Lee et al (1995). Both proposed methods are based on the Cardiff tension-field theory for intact girders and therefore reduce to a proper solution for limiting cases. The theory assumes an equilibrium stress field (tension-field) in the girder, which satisfies the theoretical conditions for a lower bound strength prediction, provided the material possesses sufficient ductility for the stress field to develop.

4.2 Square Hole Located in Corner of Tension-Field

Narayanan and Der Avanessian (1984) studied the buckling behaviour of perforated web plates using finite element analysis. For a square hole of side length \(a\) located at the corner of the tension-field, the shear resistance \(V_S\) is given as

\[
V_S = \tau_{cr}^t dt + \sigma t^2 \sin^2 \theta (d \cot \theta - b + c_r + c_t) \tau_{cr}^t \sqrt{2} \sin (45^\circ + \theta)
\]

where

\[
\tau_{cr}^t = k \frac{\pi^2 E}{12(1-v^2)} \left( \frac{t}{d} \right)^2
\]
\[ k = k_0 \left( 1 - 1.1 \sqrt{\frac{a^2}{bd}} \right) \left( 1 - 3.5 \left( \frac{a}{d} \right)^2 \sqrt{\frac{ae}{d^2}} \right) \]  

(3)

in which \( b \) is the width of the plate, \( d \) is the depth of the plate, \( e \) is the eccentricity of the square hole from the centre of the web panel, \( k_0 \) is the buckling coefficient for an intact plate having fixed edges, \( t \) is the thickness of the web, \( c_t \) and \( c_c \) are the hinge distances along the tension and compression flanges respectively and \( \sigma_{yr} \) is the tension-field stress given by

\[ \sigma_{yr} = -1.5 \tau_{cr} \sin \theta + \sqrt{\sigma_y^2 + \tau_{cr}^2 \left( 1.5 \sin \theta \right)^2 - 3} \]  

(4)

\( \theta \) is the angle of inclination of \( \sigma_{yr} \) and may be found by iteration to give the maximum value of \( V_p \). \( \sigma_y \) is the yield strength of the material.

### 4.3 Pepper-Pot Damage

Lee (1994) and Lee et al (1995) proposed an equivalent plate thickness analogy to the Cardiff tension-field theory to obtain the ultimate shear resistance \( V_p \) of a web panel having pepper-pot damage. \( V_p \) is given as

\[ V_p = \tau_{cr} dt + \sigma_{yr} \sin^2 \theta (d \cot \theta - b + c_c + c_t) t_e \]  

(5)

in which

\[ k = k_0 \left( 1 - 1.25 \sqrt{\frac{A_c}{A}} \right) \]  

(6)

where \( A \) is the total area of the intact plate \((bd)\), \( A_c \) is the area of the holes at the worst location and \( t_e \) is the equivalent thickness of the web panel. This method retains the basic form of the Cardiff tension-field theory and reduces to the original theory when there is no damage present in the web panel.

### 4.4 Comparison between Theoretical and Experimental Results

For girder M1, the residual shear strength evaluated using the Narayanan and Der Avanessian (1984) method was calculated to be 741 kN, which overestimates the test result by 20%. The overestimation is probably due to the assumption of a plastic collapse mechanism in the theoretical method, which does not reflect the collapse behaviour of the test girder. Furthermore, the analytical method was originally developed for the analysis of square web panels containing square web holes. Although the geometry of the test panel showed modest departures from the parameters assumed in the analytical method, they should have a negligible effect on the predicted results.

For girder M2, the residual shear capacity of a web panel containing randomly scattered web holes (Lee 1994 and Lee et al 1995) was calculated to be 1089 kN, which underestimates the test result by 6%. Although the analytical method accurately predicted the failure load of the pepper-pot damaged panel, it should be noted that failure of the test panel was triggered by sudden web fracture, and was thus different from the theoretically assumed plastic collapse mechanism.
5. DISCUSSION AND CONCLUSIONS

The results of two static tests on maraging steel bridge girder panels, containing a discrete web panel hole and randomly scattered web holes, subjected to predominantly shear loading have been reviewed. Failure of the damaged panels was sudden and explosive and occurred due to extensive fracture of the web panel, initiated by stress concentrations at the edges/corners of the web holes.

Analytical procedures for determining the residual shear capacity of damaged web panels, which were based on an assumed plastic collapse mechanism, were found to be unrepresentative of the fracture failures exhibited by the maraging steel test girders. Further development of the analytical procedures will probably require consideration of material fracture. Areas of particular interest include the magnitude of the applied stress field in the web panel (accounting for damage) and appropriate stress intensity factors due to the approximate size and type of crack.

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REFERENCES


9 LOW CYCLE FATIGUE AND FRACTURE
LOW CYCLE FATIGUE FRACTURE LIMIT
AS THE EVALUATION BASE OF DUCTILITY

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ABSTRACT

As the evaluation base of ductility of structures, the author [1] had already proposed at the 4th WCEE 1964 to employ their low cycle fatigue fracture limits. In this report the low cycle fatigue fracture limits of steel beam-columns and steel unit rigid frames with and without bracing members are presented experimentally. Thus the practical quantitative evaluation of ductility of steel structures may become actually possible for the aseismic design of steel structures.

KEY WORDS

ductility, low cycle fatigue, fatigue fracture, steel beam-columns, rigid frames, bracing, evaluation of ductility, aseismic design,

1. INTRODUCTION

In order to evaluate the ductility of structures, the author [1] had already proposed at the 4th WCEE 1964 to employ the low cycle fatigue fracture limit of structural elements, especially of their beam-columns. In this report, the low cycle fatigue fracture limit of steel beam-columns and unit rigid frames with and without bracing members are presented experimentally as the ductility evaluation base of steel structures. Computed results are compared with test results and some empirical formulae of low cycle fatigue fracture limits of steel beam-columns are presented for the ductility evaluation at the practical structural design.
2. DUCTILITY

The importance of ductility of structures, especially against earthquake excitation were already discussed and some hypothetical researches or aseismic design principles based upon ductility were at first presented by Tanabashi [2] in 1937 and then by Housner [3] at the 1st WCEE 1956. However, for the practical structural design there are yet only very few documents to make possible the practical application of the quantitative evaluation of ductility of structures.

The technical term, ductility factor, of which concept was introduced at first by Newmark [4] at the 2nd WCEE 1960 for the base of practical structural aseismic design, is a theoretically very clear but experimentally very obscure concept, because of the ductility to determine the plastic flow limit. Plastic flow under bending moment occurs usually by the yielding of tensile steel element in the cross section and continues the plastic rotation of hinged region of the member until the loss of compressive resistance through the initiation of unstable state of resistance in compressive element. The decrease of plastic moment does not show usually clear limit but gradually decreasing process.

In order to make clear the ductility factor experimentally, the author [1] had already proposed at the 14th WCEE 1969 to employ the low cycle fatigue fracture limit for the experimental evaluation of ductility factor. Under the repeated cyclic loading the structural member shows very clear fracture limit, especially beam-column member under a certain prescribed constant axial compression, the fracture is defined as the loss of axial resistance very clearly. In the field of gravity it is very significant too. Thus the low cycle fatigue fracture limit may give a very clear experimental base to the evaluation of ductility and it will be presented this low cycle fatigue fracture limit of not only steel members but also steel unit rigid frames with and without bracing members in this report.

3. LOW CYCLE FATIGUE FRACTURE LIMIT OF BEAM-COLUMNS

Ductility of beam-columns under cyclic bending may be clarified fundamentally only by low cycle fatigue fracture tests, carried out by constant deflection amplitude tests under the action of prescribed constant axial loads and test results are illustrated usually through the relationships between deflection (or story sway displacement) amplitudes $\delta$, or story sway angle amplitude $R_\delta (=\delta/h)$ of columns in ordinate versus number of cycle until fracture $N_b$ in abscissa in log-log scale by Yamada [1] like another fatigue tests.

There are many factors influencing on the low cycle fatigue fracture limits of steel beam-columns such as axial load level ratios $n (=N/N_y)$, flange width to thickness ratios $(b/t)$ or web depth to thickness ratios $(d/w)$ etc. The most important influencing factor on the ductility, axial level ratios, had been reported by Yamada [1] 1969, Yamada and Shirakawa [5] and presented empirical formulae for wide flange beam-columns as follows by Yamada [6]:

\begin{align*}
\text{for } n=1/3 ; \quad & \log R_\delta = -0.54 \log N_b - 0.51 \\
\text{for } n=1/2 ; \quad & \log R_\delta = -0.63 \log N_b - 1.13
\end{align*}

(1)
(2)

for fairly stocky rolled cross section with $(b/t) = 7$. 

Analytically had proposed Yamada and Kawamura [7] the ductility of wide flange steel beam-columns in cyclic bending under the assumptions of fundamental low cycle fatigue fracture limit of materials as cumulative damage,

\[ \tau \sqrt{N_B} = K. \quad 1/2 < \alpha < 1 \quad \text{(Manson-Coffin: } \alpha = 1/2) \]  

(3)

with the coefficient \( \alpha = 3/4 \). The low cycle fatigue fracture limit of wide flange beam-columns are calculated with the length of plastic hinge region of 1/6 of member length as the following limits:

a) without axial load; \( n = 0 \) (beam), the fatigue fracture limit are assumed to occur at the arrival of tensile fracture \( B \) in tensile flange.

b) under lower axial load levels; \( n < 1/2 \), the low cycle fatigue fracture occurs at the arrival of compressive strain to the ultimate state \( (D_u) \) in web center.

c) under higher axial loads; \( n > 1/2 \), the low cycle fatigue fracture occurs at the arrival of compressive strain to the ultimate state \( (D_u) \) in compression flange.

The calculated values of this low cycle fatigue fracture limits are compared with the test results by Yamada and Shirakawa [5] on the low cycle fatigue fracture limits of wide flange steel beam-columns under various axial load with various prescribed constant deflection amplitudes and shown fairly good coincidences between them such as shown in Fig. 1.
The other influencing factor, flange width to thickness ratios \((b/t)\) and web depth to thickness ratios \((d/w)\) on the low cycle fatigue fracture limits of wide flange and box cross section beam-columns were presented by Yamada, Kawabata and Yamanaka [8] for various values of \((b/t) = 10, 20\) and 30 for wide flange, and \((b/t) = 17, 24\) and 3 for box beam-columns under an axial load level ratios of \(n = 1/3\). Influences of \((d/w)\) values are also plotted in the same figure \((d/w) = 12, 19, 25\) and 37 such as shown in Figs.3 and 4.

These test data on the influences of \((b/t)\) and \((d/w)\) values upon the low cycle fatigue fracture limits under an axial load level ratio of \(n = 1/3\) are summarized by the following empirical formulae as a function of \((b/t)\) as follows:

a) for wide flange cross section,
\[
\log R^* = -0.50 \log N^* - [0.700 + 0.032(b/t)]
\] (4)
b) for box cross section,
\[
\log R_s = -0.50 \log N_b - [0.300 + 0.040(b/t)]
\]  
(5)
and illustrated in Figs. 5 and 6 for practical use.

\[
\begin{align*}
\text{Fig.5 Low Cycle Fatigue Fracture Limits of Wide Flange Beam-Columns} \\
\text{Fig.6 Low Cycle Fatigue Fracture Limits of Box Beam-Columns}
\end{align*}
\]

4. LOW CYCLE FATIGUE FRACTURE LIMIT OF BRACING ELEMENT

Bracing elements are undergone usually axial tension compression. Under cyclic loading, bracing elements undergo alternately repeated axial tension and compression. There are only few test data on such case. Yamada and Hishimoto [10] had reported on the low cycle fatigue fracture limits of steel bars with rectangular cross section under alternately repeated cyclic axial tension and compression with prescribed constant axial deformation amplitudes. Test data are summarized as the relationship between mean axial strain amplitude \( \varepsilon_a \) and number of cycles to fracture \( N_b \):
\[
\log \varepsilon_a = -0.56 \log N_b - 0.63
\]  
(6)
and illustrated in Fig. 7.

5. LOW CYCLE FATIGUE FRACTURE LIMIT OF BEAM TO COLUMN JOINTS

Fatigue fracture of steel beam to column joints are influenced by the welding details of joints, i.e. size and shapes of scallops in web of beam ends. Yamada and Masuda [11] had reported test data on some low cycle fatigue fracture limits with special references to the effects of scallops in web such as illustrated in Fig. 8. This results show clearly the fact that welded beam to column joins without scallops are far better than with scallops. The existence of scallop itself is questionable. This facts were already indicated by Stallmeyer, Munse and
Goodall [12] in high cycle fatigue fracture limit of welded slices too. Many cutted off fracture of flange ends by very low cycle fatigue at wide flange beam ends with scallops were really found at the recent Hanshin-Awaji-Earthquake, Kobe, Japan 17. Jan. 1995 [13].

6. LOW CYCLE FATIGUE FRACTURE LIMIT OF UNIT RIGID FRAME

Ductility assurance of structural system must be based upon the story ductility, and the relationships among the ductility of structural elements, stories and whole structures are already discussed by Yamada [14]. Story ductility is composed from the component elements consisting the concerned story as the total resisting behavior of elements. Resistance-sway relationship of each story may be derived from the superposed total resistance of consisting elements at the same sway drifting value of the story.

Low cycle fatigue fracture tests on steel unit rigid frame composed of wide flange steel profile with columns around weak axis - column yielding type - were carried out under a constant axial load level of $N^a/3$ by Yamada, Tsuji, Murazumi and Asagawa [15] [16]. Frame fatigue fracture occurs by the failure of columns through the deep extension of flange local buckling or bifurcation of columns, which were verified theoretically by Yamada and Iwanaga [17]. The fatigue fracture limits of such unit rigid frames are illustrated in Fig. 9 by H marks with a solid line. An empirical relationship between story sway (drift) angle amplitude $R_s$ and number of cycles to fracture $N_b$ of such unit rigid frame may be expressed by:

$$\log R_s = -0.27 \log N_b - 0.78$$

(7)

7. LOW CYCLE FATIGUE FRACTURE LIMIT OF BRACED RIGID FRAME

Low cycle fatigue fracture tests on braced steel unit rigid frame composed of wide flange steel profile with
columns and a single diagonal bracing around weak axis were carried out under a constant axial load level of Ny/3 by Yamada, Tsuji, Asagawa and Tubakimoto [16][18]. Fracture occurs through the fatigue tensile cut off fracture of bracing elements after buckling. The low cycle fatigue fracture limits of such braced steel unit rigid frames are illustrated in Fig. 10 by X marks with a dotted line. An empirical relationship between story sway (drift) angle amplitude $R_a$ and number of cycles to fracture $N_b$ may be expressed by:

$$\log R_a = -0.44 \log N_b - 0.87$$

Fig. 9 Low Cycle Fatigue Fracture Limits of Steel Unit Rigid Frames [15][16]

Fig. 10 Low Cycle Fatigue Fracture Limits of Steel Unit Rigid Frames With and Without Bracing [16][18]

8. CONCLUDING REMARKS

For the practical ductility evaluation base of steel structure, the low cycle fatigue fracture limits of steel beam-columns, bracing elements, unit rigid frames and unit braced rigid frames are presented. Empirical formulae between sway deflection amplitudes $R_a$ and number of cycles to fracture $N_b$ of them are proposed.

The contents of this paper was at first prepared for a part of Tall Building Committee SB18, Topical Volume: Fatigue Assessment and Ductility Assurance, Chapter 5, Ductility Assurance of Structural Components, Joints and Systems, but was not published.
REFERENCES


ABSTRACT

Steel moment frames sustained a large amount of cracking during the Northridge Earthquake. The evolutionary trends in the design and construction of these frames are discussed, and the consequences of these trends in seismic damage are noted. Two typical buildings which had significant cracking into the columns were analyzed in considerable detail. It is shown that a large amount of panel zone yield deformation should have occurred in these buildings, and their dynamic response was largely influenced by offsets and nonstructural masonry walls. The limited redundancy utilized in these buildings resulted in relatively large members. The computed behavior of the buildings is correlated to the results of past experiments. Finally a limited experimental study of the ductility and strain rate effects for welded connections is presented. Many factors influenced the cracking noted after the Northridge Earthquake, and some of these are discussed in this paper.

KEYWORDS

Cracking, Connections, Ductility, Earthquakes, Inelastic Behavior, Moment Frames, Steel Structures, Shear Yielding, Welded Connections

1. HISTORY OF STEEL MOMENT FRAMES IN SEISMIC DESIGN

Steel moment frames have been used in the United States since the start of the 20th Century. Prior to the 1920's, these frames were constructed with built up members with gusset plate connections as illustrated in Figure 1. These members and connections were riveted, and the entire steel frame was encased in concrete for fire protection. Few of these early steel structures were designed for seismic loading, since only wind load was considered prior to about 1930. These buildings invariably included many stiff, strong unreinforced masonry architectural walls and partitions. Structural engineers relied upon these walls and partitions to help resist lateral loads, since they did not have the capacity for extensive calculations at that time. Instead engineers employed observations of the past performance of these buildings in the design, and they were less likely to design a building outside the normal practice then they are today.

AISC Specifications (1928) were first developed in the 1920's, and standard hot rolled shapes with riveted connections such as those illustrated in Figure 2 became the common practice. Angles and T-sections were used to form the connection, and the member and the connection were encased in concrete for fire protection. Unreinforced masonry curtain walls and partitions were still used. Seismic design forces were considered in these structures, but the seismic design forces were simplified and usually smaller than those used today. These early structures were highly redundant in that every beam-column connection was a moment resistant connection, and a large but uncalculated stiffness and resistance was provided by nonstructural elements.
The riveted connections and construction described above and illustrated in Figure 2 were used until the mid-1950's or early 1960's. At that time, high strength bolts replaced the rivets, but connections such as illustrated in Figure 2 were still employed. Concrete encasement was also discontinued in favor of other lighter fire protection materials. By the late 1950's, the seismic design procedures had evolved to a period and mass dominated procedure, and therefore engineers began to reduce the mass and stiffness of the structure. However, buildings of this era still had a substantial uncalculated strength and stiffness due to nonstructural elements, and they were very redundant since moment resisting connections were used at every beam column joint. This construction continued into the early 1970's.

In the late 1960's and early 1970's, the seismic practice for steel moment frames evolved to the fully restrained (FR) bolted web-welded flange moment resisting connection illustrated in Figure 3. These connections have full penetration welds connecting the beam flange to the column, and an erection plate bolted to the web to transfer shear force. Stiffeners or continuity plates are often required to prevent local damage to the connection, and panel zone stiffeners or doubler plates may be required to control panel zone yield and deformation. This connection was chosen because research by Popov and Pinkney (1969), Popov and Stephen (1970), Krawinkler et al (1971), and Bertero et al (1973) showed that better inelastic cyclic behavior as illustrated in Figure 4 was achieved with the fully welded flanges and bolted webs than with bolted connections such as used in earlier structures. The hysteresis of the FR welded flange-bolted web connection tests were full, and the strength and stiffness remained stable through large inelastic deformation. Further, this connection developed the full plastic capacity of the beam rather than a brittle failure in the net section. The experiments used to justify this FR connection were on W24 beams and shallower, however this was not a serious limitation since steel frames of the early 1970's seldom had beams greater than this depth. These FR connections were used almost exclusively until the Northridge earthquake.
While this early research established the general directions of seismic design, a number of changes in the design specifications and professional practice occurred during the years that followed. In 1988, the Uniform Building Code (1988) changed to increase the shear strength of panel zones. This increase was based on observations of the excellent ductility provided by panel zone yielding in tests by Bertero (1973), Krawinkler (1971), Popov and others (1986) and in recognition of the added resistance due to strain hardening. The increased panel zone strength rating meant that steel frames built since 1988 may sustain larger inelastic deformation in the panel zone during an earthquake, since they initially yield at a smaller lateral force. Another change to the Uniform Building Code (1988) required supplemental welding of the beam web to the shear plate, because of a test program by Tsai and Popov (1988).

There were substantial changes in the professional practice beyond those seen in the design specifications. Until the late 1970's, FR connections were used at all beam-column connections in the structural system. This resulted in good distribution of lateral stiffness and resistance, and member were relatively small. However, the FR connections are relatively costly, and engineers began to minimize their use. At first, perimeter frames replaced frames with FR connections at all beam-column connections, since perimeter frames resulted in similar translational and torsional stiffness while significantly reducing the number of FR connections. The total seismic resistance of the structure does not decrease when this concentration of seismic resistance is employed, and so increased bending moments and stiffness must be developed within individual members and connections. In later years, even more dramatic increases in member, flange and weld sizes were produced by engineers, who concentrated the seismic resistance into individual isolated frames or bays of frames.

The consequences of the reduced redundancy years can be illustrated by comparing the designs of four steel frame buildings built in California over a 70 year period. Building A is a 26 story steel frame building constructed in downtown San Francisco in the mid 1920's with riveted connections such as those illustrated in Figure 2. Typical column spacing for this building was in the order of 5 m. (17 ft), and typical beams for these spans were no larger than W21x68. The very longest column spacing in this building were approximately 9 m. (30 ft.) with beams no larger than W30x99. Building B was built in San Francisco in the mid 1960's. This building is 22 stories and has similar connection details to those used in Building A except that high strength bolts were employed. The typical column spacing was approximately 8 m. (27 ft.) and the heaviest beams are in the order of W27x102. Building C was built in mid-1970's and is 30 stories tall with FR connections at all beam-column joints. The beam spans are approximately 9 m. (30 ft.). This is the tallest building of the four and it has the largest column spacing, and so the beam sizes are expected to be somewhat larger that those used in Buildings A and B. However, beam depths vary between W24's and W36's with typical beams being smaller than a W30x99. The very heaviest beams in the bottom stories of this 30 story structure are W36x260. These heaviest sections are somewhat an anomaly, since are they only used on the first story where the story height is taller than that used in any of the other buildings. Building D is a 17 story building located in the San Fernando Valley, and is described in the analytical studies [Paret and Sasaki (1995)] completed as part of the SAC Phase I program. The building was built in the mid 1980's, and 2 bays of seismic framing are located on each of the four sides. This building also has column spacing similar to that used in B and C, but it is the shortest and lightest building of the four buildings. Beam sizes in the moment frame are W36x300 for the bottom framing and even the top story requires a W36x150 section. The building is shorter and lighter than the earlier examples, and so its seismic design forces should be smaller than these older buildings, but the reduced redundancy resulted in significant increases in the depth and weight of the beams in the moment frames. These comparisons show that for 50 years of seismic design practice the member sizes remained quite stable because of the redundant structural system. However, in the most recent 10 to 15 years dramatic increases in members sizes have been noted because of equally dramatic reductions in redundancy of the structural system.

Similar evolutionary changes can be noted with the welding process and electrodes. The welded flange-bolted web connection was first used in the mid 1960's and early 1970's. In these early buildings, the welds were made by the shielded metal arc welding (SMAW) process, but in the early 1970's the mechanical wire feed, self shielded, flux core arc welding (FCAW) process became the normal welding method. The FCAW process is more economical because of reduced interruptions due to changing the weld electrode, and the E70T-4 electrode became the electrode of choice because of its rapid deposition rate. This electrode has no minimum notch toughness requirement, but it should be noted that the codes and specifications did not require minimum
ductility of the weld metal during this period. Further, the E70T-4 electrode was used in increasingly larger diameters (up to 3mm or 0.12 inches) in recent years, since this further increased the deposit rate.

The Northridge Earthquake occurred on January 17, 1994, and many steel frames experienced cracking during the earthquake. The cracking had a number of different variations, and many of the variations had not been observed in past experiments.

2. DAMAGE DURING NORTHRIDGE EARTHQUAKE

Present estimates indicate that cracking was noted in more than 100 steel frame buildings since the Northridge Earthquake. A building damage survey [Youssef, Bonowitz, and Gross (1995)] and database [Bonowitz and Youssef (1995)] have been accumulated, and they provide useful information regarding the extent and type of cracking. The database indicates that approximately 70% of the damaged frames had cracking in the welds. Many of these weld cracks were visible cracks, but others were detectable only by nondestructive evaluation methods. Weld cracking was noted in the top flange weld for approximately 31% of the frames and in the bottom flange welds for approximately 85% of the damaged frames. Cracking in the column was noted in approximately 26% of the damaged frames, and the vast majority of this column damage was in the vicinity of the bottom beam flange weld. The crack usually initiated near the flange weld, and progressed through the column flange into the panel zone of the beam column connection as illustrated in Figure 5. Cracking in the beams outside the welded region were noted in only 2% of the damaged frames.

Figure 5. Photograph of Crack Through Column Flange and Into Panel Zone

The database indicates that cracking was more common in newer buildings. Approximately 32% of the frames inspected had cracking in the weld, beam or column, and 15.7% of the inspected frames had cracking in the beam or column. However, approximately 50% of the inspected frames designed after 1990 had cracking in the welds, beams, or columns, and 27.7% of these newer frames had cracking in the beams or columns. This comparison suggests that buildings designed since 1990 were much more susceptible to cracking damage than the average. The database shows that buildings designed before 1980 had cracking in beams, columns, and welds in approximately 24.5% of the frames inspected, and approximately 12.5% had cracking in beams and columns. This indicates that older structures had less tendency toward cracking than average. In fact, the analogy may be stronger than suggested by this statistic, since most of the cracking observed in buildings designed prior to 1975 are concentrated in a single building. If this building is not included in the data, buildings designed before 1975 had cracking in the welds in approximately 3% of the frames inspected, and none of the frames had cracking in the beams and columns. Statistics of this type must be used with some care, however the data suggests that recent changes in the specifications and practice contribute to the problem. These changes include the relaxation of panel zone strength provisions in the 1988 UBC, changes in the properties of the steel, the reduced redundancy that has occurred in recent years, and changes in the weld electrodes.

The database also shows that the cracking damage was more significant in steel frames with deep beams and thick beam and column flanges. An average of approximately 15.7% of the frames inspected as part of the survey had cracking in the beams or columns. However, none of the frames with beam depths less than W21
had cracking of this type, and approximately 18.5% of the floor frames with beams W30 or deeper had these types. This statistic is quite important because frames with cracking in the beams and columns usually were more severely damaged than those with weld cracking only. Approximately 49% of the inspected frames had bottom flange weld cracking. However, only 17% of those frames with beam depths less than W21 experienced this cracking, while 52% of those with beam depths W30 and greater had bottom weld cracking. The data for the W21, W24, and W27 beams are intermediate, but most of these intermediate depth beams with cracking were the very heavy W24 sections with very thick flanges. It is quite probable that other factors such as span length, flange thickness, panel zone yielding and column orientation also contribute to this observed behavior. However, the concentration of damage in deeper and heavier beams must be a matter of some concern, since deeper beams are a natural consequence of the reduced redundancy and reduced number of FR moment connections used in recent years.

3. CASE STUDIES

In view of the general damage patterns noted earlier, two buildings with significant column cracking were analyzed in detail to evaluate potential causes of the cracking. Detailed design and construction information was obtained for the two buildings. Both buildings were designed and constructed less than five years before the earthquake, were of modest height and were located in regions with strong shaking.

Building One [Harrigan (1996)] was two stories tall with the floor plan illustrated in Figure 6. Its lateral resistance was provided by a single bay moment frame on each of the four sides. The moment frames are identified by the arrows at the beam column connection in the figure, and all other beam column joints were designed as pinned connections. The north and west frames had a setback between the first and second story. The building was open, and there was very little lateral resistance other than that provided by the 4 single bay steel frames. Continuity plates were used at the beam column connections, but there were no doubler plates. Cracking started in the vicinity of the bottom beam flange weld to the column, and it progressed through the column flange into the column panel zone. Ground accelerations were measured less than 2 miles from the building site, and the peak accelerations were approximately 0.57g and 0.59g in the two horizontal directions, and approximately 0.55g in the vertical direction.

![Figure 6. Floor Plan and Framing Layout of Building One](image)

Building Two [Pehlivanian (1997)] had a larger floor plan (illustrated in Figure 7) than Building One. The N-S lateral loads were resisted by 5 bay moment frames on the east and west sides of the building. Seismic loads in the E-W direction were resisted by two single bay moment frames at the N end, a 3 bay moment frames at N and S ends, and a 4 bay moment frame near the center of the structure. The beams of the moment frames
are heavy W36 sections, and the columns were typically grade 50 W21x333 sections. Continuity plates and doubler plates were noted in the drawings. However, the thickness of doubler plate was not specified, and photographs of several damaged connections show no evidence of the doubler plates. The N-S frames of this building suffered significant damage, but there was no damage noted in the E-W frames. The most serious damage was noted in the N-S frame on the east side of the building where cracking was noted in all six columns at the first story beam to column connection. The cracks started at the bottom flange welds and progressed into the column. One crack progressed nearly all the way through the column. The west side of the building had a few beam weld cracks but they were less serious and were not noted for all connections.

![Figure 7. Floor Plan and Framing Layout of Building Two](image)

The lateral load frames of Building Two continued vertically for two stories without offset or interruption, but several reinforced masonry walls were attached to the structural framing. A fairly long wall was located in the center of the building in the EW direction, and shorter walls are located in both directions in the northwest corner of the building. Two acceleration records were recorded within approximately 5 miles on either side of the building site. One record had peak accelerations of approximately 1.8g and 0.98g in the EW and NS directions, and the other was approximately 0.45g and 0.40g in the EW and NS directions. The smaller accelerations are believed to be more representative of those experienced by Building Two because there was greater similarity between the soil and geology at these two locations. However, it should be noted that the E-W accelerations were larger at both sites even though all damage occurred in the N-S direction.

Three dimensional analyses were used for both buildings. All columns, beams, diaphragms and related elements and the best available material property information were included in the model. Significant plastic deformations were computed for both buildings. The computed plastic deformation was largest in the panel zone, but flexural yielding was predicted for the columns in both buildings. However, Building One had significantly larger predicted plastic deformation than Building Two. The vertical setbacks in Building One also adversely affected the response, and concentrated the damage in the structure. The strongest shaking for Building Two was in the EW direction, but the greatest damage to the building was in the NS direction, because the masonry walls reduced E-W response. The east frame in the NS direction of Building Two was
more severely damaged than the west frame, because the walls in the northwest corners reduced the dynamic response of the west frame.

Vertical response was considered for both buildings, but it did not have an impact on the cracking despite the relatively high vertical accelerations. Although interior columns do not provide lateral resistance to the structure, they do support a proportional share of the gravity loads and vertical mass, and they provide substantial vertical stiffness. The dynamic periods for vertical response are short and there is no amplification of the vertical response. Further, columns are designed with large load factors against gravity load, while columns in the lateral load frames typically support very small gravity loads. While the vertical accelerations are unimportant, time-dependent axial loads in the columns may be significant for edge columns of the lateral load frames. These cyclic axial loads were sometimes in tension because of the small gravity loads on the columns, and this axial load may influence the cracking.

Figure 8. Detailed Finite Element Model Used to Analyze Connection Behavior

![Figure 8](image)

Note that the beam bending stress at the face of the column is significantly smaller than the integration of the local stress.

Distance Across Width of the Flange

Figure 9. Distribution of Axial Stress in the Weld at the Face of the Column

Detailed analyses were performed for critical joints with bending moments and forces obtained from dynamic analyses. The detailed models divided the beam, column, continuity plates, panel zone including doubler plates and erection plates into many shell elements as illustrated in Figure 8. Bolts were simulated by joint constraints and short, stiff beam links. The forces and moments were applied to the ends of the beam and column stubs in accordance with the elastic beam theory stress distribution. These local analyses show that there is a high concentration of tensile stress in the center of the beam flange as illustrated in Figure 9. This occurs because the column web provides greater stiffness to the center of the weld, while the column flange
provides less stiffness at other locations. Large tensile bending stresses occur in the center of the column flange at this same location. These biaxial stresses can be visualized by noting the curvature of the beam and column flange within the circled area of the deformed model illustrated in Figure 10. Transverse strains in the beam and column flange are limited by the surrounding steel, and as a result hydrostatic tensile stress develops at the center of the flange weld. Hydrostatic tensile stress does not contribute to yielding. The presence of this stress state contributes to the development of large principle stresses and these combine with the weld flaws to make this a prime location for cracking.

Calculations were also performed simulating shear yield of the panel zone. Figure 11 shows the deformed shape of the same connection with panel zone yielding and the same forces as used for Figures 8 and 10. The deflections are much larger with panel zone yield, and the focus of comparison between Figures 10 and 11 should be the shape of the deformation rather than the magnitude of the deformation. The circled area of Figure 11 shows even more severe local curvature of the beam and column flange than Figure 10. The hydrostatic tensile stress state is more severe after panel zone yield, and this further increases the potential for cracking.

4. CORRELATION WITH PAST EXPERIMENTS

The results from more than 120 past experiments on steel moment frame connections [Roeder and Foutch (1996)] were analyzed and compared to the observations and calculations of building behavior. The data shows large scatter of test results with ductility ratios varying from less than 1.0 to nearly 16. However, beam
depth appears to be one parameter with a strong influence on ductility. Figure 12 shows the total ductility as a function of depth of all unreinforced specimens with strong axis column orientation. The ductility ratio is defined as the maximum tip deflection of a cantilever divided by the elastic tip deflection at the plastic moment capacity. Thick flanges require larger full penetration welds at the column flange, and this may contribute more potential flaws which also reduce ductility for thicker flanges as illustrated in Figure 13. Finite element analyses have shown that shear yield of the panel zone increases the hydrostatic tensile stress at the center of the welded flange connection. Examination of past experiments [Roeder and Foutch (1996)] supports the hypothesis that panel zone yielding reduces the flexural ductility, but panel zone yield deformation adds to the total inelastic deformation so that total ductility may be increased by panel zone yield.

![Figure 12. Total Ductility as a Function of Beam Depth](image1)

![Figure 13. Flexural Ductility as a Function of Flange Thickness](image2)

In recent years, seismic resistance has been concentrated into the perimeter frames, isolated plane frames or bays within plane frames. The total seismic resistance required of the structure does not decrease with concentration of seismic resistance. Therefore, individual members and connections must develop increased resistance and stiffness as noted earlier. As a result, beams in steel moment frames are typically deeper today than those used many years ago. Figure 12 shows that moderate ductility is expected for beams in the W18 to W27 range, but minimal ductility can be expected for beams deeper than W30. Most modern steel frames are using these deep beams, and this partially explains the concentration of damage in newer buildings.
The large hydrostatic tensile stress at the center of the welded flange make the weld interface a location of considerable interest. Buildings damaged in the Northridge Earthquake were welded with the self shielded FCAW process with E70T-4 electrodes, which has no required minimum notch toughness. Some engineers have postulated that the rapid strain rate produced by many Northridge Earthquake acceleration records and the minimal notch toughness of the weld metal caused the cracking. A few experiments were conducted to evaluate these concerns. A beam column subassemblage with a W12x136 column and W24x55 beams was constructed as illustrated in Figure 14. One beam was welded with the SMAW and E7018 electrode and the other was welded with the self shielded FCAW process with the E70T-4 electrode.

Tension coupons were then cut from the flange as illustrated in Figure 15. Some specimens were tested under static (ASTM E-8) test conditions and others were tested with strain rates of up to 0.3 in/in/sec, since this exceeds the maximum predicted strain rate for the Northridge Earthquake. A number of specimens were tested for each condition, and so the results provide statistical data as well as direct comparison of the weld metal and strain rate effect. The tension specimen gage length included a short length of the beam flange, the entire flange weld, the entire through thickness of the column flange, the continuity plate weld, and a short length of the continuity plate as illustrated in Figure 15. Therefore, the final stress-strain curve is influenced by the five different metals acting in series, and longitudinal properties of all 3 steel types were determined to help in the evaluation of the different material behaviors. The diameter of each specimen was measured before and after testing at key points (each of the five different metal locations) along the gage length so that the relative amount of inelastic deformation in the 5 different metals can be determined.

The tests showed a large variation for ductility of both weld types, but the E7018 electrode had on average a larger ductility than the E70T-4 electrodes. Figure 16 shows the static test results for the E7018 SMAW welds and Figure 17 shows the results for the E70T-4 self shielded FCAW welds. While on average, the SMAW E7018 had greater ductility, the best and worst cases are not significantly different for the two processes. Further, the specimens with no inelastic elongation were predictable for both weld types, since visible flaws were observed in all cases. Fracture always occurred at the flaw. All welds have some flaws, but the large diameter (3 mm or 0.12 inch) E70T-4 electrode appeared to result in a larger number of flaws than the shielded arc welding, and this appeared to be the source of brittle behavior.
Specimens, which achieved significant ductility, achieved most of this ductility with through thickness yielding of the column flange. The through thickness yield stress was estimated as between 290 to 310 MPa (42 to 45 ksi) as compared to 330 to 345 MPa (48 to 50 ksi) in the longitudinal direction. This through thickness was approximately 90% of the longitudinal value. Strain rate effects did not have a significant impact on the test results. Slight increases in yield stress were noted with some of the more rapid tests, but this was inconsistently noted. This inconsistency is consistent with past test results [Manjoine (1944)], since the most rapid earthquake strain rate is at the threshold where significant strain rate effects are expected to start.

6. SUMMARY AND CONCLUSIONS

This paper has summarized a research study into the causes of cracking in steel columns from the Northridge Earthquake. An overview of the cracking and discussion of some observations from the data compiled on the cracking were provided. Case study analysis of two buildings which experienced significant cracking were described and analyzed. The observations were then compared and correlated to the results of past experimental studies, and a test program of welds was summarized. It is unlikely that a single cause will ever be found for the steel frame damage noted after the Northridge Earthquake. It is probable that the causes of
damage were different for different buildings. However, there is strong evidence that several factors made a significant contribution to the damage.

The lack of redundancy in modern steel frame buildings is one issue which played a major role in the Northridge damage. Redundancy has been significantly reduced in recent years, since moment frame connections are used in only plane frames or individual bays of plane frames rather than distributed throughout the structure. In addition, steel frame buildings have less secondary resistance from nonstructural components. This reduction in redundancy has resulted in much larger member sizes. This paper has shown that newer buildings and larger member sizes both resulted in larger statistical damage ratios than noted for older buildings and buildings with lighter member framing. In addition, the examination of past experimental results has shown a significant trend toward reduced ductility for moment frame connections with larger members.

Shear yield of panel zone is a very desirable yield mechanism in that it dissipates large amounts of energy and sustains large inelastic deformation. However, this paper has shown evidence that panel zone yielding contributes to increased potential for connection fracture at lower flexural deformation. It is shown that panel zone yielding causes a concentration of strain and local inelastic deformation near the flange weld. The analysis indicates that the hydrostatic and principle stresses increase more rapidly than the shear stress state after panel zone yielding occurs, and this may result in reduced ductility. This problem is exacerbated by the fact that shear yield has much larger apparent strain hardening than flexural yielding, and so panel zone yielding is unable to shield the critical weld regions from stress increases as does flexural yielding of the beam. Analysis of individual buildings with significant cracking in the column show that panel zone yielding was the expected behavior in both of these buildings and the damage noted in these buildings further corroborates this observation. Further, examination of past experimental results indicates that past experiments shown significant reductions in flexural ductility when large panel zone yielding occurred.

The weld electrode and weld process also contributed to the Northridge damage. The E70T-4 electrode has relatively little notch toughness, and the 3 mm diameter version of this electrode was the standard choice for steel moment frame construction prior to the Northridge earthquake. Tests described in this paper show that this electrode has on average less ductility than the E7018 electrode which was used in the earlier years of steel moment frame construction. The tests also suggest that the large diameter wire may provide a major part of this contribution, since the reduced ductility could invariably be traced to internal flaws which were detected as the specimens were cut. Finally, the statistical observation that newer buildings had more damage than older buildings, supports this observation.

The paper would suggest that other factors also contributed to the Northridge damage. First, it was noted that relatively high tensile stresses occurred in the columns of the two buildings analyzed in the course of this study. These stresses occurred because of both large bending moments and axial forces induced by overturning of the narrow moment frames. Second, the high strength of the beams was not planned in the original design of both buildings. The higher beam capacity clearly contributed to the increased panel zone demand, the increased stress on the flange weld and the increased tensile stress in the columns. Third, irregularities in the two buildings clearly contributed to the locations and concentrations of damage. There is evidence to support the contention that these factors also contributed to the damage, but the evidence for these factors is not as broadly based as the earlier three issues and are given less emphasis here. As noted earlier, it is unlikely that a single cause will ever be found for the steel frame damage noted after the Northridge Earthquake. This paper has addressed a number of these causes and hopefully will enhance the understanding of this complex problem.

7. ACKNOWLEDGMENTS

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Garth Berninghaus helped in the preparation of this paper through the analysis and experiments they performed.

8. REFERENCES


EFFECTS OF CYCLIC PLASTIC STRAINS
ON FRACTURE TOUGHNESS OF STRUCTURAL STEELS

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ABSTRACT

The effects of cyclic plastic strains on the fracture toughness of structural steels are investigated by Charpy V-notch impact tests. It is shown that strain aging reduces the fracture toughness of structural steels subjected to cyclic plastic strains. Charpy absorbed energy is examined on the top and bottom flanges of a girder of a highway bridge hit by the January 17, 1995 Kobe Earthquake. It is shown that the girder was not subjected to cyclic plastic strains during the Earthquake.

KEYWORDS

brittle fracture, fracture toughness, strain aging, cyclic plastic strain, Charpy V-notch impact test, girder, earthquake

1. INTRODUCTION

The January 17, 1995 Kobe Earthquake caused severe damage to elevated highway bridges. The cracks at the corner welds of steel piers with a box-shaped cross section, those at the connections of a beam to a column in portal frame type steel piers and those at the middle part of steel piers with a circular cross section demonstrate a fracture pattern which cannot be explained by inference from the present knowledge.

It is said that the repetition of large loads during the Earthquake was roughly ten times. To explain the mechanism of the fracture observed in the bridge steel piers, the effects of cyclic plastic strains on the fracture toughness of steel members subjected to repeated loading must be made clear. The effects of plastic strains on the fracture toughness of structural steels experiencing cold forming, that is, the effects of plastic strains imposed by monotonic loading on the fracture toughness were investigated in the past (Horikawa 1980), and the research results were introduced into the Japanese Highway Bridge Specifications
Slightly damaged steel members were reused in repairing the bridges. This throws some doubt whether the steel members subjected to repeated loading will decrease their fracture toughness due to strain aging.

The above let us investigate the effects of cyclic plastic strains on the fracture toughness of structural steels by Charpy V-notch impact test, considering strain aging. We also examined Charpy absorbed energy on the top and bottom flanges of a girder of a highway bridge hit by the Earthquake.

2. EFFECTS OF CYCLIC PLASTIC STRAINS ON FRACTURE TOUGHNESS OF STRUCTURAL STEELS

2.1 Definition of Cycles of Plastic Strains

Fig. 1 presents the definition of cycles of plastic strains. Figs. 1(a) and (b) are for monotonic loading. They are defined as 0.5 cycle. In Fig. 1(a) a tensile plastic strain is induced, and in Fig. 1(b) a compressive plastic strain is induced. Figs. 1(c) – (f) are for repeated loading. Those are applied to the definition for cycles greater than or equal to two. In Figs. 1(c) and (d) the sign of the plastic strain given by the first loading is different from that by the last loading. In Figs. 1(e) and (f) the sign of the plastic strain given by the first loading is the same as that by the last loading.

![Figure 1: Definition of cyclic plastic strain](image-url)
2.2 Inducing Cyclic Plastic Strains

As shown in Fig. 2, the upper beam (loading beam) brings about plastic strains into the flanges of the lower beam (test beam), from which Charpy V-notch impact test specimens are taken. The steel used for the flanges of the test beam is JIS SM400B with a guaranteed tensile strength of 400 MPa. Table 1 lists the mechanical property and the chemical composition mentioned in the mill sheet for the flanges and also the mechanical property obtained by tensile tests.

To avoid axial forces in the test beam, round bars are put between the loading beam and the test beam and between the test beam and the floor. A uniform bending moment is created in the test beam between the two round bars under the loading beam. Strain gauges were glued on the top and bottom flanges at three locations dividing equally between the two round bars. After unloading, the test beam was reversed, and then it was reloaded, which produced cyclic strains in the flanges of the test beam.

![Figure 2: Inducing cyclic plastic strains](image_url)

**TABLE 1**

MECHANICAL PROPERTY AND CHEMICAL COMPOSITION

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<tr>
<th></th>
<th>Yielding stress (MPa)</th>
<th>Tensile stress (MPa)</th>
<th>Elongation (%)</th>
<th>Young's modulus (MPa)</th>
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<table>
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<th>Chemical composition (Mill sheet) (%)</th>
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<tr>
<td>---</td>
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<tr>
<td>0.15</td>
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</table>
The maximum strains 0.5 % and 1.0 % and the repeated loading of 0.5, 3, 5.5 and 10.5 cycles were considered. As an example, Fig. 3 shows load versus strain curves for the top and bottom flanges of the test beam with the maximum strain 0.5 % and the repeated loading of 10.5 cycles. When the flange is in the lower side, the load is represented with the plus sign. When the flange is in the upper side, it is represented with the minus sign. When one of three strain gauges on the flange in the lower side reached the strain 0.5 %, the test beam was unloaded.

### 2.3 Charpy Impact Tests

Charpy V-notch impact test specimens were taken from the top and bottom flanges of the test beam between the two round bars under the loading beam. The specimens were situated in the longitudinal direction of the flanges. A V-notch was made in the direction of plate thickness. The aging treatment used was 250 °C and one hour. The impact tests were carried out at 0 °C.

Fig. 4 presents the relation between the absorbed energy and the number of cycles of repeated loading. Each value of the absorbed energy is the mean of the values for six impact test specimens. The values at zero cycle are the results without loading. The strains are at the last loading. The following can be seen from the figures:

1) The absorbed energy for the maximum strain 1.0 % is smaller than that for 0.5 %.
2) In the maximum strain 0.5 % in Fig. 4(a), the absorbed energy for the tensile strain is lower than that for the compressive strain. In the maximum strain 1.0 % in Fig. 4(a) and 0.5 % and 1.0 % in Fig. 4(b), however, the absorbed energy for the compressive strains is almost the same as that for the tensile strains.
3) The reduction in the absorbed energy is small between zero cycle (non loading) and 0.5 cycle (monotonic loading). In the maximum strain 1.0 % in Fig. 4(a) and 0.5 % and 1.0 % in Fig. 4(b), the absorbed energy drops sharply between 0.5 cycle and 3 cycles for the maximum strain 1.0 %, and between 0.5 cycle and 5.5 cycles for the maximum strain 0.5 %.

Fig. 5 shows the effects of the aging treatment on the absorbed energy. The results in the figure are for the tensile strains at the last loading. It is seen that the reduction in the absorbed energy due to aging treatment is small at zero cycle (non loading) and 0.5 cycle (monotonic loading), but that it is large for repeated loading.
2.4 Relation between Absorbed Energy and Plastic Strains

In this research, a plastic skeleton-strain and an accumulated plastic strain are considered to evaluate the reduction in the absorbed energy. Some researchers insist that the reduction in the fracture toughness of structural steels subjected to repeated loading can be related to plastic skeleton-strains (Building Research Institute, Ministry of Construction and Steel Material Club 1995). As shown in Fig. 6(a), in the tension stress side, adding the strain corresponding to the stress which exceeds the maximum stress in the preceding cycle makes a skeleton curve. Likewise, such a skeleton curve is made in the compression stress side. The larger of the absolute values of the plastic strain components of the skeleton curves in the tension and compression stress sides is defined as a plastic skeleton-strain. As shown in Fig. 6(b), an accumulated plastic strain is defined as the sum of plastic tension and compression strains in a cyclic stress-strain curve.
Fig. 7(a) presents the relation between the absorbed energy and the plastic skeleton-strain for the present impact test results. The absorbed energy does not seem to be connected with a plastic skeleton-strain. Fig. 7(b) shows the relation between the absorbed energy and the accumulated plastic strain. It is seen that the absorbed energy has a high correlation with the accumulated plastic strain.

\[ \varepsilon_{psk} = \text{Maximum}\left(\sum|\varepsilon^+|_p, \sum|\varepsilon^-|_p\right) \]

(a) Plastic skeleton-strain

\[ \varepsilon_{ap} = \sum|\varepsilon^+_p| + \sum|\varepsilon^-_p| \]

(b) Accumulated plastic strain

Figure 6: Definition of plastic skeleton-strain and accumulated plastic strain
EFFECTS OF CYCLIC PLASTIC STRAINS

Figure 7: Relation between absorbed energy and plastic strains

3. FRACTURE TOUGHNESS OF TOP AND BOTTOM FLANGES OF GIRDER HIT BY KOBE EARTHQUAKE

3.1 Examined Girder

The examined bridge is located between Wakihama and Iwaya on the Kobe Route of the Hanshin Expressway. It is an elevated bridge consisting of five 34.396 m simple span composite plate girders. Since the flaking of paint due to large local deformation of steel plates was not observed in the girders except their ends, the girders were expected to be reused, excepting the girder ends.

As shown in Fig. 8, Charpy V-notch impact test specimens were taken from the top flange at the locations A – F (base metal) and M, N (butt weld) and from the bottom flange at the locations G – L (base metal) and O, P (butt weld). Table 2 lists the chemical composition and the mechanical property at each location. The girder meets the guaranteed values for the chemical composition and the mechanical property at each location, except that the carbon C exceeds the guaranteed value by 0.01 % at the locations H, K. The tensile test specimens with a width smaller than that specified in the JIS (Japanese Industrial Standard 1980) give us elongation larger than usual.

Figure 8: Examined girder
## Table 2
Chemical Composition and Mechanical Property at Each Location

<table>
<thead>
<tr>
<th>Location</th>
<th>Plate thickness (mm)</th>
<th>Steel</th>
<th>Chemical composition (%)</th>
<th>Yielding stress (MPa)</th>
<th>Tensile stress (MPa)</th>
<th>Elongation (%)</th>
<th>Charpy absorbed energy* at 0 °C (J)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>C  Si  Mn  P  S  Cu  Ni  Cr  Mo</td>
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<td>A</td>
<td>20</td>
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<td>0.25 0.20 0.56 0.020 0.012 0.08 0.03 0.04 0.00</td>
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<td>300</td>
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<td>24.66</td>
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<tr>
<td>B</td>
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<td>SM490A</td>
<td>0.18 0.37 1.32 0.024 0.015 0.10 0.03 0.06 0.01</td>
<td>353</td>
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<td>48.0</td>
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<td>557</td>
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<td>H</td>
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<td>565</td>
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<td>K</td>
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<td>0.19 0.38 1.40 0.027 0.018 0.09 0.04 0.06 0.01</td>
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<td>578</td>
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<td>76.81</td>
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<td>SM490A</td>
<td>≤0.23 - ≤2.5×C ≤0.035 ≤0.035</td>
<td>- - - - 235 400～510 ≥22 - -</td>
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<td>SM490A</td>
<td>≤0.20 ≤0.55 ≤1.50 ≤0.035 ≤0.040</td>
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<td></td>
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<td>- - - - 314 490～608 ≥21 27 -</td>
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*Aging treatment not performed
3.2 Charpy Impact Tests

Charpy V-notch impact test specimens for the base metal are laid in the longitudinal and transverse directions for the top flange and in the longitudinal direction for the bottom flange. A V-notch was provided in the direction of plate thickness. The butt welds had an X-shaped groove. Charpy V-notch impact test specimens for the butt welds were situated in the longitudinal direction (crosswise to the welding line). A V-notch was made parallel to the plate surface. The aging treatment used was 250 °C and one hour. The impact tests were carried out at 0 °C.

Fig. 9 shows a difference in the absorbed energy between the aging treatment performed and not performed. The absorbed energy is the mean of the values for three impact test specimens. The circles are distributed

Figure 9: Difference in absorbed energy between aging treatment performed and not performed

Figure 10: Difference in absorbed energy between directions
around the straight line with 45 degrees except three circles. This means that there is no reduction in the fracture toughness due to aging treatment throughout the girder. When steel members subjected to cyclic plastic strains experience an aging treatment, their absorbed energy decreases. Thus this girder was not subjected to cyclic plastic strains during the Earthquake.

Fig. 10 shows a difference in the absorbed energy between the directions where the impact test specimens are laid. The absorbed energy for the transverse direction is lower than that for the longitudinal direction. This is because the absorbed energy for the direction perpendicular to the rolling direction is generally lower than that for the rolling direction.

4. CONCLUSION

In this paper, the effects of cyclic strains on the fracture toughness of structural steels were investigated by Charpy V-notch impact tests. The absorbed energy decreases with the increase in repetition of plastic strains. Especially, the reduction in the absorbed energy is large on early cycles. The larger plastic strains, the larger reduction in the absorbed energy occurs. The effects of strain aging on the fracture toughness are small for non-loading and monotonic loading, but they are large for cyclic loading. In cyclic loading, the absorbed energy with an aging treatment is much lower than that without an aging treatment. The reduction in the absorbed energy due to repeated loading has a higher correlation with an accumulated plastic strain than a plastic skeleton-strain.

Charpy absorbed energy was examined on the top and bottom flanges of a girder of a highway bridge hit by the Kobe Earthquake. It was shown that the girder had not been subjected to cyclic plastic strains during the Earthquake.

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